An Empirical Equilibrium Model of a Decentralized Asset Market*

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Abstract

I estimate a search-and-bargaining model of a decentralized market to quantify the effects of trading frictions on asset allocations and asset prices, and to quantify the effects of intermediaries that facilitate trade. Using business-aircraft data, I find that, relative to the Walrasian benchmark, 20.3 percent of the assets are misallocated, and prices are 6.61-percent lower. Dealers play an important role in reducing frictions: In a market with no dealers, 32.3 percent of the assets would be misallocated, and prices would decrease by 5.25 percent.

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1 Introduction

How large are trading frictions? How do they affect the allocations and prices of assets? What is the role of intermediaries in reducing trading frictions? This paper estimates a structural model of a decentralized market to provide quantitative answers to these questions.

Many assets trade in decentralized markets. Classic examples are financial assets such as bonds and derivatives, consumer durable goods such as cars and houses, and firms' capital assets such as plants and equipment. The fundamental characteristics of decentralized markets are that agents must search for trading partners and that, once a buyer and a seller meet, they must bargain to determine a price. Moreover, in response to trading frictions, almost all decentralized markets have intermediaries. Indeed, starting with Demsetz (1968), trading frictions have been used to explain the existence and behavior of intermediaries. The key role of intermediaries in such markets is to reduce frictions, thereby facilitating trade.

In this paper, I lay out a model of trading in decentralized markets with two-sided search and bilateral bargaining. The model formalizes the effects of trading frictions on asset allocations and prices, and the effects of intermediaries in alleviating these frictions. I then quantify the role of frictions and of dealers, estimating this model using data on business jet aircraft.

The theoretical framework combines elements from Rubinstein and Wolinsky (1987) and Duffie, Gârleanu and Pedersen (2005), extending them to capture key features of real asset markets, such as the depreciation of assets. A flow of agents enters the market in every period, seeking to acquire an aircraft. They contact sellers at a rate that depends on the mass of aircraft for sale and on traders' search ability, and they contact dealers at another rate that reflects dealers' inventories and search ability. Dealers hold inventories because they do not meet buyers and sellers simultaneously (Grossman and Miller, 1988), and they extract surplus by shortening the time that buyers and sellers have to wait in order to trade (Rubinstein and Wolinsky, 1987). When agents meet or meet dealers, they bargain over the terms of trade. Gains from trade arise from heterogeneous valuations of holding the aircraft. For example, an aircraft owner, usually a corporation, wishes to sell because its valuation for the aircraft has dropped, perhaps because the firm's profitability has declined. Equilibrium allocations and prices depend in an intuitive way on agents' and dealers' search abilities, bargaining powers and valuations.

I estimate the model by using data on the secondary market of business jet aircraft—a typical decentralized market. The data are well-suited to studying the effects of search

frictions and the role of intermediaries. In particular, they report the number of aircraft for sale and number of aircraft transactions in each month, and their ratio is informative on the magnitude of trading delays. Similarly, the data report dealers' inventories and dealers' transactions, and their ratio is informative on the role of dealers in reducing delays. In addition, the data report two series of prices: retail prices between final users of the aircraft; and wholesale prices between aircraft owners (as sellers) and dealers (as buyers). Their differences are useful in understanding how much dealers are able to command by supplying immediacy of trade (and, thus, sellers are willing to forego).

The estimation reveals that trading delays are non-trivial: On average, aircraft stay on the market approximately five months before a seller is able to finalize a sale. The quantitative importance of these delays depends on how frequently agents seek to trade, determined by a drop in their valuations; this happens, on average, every four and a half years. Moreover, the estimation implies that the dealers enjoy strong bargaining powers, capturing almost the entire surplus of transactions.

I use the estimated parameters to simulate two counterfactual scenarios. In the first one, I quantify the role of trading frictions on asset allocations and prices by computing a Walrasian market equilibrium. The estimates imply that trading frictions generate moderate inefficiencies. Compared to the Walrasian benchmark, aircraft prices are 6.61-percent lower, and 20.3 percent of all business aircraft are misallocated: 11.6 percent are on the market for sale, whereas, for 8.7 percent of all aircraft, the trading frictions are larger than sellers' expected gross gains from trade; thus, their low-valuation owners prefer to keep them rather than put them on the market for sale. Two forces affect allocations and prices in the estimated model relative to this Walrasian benchmark: search costs and trading delays. The parameter estimates imply that trading delays quantitatively account for almost all differences, and the effect of search costs on allocations and prices is small.

In the second counterfactual, I quantify the effect of dealers on the decentralized market equilibrium, computing a decentralized market with no dealers. The estimates imply that dealers play an important role in reducing frictions: In a market without them, 32.3 percent of the assets would be misallocated, and prices would decrease by 5.25 percent. Thus, the effect on allocations is larger than the effect on prices. The reason is that sellers' outside options improve relative to buyers,' thus partially offsetting the effects of higher search costs and slower trade on asset prices. Therefore, an interesting conclusion of this counterfactual analysis is that it may be difficult to infer exclusively from the magnitude of changes in asset prices the magnitude of changes in trading frictions (or, more generally, changes in trading

mechanisms and institutions) on market inefficiencies.

This paper makes two main contributions. First, it provides a framework suited to empirically analyzing decentralized asset markets. Search models have proved useful in understanding key features of labor markets, and, more recently, researchers have started to apply search models to financial markets. To my knowledge, this paper is the first to estimate a bilateral search model that investigates the microstructure of the market of a capital asset/durable good, quantifying the effects of trading delays and of intermediaries that facilitate exchange. Second, the empirical findings suggest that, even within a well-defined asset class such as business aircraft, trading frictions are a non-trivial impediment to the efficient allocation of assets and have significant effects on asset prices. Thus, the paper innovates on recent works that study the process of asset reallocations (Ramey and Shapiro, 1998, 2001; Maksimovic and Phillips, 2001; Schlingemann et al., 2002; Eisfeldt and Rampini, 2006; Gavazza, 2011a and 2011b. For an empirical model of business transfers, see, also, Holmes and Schmitz, 1995) by quantifying its inefficiencies. This is a necessary step to understanding how asset markets work and, therefore, to the design of any policy that affects them.

The paper proceeds as follows. Section 2 reviews the literature. Section 3 presents some institutional details on the business-aircraft market. Section 4 introduces the data. Section 5 presents the theoretical model, and Section 6 estimates it. Section 7 performs counterfactual analysis. Section 8 concludes. The appendices present the analytical solution of the model with the assumption of no depreciation and the estimates of the model with different assumptions on the matching function, respectively.

2 Related Literature

This paper contributes to the important literature that analyzes decentralized markets. The theoretical literature is vast. The most closely related papers examine bilateral search markets, in which both buyers and sellers search for a trading counterpart, and prices are determined through bilateral bargaining (Rubinstein and Wolinsky, 1985 and 1987; Gale, 1987; Mortensen and Wright, 2002; Duffie, Gârleanu and Pedersen, 2005 and 2007; Miao, 2006). The main focus of these theoretical papers is to investigate whether the equilibrium converges to the competitive outcome as frictions vanish. To my knowledge, this paper is

the first to estimate a bilateral search model of a real asset market to quantify the effects of trading frictions—i.e., the focus of the theoretical literature.

The paper further contributes to the literature on intermediaries. Several papers investigate the role of brokers/dealers in financial markets and their inventory-management policies; for a survey, see O'Hara (1995). Spulber (1999) presents a thorough analysis of intermediaries between customers and suppliers. Weill (2007) presents a search-and-bargaining model to understand how intermediaries provide liquidity by accumulating inventories when selling pressure is large, and then dispose of those inventories after that selling pressure has subsided. Recent empirical analysis of non-financial intermediaries include Hall and Rust (2000), who focus on the inventory investment of a single steel wholesaler, and Gavazza (2011a), who focuses on the role of lessors in reallocating commercial aircraft. To my knowledge, this paper presents the first structural empirical analysis of the role of intermediaries in a search-and-bargaining framework.

This paper is also related to the empirical literature on the structural estimation of search models. Most applications focus on labor markets. Eckstein and Wolpin (1990) pioneered this literature by estimating the model of Albrecht and Axell (1984). Eckstein and van der Berg (2006) provide a thorough survey of this literature. One key difference between the current paper and previous research is that this paper seeks to understand how search frictions affect the *level* of asset prices and of asset allocations, while most papers that structurally estimate labor-market search models focus on how search frictions affect wage *dispersion* across workers (notable exceptions are Gautier and Teulings, 2006 and 2010). Search models have also been applied to housing markets, and Carrillo (2012) is the closest empirical paper.

Finally, this paper is related to a few that investigate aircraft transactions. Using data on commercial-aircraft transactions, Pulvino (1998, 1999) finds that airlines under financial pressure sell aircraft at a 14-percent discount, and that distressed airlines experience higher rates of asset sales than non-distressed airlines do. Using data on business jets similar to the data that I use in this paper, Gilligan (2004) finds empirical evidence consistent with the idea that leasing ameliorates the consequences of information asymmetries about the quality of used aircraft. Gavazza (2011b) empirically investigates whether trading frictions vary with the size of the asset market in commercial-aircraft markets. However, none of these papers quantifies the importance of trading frictions by estimating a structural model.

3 Business-Aircraft Markets

For several reasons, the business-aircraft market is an interesting context in which to investigate the effects of search frictions and the role of intermediaries.

First, used business aircraft trade in decentralized markets, organized around privately-negotiated transactions. Almost all buyers (and sellers) are wealthy individuals or corporations, that use the aircraft to fly their executives. To initiate a transaction, a prospective seller must contact multiple potential buyers or sell its aircraft to a dealer. For buyers, comparing two similar aircraft is costly since aircraft sales involve the material inspection of the aircraft, which could be in two different locations. Thus, aircraft markets share many features with other over-the-counter markets for financial assets (mortgage-backed securities, corporate bonds, bank loans, derivatives, etc.) and for real assets (real estate), in which trading involves material and opportunity costs (Duffie, Gârleanu and Pedersen, 2005 and 2007). Moreover, compared to financial markets and other equipment markets, business-aircraft markets are "thin": Slightly more than 17,000 business jet aircraft were operated worldwide in December 2008. In thin markets, the search costs to find high-value buyers are usually large (Ramey and Shapiro, 2001; Gavazza, 2011b).

Second, intermediaries play an important role in mediating transactions. Most intermediaries operate as brokers who match buyers and sellers. Some larger intermediaries operate also as dealers, acquiring aircraft for inventories.

Finally, business aircraft are registered goods with all major "life" events (date of first flight, maintenance, scrappage, etc.) recorded, so detailed data are available. In the next section, I describe them.

4 Data

Patterns in the data suggest that trading delays are an important feature of aircraft markets. Moreover, the available data dictate some of the modeling choices of this paper. Hence, I describe the data before presenting the model. This description also introduces some of the identification issues that I discuss in more detail in Section 6.

4.1 Data Sources

I combine two distinct datasets. The first is an extensive database that tracks the history of business-aircraft transactions. The second reports the average values of several aircraft models, similar to "Blue Book" prices. I now describe each dataset in more detail.

Aircraft Transactions—This database is compiled by AMSTAT, a producer of aviation-market information systems.[1] It provides summary reports that track business-aircraft market transactions. For each month from January 1990 to December 2008 and for each model (e.g., Cessna Citation V or VI), the dataset reports information on the active fleet (e.g., the number of active aircraft; number of new deliveries); information on aircraft for sale (e.g., the number of aircraft for sale, by owners and by dealers; the average vintage); and information on completed transactions (e.g., the total number of transactions; the number of retail-to-retail, retail-to-dealer, dealer-to-dealer and dealer-to-retail transactions). The dataset also reports detailed characteristics of each aircraft model (e.g., average number of seats, maximum range, fuel consumption). I restrict the analysis to business jets, thus excluding turbo-propellers.

Aircraft Prices—I obtained business-aircraft prices from the *Aircraft Bluebook Historical Value Reference*.[2] This dataset is an unbalanced panel, reporting quarterly historic values of different vintages for the most popular business-aircraft models during the period 1990-2008. Two series are reported: average retail prices and average wholesale prices. Average retail prices report prices between final users of the aircraft, and average wholesale prices report average transaction prices between an aircraft owner (as a seller) and a dealer (as a buyer). All prices are based on the company's experience in consulting, appraisal and fleet evaluation. All values are in U.S. dollars, and I have deflated them using the GDP Implicit Price Deflator, with 2005 as the base year.

It is important to note that the construction of the *Aircraft Bluebook Historical Value Reference* implies that the retail and wholesale price series are free of several biases. First, wholesale prices refer to sales *to* dealers—thus *before* dealers could make any improvement to the aircraft. Second, the database reports historical retail and wholesale prices, even for

[1] `http://www.amstatcorp.com/pages/pr_stat.html`. The website mentions that: "AMSTAT's customers are aircraft professionals, whose primary business is selling, buying, leasing and/or financing business aircraft, as well as providers of related services and equipment."

[2] The dataset is available at `http://www.aircraftbluebook.com`. The website describes the dataset as: "The Aircraft Bluebook Historical Value Reference is specifically designed for lease companies, bankers, aircraft dealers, or anyone who needs to know the pricing history of an individual aircraft."

