



ELSEVIER

Journal of Monetary Economics 49 (2002) 215–228

Journal of
MONETARY
ECONOMICS

www.elsevier.com/locate/econbase

Are behavioral asset-pricing models structural? ☆

Stanley E. Zin^{a,b,*}

^a *Graduate School of Industrial Administration, Carnegie Mellon University, Pittsburgh, PA 15213-3890, USA*

^b *National Bureau of Economic Research, Cambridge, MA 02138, USA*

Received 12 April 2001; received in revised form 22 August 2001; accepted 23 August 2001

Abstract

The recent increase in interest in so-called behavioral models of asset-pricing is motivated partly by the desire to have models that appear realistic in light of experimental evidence, and partly by their success in moment-matching exercises. This paper argues that the attention given to these two criteria misses perhaps the most important aspect of the modeling exercises. That is, the search for parameters that are invariant to changes in the economic environment. It is precisely this invariance that motivates the use of a tightly parameterized general equilibrium model. Assessing a model on this dimension is difficult and, as the paper argues through the use of suggestive examples, will undoubtedly require strong subjective judgments about the reasonableness of preference assumptions. Such judgments are routinely made about the reasonableness of assumptions about stochastic endowments. The paper suggests that more effort be applied to understanding aggregation in these models and to the exploration of behavioral assumptions in a less flexible but less corruptible time-stationary recursive class of preferences. © 2002 Published by Elsevier Science B.V.

JEL classification: B41; E44; G12

Keywords: Asset-pricing; Behavioral finance; Structural models

To put the point less paradoxically, the relevant question to ask about the “assumptions” of a theory is not whether they are descriptively “realistic”, for they never are, but whether they are sufficiently good approximations for the

☆ I would like to thank Bryan Routledge, Tom Tallarini, Rick Green and Peter Bossaerts for stimulating conversations, and Andy Abel and Jessica Wachter for helpful comments.

*Corresponding author. Graduate School of Industrial Administration, Carnegie Mellon University, Pittsburgh, PA 15213-3890, USA. Tel.: +1-412-268-3700; fax: +1-412-268-7357.

E-mail address: zin@bobbyorr.gsia.cmu.edu (S.E. Zin).

purpose in hand. And this question can be answered only by seeing whether the theory works, which means whether it yields sufficiently accurate predictions.

Friedman (1953)

On this general view of the nature of economic theory then, a “theory” is not a collection of assertions about the behavior of the actual economy but rather an explicit set of instructions for building a parallel or analogue system—a mechanical, imitation economy. A “good” model, from this point of view, will not be exactly more “real” than a poor one, but will provide better imitations. Of course, what one means by a “better imitations” will depend on the particular questions to which one wishes answers.

Lucas (1980)

1. Introduction

The recent successes of behavioral asset-pricing models provide new hope for the quantitative research program started by Mehra and Prescott (1985) following the theoretical work of Lucas (1978). That is, there is a renewed interest in the ability of a tightly parameterized, representative-agent, general-equilibrium model to explain the salient features of historical asset-market data (e.g., large equity premium, excess volatility, etc.). What makes an asset-pricing model “behavioral” can itself be the subject of debate. For the purposes of this paper, I will lump all asset-pricing models that endow agents with preferences that do not adhere to the assumption of time-stationary expected utility (i.e., “Savage rationality”), into the category of “behavioral”. Many of these preference assumptions are directly motivated by evidence from experimental psychology and behavioral decision theory, e.g., *loss aversion* (Epstein and Zin, 1990; Benartzi and Thaler, 1995; Barberis et al., 1999), or *hyperbolic discounting* (Luttmer and Mariotti, 2000; Krusell and Smith, 2000). Also falling within this broad definition, however, are models that may depart from classical assumption by allowing for state-dependent utility functions, but that are less formally motivated by behavioral evidence, e.g., *habit formation* (Abel, 1990; Constantinides, 1990; Campbell and Cochrane, 1995; Wachter, 2001). These examples are suggestive and are in no way an exhaustive list of behavioral asset-pricing models. Indeed as more experimental evidence filters into economics from various fields of psychology, this list continues to grow at a rapid rate.

This paper takes a sympathetic view of these recent behavioral approaches and tries to identify what these models have yet to accomplish before they can claim success and presumably supplant more traditional approaches. Particular attention is paid to the need for *structural* models, and whether behavioral models are more or less likely to achieve the sort of “deep structural excavation” called for by the rational expectations revolution in dynamic macroeconomics.

The methodological guidelines laid out by Friedman and Lucas in the two quotes above, cast a very different light on the current debate about the usefulness of *behavioral* versus more *traditional* models of asset markets, than what one might hear in academic circles and even in the popular press. From this perspective, traditional models cannot be viewed as inherently better because of their reliance on well-understood Savage rationality. Likewise, behavioral models cannot claim superiority simply based on experimental evidence of individual departures from this definition of rationality, and the sense of modeling realism that this evidence invokes. If this debate can be settled, then following Lucas' advice, it can only be settled by determining which approach to building a mechanical, imitation economy provides "better imitations" of real asset markets.

