On the Need for a New Approach to Analyzing Monetary Policy

Andrew Atkeson and Patrick J. Kehoe

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ABSTRACT

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*Atkeson, UCLA, Federal Reserve Bank of Minneapolis, and NBER; Kehoe, Federal Reserve Bank of Minneapolis and University of Minnesota. The views expressed here are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.
Modern models of monetary policy start from the assumption that the central bank controls an asset price, namely the short rate, as its policy instrument. In these models this asset price is then linked to the economy through agents’ Euler equation for nominal bonds. More abstractly, this Euler equation links the policy instrument to the economy through the pricing kernel. To be useful, a model of how monetary policy affects the economy should account for how the pricing kernel moves with the short rate\footnote{Throughout this paper we consider models in which all variables are conditionally log-normal and we use the term \textit{pricing kernel} as short-hand for the log of the pricing kernel.}.

Decades of empirical work in economics and finance have uncovered some robust regularities about the dynamics of interest rates and risk that we use to shed light on how the pricing kernel moves with the short rate. For postwar U.S. data these regularities are as follows. The dynamics of the term structure of interest rates can be usefully summarized as being driven by a secular component and a business cycle component. The secular component is essentially a random walk and closely tracks the long rate. This component accounts for about 90\% of the movements in the short rate and over 97\% of the movements of the term structure of interest rates. The business cycle component accounts for most of the rest of the movements of the short rate and the term structure. It closely tracks the yield spread and is associated with movements in risk of roughly the same magnitude as movements in the yield spread.

We develop a model of the pricing kernel consistent with this evidence on the dynamics of interest rates and risk. This pricing kernel builds on the work of Backus, Foresi, Mozumdar, and Wu (2001) and Backus, Foresi, and Telmer (2001). We use this model to measure the comovements of the secular and business cycle components of the short rate with the conditional mean and the conditional variance of the (log of the) pricing kernel. We find that movements in the secular component correspond to movements in the conditional mean of the pricing kernel while movements in the business cycle component correspond to movements in the conditional variance of the pricing kernel. Standard monetary models assume that the conditional variance of the pricing kernel is constant and necessarily miss these facts. Thus a new approach to monetary policy analysis is needed. We use these results to shed light on what new models of monetary policy should look like.
To sketch out a research program we develop a simple economic model that is consistent with the pricing kernel we used to decompose the dynamics of interest rates and risk. We use this model to interpret monetary policy over the postwar period. In our economic model we interpret movements in the secular component of interest rates as driven by movements in the Fed’s long run inflation target which follows a random walk. We interpret movements in the business cycle component as driven by the Fed’s endogenous policy response to exogenously moving real risk.

In our economic model we mechanically describe the secular movements in Fed policy as arising from a random walk inflation target. Our approach here is similar to that followed in many recent monetary models. The main problem we see with this approach is that it attributes the vast bulk of the movements in the Fed’s policy instrument to a purely mechanical factor. Thus while this approach seems adequate as a statistical description of Fed policy, it seems useless for answering fundamental questions at any more than a superficial level: Why did the great inflation of the 1970s occur? Why did it end? Is it likely to occur again? and How can we change institutions to reduce that likelihood?

We argue that to answer such questions a deeper model of the forces driving the secular component of policy are needed. We briefly discuss some ambitious attempts by Orphanides (2002), Sargent, Williams, and Zha (2005), and Primiceri (2006) at modeling these forces but find them wanting. We are led to call for a new approach to modeling the economic forces underlying the secular movements in Fed policy.

In our economic model what the Fed is doing over the business cycle is simply responding to exogenous changes in the real risk. Specifically the Fed is responding to exogenous changes in the conditional variance of the real pricing kernel with the aim of maintaining inflation close to a target level. Clearly, this view differs radically from the standard view of what the Fed does over the business cycle. In the standard view risk plays no role. Instead the Fed’s policy is a function of its forecasts of economic variables that enter the mean of the pricing kernel, such as expected real growth and expected inflation. This policy is often summarized by a Taylor rule. Our interpretation of the historical record is that over the business cycle what the Fed actually did has little to do with these forecasts about changes in conditional means of growth and inflation. Instead, policy mainly responded to exogenous
changes in real risk.