Table 1: Summary statistics

Panel A: Aircraft Transactions	Obs.	Mean	St. Dev.	Median
Models of Aircraft	26,237	161		
Active Aircraft	26,237	89.79	107.28	49
Aircraft for Sale	26,237	10.91	15.11	6
–by Owners	26,237	7.75	11.94	4
–by Dealers	26,237	3.16	4.78	2
Average Age, Aircraft for Sale	23,225	18.43	11.09	19
Retail-to-Retail Transactions	26,237	0.58	1.12	0
Retail-to-Dealer Transactions	26,237	0.83	1.48	0
Dealer-to-Dealer Transactions	26,237	0.28	0.78	0
Dealer-to-Retail Transactions	26,237	0.83	1.50	0
Total Number of Transactions	26,237	1.42	2.29	0
Panel B: Aircraft Prices				
Models of Aircraft	31,524	72		
Retail Price (in $1,000)	31,524	7,607	8,534	4,343
Wholesale Price (in $1,000)	31,524	6,731	7,555	3,849
Age (in Years)	31,524	14.43	9.69	13.25

Notes—This table provides summary statistics of the variables used in the empirical analysis. Panel A presents summary statistics for the Aircraft Transactions dataset. Each observation represents a model-month pair. Panel B presents summary statistics for the Blue Book prices dataset. Each observation represents a model-vintage-quarter tuple. Aircraft prices are in thousands of U.S. dollars and have been deflated using the GDP Implicit Price Deflator, with 2005 as the base year.

those model-vintage pairs for which only one unit (i.e., one serial number) exists, suggesting that the price series are not affected by sellers' selection based on aircraft quality (observable or unobservable to both trading parties) or on their valuations

4.2 Data Description

Table 1 provides summary statistics of the main variables used in the empirical analysis. Panel A refers to the Aircraft Transactions Dataset, a sample containing 161 models (the definition of a model is quite fine in this dataset), comprising a total of 26,237 aircraft model-month observations.[3] For each model, the stock of Active Aircraft equals approximately 90 units, of which approximately ten are on sale in a given month. The number of transactions is small relative to the number of aircraft on sale: On average, there are

[3]The quantitative analysis in Section 6 does not exploit the cross-sectional differences across models.

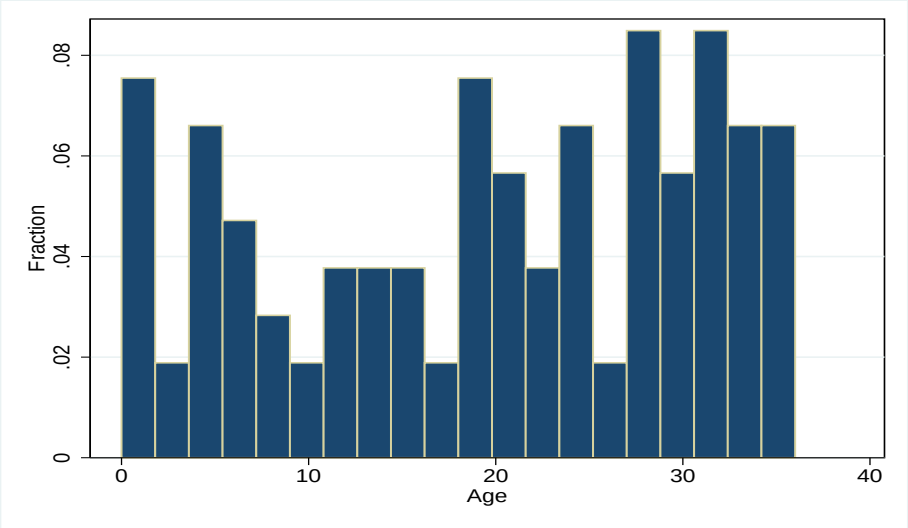

FIG. 1: Histogram of the average ages across models of aircraft on the market in January 2001.

0.58 RETAIL-TO-RETAIL TRANSACTIONS and 0.83 DEALER-TO-RETAIL TRANSACTIONS per model-month pair (I include Lease transactions in DEALER-TO-RETAIL TRANSACTIONS). There are also a few DEALER-TO-DEALER TRANSACTIONS—0.28 per month, on average—suggesting that dealers smooth their inventories by trading with other intermediaries. The TOTAL NUMBER OF TRANSACTIONS, defined as the sum of RETAIL-TO-RETAIL TRANSACTIONS and DEALER-TO-RETAIL TRANSACTIONS, indicates that, on aggregate, approximately 1.5 aircraft per model trades in a given month and that a dealer is the seller to the final retail user in approximately 60 percent of these transactions.

The average age of aircraft for sale is 18.43 years, with a large dispersion across observations (model-month pairs).[4] This dispersion indicates that aircraft of many vintages are on the market simultaneously. For example, Figure 1 plots the histogram of the average ages across models of aircraft on the market in January 2001, showing that the youngest vintages are frequently for sale. This pattern is in stark contrast with the evidence from other asset markets in which replacement purchases are the main motives for trade, such as, for example, the car market. In particular, in the car market, the fraction of cars sold is lowest for the youngest vintages (Stolyarov, 2002; Gavazza, Lizzeri and Rokestkiy, 2012). The reason is that, because of transaction costs, few households sell their cars one year after purchasing them, so the resale rates are increasing when cars are relatively new. Hence, Figure 1 suggests that replacement is not the first-order motive for trade.

[4]The number of observations of AVERAGE AGE, AIRCRAFT FOR SALE is lower than all other variables because there are no aircraft for sale in some model-month pairs.

Panel B provides summary statistics for the Aircraft Prices dataset. This sample contains 72 models (indicating that the definition of a model is coarser than in the Aircraft Transactions Dataset), comprising a total of 31,524 aircraft model-vintage-quarter observations. The average Retail Price of an aircraft in the sample is 7.6 million (year 2005) dollars, and the average Wholesale Price is 6.7 million (year 2005) dollars. Moreover, there is substantial variation in both prices: The standard deviation of retail prices is 8.5 million dollars and of wholesale prices is 7.5 million dollars.

The two datasets provide a rich description of the business-aircraft market and are well-suited to investigating the importance of frictions and the role of dealers. Specifically, three key patterns suggest that trading delays may be non-trivial and that dealers reduce them. First, the difference between the number of Aircraft for Sale (on average, 11 aircraft per model-month) and the Total Number of Transactions (on average, 1.5 per model-month) means that aircraft stay on the market for several months before selling, indicating that trading delays are substantial. Second, the ratio between Retail-to-Dealer Transactions and Aircraft for Sale by Dealers is higher than the ratio between Total Number of Transactions and Aircraft for Sale, suggesting that dealers are faster than owners at turning aircraft over.[5] Third, the difference between the Retail Price and the Wholesale Price is quite large (on average, 13 percent), corroborating that frictions are relevant in this market and that dealers are able to command a substantial markup by supplying immediacy of trade.

With all their advantages, however, the datasets pose some challenges. In my view, the main limitation is that both datasets provide aggregate statistics of the market for different models. This limitation implies that a model with rich heterogeneity of agents and dealers, while theoretically feasible, would not be identified with these aggregated data. For example, the aircraft-price dataset does not allow the identification of rich heterogeneity in asset valuations, which would be possible if transaction prices were available. Therefore, the model admits a parsimonious binary distribution of valuations, high and low. In Section 8, I discuss the implications of this limited heterogeneity for the interpretation of the empirical results. Similarly, the aircraft-transaction dataset reports only aggregate dealers' inventories, limiting the possibility of identifying heterogeneity across dealers. Finally, an additional limitation is that the aircraft-transaction dataset does not report whether retail buyers or

[5]These differential trading patterns are difficult to explain through a model of "thin" markets—i.e., markets with a very small number of buyers and sellers wishing to trade at non-perfectly synchronized times. The differential patterns are also comparable between more- and less-popular aircraft.

sellers hired a broker to search for trading counterparts, although these intermediaries are popular in business-aircraft markets.

5 Model

In this section, I lay out a model of a decentralized market with two-sided search to theoretically investigate the effects of search frictions on asset allocations and prices. The model combines elements from Rubinstein and Wolinsky (1987) and Duffie, Gârleanu and Pedersen (2005), extending them to capture key features of real asset markets, such as the depreciation of assets.

I model frictions of reallocating assets explicitly. In particular, each agent contacts another agent randomly, and this is costly for two reasons: 1) There is an explicit cost c_s of searching; and 2) there is a time cost in that all agents discount future values by the discount rate $\rho > 0$.

5.1 Assumptions

Time is continuous and the horizon infinite. A mass μ of risk-neutral agents enters the economy at every instant. All entrants have an exogenous parameter $z = z_h > 0$ that measures their valuation for an aircraft. This valuation parameter is a Markov chain, switching from z_h to $z_l = 0$ with intensity λ; z_l is an absorbing state. The valuation processes of any two agents are independent. Hence, the mass of high-valuations agents is equal to $\frac{\mu}{\lambda}$.

There is a constant flow x of new aircraft entering the economy in every instant. Aircraft depreciate continuously over time and are exogenously scrapped when they reach age T, but could be endogenously scrapped earlier. Thus, the endogenous total mass A of aircraft is, at most, equal to xT. I assume that $\frac{\mu}{\lambda} > xT$, implying that $\frac{\mu}{\lambda} > A$, which means that the "marginal" owner in a Walrasian market is a high-valuation agent. An aircraft of age a generates an instantaneous flow of utility equal to $\pi(z, a)$ to its owner with valuation z. The function $\pi(z, a)$ satisfies the following: 1) It is increasing in the valuation z, $\pi(z_h, a) > \pi(z_l, a)$; 2) it is decreasing in the age a of the asset, $\frac{\partial \pi(z,a)}{\partial a} < 0$; and 3) it exhibits negative complementarity between the valuation z and the age a of the asset, $\frac{\partial \pi(z_h,a)}{\partial a} < \frac{\partial \pi(z_l,a)}{\partial a}$.

Each agent can own either zero or one aircraft.[6] Agents can trade aircraft: A given agent

[6]Hence, I do not consider quantity decisions, as do Duffie, Gârleanu and Pedersen (2005 and 2007), Miao (2006), Vayanos and Wang (2007), Vayanos and Weill (2008), and Weill (2007 and 2008). See Lagos and

wishing to trade (either a buyer or a seller) pays a flow cost c_s to search for a counterparty.[7] While searching, he makes contact with other agents pairwise independently at Poisson arrival times with intensity $\gamma > 0$. Thus, given that matches are determined at random, the arrival rate γ_s for a seller (the rate at which he meets buyers) is $\gamma_s = \gamma\mu_b$, and the arrival rate $\gamma_b(a)$ for a buyer (the rate at which he meets sellers of an age-a aircraft) is $\gamma_b(a) = \gamma\mu_s(a)$, where μ_b and $\mu_s(a)$ are the endogenous equilibrium masses of buyers and sellers of an age-a aircraft, respectively.[8]

In addition, there is an endogenous mass μ_d of independent used-aircraft dealers that meets traders also through a search process. Each dealer has a flow cost equal to k and has, at most, one unit of inventory. An agent wishing to trade meets dealers pairwise independently at Poisson arrival times with intensity $\gamma' > 0$. Thus, a buyer meets a dealer with a vintage-a aircraft to sell at rate $\gamma_{bd}(a) = \gamma'\mu_{do}(a)$, and a dealer with an aircraft to sell meets a buyer at rate $\alpha_{ds} = \gamma'\mu_b$, where $\mu_{do}(a)$ is the endogenous mass of dealers with a vintage-a aircraft to sell. The rates $\gamma_{bd}(a)$ and α_{ds} represent the sum of the intensity of buyers' search for dealers and dealers' search for buyers. A seller meets a dealer willing to buy an aircraft at rate $\gamma_{sd} = \gamma'\mu_{dn}$, and a dealer willing to buy an aircraft meets a seller of an aicraft of age a at rate $\alpha_{db}(a) = \gamma'\mu_s(a)$, where μ_{dn} is the endogenous mass of dealers with an aircraft to sell. There is free entry into the dealers' market.

Once a buyer and a seller meet, or they meet a dealer, parties negotiate a price to trade. I assume that a buyer and a seller negotiate a price according to generalized Nash bargaining, where $\theta_s \in [0,1]$ denotes the bargaining power of the seller. Similarly, when an agent meets a dealer, they negotiate a price, and $\theta_d \in [0,1]$ denotes the bargaining power of the dealer.[9]

Rocheteau (2009) for a model that considers quantity decisions.

[7]The main role of the search cost c_s is to avoid swaps of assets. For example, a low-valuations agent with a young aircraft and a high-valuation agent with an old aircraft may want to swap their assets. However, a positive flow cost of search c_s eliminates these trades with small gains from trade.

[8]Thus, the matching function exhibits increasing returns to scale. Appendix A reports on the estimates of the model and on the counterfactual analyses under the assumption that the matching function exhibits constant returns to scale.

[9]The model assumes symmetry of information about the quality of the asset. Several institutional features of aircraft markets support this assumption. First, the aviation authorities often regulate aircraft maintenance. Second, maintenance records are frequently available, and all parties can observe the entire history of owners of each aircraft. Finally, all transactions involve a thorough material inspection of the aircraft.

5.2 Solution

There are four types of agents in the model economy and two types of dealers: high- and low- valuation owners and non-owners, and dealers with and without an aircraft for sale. I denote their types ho, lo, hn, ln, do, dn, respectively.