It is now common for behavioral models to adopt the dynamic stochastic general equilibrium approach of Lucas (1978) and Mehra and Prescott (1985) as a framework for understanding the consequences of alternative behavioral assumptions on observables in asset markets. In other words, the basic difference between the two approaches can be thought of as differences in assumptions about agents' preferences—most often a hypothetical representative agent. Therefore, the common use of dynamic general equilibrium endowment economies by both approaches provides a common framework for comparison.

The use of these general equilibrium models highlights an implicit desire to obtain structural models. In this case, "structural" is used to differentiate between purely statistical descriptions of empirical evidence (which may or may not use economic theory to suggest functional forms and factors), from models that derive their empirical predictions directly from the structure of a parametric version of an economic theory. The obvious implication being that like more traditional models, behavioral models can potentially provide useful guidance for understanding the likely consequences of changes in the economic environment. That is, they can be used to make forecasts about situations in which we have no (or at least very little) historical evidence. If that was not the desire of behavioralists in finance, then behavioral arguments could be safely relegated to a relatively minor role in the design and interpretation of reduced-form econometric models. Predicting responses to changes in the economic environment, e.g., changes in government policy, therefore, are precisely the "particular questions" for which we seek "better imitations", using Lucas' words, or the "sufficiently accurate predictions", using Friedman's words. The primary reason for using tightly parameterized general equilibrium models to characterize asset-market outcomes is precisely the need for identifying policy-invariant structural parameters.

Behavioral models have an obvious and important advantage over traditional models: their parameters can be calibrated so that various moments of particular interest of the distribution of asset prices generated by these models, will closely match their sample counterparts in historical data. That is, they do a better job imitating the large equity premiums, volatility, and persistent dynamics that are so puzzling from the perspective of traditional models. Clearly, this data-fitting exercise seems like a necessary condition for evaluating the usefulness of any equilibrium model. We would have little confidence in any model's ability to forecast outcomes

in new environments when it is incapable of forecasting in the current environment. These conclusions, however, require a fair bit of judgment on the part of the modeler. Section 2 outlines judgments that modeler's working in this area typically make about the *reasonableness* of assumptions about the stochastic properties of exogenous variables. A simple example demonstrates that poorly fitting Savage-rational models can be made to fit empirical evidence by making assumptions about higher-order moments of the distribution of endowments. Most people would object to this strategy, however, since these assumptions might not seem *reasonable* given their prior beliefs. Where these beliefs come from is unspecified. They may derive from beliefs about the deeper micro-foundations of production, or they may derive from personal experience, or they may be purely whimsical. Direct statistical evidence about these higher-order-moment assumptions is likely to be misleading, or at best inconclusive, so we are left with non-sample-based judgments about the reasonableness of these assumptions.

Section 3 of the paper looks at a similar sort of reasonableness criterion for preference assumptions. Unfortunately, unlike the case of assumptions about endowments, we are not yet at the stage where there is a consensus about what types of preference assumptions are unreasonable. The examples in Section 3 demonstrate that virtually any well-fitting, reduced-form empirical model of asset pricing can be incorporated into the representative agent's preferences so that a purely statistical model could be viewed as the outcome of what one could claim to be a structural general equilibrium model. Naturally, there is no guarantee that a model constructed in this way will have parameters that are invariant to the types of structural change for which these models must provide guidance. Analogous to the case for reasonableness of endowments, the modeler is forced to make judgments about the reasonableness of the preference assumptions. It is clear that fitting historical data is not a sufficient criterion for determining the usefulness of particular preference assumptions for delivering a structurally stable model. Likewise, without some explicit aggregation results, individual-level experimental evidence will also be insufficient.

The main conclusion of this paper is straightforward: the parameters of asset-pricing models including behavioral models must be invariant to changes in the economic environment. This is not a very original conclusion and it is not likely to be very controversial. What is controversial and quite difficult, is assessing whether this goal has been met. Econometric testing for structural stability is notoriously problematic, especially in small samples. Moreover if the type of structural change under investigation has no historical counterpart, then purely statistical testing will be uninformative. The examples in Sections 2 and 3 of the paper suggest that any assessment of the structural stability of a model will require the use of both non-sample information and the researcher's judgment. Accounting for historical evidence is not enough. The researcher is forced to evaluate the reasonableness of the assumptions on preferences and technologies under which the asset-pricing model can both account for historical evidence and maintain its basic structure in the face of significant changes in the economic environment. If aggregation results are available, then preferences of the representative agent that appear unreasonable

given individual-level evidence would certainly be a cause for concern. These results also suggest that maintaining time-consistency as a basic axiom for preferences, and avoiding the introduction of arbitrary state variables in the utility function, will help eliminate much of the scope for well-fitting but non-structural empirical asset-pricing models to pass as deeper structural models.