In modern monetary models the policy instrument enters the economy through the Euler equation that links the short rate to expectations of growth in the log of the marginal utility of consumption and inflation. Canzoneri, Cumby, and Diba (2007) document that this Euler equation in standard models does a poor job of capturing this link between policy and the economy at business cycle frequencies. We offer a potential explanation for the failure of the Euler equation. Existing research nearly universally imposes that the conditional variances of these variables that enter the Euler equation are constant. Thus, all the movements in the pricing kernel in these models arise from movements in the conditional means. With our model of the pricing kernel we find precisely the opposite, at least for the business cycle. That is, over the business cycle nearly all of the movements in the Euler equation come from movements in conditional variances and not from conditional means.

Given this finding we argue that recent attempts to fix this Euler equation by making the conditional means of the pricing kernel more volatile while continuing to assume that the conditional variances are constant are misguided. We argue that instead researchers should be looking for a framework that delivers smooth conditional means and volatile conditional variances of the pricing kernel at business cycle frequencies. That is, researchers should come to terms with the fact that at business cycle frequencies interest rates move one for one with risk.

1. The Behavior of Interest Rates and Risk

The main instrument of monetary policy is the short term nominal interest rate, or simply the short rate. Movements in the short rate are associated with movements in interest rates of longer maturities and with movements in the expected excess returns on risky assets, or simply movements in risk. Empirical work in finance over the last several decades has established several regularities regarding the dynamics of interest rates and risk that any useful analysis of monetary policy must address.

In this section we discuss four regularities, two regarding the dynamics of interest rates and two regarding the comovements of interest rates and risk. For the dynamics of interest rates we use a traditional principal components analysis to decompose the dynamics of the
yield curve. This analysis reveals the following two regularities.

1. The first principal component accounts for a large majority of the movements in the yield curve. Because it is associated with similar movements in the yields on all maturities (essentially parallel shifts in the term structure), it is commonly referred to as the level factor in interest rates. This first principal component also has the property that it is (nearly) permanent and is well modeled by a random walk. We refer to the first principal component as the secular component of interest rates in order to emphasize this property. In the data this secular component corresponds closely to the long rate.

2. The second principal components accounts for most of the remaining movements in the yield curve. Because it is associated with changes in the short rate relative to long rates, namely, with changes in the slope of the yield curve it is commonly referred to as the slope factor in interest rates. This component also has the property that it captures most of the movements in interest rates at business cycle frequencies. We refer to this component as the business cycle component of interest rates in order to emphasize this property. In practice this business cycle component is essentially the yield spread between the long rate and the short rate.

We document these regularities here. We use monthly data on U.S. interest rates of maturities 3 months and imputed zero coupon yields for maturities $k = 1, \ldots, 13$ years over the postwar period from 1946:12 to 2007:12. For the period 1946:12 to 1991:2 we use the data from McCulloch and Kwon (1991) for these series. For the period 1991:3 to 2007:12 we use CRISP data for the 3-month Tbill rate and the data from Gurkaynak, Sack, and Wright (2006) for the other zero coupon rates. For the remainder of the paper we use the 3 month rate as our measure of the short rate and the 13 year zero coupon rate as our measure of the long rate.

To illustrate the regularities regarding the dynamics of interest rates we perform a traditional principal components analysis of the yield curve. (We closely follow the procedure in Piazzesi 200x, Section 7.2.) We focus on the first two principal components, which together account for over 99% of the variance of the short rate and over 99.8% of the total variance of all yields. In Figure 1 we plot the short rate and the first two principal components of the
yield curve\(^2\).

To document our first regularity, we note that the first principal component accounts for over 90% of the variance of the short rate as measured by the 3-month Tbill rate (as well as over 97% of the total variance of all yields). This first principal component has a monthly autocorrelation over .993. From Figure 1 it is clear visually that this component captures the long secular swings in the short rate. As we show in Figure 2 this first principal component corresponds closely to the long rate.