The owner of an age-a aircraft with valuation z can keep operating it, put it up for sale, or scrap it. In the first case, he enjoys the flow profit $\pi(z, a)$. In the second case, he meets potential trading partners at rate γ_s and a dealer at rate γ_{sd}. In the last case, he becomes an agent with no aircraft. Intuitively, owners prefer to sell their assets when their valuations are low and the assets are relatively young since the complementarity between valuation z and age a of the flow utility $\pi(z, a)$ means that the gains from trade between buyers and sellers are larger for younger aircraft. Moreover, high-valuation owners scrap their old aircraft, seeking to purchase newer ones. Similarly, an agent with no aircraft can meet active sellers of age-a aircraft at rate $\gamma_b(a)$ and a dealer at rate $\gamma_{bd}(a)$, or he can exit the market. Non-owners choose to search when their valuation is high, and they purchase only relatively young assets. Dealers without inventories operate similarly to buyers, and dealers with inventories operate similarly to sellers, with the key difference that dealers' opportunity costs differ from buyers' and sellers' and, thus, their choice of which assets (i.e., vintages) to purchase differs.

I now formally derive the value functions for all agents of the economy and the transaction price at which trade occurs. These value functions allow us to pin down the equilibrium conditions and derive the endogenous distribution of owners' valuations. This distribution describes in an intuitive way how frictions generate allocative inefficiencies and affect aircraft prices.

5.2.1 Agents' Value Functions

Let $U_{ho}(a)$ be the value function of an agent with valuation z_h who owns an aircraft of age a and is not seeking it to sell it. $U_{ho}(a)$ satisfies:

$$\rho U_{ho}(a) = \pi(z_h, a) + \lambda(V_{lo}(a) - U_{ho}(a)) + U'_{ho}(a). \tag{1}$$

Equation (1) has the usual interpretation of an asset-pricing equation. An agent with valuation z_h enjoys the flow utility $\pi(z_h, a)$ from an aircraft of age a. At any date one possible event, at most, might happen to him: At rate λ, his valuation drops to z_l, in

which case he chooses between continuing to operate the aircraft (enjoying the value $U_{lo}(a)$) and actively seeking to sell it (enjoying value $S_{lo}(a)$). Thus, the agent obtains a value $V_{lo}(a) = \max\{U_{lo}(a), S_{lo}(a)\}$ and a capital loss equal to $V_{lo}(a) - U_{ho}(a)$. Moreover, the aircraft depreciates continuously, so that the agent has a capital loss equal to $U'_{ho}(a)$.

Similarly, the value function $S_{ho}(a)$ of an agent with valuation z_h who owns an aircraft of age a and is actively seeking to sell it satisfies:

$$\begin{aligned}\rho S_{ho}(a) &= \pi(z_h, a) - c_s + \lambda(V_{lo}(a) - S_{ho}(a)) + \gamma_s \max\{p(a) + V_{hn} - S_{ho}(a), 0\} + \\ &\quad \gamma_{sd} \max\{p_B(a) + V_{hn} - S_{ho}(a), 0\} + S'_{ho}(a). \end{aligned} \tag{2}$$

An agent with valuation z_h enjoys the flow utility $\pi(z_h, a)$ from an aircraft of age a, but he pays the flow cost c_s while he actively seeks to sell it. At any date, one of three possible events, at most, might happen to him: 1) At rate λ, his valuation drops to z_l. In this case, the agent chooses between keeping the aircraft (enjoying the value $U_{lo}(a)$) and actively seeking to sell it (enjoying value $S_{lo}(a)$). Hence, the agent obtains a capital loss equal to $V_{lo}(a) - S_{ho}(a)$. 2) At rate γ_s, the agent meets a potential buyer and chooses between trading the aircraft or keeping it. If he trades it at price $p(a)$, he becomes a high-valuation non-owner with value V_{hn}, thus obtaining a capital gain equal to $p(a) + V_{hn} - S_{ho}(a)$. If he keeps it, he obtains a capital gain of zero. 3) At rate γ_{sd}, he meets a dealer and chooses between trading the aircraft or keeping it. If he trades it at price $p_B(a)$, he then chooses between actively searching for another aircraft or not, thus obtaining a capital gain equal to $p_B(a) + V_{hn} - S_{ho}(a)$. If he keeps it, he obtains a capital gain of zero. Moreover, the aircraft depreciates continuously, so that the agent has a capital loss equal to $S'_{ho}(a)$.

The value functions $U_{lo}(a)$ and $S_{lo}(a)$ of an agent with valuation z_l who owns an aircraft of age a satisfy the following Bellman equations, respectively:

$$\rho U_{lo}(a) = \pi(z_l, a) + U'_{lo}(a), \tag{3}$$

$$\begin{aligned}\rho S_{lo}(a) &= \pi(z_l, a) - c_s + \gamma_s \max\{p(a) + V_{ln} - S_{lo}(a), 0\} + \\ &\quad \gamma_{sd} \max\{p_B(a) + V_{ln} - S_{lo}(a), 0\} + S'_{lo}(a). \end{aligned} \tag{4}$$

The interpretation of equations (3) and (4) is now straightforward. An agent with valuation z_l enjoys the flow utility $\pi(z_l, a)$ from an aircraft of age a. If he does not seek to sell the aircraft, the only event that affects his utility is the depreciation of the aircraft, generating

a capital loss equal to $U'_{lo}(a)$. If he seeks to sell the aircraft, his flow utility is reduced by the search cost c_s. Then, at any date, he meets a potential buyer at rate γ_s, in which case he chooses between selling the aircraft at price $p(a)$—thus obtaining a capital gain equal to $p(a) + V_{ln} - S_{lo}(a)$—or keeping it—thus obtaining a capital gain of zero. Similarly, he meets a dealer at rate γ_{sd}, in which case he chooses between trading the aircraft at price $p_B(a)$—thus, obtaining a capital gain equal to $p_B(a) + V_{ln} - S_{lo}(a)$—or keeping it—thus obtaining a capital gain of zero. Moreover, the aircraft depreciates continuously, so that the agent has a capital loss equal to $S'_{lo}(a)$.

The value functions of high- and low-valuation agents with no aircraft satisfy:

$$\begin{aligned}
\rho S_{hn} &= -c_s + \lambda(V_{ln} - S_{hn}) + \int \gamma_b(a) \max\{V_{ho}(a) - p(a) - S_{hn}, 0\} da + \qquad (5)\\
&\quad \int \gamma_{bd}(a) \max\{V_{ho}(a) - p_A(a) - S_{hn}, 0\} da,\\
\rho U_{hn} &= 0, \qquad (6)\\
\rho S_{ln} &= -c_s + \int \gamma_b(a) \max\{V_{lo}(a) - p(a) - S_{ln}, 0\} da + \qquad (7)\\
&\quad \int \gamma_{bd}(a) \max\{V_{lo}(a) - p_A(a) - S_{ln}, 0\} da,\\
\rho U_{ln} &= 0. \qquad (8)
\end{aligned}$$

Equation (5) says that a high-valuation agent with no aircraft who is paying the search cost c_s has a capital loss equal to $V_{ln} - S_{hn}$ when, at rate λ, his valuation drops from high to low; has a capital gain equal to $\max\{V_{ho}(a) - p(a) - S_{hn}, 0\}$ when, at rate $\gamma_b(a)$, he meets a seller of an aircraft of age a; and has a capital gain equal to $\max\{V_{ho}(a) - p_A(a) - S_{hn}, 0\}$ when, at rate $\gamma_{bd}(a)$, he meets a dealer selling an aircraft of age a, where $V_{ho}(a) = \max\{U_{ho}(a), S_{hn}\}$. Since the hn-agent does not know the age of the aircraft that the counterparty will have, he takes expectation over the possible capital gains that arise from the different vintages. Similarly, equation (7) says that a low-valuation agent with no aircraft who is paying the search cost c_s to search for a counterparty has an expected capital gain equal to $\max\{V_{lo}(a) - p(a) - S_{ln}, 0\}$ when he meets a potential seller of an aircraft of age a, and an expected capital gain equal to $\max\{\max\{V_{lo}(a), W_{lo}(a)\} - p_A(a) - W_{ln}, 0\}$ when he meets a dealer. Equations (6) and (8) say that agents without aircraft who are not searching have a zero value.

5.2.2 Dealers' Value Functions

As in Rubinstein and Wolinsky (1987), dealers can extract surplus by shortening the time that buyers and sellers have to wait in order to trade. They are capacity-constrained and cannot store more than one aircraft. Thus, the value functions $J_{do}(a)$ and J_{dn} of dealers with and without an aircraft for sale, respectively, satisfy:

$$\rho J_{do}(a) = \max\left\{-k + \alpha_{ds}\left(p_A(a) + J_{dn} - J_{do}(a)\right) + J'_{do}(a), \rho J_{dn}\right\}, \tag{9}$$

$$\rho J_{dn} = -k + \int \alpha_{db}(a) \max\left\{J_{do}(a) - p_B(a) - J_{dn}, 0\right\} da. \tag{10}$$

Equation (9) says that a dealer who owns an aircraft of age a chooses between two alternatives: 1) He can actively seek to sell it, meeting a potential buyer at rate α_{ds}. In this case, the dealer trades the aircraft at a negotiated price $p_A(a)$, obtaining a capital gain equal to $p_A(a) + J_{dn} - J_{do}(a)$. Moreover, the dealer pays the flow cost k and its aircraft depreciates continuously, so the dealer has a capital loss equal to $J'_{do}(a)$; and 2) he can scrap the aircraft, thereby becoming a dealer without aircraft for sale.

Similarly, equation (10) says that a dealer without aircraft pays the flow cost k while he actively seeks to purchase one. At rate $\alpha_{db}(a)$, this dealer meets a potential seller of an aircraft of age a, in which case the dealer decides whether or not to trade, thus enjoying a capital gain equal to $\max\left\{J_{do}(a) - p_B(a) - J_{dn}, 0\right\}$. Since the dn-dealer does not know the age of the aircraft that the counterparty will have, he takes expectation over the possible capital gains that arise from the different vintages.

Dealers' free entry requires that $J_{dn} = 0$—i.e., dealers' expected capital gain is exactly equal to their fixed operating cost.

5.2.3 Prices

When a buyer and a seller meet and agree to trade, the negotiated price

$$p(a) = (1 - \theta_s)\left(S_{lo}(a) - V_{ln}\right) + \theta_s\left(U_{ho}(a) - S_{hn}\right) \tag{11}$$

is the solution to the following symmetric-information bargaining problem:

$$\begin{aligned} & \max_{p(a)} \left[U_{ho}(a) - p(a) - S_{hn}\right]^{1-\theta_s} \left[p(a) + V_{ln} - S_{lo}(a)\right]^{\theta_s} \\ \text{subject to:} \quad & U_{ho}(a) - p(a) - S_{hn} \geq 0 \text{ and } p(a) + V_{ln} - S_{lo}(a) \geq 0. \end{aligned}$$

Similarly, the ask and bid prices $p_A(a)$ and $p_B(a)$ satisfy:

$$p_A(a) = (1-\theta_d)(J_{do}(a) - J_{dn}) + \theta_d(U_{ho}(a) - S_{hn}), \quad (12)$$

$$p_B(a) = (1-\theta_d)(J_{do}(a) - J_{dn}) + \theta_d(S_{lo}(a) - V_{ln}). \quad (13)$$

5.2.4 Agents' and Dealers' Policies

We can simplify the value functions of all types of agents $\{ho, lo, hn, ln\}$ recognizing that gains from trade may arise only when high-valuation non-owners meet low-valuation owners. Moreover, since the flow payoff $\pi(z, a)$ exhibits complementarity between agents' valuation z and the age a of the asset, the gains from trade are larger for younger assets. In turn, this implies that, if the assets are sufficiently heterogeneous between them (i.e., either δ_2 or T is large), not all assets trade in equilibrium.

More specifically, since $U_{ho}(a)$ is intuitively decreasing in a, there exists a cutoff age a_h^* such that *ho*-agents scrap their assets when it reaches age a_h^*—i.e., a_h^* satisfies $U_{ho}(a_h^*) = S_{hn}$. Moreover, this same cutoff age a_h^* determines whether or not *hn*-agents purchase an age-a asset. To understand this, note that *hn*-agents purchase an age-a asset if their gains from trade are positive: $U_{ho}(a) - p(a) - S_{hn} \geq 0$, or $0 \leq p(a) \leq U_{ho}(a) - S_{hn}$. Since the negotiated prices (11) are lower than the prices that sellers would command if they captured all surplus (i.e., $\theta_s = 1$), then the condition $p(a) \leq U_{ho}(a) - S_{hn}$ always holds, as long as *lo*-owners of these assets are willing to sell them (see next paragraph). An identical argument implies that *hn*-agents purchase assets of age $a \leq a_h^*$ from dealers as long as dealers have inventories of these assets.