2. Endowments: never a dull moment

Resolving the equity premium puzzle has been a goal of researchers in finance since it was first declared a puzzle by Mehra and Prescott in 1985. In this section, we use the Mehra and Prescott model and the equity premium puzzle as a canonical general equilibrium calibration exercise.

The Mehra–Prescott model is so well-known that there is no great need to review all of its details here. However, to set notation and terminology, it is worth specifying a few of its key equations. In particular, the price of equity is found by first solving for the price–dividend ratio in the system of equations

$$P_i = \beta \sum_{j=1}^s \lambda_j^{\alpha+1} (1 + P_j) \pi_{ij}, \quad (1)$$

where $\{\lambda_1, \lambda_2, \dots, \lambda_s\}$ are the discrete values for the growth rate of the aggregate dividend/endowment process, $[\pi_{ij}]$ are the probabilities that these growth rates transition from state- i to state- j , $\{P_1, P_2, \dots, P_s\}$ are the discrete values of the equilibrium equity price–dividend ratio, and α and β are parameters of the representative agent's preferences. The return on equity is given by

$$(1 + R_{ij}^e) = \left(\frac{P_j + 1}{P_i} \right) \lambda_j. \quad (2)$$

The return on a risk-free bond is given by

$$(1 + R_i^f) = \left[\beta \sum_{j=1}^s \lambda_j^{\alpha} \pi_{ij} \right]^{-1}. \quad (3)$$

Assume that in this example we can reparameterize the probability model for the endowment growth process in terms of the moments of the distribution. That is, since the first $s^2 - s$ moments of the distribution, denoted as $\mu_{(k)}^{\lambda}$, $k = 1, 2, \dots, s^2 - s$, are nonlinear functions of the s states, and $(s - 1)s - s$ transition probabilities, we could imagine in principle inverting this just-identified system of equations and rewriting the states and probabilities as functions of these moments. Therefore, the parameters of the distribution of the equilibrium equity return can be written as function of two different sets of parameters: the preference parameters and the moments of this endowment growth distribution. This also implies that all of the moments of the equity return distribution, denoted as $\mu_{(k)}^e$ are functions of these two sets of parameters:

$$\mu_{(k)}^e = F_k^e(\alpha, \beta, \mu^{\lambda}). \quad (4)$$

A similar argument can be made for the moments of the risk-free return:

$$\mu_{(k)}^f = F_k^f(\alpha, \beta, \mu^{\lambda}). \quad (5)$$

The calibration exercise proceeds as follows. The parameters of the exogenous dividend growth process are specified so that some of the moments of this process match their empirical analogs in historical data. The moments used for calibration are denoted by an n_1 -vector μ_1^{λ} and their sample analogs by $\bar{\mu}_1^{\lambda}$. If n_1 is less than $s^2 - s$, the number of free parameters in the probability model, then there will be $n_2 = s^2 - s - n_1$ moments of the distribution that are unaccounted for. Denote this vector of moments as μ_2^{λ} . That is, since it is not possible to identify $s^2 - s$ parameters with only n_1 moments, there must be an implicit assumption that μ_2^{λ} is a known function of μ_1^{λ} , i.e., $\mu_2^{\lambda} = f(\mu_1^{\lambda})$. Implicitly, this second set of moments is calibrated using this assumption so that $\bar{\mu}_2^{\lambda} = f(\bar{\mu}_1^{\lambda})$.

The moments of the equilibrium return distributions are completely determined by this calibration:

$$\begin{aligned} \mu_{(k)}^c &= F_k^c(\alpha, \beta, \bar{\mu}_1^{\lambda}, f(\bar{\mu}_1^{\lambda})), \\ \mu_{(k)}^f &= F_k^f(\alpha, \beta, \bar{\mu}_1^{\lambda}, f(\bar{\mu}_1^{\lambda})). \end{aligned} \quad (6)$$

With this notation, the equity premium puzzle can be characterized by setting some moments of the returns distributions to the values of their sample analogs, then asking whether there are reasonable values of the preference parameters that solve these equations. That is, by using sample means of the two returns, find values $\bar{\alpha}$ and $\bar{\beta}$ that solve the equations

$$\begin{aligned} \bar{\mu}_{(1)}^c &= F_1^c(\bar{\alpha}, \bar{\beta}, \bar{\mu}_1^{\lambda}, f(\bar{\mu}_1^{\lambda})), \\ \bar{\mu}_{(1)}^f &= F_1^f(\bar{\alpha}, \bar{\beta}, \bar{\mu}_1^{\lambda}, f(\bar{\mu}_1^{\lambda})), \end{aligned} \quad (7)$$

where in obvious notation, $\bar{\mu}_{(1)}^c$ and $\bar{\mu}_{(1)}^f$ are the sample means of the two returns. The puzzle arises when the values $\bar{\alpha}$ and $\bar{\beta}$ are *unreasonable*.