To document our second regularity, we show in Figure 3 that the second principal component is very similar to the yield spread between the short and the long rate. This second principal component has a monthly correlation of .957. From Figure 1 it is clear visually that, barring one exception in the early 1980s, this component captures well the business cycle movements in the short rates.

With regard to the dynamics of interest rates and risk, decades of empirical work in economics and finance reveals that movements in the business cycle component of interest rates are associated with substantial movements in risk. Specifically, this work finds two regularities regarding the comovements of interest rates and risk.

3. Movements in the short rate relative to the long rate, that is, the yield spread, are associated with movements in the expected excess returns to holding long term bonds of a similar magnitude.

4. Movements in the short rate relative to foreign-currency short rates are associated with movements in the expected excess returns to holding foreign-currency bonds of a similar magnitude.

Our third and fourth regularities, regarding movements in the business cycle component of interest rates and risk have been well-documented in the literature. We begin with the regularity on the yield spread and the expected excess returns to holding long bonds. We use the following notation to describe these empirical results more precisely. Let \( P^k_t \) denote the price in period \( t \) of a zero-coupon bond that pays off one dollar in period \( t + k \) and let

\(^2\)We have scaled these principal components so that the short rate's loadings on each of these components are equal to one.
\( p_k^t = \log P_k^t \). Then the return to holding this \( k \) period bond for one period is \( r_{kt+1}^k = p_{kt+1}^k - p_k^t \) and the (log) excess return to holding this bond over the short bond is \( r_{xt+1}^k = r_{kt+1}^k - i_t \).

The **risk premium** on long bonds is the expected excess return \( E_t r_{xt+1}^k \). Many authors have run return forecasting regressions of excess returns against the yield spread similar to the regression

\[
(1) \quad r_{xt+1}^k = \alpha^k + \beta^k (y_t^L - i_t) + \varepsilon_{t+1}^k
\]

Regressions this form have been run for 20 years, starting with the work of Fama and Bliss (1987). (See also, Campbell and Shiller 1991, and Cochrane and Piazzesi 2005.)

Note that under the hypothesis that the risk premia on long bonds are constant over time, the slope coefficient \( \beta^k \) in this regression should be zero. In the data, however, these regressions yield estimates of \( \beta^k \) that are significantly different from zero with point estimates typically greater than 1 for moderate to large \( k \).

We emphasize the magnitude of this slope coefficient here because these regression results thus imply that the risk premium on long bonds moves more than one for one with the yield spread. More precisely, note that a finding that the slope coefficient \( \beta^k \geq 1 \) implies that

\[
(2) \quad Cov(E_t r_{xt+1}^k, y_t^L - i_t) \geq Var(y_t^L - i_t)
\]

which, using simple algebra, implies that the variance in the risk premium on long bonds is greater than the risk premium

\[
(3) \quad Var(E_t r_{xt+1}^k) \geq Var(y_t^L - i_t).
\]

Our fourth regularity regarding movements in the spread between the short rate and foreign currency denominated short rates and the expected excess returns to holding foreign currency denominated bonds is simply a consequence of the empirical finding that exchange rates are well-approximated by random walks as documented in Meese and Rogoff (19??) and much subsequent work. To see this, let

\[
(4) \quad r_{xt+1}^* = i_t^* + e_{t+1} - e_t - i_t
\]
denote the (log) excess return on a foreign short bond with rate \( i_t^* \) where \( e_t \) is the log of the exchange rate. Since exchange rates are a random walk \( E_t e_{t+1} = e_t \) so that

\[
E_{t,x_{t+1}} = i_t^* - i_t.
\]

(5) that is, the expected excess return on a foreign bond is simply the interest differential across currencies.

2. Towards an Economic Model

Modern analyses of monetary policy start from the assumption that the central bank controls an asset price, namely the short rate, as its policy instrument. In our models this asset price is then linked to the economy through agents’ Euler equation for nominal bonds. More abstractly, this Euler equation links the policy instrument to the economy through the pricing kernel. The comovements of the pricing kernel and the short rate are important clues of how monetary policy is related to the economy.