The policy of *lo*-owners satisfies a cutoff rule, as well. Specifically, since the flow utility $\pi(z_l, a)$ is decreasing in the age a of the asset, whereas the flow search cost c_s is constant, there exists a cutoff age a_l^* such that *lo*-agents sell their aircraft if it is younger than a_l^*, but keep it if it is older—i.e., $U_{lo}(a) > S_{lo}(a)$ if $a < a_l^*$, $U_{lo}(a_{lo}^*) = S_{lo}(a_{lo}^*)$ and $U_{lo}(a) < S_{lo}(a)$ if $a > a_l^*$. Equilibrium requires that *lo*-agents sell assets that *hn*-agents are willing to purchase—thus, $a_l^* \leq a_h^*$. Moreover, since *lo*-agents enjoy positive utility from these old assets, and this utility is greater than the opportunity cost of searching for a younger aircraft, they keep these old assets until they reach the scrappage age T.

Thus, the agents' value functions are:

$$\begin{aligned} V_{ho}(a) &= \begin{cases} U_{ho}(a) & \text{for } a < a_h^*, \\ S_{hn} & \text{for } a \geq a_h^*, \end{cases} \\ V_{lo}(a) &= \begin{cases} S_{lo}(a) & \text{for } a < a_l^*, \\ U_{lo}(a) & \text{for } a \leq a_l^* < T, \\ V_{ln} & \text{for } a = T, \end{cases} \\ V_{hn} &= S_{hn}, \\ V_{ln} &= 0. \end{aligned}$$

Similar arguments apply to dealers, as well. Their value function $J_{do}(a)$ is intuitively decreasing in a because younger aircraft sell at greater margins, as the gains from trade are larger. Thus, since dealers' fixed cost k is independent of the age of the asset, there exist two cutoff ages a_{dn}^* and a_{do}^* such that dealers do not purchase aircraft older than a_{dn}^* and scrap aircraft when they reach age a_{do}^*. Since the asset purchase price is sunk at the time of scrapping it, but not at the time of buying it, $a_{dn}^* < a_{do}^*$. Moreover, equilibrium requires that dealers sell assets that buyers are willing to purchase. Hence, $a_{do}^* \leq a_h^*$.

5.2.5 Distribution of Agents

Let $\mu_i(a)$ be the masses of owners of age-a aircraft whose state is $i \in \{ho, lo, do\}$, and let μ_i be the masses of non-owners whose state is $i \in \{hn, ln, dn\}$. The masses of owners evolve over time according to the following system of differential equations:

$$\dot{\mu}_{ho}(a) = \gamma_b(a)\mu_{hn} + \gamma_{bd}(a)\mu_{hn} - \lambda\mu_{ho}(a) \text{ for } a < a_h^*, \tag{14}$$

$$\begin{aligned} \dot{\mu}_{lo}(a) &= \lambda 1(a < a_h^*)\mu_{ho}(a) - \gamma_s 1(a < a_h^*)1(a < a_l^*)\mu_{lo}(a) \\ &\quad -\gamma_{sd}1(a < a_{dn}^*)1(a < a_l^*)\mu_{lo}(a) \text{ for } a < T, \end{aligned} \tag{15}$$

$$\dot{\mu}_{do}(a) = \alpha_{db}(a)1(a \leq a_{dn}^*)\mu_{dn} - \alpha_{ds}\mu_{do}(a) \text{ for } a < a_{do}^*, \tag{16}$$

with initial conditions $\mu_{ho}(0) = x$ and $\mu_{lo}(0) = \mu_{do}(0) = 0$, and terminal conditions $\mu_{ho}(a) = 0$ for $a_h^* < a < T$ and $\mu_{do}(a) = 0$ for $a_{do}^* < a < T$. The notation $1(Y)$ represents an indicator function equal to one if the event Y is true, and zero otherwise.

The intuition for these equations is simple. Specifically, equation (14) states that the mass of high-valuation agents with an age-a asset is the result of flows of three sets of agents: 1)

the inflow of high-valuation non-owners that found a seller of an age-a aircraft—the term $\gamma_b(a)\,\mu_{hn}$; 2) the inflow of high-valuation non-owners that found a dealer selling an age-a aircraft—the term $\gamma_{bd}(a)\,\mu_{hn}$; and 3) the outflow of high-valuation age-a aircraft owners whose valuation just dropped—the term $\lambda\mu_{ho}(a)$. Moreover, equation (14) incorporates the equilibrium outcomes that low-valuation owners of age-a aircraft are willing to sell it rather than keep it (i.e., $\gamma_b(a) > 0$) and dealers have inventories of age-a aircraft (i.e., $\gamma_{bd}(a) > 0$) only if high-valuation non-owners are willing to buy them (i.e., $a < a_h^*$). Equation (15) states that the mass of low-valuation agents with an age-a asset is the result of flows of three sets of agents: 1) the inflow of high-valuation agents whose valuation just dropped. Since high-valuation agents scrap their assets when they reach age a_h^*, this inflow applies only to owners of assets younger than a_h^*. Hence, the term $\lambda 1(a < a_h^*)\,\mu_{ho}(a)$; 2) the outflow of low-valuation aircraft owners who prefer to sell their aircraft rather than keep it (i.e., an aircraft of age $a < a_l^*$) and found a buyer willing to purchase it (i.e., an aircraft of age $a < a_h^*$). Hence the term $\gamma_s 1(a < a_h^*)\,1(a < a_l^*)\,\mu_{lo}(a)$; and 3) the outflow of low-valuation aircraft owners that prefer to sell their aircraft rather than keep it (i.e., an aircraft of age $a < a_l^*$) and that found a dealer willing to purchase it (i.e., an aircraft of age $a < a_{dn}^*$). Hence the term $\gamma_{sd} 1(a < a_{dn}^*)\,1(a < a_l^*)\,\mu_{lo}(a)$. The intuition for equation (16) is similar.

Moreover, in a small time interval of length ϵ, up to terms in $o(\epsilon)$, the masses of high-valuation non-owners and dealers without inventories evolve from time t to time $t+\epsilon$ according to:

$$\begin{aligned}
\dot{\mu}_{hn} &= (\mu - x) + \mu_{ho}(a_{ho}^*) - \lambda\mu_{hn} - \mu_{hn}\int_0^{a_l^*}\gamma_b(a)\,da - \mu_{hn}\int_0^{a_{dn}^*}\gamma_{bd}(a)\,da, \\
\dot{\mu}_{dn} &= \alpha_{ds}\int_0^{a_{do}^*}\mu_{do}(a)\,da - \mu_{dn}\int_0^{a_l^*}\alpha_{db}(a)\,1(a \le a_{dn}^*)\,da + \mu_{do}(a_{dn}^{**}),
\end{aligned}$$

whereas, in each instant, the mass of ln-agents equals:

$$\mu_{ln} = \lambda\mu_{hn} + \gamma_s\int_0^{a_l^*}\mu_{lo}(a)\,1(a \le a_h^*)\,da + \gamma_{sd}\int_0^{a_l^*}\mu_{lo}(a)\,1(a \le a_{dn}^*)\,da + \mu_{lo}(T).$$

Steady state imposes the following constraints on the evolution of these masses: 1) The aggregate masses of owners $\int_0^T \mu_i(a)\,da$ for $i \in \{ho, lo, do\}$ and the aggregate masses μ_{hn} and μ_{dn} are constant over time; 2) the mass μ_{ln} of exiters equals the mass μ of new entrants; 3) the total mass of agents with high valuation $\mu_{hn} + \int_0^T \mu_{ho}(a)\,da$ equals $\frac{\mu}{\lambda}$; 4) the total mass of dealers μ_d equals $\int_0^T \mu_{do}(a)\,da + \mu_{dn}$; 5) the aggregate masses of assets sold and pur-

chased by dealers equal the aggregate masses of assets purchased from and sold to dealers: $\mu_{hn}\int_0^{a^*_{do}}\gamma_{bd}(a)\,da = \alpha_{ds}\int_0^{a^*_{do}}\mu_{do}(a)\,da$ and $\mu_{dn}\int_0^{a^*_{dn}}\alpha_{db}(a)\,da = \gamma_{sd}\int_0^{a^*_{dn}}\mu_{lo}(a)\,da$; and 6) dealers' aggregate inventories are constant over time: $\mu_{hn}\int_0^{a^*_{do}}\gamma_{bd}(a)\,da = \alpha_{ds}\int_0^{a^*_{do}}\mu_{do}(a)\,da = \mu_{dn}\int_0^{a^*_{dn}}\alpha_{db}(a)\,da = \gamma_{sd}\int_0^{a^*_{dn}}\mu_{lo}(a)\,da$. In addition, equilibrium requires that the total mass of owners of age-a aircraft $\mu_{ho}(a)+\mu_{lo}(a)+\mu_{do}(a)$ equals the mass x of aircraft for $a \leq \min\{a^*_{do}, a^*_h\}$, and $\mu_{ho}(a)+\mu_{lo}(a)+\mu_{do}(a) < x$ for $a > \min\{a^*_{do}, a^*_h\}$.

All these steady-state equalities and the equilibrium condition $\mu_{ho}(a)+\mu_{lo}(a)+\mu_{do}(a) \leq x$ allow us to solve for the endogenous masses as a function of the exogenous parameters x, T, μ and λ, and the exogenous parameters of the matching functions.

Letting γ increase, $\lim_{\gamma\to+\infty}\mu_{lo}(a)$ converges to 0: When frictions vanish, no low-valuation agent owns an aircraft. Similarly, $\lim_{\gamma'\to+\infty}\mu_{do}(a)$ converges to 0. Thus, the masses $\int_0^T\mu_{lo}(a)\,da$ and $\int_0^T\mu_{do}(a)\,da$—i.e., the masses of low-valuation agents and dealers with inventories—are measures of assets inefficiently allocated in the economy. Based on the parameters estimated from the data, in Section 7.1, I compute these measures to quantify the effects of trading frictions on asset allocations.

6 Quantitative Analysis

The model does not admit an analytic solution for the asset allocations and asset prices as a function of all the model primitives, except in the special case when assets are homogeneous—i.e., $\delta_2 = 0$. Appendix B reports this special case. Hence, the goal of the quantitative analysis is to choose the parameters that best match moments of the data with the corresponding moments computed from the model's numerical solution. Since δ_2 is estimated to be small (approximately equal to .02), the analytic solution of the model under the assumption that $\delta_2 = 0$, presented in Appendix B, represents a good approximation of the general model, thus making the intuition for the identification of the parameters more transparent.

6.1 Estimation and Identification

I estimate the model using the data on business aircraft described in Section 4, assuming that they are generated from the model's steady state. I set the unit of time to be one quarter.

Unfortunately, the data lack some detailed information to identify all parameters. Therefore, I have to fix some values. Specifically, the discount rate ρ is traditionally difficult to

identify, and I fix it to $\rho = .015$. Moreover, I fix the useful lifetime of an aircraft to be equal to $T = 160$ quarters. Furthermore, I use a directory of aircraft dealers to "estimate" total dealers' capacity to be equal to $\mu_d = 1000$. Moreover, I assume that aircraft owners' flow payoff equals:

$$\pi(z, a) = z\left(\delta_0 + \delta_1 e^{-\delta_2 a}\right), \tag{17}$$

further imposing that $\delta_1 = 1$, as this parameter is not separately identified from the baseline valuation $z_l\delta_0$ (this is just a normalization).

I estimate the vector of other parameters $\psi = \{\lambda, \gamma_s, \gamma_{sd}, \alpha_{ds}, \mu, z_h, z_l, \delta_0, \delta_2, c_s, k\}$ using a minimum-distance estimator that matches key moments of the data with the corresponding moments of the model. More precisely, for any value of these parameters, I solve the model described in Section 5 to find agents' and dealers' policy functions, and agents' and dealers' distributions that are consistent with each other. Based on the model's solution, I calculate the vector $m(\psi)$ composed by the following moments:

1. The fraction of aircraft for sale, which, in the model, equals:

$$\frac{\int_0^{a_l^*} \mu_{lo}(a)\, da + \int_0^{a_{do}^*} \mu_{do}(a)\, da}{A}. \tag{18}$$

2. The fraction of aircraft for sale by dealers (i.e., aggregate dealers' inventories), which equals:

$$\frac{\int_0^{a_{do}^*} \mu_{do}(a)\, da}{A}. \tag{19}$$

3. The fraction of retail-to-retail transactions to total aircraft, which equals:

$$\frac{\gamma_s \int_0^{a_l^*} \mu_{lo}(a)\, da}{A}. \tag{20}$$

4. The fraction of dealer-to-retail transactions to total aircraft, which equals:

$$\frac{\alpha_{ds} \int_0^{a_{do}^*} \mu_{do}(a)\, da}{A}. \tag{21}$$

5. The average age of aircraft for sale, which equals:

$$\frac{\int_0^{a_l^*} a\mu_{lo}(a)\,da + \int_0^{a_{do}^*} a\mu_{do}(a)\,da}{\int_0^{a_l^*} \mu_{lo}(a)\,da + \int_0^{a_{do}^*} \mu_{do}(a)\,da}.$$

6. The total mass of assets A, which equals:

$$\int_0^T (\mu_{ho}(a) + \mu_{lo}(a) + \mu_{do}(a))\,da.$$

7. The mass of dealers μ_d, which equals:

$$\mu_{dn} + \int_0^{a_{do}^*} \mu_{do}(a)\,da.$$

8. For each age $a \in [0, 120]$, the price $p(a)$, which equals:

$$p(a) = (1-\theta_s)(S_{lo}(a) - V_{ln}) + \theta_s(U_{ho}(a) - S_{hn}). \tag{22}$$

9. For each age $a \in [0, 120]$, the bid price $p_B(a)$, which equals:

$$p_B(a) = (1-\theta_d)(J_{do}(a) - J_{dn}) + \theta_d(S_{lo}(a) - V_{ln}). \tag{23}$$

10. Dealers' free-entry conditions:

$$J_{dn} = 0.$$

The minimum-distance estimator minimizes the criterion function

$$(m(\psi) - m_S)'\,\Omega\,(m(\psi) - m_S),$$

where m_S is the vector of sample moments corresponding to the vector $m(\psi)$ of moments, and Ω is a symmetric, positive-definite weighting matrix. In practice, I choose Ω to be a diagonal matrix with diagonal elements that are proportional to the inverse of $m_S'm_S$ to make the scale comparable across moments. Moreover, since I am matching the moments $p(a)$ and $p_B(a)$ for every age $a \in [0, 120]$, I weight each of these price moments by $\frac{1}{121}$ to make their aggregate weight in the criterion function comparable to the weight of each of the non-price moments.