We defer further discussion of this reasonableness criterion for the next section. In the meantime, let us focus on the calibrated endowment process. What this notation makes clear is that the researcher has the discretion of setting the preference parameters at what they determine to be reasonable values, say α^* and β^* , and then fitting the sample means of returns by relaxing the assumptions on μ_2^{λ} . That is, rather than using the implied moments $f(\bar{\mu}_1^{\lambda})$, choose values $\hat{\mu}_1^{\lambda}$, to solve¹

$$\begin{aligned} \bar{\mu}_{(1)}^c &= F_1^c(\alpha^*, \beta^*, \bar{\mu}_1^{\lambda}, \hat{\mu}_2^{\lambda}), \\ \bar{\mu}_{(1)}^f &= F_1^f(\alpha^*, \beta^*, \bar{\mu}_1^{\lambda}, \hat{\mu}_2^{\lambda}). \end{aligned} \quad (8)$$

This exercise is similar to the resolution of the equity premium puzzle proposed by Rietz (1988). He noted that by using a simple two-state distribution, Mehra and Prescott were implicitly ruling out skewness in the endowment distribution. He found that sufficient skewness that took the form of a disastrous state for dividend

¹ Note that this argument essentially requires the invertibility of the F functions, which is a condition that may not always be satisfied.

growth would allow the calibrated model to generate a sizeable equity premium while assuming reasonable values of the preference parameters. It is also analogous to the so-called *peso problem*: agents in the economy are demanding a premium for risks that they know exist but that have not yet been accurately reflected in historical data. Along similar lines but in a more general model, Bansal and Yaron (2000) have shown that when the autoregressive polynomial has a root that is close in value to a root of the moving average polynomial (i.e., near root cancellation) in an ARMA specification for dividend growth, then equity may have long-run risks that agents care about, but that finite-sample reduced-form econometrics will have difficulty identifying.

This basic argument could be extended to fitting other moments of the returns distribution, up to the limits of the size of the discretization and the ability to impose regularity conditions on the implied probabilities. Freeing up the parameters μ_2^z can potentially generate a calibrated model that fits a large number of sample moments.

Obviously, $\hat{\mu}_2^z$ must be significantly different than $f(\hat{\mu}_1^z)$, or there would not have been a puzzle to start with. On the other hand, if the data were sufficiently informative to accurately identify these moments, then there would not be any meaningful distinction between μ_1^z and μ_2^z . Unfortunately, this is not generally the case. We must, therefore, make a non-sample-based evaluation of the parameter values $\hat{\mu}_2^z$. For the puzzle to persist as it has for many years, these values must be judged to be unreasonable. This is the essence of the argument made by Mehra and Prescott (1988) in response to Rietz's example.

The point of this example is not to claim that these calibration exercises are tautological, but rather to highlight one of the main sources for the empirical rejections of the model, namely, difficult-to-test restrictions on higher-order moments of the distribution of the exogenous endowment process. For example, the typical economist working in this area would be thoroughly dissatisfied with an explanation of a phenomenon as basic as a large risk premium that relied on an extreme value for a parameter like the fifth-order moment, a moment that is difficult to visualize or even to describe. Moreover, it is difficult to imagine that such explanations would be deemed structural. If we have no idea what these moments really mean, or what real-world phenomenon could have generated particular values for these moments, then it is difficult to conclude that these parameters would be invariant to the types of changes in the economic environment that the model is designed to help us understand. In other words, the parameterization would be judged to be unreasonable.

Generally, these types of judgments are not terribly controversial. A strong consensus has developed regarding reasonable choices for the exogenous endowment process in these types of economies. These reasonable choices have become part of the landscape and are rarely debated. On the other hand, what the behavioral asset-pricing literature highlights is that the perceived reasonableness of preference assumptions is something that we are just beginning to come to grips with. There is at this point no consensus as to what is reasonable and what is unreasonable as a specification for the preferences of a representative agent. The next section highlights that, like the reasonableness assumptions that are commonly made when

specifying endowments, comparable non-sample-based judgments are also necessary when specifying preferences.

3. Preferences: reverse engineering

The examples in this section demonstrate the potential problems that asset-pricing models can encounter when preference parameters are added with the goal of fitting observed prices. The examples are extreme and serve as a metaphor for the general over-fitting problem. The models in this section relax stationarity as a basic feature of preferences and the implied time-inconsistency problem is solved by backward recursion. The benefit of this preference specification is an improved empirical fit. The cost, however, is that when taken to a logical extreme, the examples show that virtually any well-fitting empirical model can be the equilibrium outcome of an apparently structural model, irrespective of the actual structural stability of the model's parameters.

3.1. Time-varying discount factors I: deterministic

We now develop a simple and stylized model of the term structure of interest rates that incorporates non-geometric discounting.