Here we use the evidence that we have just discussed on the behavior of interest rates and risk to uncover these critical comovements. We begin by arguing that standard monetary models necessarily miss these comovements. We then develop a simple model of the pricing kernel consistent with our evidence of the dynamics of interest rates and risk. In the next section we use this model to give an economic interpretation of our decomposition of interest rates into secular and business cycle components. We use this interpretation to shed light on what new models of monetary policy should look like.

A. The Short Rate and the Standard Euler Equation

Consider first the link between the short rate and macroeconomic aggregates built into standard monetary models. We begin with representative agent models. The short term nominal interest rate enters standard representative consumer models through an Euler equation of the form

\[
\frac{1}{1+i_t} \equiv \exp(-i_t) = \beta E_t \left[ \frac{U_{c_{t+1}}}{U_{c_t}} \frac{1}{\pi_{t+1}} \right],
\]

(6) where \( i_t \) is the logarithm of the short term nominal interest rate \( 1+i_t \), \( \beta \) and \( U_{c_t} \) are the discount factor and the marginal utility of the representative consumer, and \( \pi_{t+1} \) is the
inflation rate. Analysts then commonly assume that the data are well-approximated by a conditionally log-normal model so that this Euler equation can be written as

\[ i_t = -E_t \left[ \log \frac{U_{ct+1}}{U_{ct}} \frac{1}{\pi_{t+1}} \right] - \frac{1}{2} \text{var}_t \left[ \log \frac{U_{ct+1}}{U_{ct}} \frac{1}{\pi_{t+1}} \right]. \]

The critical question in monetary policy analysis is what terms on the right hand side of (7) change when the monetary authority changes the interest rate \( i_t \). The traditional assumption is that conditional variances are constant, so that the second term in (7) is constant. This leaves the familiar version of the Euler equation:

\[ i_t = -E_t \log \frac{U_{ct+1}}{U_{ct}} + E_t \log \pi_{t+1} + \text{constant}. \]

Thus, by assumption, standard monetary models imply that movements in the short rate are associated one-for-one with the sum of the movements in the expected growth of the log of marginal utility for the representative consumer and expected inflation. The debate in the literature on the effects of monetary policy might be summarized roughly as a debate over how much of the movement in the short rate is reflected in the expected growth of the log of marginal utility of consumption (representing a real effect of monetary policy) and how much of the movement is reflected in expected log inflation (representing a nominal effect of monetary policy). The answer to this question in the context of a specific model depends on the specification of the other equations of the model. However, virtually universally, the possibility that movements in the short rate might be associated with changes in the conditional variances of these variables is ruled out by assumption.

We have described the standard Euler equation in the context of a model with a representative consumer. Our discussion also applies to more general models which do not assume a representative consumer. To see this note that we can write equations (7)-(8) more abstractly in terms of a nominal pricing kernel (or stochastic discount factor) \( m_{t+1} \) as

\[ \exp(-i_t) = E_t \exp m_{t+1}. \]

In a model with a representative agent this pricing kernel is given by \( \exp(m_{t+1}) = \beta U_{ct+1}/(U_{ct} \pi_{t+1}) \) and (7) is the representative agent’s first order condition for optimal bond holdings. In some segmented market models (9) is first order condition for the subset of agents who actually...
participate in the bond market while in others (??) is no single agent’s first order condition. In general equation (??) is implied by lack of arbitrage possibilities in financial market.

Using conditional log-normality (??) implies

\[ i_t = -E_t [m_{t+1}] - \frac{1}{2} \text{var}_t [m_{t+1}] \]

and with constant conditional variances we have

\[ i_t = -E_t m_{t+1} + \text{constant}. \]  

Thus the more general assumption made in the literature is that movements in the short term interest rate are associated with movements in the conditional mean of the log of the pricing kernel and not with movements in its conditional variance.

It is clear that standard monetary models with constant conditional variances are inconsistent with the evidence from finance of time-varying risk premia. To see this consider the

**Proposition 1.** In any model in which variables are conditionally log normal and conditional second moments are constant, lack of arbitrage implies that risk is constant.