Although the model is highly nonlinear, so that (almost) all parameters affect all outcomes, the identification of some parameters relies on some key moments in the data. Specifically, equations (18)-(21), along with the differential equations (14)-(16) that define the functions $\mu_{ho}(a)$, $\mu_{lo}(a)$ and $\mu_{do}(a)$, identify the parameters λ, γ_s, γ_{sd} and α_{ds}. Table 1 reports that the fraction of aircraft for sale is non-trivial, thus indicating that trading delays are relevant. Hence, the model can fit this key feature of the data very well. Similarly, Table 1 reports that dealers are faster than sellers at trading aircraft, and the model captures this difference through the trading parameters γ_s, γ_{sd} and α_{ds}.

From the estimates of the parameters λ, γ_s, γ_{sd}, α_{ds}, together with A and μ_d, we can recover μ, γ and γ' from the arrival rates (24)-(29) and from the steady-state condition:

$$\frac{\mu}{\lambda} = \mu_{hn} + \int_0^T \mu_{ho}(a)\, da.$$

For any value of the bargaining parameters θ_s and θ_d, from the price equations (22) and (23), we can recover the values of the parameters z_h, z_l, c_s, δ_0 and δ_2. The price differences across vintages identify the parameter δ_2, whereas the level of prices identifies the parameters z_h, z_l, c_s, δ_0. Moreover, the separate identification of z_h, z_l, and c_s, δ_0 relies on the assumptions that the valuations z_h and z_l interact with the age of the aircraft in the flow utility (17), whereas the search cost c_s and δ_0 do not. Finally, I can recover k from dealers' free-entry condition $J_{dn} = 0$.

The bargaining parameters θ_s and θ_d are not point-identified. The reason is that, conditional on the trading frictions, prices depend on the gains from trade (i.e., agents' valuations) and how these gains are split between the parties (i.e., agents' bargaining parameters). Thus, the prices alone do not separately identify agents' valuations and bargaining parameters. Therefore, I estimate an interval for their values by imposing that the estimated parameters satisfy some restrictions dictated by the model. Specifically, I require the following

inequalities to hold at the estimated parameters:

$$\begin{aligned} z_h &\geq z_l, \\ z_l &\geq 0, \\ c_s &\geq 0, \\ k &\geq c_s, \\ W_{hn} &\geq 0. \end{aligned}$$

Thus, I impose that: buyers' valuation is higher than sellers'; sellers have non-negative valuations for the assets; the cost of search is non-negative; dealers' fixed cost is greater than agents' search cost; and high-valuation agents with no aircraft have a positive value of searching. Since the bargaining parameters are set-identified only, the other parameters also are set-identified only.[10]

6.1.1 Estimates

Figure 2 displays the set of values of the bargaining parameters consistent with the constraints imposed by the model. The set rules out that the "short" side of the market—i.e., sellers and dealers, in particular—captures the minimal share of the surplus. Table 2 reports estimates of all other parameters for selected values of the bargaining parameters: the approximate vertices and the approximate centroid of the set. Since some parameters are combinations of other parameters, I compute standard errors by bootstrapping the data using 100 replications.

The magnitude of the parameter λ indicates that, on average, valuations switch from high to low approximately every four years. The magnitude of the sum $\gamma_s + \gamma_{sd}$ indicates that aircraft stay on the market approximately six months before a low-valuation owner is able to sell it. The magnitude of the parameter α_{ds} indicates that, on average, it takes slightly less than three months for a dealer to find a buyer. These parameters γ_s, γ_{sd} and

[10]Some papers in the labor literature attempt to estimate workers' and firms' bargaining parameters; for recent contributions within the labor-search literature, see Cahuc, Postel-Vinay and Robin (2006) and Flinn (2006). The key advantage of those papers is that they can use additional information on labor demand or on firms' production function. With this additional information, they can use the observed wages to infer workers' bargaining parameter. In the current setting, it would correspond to having additional information on aircraft demand that could be used to infer agents' valuations z_h and z_l. Instead, in the absence of this information, I am using prices to recover agents' valuations and other restrictions of the model to rule out some values of the bargaining parameters.

TABLE 2: Parameter Estimates

	$\theta_s = .25$ $\theta_d = .45$	$\theta_s = .97$ $\theta_d = .75$	$\theta_s = .49$ $\theta_d = .75$	$\theta_s = .27$ $\theta_d = .99$	$\theta_s = .71$ $\theta_d = .99$
λ	0.0759	0.0582	0.0674	0.0780	0.0609
	[0.0754; 0.0761]	[0.0564; 0.0592]	[0.0666; 0.0678]	[0.0773; 0.0786]	[0.0588; 0.0613]
γ_s	0.2296	0.2297	0.2351	0.2324	0.2348
	[0.2212; 0.2298]	[0.2154; 0.2351]	[0.2285; 0.2363]	[0.2215; 0.2324]	[0.2250; 0.2401]
γ_{sd}	0.3295	0.3327	0.3300	0.3506	0.3487
	[0.3280; 0.3305]	[0.3084; 0.3399]	[0.3144; 0.3310]	[0.3476; 0.3506]	[0.3207; 0.3515]
α_{ds}	1.0679	0.7547	0.9011	1.1546	0.7906
	[1.0642; 1.0702]	[0.7334; 0.7821]	[0.8835; 0.9086]	[1.1402; 1.1582]	[0.7636; 0.8041]
γ	$0.0943 * 10^{-3}$	$0.1602 * 10^{-3}$	$0.1223 * 10^{-3}$	$0.0910 * 10^{-3}$	$0.1601 * 10^{-3}$
	$[0.0922; 0.0957] * 10^{-3}$	$[0.1409; 0.1674] * 10^{-3}$	$[0.1177; 0.1229] * 10^{-3}$	$[0.0872; 0.0927] * 10^{-3}$	$[0.1411; 0.1655] * 10^{-3}$
γ'	$0.4452 * 10^{-3}$	$0.4931 * 10^{-3}$	$0.4685 * 10^{-3}$	$0.4601 * 10^{-3}$	$0.5404 * 10^{-3}$
	$[0.4430; 0.4474] * 10^{-3}$	$[0.4931; 0.5392] * 10^{-3}$	$[0.4508; 0.4732] * 10^{-3}$	$[0.4570; 0.4613] * 10^{-3}$	$[0.4997; 0.5458]$
μ	684.5	586.1	620.6	703.2	593.1
	[679.0; 686.2]	[557.3; 624.2]	[606.9; 631.5]	[691.8; 705.7]	[567.2; 623.8]
δ_2	0.0198	0.0201	0.0197	0.0176	0.0197
	[0.0198; 0.0200]	[0.0201; 0.0207]	[0.0196; 0.0198]	[0.0175; 0.0178]	[0.0197; 0.0202]
δ_0	0.0195	0.0115	0.0130	0.0123	0.0101
	[0.0195; 0.0203]	[0.0109; 0.0118]	[0.0130; 0.0135]	[0.0123; 0.0128]	[0.0095; 0.0104]
z_h	715, 940	612, 670	666, 320	682, 850	612, 050
	[706, 590; 717, 900]	[610, 260; 638, 050]	[633, 390; 668, 680]	[677, 500; 686, 480]	[606, 120; 629, 450]
z_l	342, 320	303, 460	319, 960	323, 780	347, 830
	[335, 560; 347, 860]	[288, 060; 314, 510]	[315, 530; 328, 030]	[319, 520; 325, 630]	[347, 230; 369, 630]
c_s	49, 849	17, 323	36, 502	19, 983	18, 833
	[49, 849; 51, 143]	[16, 866; 18, 513]	[36, 406; 38, 837]	[19, 983; 20, 400]	[18, 758; 19, 773]
k	73, 434	22, 837	103, 690	381, 360	79, 133
	[71, 977; 76, 568]	[20, 196; 32, 255]	[97, 290; 107, 480]	[376, 310; 195, 460]	[67, 752; 85, 356]

Notes—This table reports the estimates of the parameters. 95-percent confidence intervals in brackets are obtained bootstrapping the data using 100 replications.

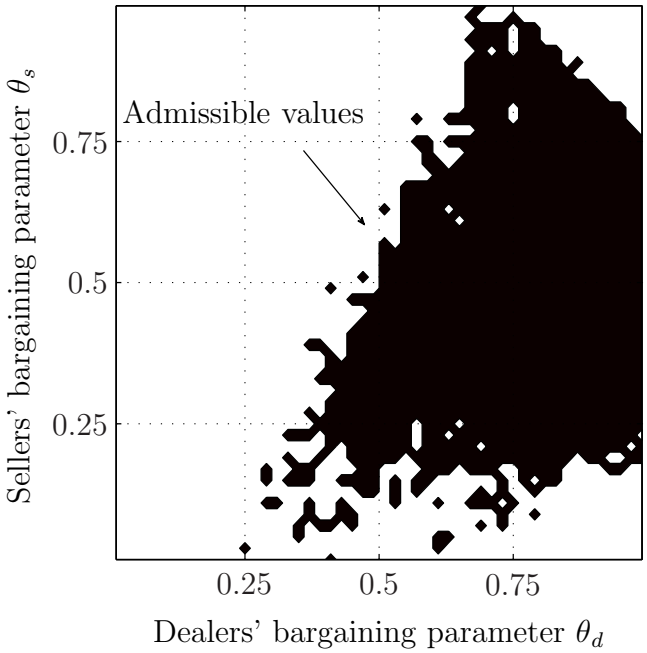

FIG. 2: Admissible values of the bargaining parameters.

α_{ds} imply that trading delays are non-trivial in this market, and dealers play an important role in reducing them.

The parameter δ_2 indicates that aircraft depreciate by approximately eight percent every year, a decline comparable to that of larger commercial aircraft. Indeed, business aircraft older than thirty years are common (see Figure 1). The valuations z_h and z_l indicate that the difference between buyers' and sellers' valuations are large—i.e., gains from trade are large—and this difference is slightly larger when sellers have lower bargaining parameters. The parameter c_s indicates that search costs are non-trivial, varying between \$17,000 and \$50,000 per quarter, depending on the bargaining parameters. On average, each dealer trades one aircraft every $\frac{1}{\int_0^{a^*_{do}} \alpha_{db}(a)da} + \frac{1}{\alpha_{ds}} \simeq 3$ quarters. Combined with the difference between retail and wholesale prices reported in Table 1 and dealers' free-entry condition $J_{dn} = 0$, dealers' fixed flow cost k obtains. Table 2 reports that dealers' cost k varies dramatically with the bargaining parameters—from approximately \$20,000 to approximately \$400,000 per quarter—suggesting that the costs of "market-making" could be large.

Appendix A reports on the robustness of the parameter estimates under the alternative assumption that the matching function exhibits constant returns to scale. The estimates and their implications for asset allocations and asset prices are very similar to those reported in Table 2.

7 Counterfactual Analyses

I now perform two counterfactuals to answer the questions of how frictions and how dealers affect asset allocations and prices. In both cases, I compute the equilibrium in these counterfactual scenarios using the parameter estimates reported in column (3) of Table 2 from the model with bargaining parameters $\theta_s = .49$ and $\theta_d = .75$, the approximate centroid of the set displayed in Figure 2. (These counterfactuals could be computed for any values of the bargaining parameters.)

7.1 Quantifying the Effects of Frictions

The estimates of the parameters allow me to quantify the effect of trading frictions on the allocation and the prices of assets. Specifically, I compare asset allocations and asset prices with the Walrasian efficient benchmark, which is a special case of the model presented in Section 5 when trade is frictionless—i.e., $\gamma \to +\infty$ and $c_s = 0$.

In the Walrasian market, as soon as high-valuation agents enter the market, they immediately meet low-valuation sellers or dealers. Hence, in a frictionless market, no low-valuation agents and no dealers have aircraft—i.e., $\mu_{lo}^w(a) = \mu_{do}^w(a) = 0$ for any a. Thus, the sum $\int_0^T (\mu_{lo}(a) + \mu_{do}(a))\, da$ measures the total mass of assets misallocated due to search frictions. Of these misallocated assets, the mass $\int_0^{a_l^*} \mu_{lo}(a)\, da + \int_0^{a_{do}^*} \mu_{do}(a)\, da$ is on the market for sale, whereas the mass $\int_0^T (\mu_{lo}(a) + \mu_{do}(a))\, da - \int_0^{a_l^*} \mu_{lo}(a)\, da - \int_0^{a_{do}^*} \mu_{do}(a)\, da = \int_{a_l^*}^T \mu_{lo}(a)\, da$ corresponds to the mass of aircraft older than a_l^* that low-valuation owners are keeping rather than selling since sellers' expected share of the total gains from trade are lower than their transaction costs. Overall, the allocative costs of the trading frictions, captured by the parameters γ_s, γ_{sd} and α_{ds}, depend on how frequently agents seek to trade, captured by the parameter λ.