Consider for the sake of simplicity, a three-period deterministic economy where the representative agent has preferences that are linear in consumption but exhibit time-varying discount factors. Utility in this example is given by

$$u(c_1, c_2, c_3) = (c_1 - a) + \beta_1^{(1)}(c_2 - a) + \beta_1^{(1)}\beta_1^{(2)}(c_3 - a), \quad (9)$$

where $\{c_1, c_2, c_3\}$ represents consumption in each period, $\{\beta_1^{(1)}, \beta_1^{(2)}\}$ are preference period-1 parameters, and $a > 0$ is the subsistence level of consumption.

The agent receives endowments in each of the three periods denoted by $\{e_1, e_2, e_3\}$, and can trade in one- and two-period pure-discount bonds with face values of 1 and prices given by $b_1^{(1)}$ and $b_1^{(2)}$, respectively. Short sales of both bonds are restricted to be bigger than $-\bar{q}$, and lending is constrained such that consumption never falls below the subsistence level. Denote the quantities of the bonds that the agent chooses as $\{q_1^{(1)}, q_1^{(2)}\}$. With these assumptions, the utility maximization problem in period-1 can be written as

$$\max_{\{q_1^{(1)}, q_1^{(2)}\}} (e_1 - q_1^{(1)}b_1^{(1)} - q_1^{(2)}b_1^{(2)}) + \beta_1^{(1)}(e_2 + q_1^{(1)}) + \beta_1^{(1)}\beta_1^{(2)}(e_3 + q_1^{(2)}) \quad (10)$$

subject to the restrictions

$$\begin{aligned} -\bar{q} &\leq q_1^{(1)}, \\ -\bar{q} &\leq q_1^{(2)}, \\ q_1^{(1)} + q_1^{(2)} &\leq e_1 - a. \end{aligned} \quad (11)$$

A conceptual problem can arise when trying to solve this agent's maximization problem: the optimal investment policies formed in period-1 need not be time-consistent. For example, imagine the situation where the agent is impatient in the short run but will be more patient in the future,

$$\beta_1^{(1)} < \beta_1^{(2)}. \quad (12)$$

At a broad level of generality, this assumption is consistent with the behavioral evidence (Loewenstein, 1987) that motivated the work of Laibson (1996). In addition, assume that bond prices are such that borrowing is strictly preferable in period-1:

$$\begin{aligned} b_1^{(1)} &> \beta_1^{(1)}, \\ b_1^{(2)} &> \beta_1^{(1)}\beta_1^{(2)}, \end{aligned} \quad (13)$$

which clearly implies that the agent will borrow as much as possible in period-1, which implies borrowing an amount in both bonds equal to \bar{q} , or $q_1^{(1)*} = q_1^{(2)*} = -\bar{q}$. We will also assume that $e_t - \bar{q} > a$, for $t = 2, 3$, so that the agent is unable to borrow enough to precommit future consumption to the subsistence level. The left panel of Fig. 1 depicts the period-1 optimization.

The problem of time-consistency arises when we consider the situation in which the assumptions above hold and we also assume

$$b_2^{(1)} < \beta_2^{(1)}, \quad (14)$$

where $b_2^{(1)}$ is equal to the implied forward rate $b_1^{(2)}/b_1^{(1)}$, and $\beta_2^{(1)}$ is the period-2 discount factor. In this case, regardless of how much the agent borrows in period-1, upon arriving in period-2, the agent will view the optimal decision to be to save as much as possible for period-3. In other words, the decision that left the agent short and amount of \bar{q} in the one-period bond in period-2 (i.e., the remainder of the duration of the two-period bond), will no longer be optimal. The agent can increase

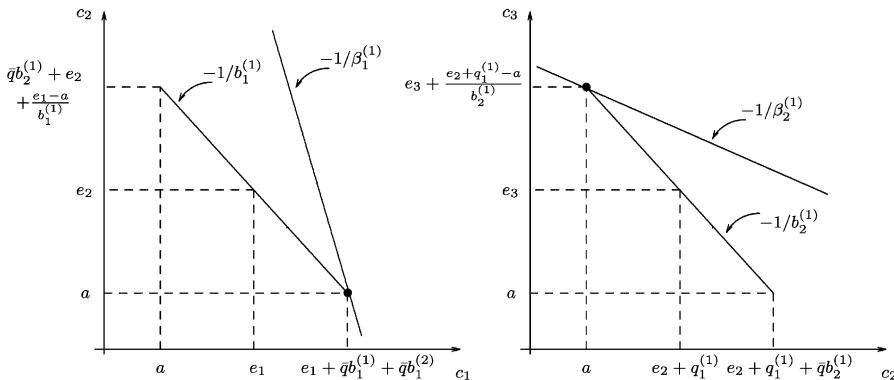


Fig. 1. Time-inconsistent choice.

utility by altering their one-period bond holdings such that

$$q_2^{(1)*} = \frac{e_2 - \bar{q} - a}{b_2^{(1)}} > -\bar{q}. \quad (15)$$

Naturally, this occurs in the absence of some mechanism that pre-commits all previous investment decisions. This decision is depicted in the right panel of Fig. 1.