**Proof.** Let \( m_{t+1} \) be (the log of the) pricing kernel and let \( r_{t+1} \) be any log asset return. Lack of arbitrage implies the standard asset pricing formula

\[ 1 = E_t \exp(m_{t+1} + r_{t+1}) \]

Taking logs of (??) and using conditional log-normality gives that

\[ 0 = E_t m_{t+1} + E_t r_{t+1} + \frac{1}{2} \text{var}_t (m_{t+1} + r_{t+1}) \]

Using (??) implies that the expected excess return on this asset

\[ E_t r_{t+1} - i_t = -\frac{1}{2} \text{var}_t (m_{t+1}) - \text{cov}_t (m_{t+1}, r_{t+1}). \]

So if conditional second moments are constant then expected excess returns are constant. Hence risk is constant. **Q.E.D.**

This proposition implies that when we log-linearize our models and impose that the primitive shocks have constant conditional variances then risk is constant. Our reading of the literature on monetary policy is that these assumptions are nearly universal. Is this a serious problem if we want to use these models to understand what in the macroeconomy moves when the short rate moves? We argue that the answer to this question is yes.
B. A Model of the Pricing Kernel

Here we present a model of the pricing kernel that is consistent with the evidence on interest rates and risk that we have discussed. This model is similar to the “negative” Cox-Ingersoll-Ross model analyzed in Backus, Foresi, Mozumdar, and Wu (2001) augmented with a random walk process and an i.i.d. shock to the pricing kernel. To analyze the expected excess returns on foreign bonds we extend the model to having two countries and two currencies in a manner similar to that in Backus, Foresi, and Telmer (2001).

The Home Country Pricing Kernel

The model has two state variables \( z_{1t} \) and \( z_{2t} \) that govern the dynamics of the pricing kernel interest rates and risk. The first state variable follows a random walk with

\[
 z_{1t+1} = z_{1t} + \sigma_1 \epsilon_{1t+1} \tag{14} 
\]

and the other state variable follows an AR1 process with heteroskedastic innovations given by

\[
 z_{2t+1} = (1 - \varphi) \theta + \varphi z_{2t} + z_{2t}^{1/2} \sigma_2 \epsilon_{2t+1}. \tag{15} 
\]

The innovations \( \epsilon_{1t+1}, \epsilon_{2t+1} \), are independent standard normal random variables. Because these state variables are independent and all yields will be linear combinations of these variables, they correspond to the principal components of the yield curve. We will show below the \( z_{1t} \) is a level factor and \( z_{2t} \) is a slope factor. To emphasize its persistence we refer to \( z_{1t} \) as the secular component of interest rates. Because it is stationary we refer to \( z_{2t} \) as the business cycle component of interest rates.

We use these two state variables to parameterize the dynamics of the pricing kernel. The (log of the) pricing kernel \( m_{t+1} \) is given by

\[
 -m_{t+1} = \delta + z_{1t} + \sigma_1 \epsilon_{1t+1} - (1 - \lambda^2/2) z_{2t} + z_{2t}^{1/2} \lambda \epsilon_{2t+1} + \sigma_3 \epsilon_{3t+1} \tag{16} 
\]

where \( \epsilon_{3t+1} \) is a third independent standard normal random variable.
The Short Rate

Given this stochastic process for the pricing kernel, we use the standard asset pricing formula

$$i_t = \log E_t \exp(m_{t+1})$$

to solve for the dynamics of the short rate. Because the pricing kernel is conditionally log-normal, we have

$$i_t = -E_t m_{t+1} - \frac{1}{2} Var_t(m_{t+1})$$  \hspace{1cm} (17)

so that movements in the short rate correspond to a combination of movements in the conditional mean of the log of the pricing kernel and movements in the conditional variance of the log of the pricing kernel. Observe that the conditional mean of the log of the pricing kernel is given by

$$E_t m_{t+1} = -\delta - z_{1t} + (1 - \lambda^2/2) z_{2t}$$  \hspace{1cm} (18)

and that the conditional variance of the log of the pricing kernel is given by

$$\frac{1}{2} Var_t(m_{t+1}) = \frac{1}{2} (\sigma_1^2 + \sigma_3^2) + \frac{\lambda^2}{2} z_{2t}.$$  \hspace{1cm} (19)

We thus have that

$$i_t = \delta - \frac{1}{2} \sigma_1^2 + z_{1t} - z_{2t}$$  \hspace{1cm} (20)

Note that the structure of this model implies that the state $z_{1t}$ is the secular component of the short rate and the state $z_{2t}$ is the business cycle component of the short rate.