Walrasian prices solve:

$$p^w(a) = \int_a^T e^{-\rho(t-a)} z_h \left(\delta_0 + \delta_1 e^{-\delta_2 t}\right) dt$$

and, thus, are equal to $p^w(a) = \frac{z_h \delta_0}{\rho}\left(1 - e^{-(T-a)\rho}\right) + \frac{z_h \delta_1 e^{\rho a}}{\rho + \delta_2}\left(e^{-a(\delta_2+\rho)} - e^{-T(\delta_2+\rho)}\right)$. Instead, in the estimated model with frictions, prices are equal to $p(a) = (1-\theta_s)(S_{lo}(a) - V_{ln}) + \theta_s (U_{ho}(a) - S_{hn})$, as in equation (11).

Columns (1) and (2) in Table 3 report the calculations of these magnitudes for the model

TABLE 3: Comparison with Walrasian Market

	(1)	(2)	(3)
	ESTIMATED MODEL	WALRASIAN MARKET	$c_s = 0$
$\int_0^T (\mu_{lo}(a) + \mu_{do}(a))\, da$	$1,851$	0	$2,313$
	$[1,805; 1,888]$	$[0; 0]$	$[2,234; 2,404]$
$\frac{\int_0^T (\mu_{lo}(a)+\mu_{do}(a))da}{A}$	0.203	0	0.253
	$[0.198; 0.203]$	$[0; 0]$	$[0.251; 0.259]$
$\int_0^{a_l^*} \mu_{lo}(a)\, da + \int_0^{a_{do}^*} \mu_{do}(a)\, da$	$1,064$	0	$1,238$
	$[1,061; 1,097]$	$[0; 0]$	$[1,183; 1,277]$
$\frac{\int_0^{a_l^*} \mu_{lo}(a)da + \int_0^{a_{do}^*} \mu_{do}(a)da}{A}$	0.116	0	0.135
	$[0.116; 0.118]$	$[0; 0]$	$[0.133; 0.137]$
$p(0)$	$18,340,857$	$19,638,883$	$18,228,984$
	$[17,559; 18,367] * 10^3$	$[18,862; 19,813] * 10^3$	$[17,478; 18,269] * 10^3$
$p(10)$	$8,278,037$	$9,063,420$	$8,178,205$
	$[8,021; 8,307] * 10^3$	$[8,921; 9,336] * 10^3$	$[7,972; 8,212] * 10^3$

Notes—This table reports counterfactual allocations and prices. 95-percent confidence intervals in brackets are obtained bootstrapping the data using 100 replications.

and the Walrasian benchmark, respectively, using the parameters estimated with $\theta_s = .45$ and $\theta_d = .75$. (Other values of the bargaining parameters have no effects on Walrasian allocations and modest effects on Walrasian prices.) The first row reports that trading frictions cause the misallocation of 1,851 aircraft, corresponding to 20.3 percent of all aircraft. The third row reports that 1,064 aircraft are on the market for sale, whereas, for the remaining $1,851 - 1,064 = 787$ aircraft, the transaction costs of selling them are larger the sellers' expected gains from trade and, thus, these aircraft are not even on the market for sale. The instantaneous welfare loss due to this misallocation is equal to the difference between the utility flows in the estimated model and in the Walrasian market, and this corresponds to $\int_0^T \left(\mu_{ho}(a) z_h + \mu_{lo}(a) z_l\right)\left(\delta_0 + \delta_1 e^{-\delta_2 a}\right) da - xT \int_0^T z_h \left(\delta_0 + \delta_1 e^{-\delta_2 a}\right) da$. Using the estimated parameters, the welfare loss equals $\$183,585,647$ per quarter, or $\frac{\int_0^T (\mu_{lo}(a)(z_h - z_l) + \mu_{do}(a) z_h)\left(\delta_0 + \delta_1 e^{-\delta_2 a}\right) da}{xT \int_0^T z_h \left(\delta_0 + \delta_1 e^{-\delta_2 a}\right) da} =$ 9.42 percent of total potential welfare. Table 3 further shows that trading frictions decrease the price of new aircraft by approximately $\frac{p(0) - p^w(0)}{p^w(0)} = -6.61$ percent (approximately $\$1,298,000$) relative to the Walrasian benchmark. The percent decrease of a ten-year-old aircraft is similar.

Two forces affect allocations and prices in the estimated model relative to the Walrasian market: search costs and trading delays. We can assess the relative contributions of these two forces by computing the equilibrium when $c_s = 0$; column (3) in Table 3 reports these

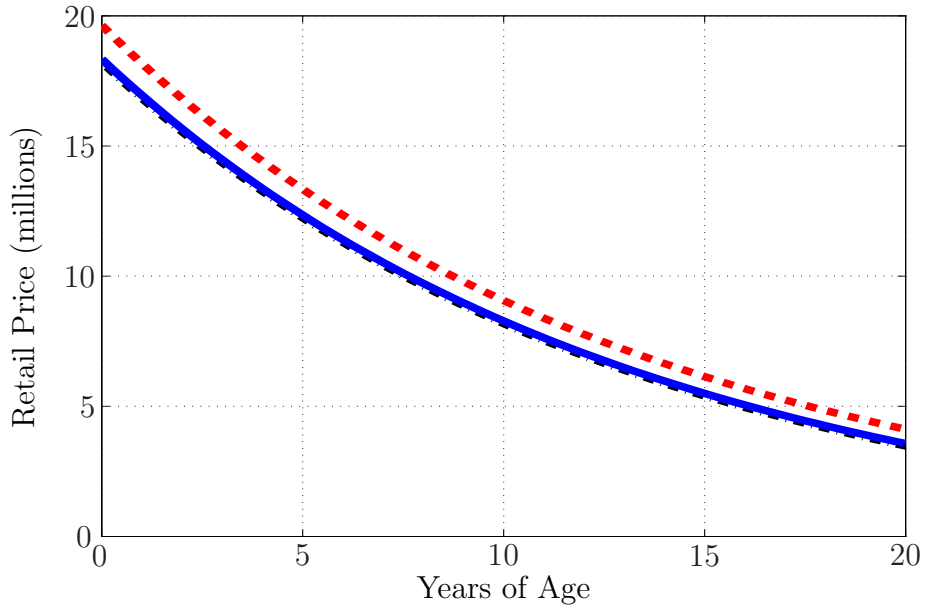

FIG. 3: Actual prices (solid line), Walrasian prices (dashed line) and prices without search costs (dash-dotted line) as a function of aircraft age.

calculations. The direct effect of the elimination of search costs is that parties' outside options in bargaining increase, thereby affecting negotiated prices. In addition, this reduction in search costs has general-equilibrium effects by affecting dealers' margins. Thus, dealers' free-entry condition implies that their mass changes, as well. Since dealers' mass determines agents' buying and selling rates, the elimination of search costs affects equilibrium allocations, further affecting equilibrium prices. More precisely, dealers' margins decrease, thereby decreasing their mass μ_d and their aggregate inventories. Hence, sellers' trading rates with dealers γ_{sd} decrease, thereby increasing the masses $\mu_{lo}(a)$ of sellers. Overall, column (3) says that the mass of misallocated aircraft increases when $c_s = 0$, indicating that inefficiencies may *increase* when search costs vanish—i.e., in contrast to a large search literature. Column (3) further shows that asset prices are essentially unchanged when $c_s = 0$. Thus, an interesting conclusion of the counterfactual analysis of column (3) is that it may be difficult to infer exclusively from asset prices the effects of changes in trading frictions on market inefficiencies.

Figure 3 plots the estimated and counterfactual prices, confirming that actual prices (the solid line) are always lower than Walrasian prices (the dashed line) and prices without search costs (the dash-dotted line). Moreover, the figure displays an additional interesting pattern: Walrasian prices decline at a faster rate than actual prices. The reason is that the willingness

to pay for a marginally younger—i.e., better—aircraft is higher if there are no frictions.

Overall, these counterfactuals imply non-trivial effects on allocations and prices, illustrating clearly the importance of frictions in this decentralized market.

7.2 Quantifying the Effects of Dealers

The estimates of the parameters allow me to quantify the effects of dealers on the allocation and prices of assets. Specifically, I compare asset allocations and asset prices with the counterfactual of no dealers—i.e., $\gamma' = 0$ or $k \to +\infty$.

If there are no dealers, the distribution of agents evolves according to:

$$\begin{aligned}
\dot{\mu}_{ho}(a) &= \gamma_b(a)\,\mu_{hn} - \lambda\mu_{ho}(a) \text{ for } a < a_h^{**}, \\
\dot{\mu}_{lo}(a) &= \lambda 1(a < a_h^{**})\,\mu_{ho}(a) - \gamma_s 1(a < a_h^{**})\,1(a < a_l^{**})\,\mu_{lo}(a) \text{ for } a < T, \\
\dot{\mu}_{hn} &= (\mu - x) + \mu_{ho}(a_{ho}^{**}) - \lambda\mu_{hn} - \mu_{hn}\int_0^{a_l^{**}} \gamma_b(a)\,da,
\end{aligned}$$

with initial conditions $\mu_{ho}(0) = x$ and $\mu_{lo}(0) = 0$ and terminal conditions $\mu_{ho}(a) = 0$ for $a_h^{**} < a < T$. Moreover, in each instant, the mass of ln-agents equals:

$$\mu_{ln} = \lambda\mu_{hn} + \gamma_s \int_0^{a_l^{**}} \mu_{lo}(a)\,1(a \leq a_h^{**})\,da + \mu_{lo}(T).$$

The cutoff ages a_h^{**} and a_l^{**} may differ from a_h^* and a_l^* defined in Section (5.2.5), as agents' value functions and, thus, policies depend on whether or not dealers are active. Moreover, the steady-state distribution of agents determines the counterfactual trading probabilities, which are key components of the equilibrium prices $p^{nd}(a)$.

Table 4 reports the calculations of these magnitudes for the model and the counterfactual market with no dealers, and Figure 4 plots the prices. The table and the figure show interesting features. The first row of the table reports that the number of misallocated assets increases to 2,977 units, or 32.3 percent of all aircraft—a 12-percentage-points increase, or a 59-percent increase, in misallocated assets relative to the estimated model. The third row reports that 988 of these 2,977 misallocated aircraft would be on the market for sale in a market without dealers. This mass is smaller than in the estimated model because the larger trading frictions increase the mass of low-valuation owners that are keeping their aircraft rather than selling them—i.e., $a_l^{**} < a_l^*$—thereby reducing the volume of trade. This effect dominates the opposing effect that longer trading delays increase the mass of assets on the

TABLE 4: Comparison with a Market with No Dealers

	ESTIMATED MODEL	NO DEALER MARKET
$\int_0^T \mu_{lo}(a)\,da + \int_0^{a^*_{do}} \mu_{do}(a)\,da$	$1,851$	$2,977$
	$[1,805; 1,888]$	$[2,196; 3,4271]$
$\frac{\int_0^T \mu_{lo}(a)da + \int_0^{a^*_{do}} \mu_{do}(a)da}{A}$	0.203	0.323
	$[0.198; 0.203]$	$[0.236; 0.358]$
$\int_0^{a^*_l} \mu_{lo}(a)\,da + \int_0^{a^*_{do}} \mu_{do}(a)\,da$	$1,064$	988.7
	$[1,061; 1,097]$	$[798.4; 1,613.8]$
$\frac{\int_0^{a^*_l} \mu_{lo}(a)da + \int_0^{a^*_{do}} \mu_{do}(a)da}{A}$	0.116	0.107
	$[0.116; 0.118]$	$[0.091; 0.172]$
$p(0)$	$18,340,857$	$17,377,089$
	$[17,559; 18,367] * 10^3$	$[16,874,000; 18,452,000]$
$p(10)$	$8,278,037$	$7,369,779$
	$[8,021; 8,307] * 10^3$	$[7,096,400; 8,445,800]$

Notes—This table reports counterfactual allocations and prices in a market without dealers. 95-percent confidence intervals in brackets are obtained bootstrapping the data using 100 replications

market for a given volume of trade.

Table 4 further shows that prices of new aircraft decrease by approximately one million dollars, a 5.25-percent decrease. Thus, the effect on allocations is substantially larger than the effect on prices. The intuition is the following: Without dealers, the volume of trade is lower. Therefore, at any point in time, the mass of high-valuation agents seeking to buy an aircraft is larger—i.e., $\mu^{nd}_{hn} > \mu_{hn}$. Hence, it is easier for sellers to find a high-valuation buyer—i.e., $\gamma^{nd}_s > \gamma_s$—thereby increasing their value function $V_{lo}(a)$ and their outside option in bargaining. This effect attenuates the opposite effects of higher search costs and slower trade on asset prices. Therefore, this counterfactual analysis reiterates that it may be difficult to infer exclusively from the magnitude of changes in asset prices the magnitude of changes in trading frictions (or, more generally, changes in trading mechanisms and institutions) on market inefficiencies.