The now widely adopted procedure for eliminating the logical paradox of time-inconsistent decisions is to restrict the agent's choice to plans that are necessarily time-consistent. That is, only plans that will be carried out in the future will be considered in the optimization problem. A simple algorithm for achieving this result in this simple three-period example, is to first optimize over the second-period choice, then use this solution to constrain the first-period optimization. In the example described above, this amounts to imposing the constraint that $q_2^{(1)*} = (e_2 - q_1^{(1)} - a)/b_2^{(1)}$ on the period-1 choice of $q_1^{(1)}$. The result is a period-1 choices that yield lower utility, but are time-consistent: $q_1^{(1)*} = -\bar{q}$, $q_1^{(2)*} = 0$. This constrained optimization is depicted in Fig. 2.

Equilibrium prices can be found in a very natural way given these time-consistent choices. If the agent is thought of as a representative agent and endowments are thought of as aggregate endowments, then equilibrium bond prices must be such that the agent willingly holds the aggregate endowment at those prices. In other words, the aggregate demand for bonds is zero. Therefore, in this simple economy, equilibrium prices can be found recursively and are given by

$$\begin{aligned} b_2^{(1)} &= \beta_2^{(1)}, \\ b_1^{(1)} &= \beta_1^{(1)}, \\ b_1^{(2)} &= \beta_1^{(1)}\beta_2^{(1)}. \end{aligned} \quad (16)$$

Note that unlike a model with constant discounting, i.e., $\beta_1^{(1)} = \beta_1^{(2)} = \beta_2^{(1)} = \beta$, this model is capable of generating a slope in the term structure of interest rates. Virtually, any term premium observed in data could conceivably be generated through differences in the parameters of the utility function. This is not too

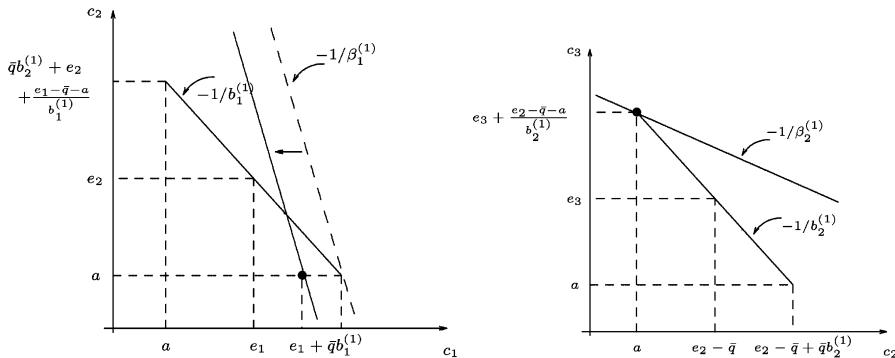


Fig. 2. Time-consistent choice.

surprising, given the way the model is set up, and the number of free parameters in the agent's preferences.

It is important to note that there was nothing specific in this argument to the assumption of only three periods. Exactly the same results would obtain in any comparable finite-horizon economy. For example, we can generalize the agents preferences to

$$U(c_t, c_{t+1}, \dots, c_T) = (c_t - a) + \sum_{\tau=t}^{T-1} \left[\prod_{i=t}^{\tau} \beta_i^{(i)} \right] (c_\tau - a), \quad (17)$$

where $\{\beta_t^{(1)}, \beta_t^{(2)}, \dots, \beta_t^{(T-t)}\}$, $t = 1, 2, \dots, T-1$, are preference parameters. If the agent has a sequence of endowments given by $\{e_1, e_2, \dots, e_T\}$, and can trade in a complete set of multi-period pure-discount bonds with a prices of $b_t^{(n)}$, where n denotes the maturity of the bond, and a face value of 1, where borrowing and lending are again constrained as in the three-period example, then time-consistency will once again be an issue. Solving this problem in an analogous fashion, and solving for equilibrium bond prices results in an equilibrium in which one-period bond prices satisfy

$$b_t^{(1)} = \beta_t^{(1)}, \quad (18)$$

for $t = 1, 2, \dots, T-1$, and multi-period bond prices satisfy

$$b_t^{(n)} = \prod_{i=t}^{t+n-1} \beta_i^{(1)}, \quad (19)$$

for $n = 2, 3, \dots, T-t$. In a deterministic setting, therefore, any term structure of interest rates can be supported by appropriate choices of the discounting parameters of the representative agent's utility function.

Note that this is true irrespective of the phenomenon that actually generated interest rates. The point worth noting is that *any* model that generates a term structure of interest rates will be observationally equivalent to a model that combines time-varying discount factors, borrowing constraints, and time-consistent planning as described above. Obviously then, the parameters of this model can be chosen to match observed interest rates exactly; yet this model could not be viewed as structural. If the structure of the true underlying economy is subject to a change, then prices could change in a way that the time-varying discount factor model cannot capture.