In contrast to the standard monetary models, this model allows for variation over time in the conditional variance of the pricing kernel. As (17) makes clear, that variation corresponds to business cycle movements in the short rate with the extent of that variation governed by the parameter $\lambda$. In particular, $\lambda$ governs how movements in the business cycle component of the short rate are divided up between movements in the conditional mean of the (log of the) pricing kernel and the conditional variance of the (log of the) pricing kernel. Specifically, the response of the conditional mean of the pricing kernel to $z_{2t}$ is $(1 - \lambda^2/2)$ and the response of $1/2$ the conditional variance is $\lambda^2/2$. Thus, if $\lambda = 0$, then, as in the
standard model, the conditional variance of the pricing kernel is constant and all movements
in $z_{2t}$ correspond to movements in the conditional mean of the log of the pricing kernel. In
contrast, if $\lambda = \sqrt{2}$, then the conditional mean of the pricing kernel does not respond to
movements in $z_{2t}$ while one half the conditional variance of the pricing kernel responds one
for one with $z_{2t}$. If $\lambda > \sqrt{2}$, then the conditional mean and the conditional variance of the
pricing kernel move in opposite directions when the business cycle component of the short
rate moves.

C. Solving for longer term interest rates

To solve for longer term interest rates we use the standard asset pricing formula

$$p_t^k = \log E_t \exp(m_{t+1} + p_{t+1}^{k-1})$$

(21)

to set up a recursive formula for the bond prices. It turns out that prices are linear functions
of the states $z_{1t}$ and $z_{2t}$. Hence we can represent these prices as

$$p_t^k = -A_k - B_k z_{1t} - C_k z_{2t}.$$  

(22)

We can use standard undetermined coefficients to derive the following proposition.

**Proposition 1.** The coefficients of the bond prices are given recursively by

$$A_k = \delta + A_{k-1} + C_{k-1}(1 - \varphi)\theta - \frac{1}{2}(B_{k-1} + 1)^2\sigma_1^2 - \sigma_3^2,$$

$$B_k = B_{k-1} + 1,$$

$$C_k = -(1 - \lambda^2/2) + C_{k-1}\varphi - \frac{1}{2}(\lambda + C_{k-1}\sigma_2)^2$$

with $A_1 = \delta - \sigma_1^2/2$, $B_1 = 1$, $C_1 = -1$.

**Proof.** To find these prices, we start with $k = 1$ to find the price of the short-term
bond, using the asset pricing condition (??) with $p_{t+1}^0 = 0$ so that

$$p_t^1 = \log E_t \exp(m_{t+1}) = E_t m_{t+1} + \frac{1}{2}\text{Var}_t(m_{t+1})$$

so plugging into both sides gives

$$-A_1 - B_1 z_{1t} - C_1 z_{2t} = -\delta - z_{1t} + \frac{1}{2}\sigma_1^2 + z_{2t}.$$
so that \( A_1 = \delta - \sigma_1^2/2, \ B_1 = 1, \ C_1 = -1. \)

For \( k > 1 \) we write the coefficients at \( k \) as functions of the coefficients at \( k - 1 \) as follows. Given our form in (??)

\[ p_{t+1}^{k-1} = -A_{k-1} - B_{k-1} z_{1t+1} - C_{k-1} z_{2t+1} \]