8 Concluding Remarks

This paper provides a framework to empirically analyze decentralized asset markets. I apply this framework to investigate the effect of trading frictions on asset allocations and asset prices in the business-aircraft market. The estimated model implies that trading frictions

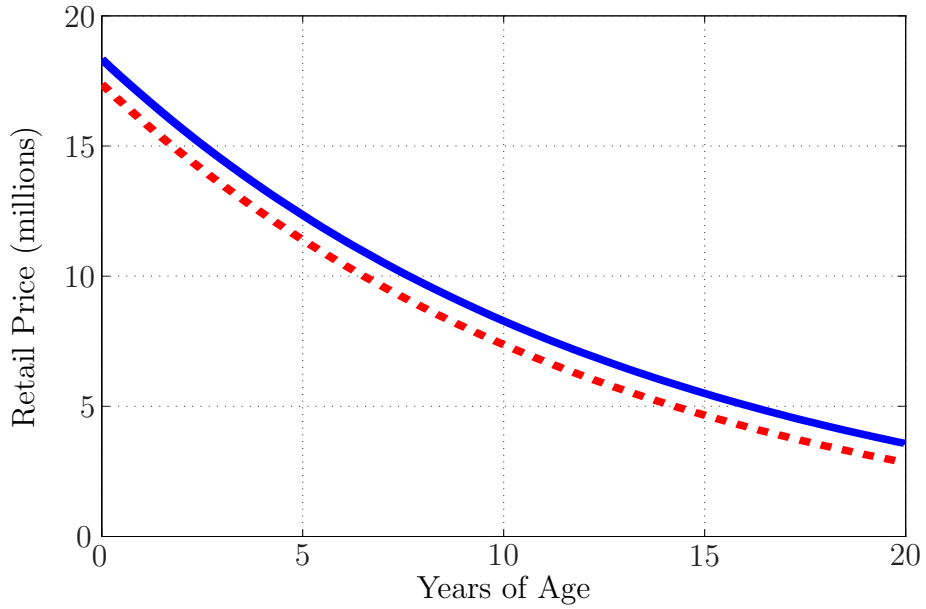

FIG. 4: Actual (solid line) and counterfactual (dashed line) aircraft prices if there were no dealers as a function of aircraft age.

generate moderate inefficiencies in such markets: Compared to the Walrasian benchmark, 20.3 percent of all business aircraft are misallocated, and aircraft prices are 6.61-percent lower. Moreover, dealers play an important role in reducing frictions: In a market with no dealers, 32.3 percent of the assets would be misallocated, and prices would decrease by 5.25 percent.

The model proposed in the paper describes the key features of asset markets, and its estimation allows me to quantify the importance of frictions and of dealers. Nonetheless, the model also has some limitations. First, the supply x of new assets is exogenous. This modeling assumption is common in many other models that focus on secondary markets and on trading frictions. A second limitation is that information on buyers is unavailable: The mass of new buyers μ is exogenous, and their meeting rates are derived from the model. In particular, the identification of trading frictions relies exclusively on the sellers' side of the market. This limitation is common to many equilibrium search models, and part of the point of this paper is to show that, under the same assumptions, it is possible to quantify the effect of trading delays on allocations and prices. A third limitation is that there are no aggregate shocks. In reality, the business-aircraft market, like most other asset markets, exhibits non-trivial fluctuations in prices and trading volume, corresponding to fluctuations

in supply and demand. The key advantage of a stationary framework is that the numerical solution of the model is simple, making the estimation simple, as well. Nonetheless, a potential concern is how sensitive the estimation results are to this stationarity assumption. Fourth, an additional limitation, quite common among search models, is that price is the only characteristic over which parties bargain, and no other characteristics, such as financing, are taken into consideration.

Finally, in my view, perhaps the main limitation is that the model allows for limited heterogeneity in agents' valuations and dealers' inventories. As explained in Section 4, this limitation stems from the aggregate nature of the data, which makes a model with richer heterogeneity difficult to identify. One important consequence of agents' limited heterogeneity is that the marginal buyer and the marginal seller in the estimated model and in the Walrasian market are the same. Instead, if agents' heterogeneity is richer, the marginal buyers and the marginal sellers differ in a model with or without frictions. More precisely, there exists a unique buyers' threshold in the valuation distribution such that an agent that does not currently own an aircraft and whose valuation jumps above the threshold chooses to acquire one. Similarly, there exists a unique sellers' threshold valuation such that an agent that currently owns an aircraft and whose valuation falls below the threshold chooses to sell it. When there are trading frictions, buyers' threshold is higher than sellers' since frictions create a wedge that prevents sellers from selling and buyers from buying. Instead, in a frictionless Walrasian market, these thresholds converge, and the marginal buyer and the marginal seller coincide. This argument suggests that the mass of misallocated young assets may be *larger* than the estimated one, since the latter does not take into account the misallocation of assets owned by agents between the thresholds that are not searching to sell or buy an aircraft, but would do so in a frictionless market. However, this argument also suggests that the mass of misallocated old assets may be *smaller* than the estimated one, since the latter does not take into account the transactions of assets between agents with more extremal valuations than the model's binary valuations. The net effect of these two approximations on the total mass of misallocated assets is ambiguous. Moreover, the previous argument implies that Walrasian prices may be lower than the counterfactual values calculated in Section 7.1 since the marginal owner in a Walrasian market has a lower valuation than in the counterfactual market of Section 7.1.

APPENDICES

A Alternative Matching Function

In this Appendix, I report on the estimation of the model under the assumptions that a constant-returns-to-scale matching function determines agents' and dealers' matching rates. More specifically, I assume that the matching functions are Cobb-Douglas: Buyers μ_{hn} and sellers $\int_0^{a_l^*} \mu_{lo}(a)\,da$ meet according to

$$m\left(\mu_{hn}, \int_0^{a_l^*} \mu_{lo}(a)\,da\right) = \gamma \mu_{hn}^{\beta_1} \left(\int_0^{a_l^*} \mu_{lo}(a)\,da\right)^{\beta_2},$$

whereas buyers μ_{hn} and sellers $\int_0^{a_l^*} \mu_{lo}(a)\,da$ meet dealers with inventories $\int_0^{a_{do}^*} \mu_{do}(a)\,da$ and without inventories μ_{dn}, respectively, according to:

$$\begin{aligned} m'(\mu_{hn}, \mu_{do}) &= \gamma' \mu_{hn}^{\beta_1} \left(\int_0^{a_{do}^*} \mu_{do}(a)\,da\right)^{\beta_2}, \\ m'(\mu_{dn}, \mu_{lo}) &= \gamma' \mu_{dn}^{\beta_1} \left(\int_0^{a_l^*} \mu_{lo}(a)\,da\right)^{\beta_2}. \end{aligned}$$

Hence, agents' arrival rates are:

$$\gamma_s = \frac{\gamma \mu_{hn}^{\beta_1} \left(\int_0^{a_l^*} \mu_{lo}(a)\,da\right)^{\beta_2}}{\int_0^{a_l^*} \mu_{lo}(a)\,da}, \tag{24}$$

$$\gamma_b(a) = \frac{\mu_{lo}(a)}{\int_0^{a_l^*} \mu_{lo}(a)\,da} \frac{\gamma \mu_{hn}^{\beta_1} \left(\int_0^{a_l^*} \mu_{lo}(a)\,da\right)^{\beta_2}}{\mu_{hn}}, \tag{25}$$

$$\gamma_{sd} = \frac{\gamma' \mu_{dn}^{\beta_1} \left(\int_0^{a_l^*} \mu_{lo}(a)\,da\right)^{\beta_2}}{\int_0^{a_l^*} \mu_{lo}(a)\,da}, \tag{26}$$

$$\gamma_{bd}(a) = \frac{\mu_{do}(a)}{\int_0^{a_{do}^*} \mu_{do}(a)\,da} \frac{\gamma' \mu_{hn}^{\beta_1} \left(\int_0^{a_{do}^*} \mu_{do}(a)\,da\right)^{\beta_2}}{\mu_{hn}}, \tag{27}$$

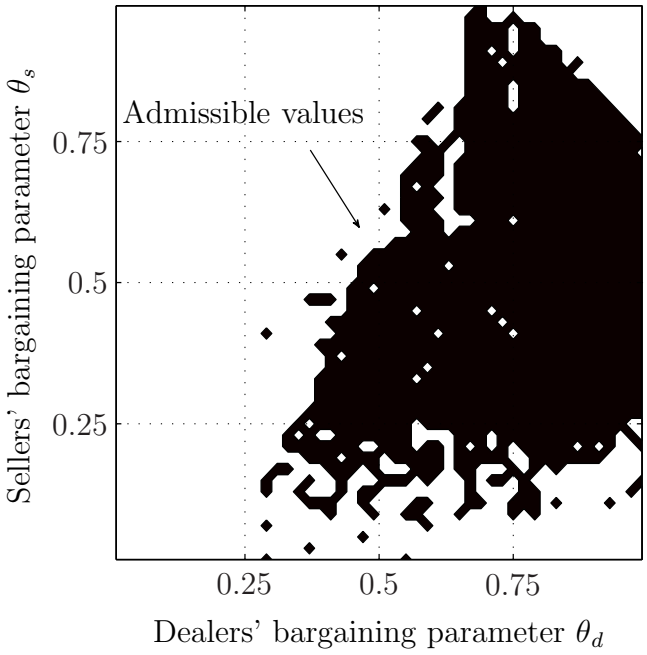

FIG. 5: Admissible values of the bargaining parameters. Matching function with constant returns to scale.

and dealers' are:

$$\alpha_{ds} = \frac{\gamma' \mu_{hn}^{\beta_1} \left(\int_0^{a_{do}^*} \mu_{do}(a)\, da \right)^{\beta_2}}{\int_0^{a_{do}^*} \mu_{do}(a)\, da}, \tag{28}$$

$$\alpha_{db}(a) = \frac{\mu_{lo}(a)}{\int_0^{a_l^*} \mu_{lo}(a)\, da} \frac{\gamma' \mu_{dn}^{\beta_1} \left(\int_0^{a_l^*} \mu_{lo}(a)\, da \right)^{\beta_2}}{\mu_{dn}}. \tag{29}$$

The model estimated in the main text corresponds to the case $\beta_1 = \beta_2 = 1$—i.e., a matching function with increasing returns.[11] In this Appendix, I report the parameters' estimates in the case $\beta_1 = \beta_2 = .5$—i.e., a matching function with constant returns to scale. While this case does not have an explicit micro-foundation in the current model, a matching function with constant returns to scale seems empirically robust in labor markets (Petrongolo and Pissarides, 2001). More generally, it is possible to estimate the model and perform the counterfactuals under any other assumption on the matching functions $m(\cdot,\cdot)$ and $m'(\cdot,\cdot)$.

Figure 5 displays the bargaining parameters consistent with the equilibrium of the model. The figure is remarkably similar to Figure 2, thus reiterating that the data are inconsistent with the idea that the "short" side of the market (sellers and dealers, in particular) enjoys

[11] Gavazza (2011b) provides evidence of increasing returns in commercial-aircraft markets.

minimal bargaining power.[12]

Table 5 reports the estimates of all other parameters for selected values of the bargaining parameters: the approximate vertices and the approximate centroid of the set. The parameters are very similar to those reported in Table 2, with the exception of the matching rates γ and γ' that are substantially higher. This is intuitive, as the data directly identify sellers' and dealers' arrival rates only. Thus, γ and γ' have to increase if the matching technology exhibits lower returns to scale.

Tables 6 and 7 report key statistics from the counterfactual Walrasian market and from the counterfactual market without dealers, respectively, with bargaining parameters $\theta_s = .49$ and $\theta_d = .75$. The magnitudes from the estimated model and the counterfactuals are quite similar to those reported in Tables 3 and 4 in the main text, indicating that these counterfactual analyses are robust to different assumptions on the returns to scale of the matching function.

[12]The similarity between Figures 2 and 5 is perhaps another indication that there is no information in the data to identify the returns to scale in the matching function.