3.2. Time-varying discount factors II: stochastic

Now consider a stochastic generalization of the finite-horizon problem outlined above. The state of the economy is a stationary stochastic process $\{x_t\}_{t=1}^T$ that, for the sake of simplicity, follows a finite-state Markov process with transition probabilities $\text{Prob}(x_{t+1} = x_j | x_t = x_i) = \pi(x_i, x_j) = \pi_{ij}$, $i, j = 1, 2, \dots, s$. Denote the agent's stochastic endowments as $\{\tilde{e}_t\}$. There is a complete set of date- and state-contingent securities that trade in competitive markets. One-period prices at date- t

are given by $b_t(i, j) = m_{ij}\pi_{ij}$, for some function $m(x_i, x_j) = m_{ij} \geq 0$ for all $i, j = 1, 2, \dots, s$.

The agent in this economy has linear expected utility with stochastic time-varying discount factors:

$$U(c_1, \tilde{c}_2, \dots, \tilde{c}_T) = (c_1 - a) + E \left\{ \sum_{t=2}^T \left[\prod_{i=1}^{t-1} \beta(x_i, x_{i-1}) \right] (\tilde{c}_t - a) \middle| x_1 \right\}. \quad (20)$$

The budget constraint for this agent is given by

$$\begin{aligned} c_1 + \sum_s \sum_{t=2}^T \left[\prod_{\tau=1}^{t-1} m(x_\tau, x_{\tau-1}) \pi(x_\tau, x_{\tau-1}) \right] (\tilde{c}_t + \tilde{q}_{t-1}) \\ \leq e_1 + \sum_s \sum_{t=2}^T \left[\prod_{\tau=1}^{t-1} m(x_\tau, x_{\tau-1}) \pi(x_\tau, x_{\tau-1}) \right] \tilde{e}_t, \end{aligned} \quad (21)$$

where \sum_s denotes the appropriate compound summations over states, and \tilde{q}_t are the agents purchases of state-contingent claims for date t .

To find the competitive equilibrium prices consistent with this specification, we impose short-sale constraints and bound consumption with its subsistence level, a , and solve the time-consistency issue recursively as before. That is, we constrain current decisions with their consistency with optimal future decisions. Given these time-consistent plans, we find equilibrium state-contingent claims prices that clear the market, i.e., prices at which the representative agent is happy consuming the aggregate endowment. Equilibrium state prices, therefore, will satisfy

$$m(x_{t+1}, x_t) = \beta(x_{t+1}, x_t). \quad (22)$$

The interpretation of this result is as before. Virtually, any choice for the process for state prices, $\{m(x_{t+1}, x_t)\}_{t=1}^{T-1}$, can be supported as a complete-markets competitive equilibrium. In addition, utility does not have to be linear in consumption to get this result. Judiciously rescaling the per-period discount factor by $u'(e_t)/u'(e_{t+1})$, where u is a nonlinear per-period utility function, will return the same state prices.

The hazard for behavioral asset-pricing models is clear from these examples. Although it is true that if time-stationarity of preferences is relaxed, we can still solve for time-consistent plans and equilibrium prices, the scope for misinterpretation of the model's predictions is greatly increased. The same is true for introducing exogenous state variables into the utility function. Even though allowing scope for these additions may constitute interesting departures from standard expected utility models, they also open the door to reverse engineering of preferences to guarantee a good fit. Simply inserting state-variables into the utility function to obtain a good fit, then labeling the model as behavioral is not likely to further our understanding of either decision making under uncertainty or of equilibrium asset-market behavior.

Once again, the point of this example is not to claim that all models that incorporate behavioral features that imply some form of time-non-stationarity or state-dependent utility are tautological. Rather it is intended to emphasize that

obtaining a good fit cannot be the sole purpose of the exercise. The specification of preferences has to be judged reasonable by some other non-sample-based criterion. Adding exogenous factors to the utility function or adopting ad hoc functional forms can be as specious as generating large equity premiums with extreme values for higher-order moments of endowments.

If the representative agent's preferences could be derived from direct aggregation of individual agent's preferences, and even if this aggregation was only approximate, then we could use experimental evidence on individual choices, and perhaps even introspection, to gauge the reasonableness of the representative agent's preferences. Work in this area is just beginning, but theoretical contributions like Constantinides and Duffie (1995), Tallarini (2001), and Telmer and Zin (2002), and experimental evidence like Bossaerts et al. (2000), suggest that having a better understanding of aggregation will be an essential component in obtaining a consensus on reasonableness.

In the absence of aggregation results, an alternative approach would be to maintain as much theoretical structure on preferences as possible. Maintaining recursivity and time-stationarity of utility rather than imposing time-consistency on choices seems like a reasonable place to start. This would severely limit the scope for the sorts of spurious fitting exercises described above. Epstein and Zin (1989, 1991) provide a recursive and time-stationary framework for incorporating many of the behavioral concepts under consideration.