TABLE 5: Parameter Estimates, Matching Function with Constant Returns to Scale

	$\theta_s = .25$ $\theta_d = .45$	$\theta_s = .97$ $\theta_d = .75$	$\theta_s = .49$ $\theta_d = .75$	$\theta_s = .27$ $\theta_d = .99$	$\theta_s = .71$ $\theta_d = .99$
λ	0.0762 [0.0758; 0.0769]	0.0586 [0.0581; 0.0587]	0.0681 [0.0676; 0.0682]	0.0786 [0.0775; 0.0795]	0.0609 [0.0608; 0.0610]
γ_s	0.2327 [0.2309; 0.2354]	0.2293 [0.2293; 0.2356]	0.2368 [0.2367; 0.2382]	0.2228 [0.2189; 0.2259]	0.2492 [0.2491; 0.2497]
γ_{sd}	0.3404 [0.3345; 0.3420]	0.3337 [0.3337; 0.3348]	0.3334 [0.3322; 0.3349]	0.3466 [0.3329; 0.3494]	0.3602 [0.3602; 0.3609]
α_{ds}	1.0597 [1.0518; 1.0608]	0.7567 [0.7561; 0.7597]	0.9039 [0.9037; 0.9096]	1.1384 [1.0904; 1.1447]	0.8080 [0.8033; 0.8093]
γ	0.1299 [0.1295; 0.1320]	0.1765 [0.1759; 0.1822]	0.1497 [0.1487; 0.1503]	0.1244 [0.1208; 0.1252]	0.1834 [0.1817; 0.1841]
γ'	0.3500 [0.3477; 0.3528]	0.3863 [0.3862; 0.3904]	0.3561 [0.3544; 0.3564]	0.3543 [0.3423; 0.3545]	0.3974 [0.3958; 0.3992]
μ	705.5 [702.9; 715.1]	590.3 [588.8; 597.4]	642.7 [640.8; 643.7]	705.4 [669.1; 717.6]	604.8 [603.3; 606.7]
δ_2	0.200 [0.0199; 0.0200]	0.0205 [0.0205; 0.0206]	0.0198 [0.0198; 0.0198]	0.0178 [0.0177; 0.0178]	0.0196 [0.0196; 0.0198]
δ_0	0.0194 [0.0194; 0.0197]	0.0113 [0.0113; 0.0114]	0.0125 [0.0125; 0.0126]	0.0133 [0.0133; 0.0139]	0.0100 [0.0100; 0.0100]
z_h	742, 150 [738, 550; 751, 970]	629, 910 [628, 290; 634, 360]	679, 070 [676, 760; 679, 390]	681, 260 [665, 080; 688, 420]	608, 220 [605, 850; 609, 180]
z_l	344, 640 [340, 280; 345, 490]	298, 450 [298, 420; 301, 920]	317, 870 [315, 750; 317, 930]	326, 570 [320, 880; 334, 250]	344, 440 [344, 440; 348, 850]
c_s	49, 032 [48, 813; 49, 559]	17, 745 [17, 738; 17, 806]	35, 441 [35, 382; 35, 565]	20, 771 [20, 758; 21, 496]	18, 745 [18, 745; 18, 764]
k	77, 568 [76, 425; 80, 542]	27, 813 [26, 213; 28, 056]	106, 310 [105, 720; 111, 090]	391, 080 [354, 440; 408, 450]	75, 034 [74, 656; 78, 058]

Notes—This table reports MSM estimates of the parameters. 95-percent confidence intervals in brackets are obtained bootstrapping the data using 100 replications.

Table 6: Comparison with Walrasian Market, CRS Matching Function

	(1)	(2)	(3)
	Estimated Model	Walrasian Market	$c_s = 0$
$\int_0^T (\mu_{lo}(a) + \mu_{do}(a))\, da$	$1,853$	0	$1,944$
	$[1,847; 1,861]$	$[0; 0]$	$[1,944; 1,985]$
$\frac{\int_0^T (\mu_{lo}(a)+\mu_{do}(a))da}{A}$	0.199	0	0.208
	$[0.197; 0.199]$	$[0; 0]$	$[0.208; 0.213]$
$\int_0^{a_l^*} \mu_{lo}(a)\, da + \int_0^{a_{do}^*} \mu_{do}(a)\, da$	$1,097$	0	$1,104$
	$[1,088; 1,098]$	$[0; 0]$	$[1,099; 1,143]$
$\frac{\int_0^{a_l^*} \mu_{lo}(a)da + \int_0^{a_{do}^*} \mu_{do}(a)da}{A}$	0.118	0	0.118
	$[0.116; 0.118]$	$[0; 0]$	$[0.117; 0.119]$
$p(0)$	$18,633,001$	$20,085,483$	$18,557,561$
	$[18,539; 18,633] * 10^3$	$[19,992; 20,095] * 10^3$	$[18,520; 18,608] * 10^3$
$p(10)$	$8,381,567$	$9,410,792$	$8,322,374$
	$[8,330; 8,382] * 10^3$	$[9,348; 9,427] * 10^3$	$[8,305; 8,364] * 10^3$

Notes—This table reports counterfactual allocations and prices. 95-percent confidence intervals in brackets are obtained bootstrapping the data using 100 replications.

Table 7: Comparison with a Market with No Dealers, CRS Matching Function

	Estimated Model	No Dealer Market
$\int_0^T \mu_{lo}(a)\, da + \int_0^{a_{do}^*} \mu_{do}(a)\, da$	$1,853$	$3,796$
	$[1,847; 1,861]$	$[3,776; 3,804]$
$\frac{\int_0^T \mu_{lo}(a)da + \int_0^{a_{do}^*} \mu_{do}(a)da}{A}$	0.199	0.404
	$[0.197; 0.199]$	$[0.399; 0.404]$
$\int_0^{a_l^*} \mu_{lo}(a)\, da + \int_0^{a_{do}^*} \mu_{do}(a)\, da$	$1,097$	$1,203$
	$[1,088; 1,098]$	$[1,182; 1,203]$
$\frac{\int_0^{a_l^*} \mu_{lo}(a)da + \int_0^{a_{do}^*} \mu_{do}(a)da}{A}$	0.118	0.128
	$[0.116; 0.118]$	$[0.125; 0.128]$
$p(0)$	$18,633,001$	$17,238,479$
	$[18,539; 18,633] * 10^3$	$[17,165,000; 17,254,000]$
$p(10)$	$8,381,567$	$7,197,330$
	$[8,330; 8,382] * 10^3$	$[7,159,800; 7,212,200]$

Notes—This table reports counterfactual allocations and prices in a market without dealers. 95-percent confidence intervals in brackets are obtained bootstrapping the data using 100 replications.

B The case $\delta_2 = 0$

When assets are not depreciating—i.e., $\delta_2 = 0$—we can obtain the analytical solution of the model (Duffie, Gârleanu and Pedersen, 2005 and 2007). Since δ_2 is estimated to be small, this restricted version of the model provides a reasonable approximation to the more-general model, thereby providing a clean intuition for the identification of the parameters.

Since assets are not depreciating, we can assume that the mass of assets is fixed and equal to A; thus, $x = 0$ and $T \to \infty$. Moreover, without loss of generality, we can set $\delta_1 = 0$ and $\delta_0 = 1$.

The steady-state evolution of the masses μ_i determines the equilibrium allocation. Up to terms in $o(\epsilon)$, in equilibrium, these masses evolve from time t to time $t + \epsilon$ according to:

$$\begin{aligned}
\mu_{lo}(t+\epsilon) &= \lambda\epsilon\mu_{ho}(t) + (1 - \gamma_s\epsilon - \gamma_{sd}\epsilon)\,\mu_{lo}(t), \\
\mu_{ho}(t+\epsilon) &= \gamma_b\epsilon\mu_{hn}(t) + \gamma_{bd}\epsilon\mu_{hn}(t) + (1 - \lambda\epsilon)\,\mu_{ho}(t), \\
\mu_{hn}(t+\epsilon) &= \epsilon\mu + (1 - \lambda\epsilon - \gamma_b\epsilon - \gamma_{bd}\epsilon)\,\mu_{hn}(t), \\
\mu_{ln}(t+\epsilon) &= \lambda\epsilon\mu_{hn}(t) + \gamma_s\epsilon\mu_{lo}(t) + \gamma_{sd}\epsilon\mu_{lo}(t), \\
\mu_{do}(t+\epsilon) &= \alpha_{db}\epsilon\mu_{dn}(t) + (1 - \alpha_{ds}\epsilon)\,\mu_{do}(t), \\
\mu_{dn}(t+\epsilon) &= \alpha_{ds}\epsilon\mu_{do}(t) + (1 - \alpha_{db}\epsilon)\,\mu_{dn}(t).
\end{aligned}$$

The intuition for these equations is similar to the intuition reported in Section 5.2.5. For example, the first equation states that the mass of low-valuation agents with an asset results from the flows of three sets of agents: 1) the inflow of high-valuation owners whose valuation just dropped—the term $\lambda\epsilon\mu_{ho}(t)$; 2) the outflow of low-valuation owners that found a dealer—the term $\gamma_{sd}\epsilon\mu_{lo}(t)$; and 3) the mass of low-valuation owners that have not found a buyer—the term $(1 - \gamma_s\epsilon)\,\mu_{lo}(t)$.

Steady state implies that the total mass of agents with high valuation $\mu_{hn} + \mu_{ho}$ is equal to $\frac{\mu}{\lambda}$, that the total mass of owners $\mu_{ho} + \mu_{lo} + \mu_{do}$ is equal to the mass of assets A, and that the total mass of dealers μ_d is equal to $\mu_{do} + \mu_{dn}$. Hence, $\mu_{hn} + A - \mu_{lo} - \mu_{do} = \frac{\mu}{\lambda}$. In turn, since $A < \frac{\mu}{\lambda}$, this implies that $\mu_{hn} > \mu_{lo} + \mu_{do}$—i.e., sellers are the "short" side of the market. Moreover, the masses of assets sold and purchased by dealers must be equal to the mass of assets purchased and sold to dealers: $\alpha_{ds}\mu_{do} = \gamma_{bd}\mu_{hn}$ and $\alpha_{db}\mu_{dn} = \gamma_{sd}\mu_{lo}$. Furthermore, steady state requires $\alpha_{ds}\mu_{do} = \gamma_{bd}\mu_{hn} = \alpha_{db}\mu_{dn} = \gamma_{sd}\mu_{lo}$ since dealers' aggregate inventories do not change over time.

Rearranging and taking the limit for $\epsilon \to 0$, the masses of agents are:

$$\begin{aligned}
\mu_{lo} &= \frac{\lambda}{\gamma_s + \gamma_{sd}} \mu_{ho}, \\
\mu_{ho} &= \frac{\gamma_b \mu_{hn} + \gamma_{bd} \mu_{hn}}{\lambda}, \\
\mu_{hn} &= \frac{\mu}{\lambda + \gamma_b + \gamma_{bd}}, \\
\mu_{ln} &= \lambda \mu_{hn} + \gamma_s \mu_{lo} + \gamma_{sd} \mu_{lo} = \mu, \\
\mu_{do} &= \frac{\alpha_{db}}{\alpha_{ds}} \mu_{dn} = \frac{\gamma_{sd}}{\alpha_{ds}} \mu_{lo}.
\end{aligned}$$

We can now calculate the moments (18)-(21) that we seek to match to their empirical analogs in the quantitative analysis. Specifically, the fraction of aircraft for sale is equal to:

$$\frac{\mu_{lo} + \mu_{do}}{A} = \frac{\lambda \left(\alpha_{ds} + \gamma_{sd} \right)}{\lambda \alpha_{ds} + \lambda \gamma_{sd} + \gamma_s \alpha_{ds} + \alpha_{ds} \gamma_{sd}}.$$

The fraction of aircraft for sale by dealers (or dealers' inventories) is equal to:

$$\frac{\mu_{do}}{A} = \frac{\lambda \gamma_{sd}}{\lambda \alpha_{ds} + \lambda \gamma_{sd} + \gamma_s \alpha_{ds} + \alpha_{ds} \gamma_{sd}}.$$

The fraction of retail-to-retail transactions to total aircraft is equal to:

$$\frac{\gamma_s \mu_{lo}}{A} = \frac{\lambda \gamma_s \alpha_{ds}}{\lambda \alpha_{ds} + \lambda \gamma_{sd} + \gamma_s \alpha_{ds} + \alpha_{ds} \gamma_{sd}}.$$

The fraction of dealer-to-retail transactions to total aircraft is equal to:

$$\frac{\alpha_{ds} \mu_{do}}{A} = \frac{\lambda \gamma_{sd} \alpha_{ds}}{\lambda \alpha_{ds} + \lambda \gamma_{sd} + \gamma_s \alpha_{ds} + \alpha_{ds} \gamma_{sd}}.$$

The equilibrium prices are determined by the solution to the following system of equa-

tions:

$$
\begin{aligned}
\rho U_{ho} &= z_h + \lambda \left(S_{lo} - U_{ho} \right), \\
\rho S_{lo} &= z_l - c_s + \gamma_s \left(p + V_{ln} - S_{lo} \right) + \gamma_{sd} \left(p_B + V_{ln} - S_{lo} \right), \\
\rho S_{hn} &= -c_s + \lambda \left(V_{ln} - S_{hn} \right) + \gamma_b \left(U_{ho} - p - S_{hn} \right) + \gamma_{bd} \left(U_{ho} - p_A - S_{hn} \right), \\
\rho V_{ln} &= 0, \\
\rho J_{do} &= -k + \alpha_{ds} \left(p_A + J_{dn} - J_{do} \right), \\
\rho J_{dn} &= -k + \alpha_{db} \left(J_{do} - p_B - J_{dn} \right), \\
p &= (1 - \theta_s) \left(S_{lo} - V_{ln} \right) + \theta_s \left(U_{ho} - S_{hn} \right), \\
p_A &= (1 - \theta_d) \left(J_{do} - J_{dn} \right) + \theta_d \left(U_{ho} - S_{hn} \right), \\
p_B &= (1 - \theta_d) \left(J_{do} - J_{dn} \right) + \theta_d \left(S_{lo} - V_{ln} \right).
\end{aligned}
$$

The first four equations are the value functions of agents; the fifth and the sixth equations are the value functions of dealers; and the the last three equations are the negotiated prices.

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