4. Conclusions

The purpose of trying to characterize asset-market data using a tightly parameterized, representative-agent, general-equilibrium model is to try to uncover deep structural parameters. This is equally true of behavioral models as it is of more traditionally expected utility models. This is not a simple task and, as the examples in this paper suggest, fitting historical data is not sufficient to insure structural stability.

Evaluating a model in this dimension will always require an element of subjective judgment of the reasonableness of the assumptions of the model. This is perhaps even more true of behavioral asset-pricing models than more traditionally expected utility models. The reason is that behavioral evidence may suggest the inclusion of state variables in the utility function and the relaxation of the stationarity assumption of intertemporal preferences. This exposes these models to the risk of being reverse engineered to fit the data, without serious consideration of whether the parameters of the model can be deemed structural.

Econometric testing for structural stability is likely to be problematic, especially in small samples. In addition, statistical test will be uninformative if the types of change to the economic environment being contemplated has no natural analog in historical experience. A better understanding of how behavioral models aggregate provides some hope for reaching a consensus about reasonableness, since this will allow inference about the representative agent's preferences based on individual-level experimental evidence.

Finally, since the use of behavioral models often opens the door for claims of reverse engineering, it seems prudent to work harder to avoid these criticisms. Maintaining assumptions on recursivity and time-stationarity of intertemporal preferences, while incorporating behavioral concepts is both feasible, as shown in Epstein and Zin (1990), and desirable given the discipline that this will naturally enforce on the modeling exercise.

References

- Abel, A.B., 1990. Asset prices under habit formation and catching up with the Jones. *American Economics Review* 80, 38–42.
- Bansal, R., Yaron, A., 2000. Risks for the long run: a potential explanation of the equity premium puzzle. Working Paper, University of Pennsylvania.
- Barberis, N., Huang, M., Santos, T., 1999. Prospect theory and asset prices. NBER Working Paper 7220.
- Benartzi, S., Thaler, R.H., 1995. Myopic loss aversion and the equity premium puzzle. *Quarterly Journal of Economics* 110, 73–92.
- Bossaerts, P., Plott, C., Zame, W., 2000. Prices and allocations in financial markets: theory and evidence. Working Paper, California Institute of Technology.
- Campbell, J., Cochrane, J., 1995. By force of habit: a consumption-based explanation of aggregate stock market behavior. NBER Working Paper 4995.
- Constantinides, G.M., 1990. Habit formation: a resolution of the equity premium puzzle. *Journal of Political Economy* 98, 519–543.
- Constantinides, G.M., Duffie, D., 1995. Asset pricing with heterogeneous consumers. Working Paper, University of Chicago.
- Epstein, L.G., Zin, S.E., 1989. Substitution, risk aversion, and the temporal behavior of consumption and asset returns I: a theoretical framework. *Econometrica* 57, 937–969.
- Epstein, L.G., Zin, S.E., 1990. ‘First-order’ risk aversion and the equity premium puzzle. *Journal of Monetary Economics* 26, 387–407.
- Epstein, L.G., Zin, S.E., 1991. The independence axiom and asset returns. NBER Working Paper T0109.
- Friedman, M., 1953. *Essays in Positive Economics*. University of Chicago Press, Chicago.
- Krusell, P., Smith Jr., A.A., 2000. Consumption and savings decisions with quasi-geometric discounting. Working Paper, Carnegie Mellon University.
- Laibson, D.I., 1996. Hyperbolic discount functions, undersaving, and savings policy. NBER Working Paper 5635.
- Loewenstein, G., 1987. Anticipation and the valuation of delayed consumption. *The Economic Journal* 97, 666–684.
- Lucas Jr., R.E., 1978. Asset prices in an exchange economy. *Econometrica* 46, 1429–1445.
- Lucas Jr., R.E., 1980. Methods and problems in business cycle theory. *Journal of Money, Credit and Banking* 12, 696–715.
- Luttmer, E.G.J., Mariotti, T., 2000. Subjective discounting in an exchange economy. Working Paper, London School of Economics.
- Mehra, R., Prescott, E.C., 1985. The equity premium: a puzzle. *Journal of Monetary Economics* 15, 145–161.
- Mehra, R., Prescott, E.C., 1988. The equity premium: a solution? *Journal of Monetary Economics* 22, 133–136.
- Rietz, T.A., 1988. The equity premium: a solution. *Journal of Monetary Economics* 22, 117–131.
- Tallarini, T., 2001. Whose habit is it anyway? Working Paper, Carnegie Mellon University.
- Telmer, C., Zin, S.E., 2002. Prices as factors: approximate aggregation with incomplete markets. *Journal of Economic Dynamics and Control* 26(7–8).
- Wachter, J., 2001. Habit formation and returns on stocks and bonds. Working Paper, New York University.