Using the form of the dynamics of the state variables (??) and (??) we have

\[ p_{t+1}^{k-1} = -A_{k-1} - B_{k-1} z_{1t} - B_{k-1} \sigma_1 \epsilon_{1t+1} - C_{k-1} (1 - \varphi) \theta - C_{k-1} \varphi z_{2t} - C_{k-1} \sigma_2 z_{2t+1}^{1/2} \epsilon_{2t+1}. \]

Note then that this bond price is conditionally log-normal. Combining this bond price with our form for \( m_{t+1} \) gives

\[
\log E_t \exp(m_{t+1} + p_{t+1}^{k-1}) = E_t(m_{t+1} + p_{t+1}^{k-1}) + \frac{1}{2} \text{Var}_t(m_{t+1} + p_{t+1}^{k-1}) =
\]

\[
-\delta - A_{k-1} - C_{k-1} (1 - \varphi) \theta + \frac{1}{2} (B_{k-1} + 1)^2 \sigma_1^2 - (B_{k-1} + 1) z_{1t} -
\]

\[
- (1 - \lambda^2/2) + C_{k-1} \varphi) z_{2t} + \frac{1}{2} (\lambda + C_{k-1} \sigma_2)^2 z_{2t} + \sigma_3^2
\]

Using

\[ p_t^k = -A_k - B_k z_{1t} - C_k z_{2t} \]

then gives recursive formulas for the coefficients of bond prices and yields. \textit{Q.E.D.}

**Level and Slope Factors**

We now show that in our model the secular component of interest rates \( z_{1t} \) corresponds to a level factor which leads to parallel shifts in the yield curve and that the business cycle component \( z_{2t} \) corresponds to a slope factor which leads to changes in the spread between the long rate and the short rate.

Since yields are related to prices by \( y_t^k = -p_t^k/k, \) then (??) implies that yields can be written as

\[ y_t^k = \frac{1}{k} (A_k + B_k z_{1t} + C_k z_{2t}). \]
Thus, the implications of this model for the yield curve and its movements depend on the behavior of the coefficients $A_k/k$, $B_k/k$, $C_k/k$. Note here that our recursion implies that $B_k = k$. Thus we can write yields as

$$y^k_t = z_{1t} + \frac{1}{k} (A_k + C_k z_{2t}).$$

Clearly, movements in the secular component $z_{1t}$ correspond to parallel shifts in the yield curve because when this component moves all yields shift by the same amount. Hence, this component corresponds to a level factor in yields.\(^3\) Note that this result follows from the fact that $z_{1t}$ is a random walk.

We next show that $z_{2t}$ corresponds to a slope factor. To do so note that $C_k$ converges to a negative constant $\bar{C}$ as $k$ grows. Hence, for large $k$, movements in $z_{2t}$ have no impact on long yields since $\bar{C}/k$ goes to zero as $k$ gets large. In particular, since $C_1 = -1$, we have that any yield differential is given by

$$y^k_t - i_t = \text{constant} + (C_k/k + 1) z_{2t}$$

and the observation that $C_k/k$ converges to zero as $k$ gets large implies that the yield differential converges to

$$y^k_t - i_t = \text{constant} + z_{2t}$$

as $k$ gets large. Thus, $z_{2t}$ is a slope factor in that movements in it correspond to movements in the spread between the long rate and the short rates for long enough maturity bonds.

**Expected Excess Returns**

We now turn to our model’s implications for expected excess returns on both long term bonds and foreign currency denominated bonds.

\(^3\)Note that theoretically, the inclusion of a random walk component of the short rate leads to counterfactual implications for the average value of very long yields. This is because $A_k$ is has a component that grows linearly with $k$ as $k$ gets large (coming from $\delta$ and $\bar{C}(1 - \varphi)\theta$ and then a component that grows with $k^2$ coming from $B_{k-1}^2$. This implies that for large $k$, the constant $A_k/k$ goes to negative infinity fast. We will not worry about this limiting implication. Instead, we imagine that the random walk component of interest rates is in fact stationary, but that it appears to be a random walk over a 30 year horizon.
Long Term Bonds  We begin with the excess returns to holding a long term bond for one period. To computed these in our model we use the asset pricing formula (??). Since bond prices and the kernel are conditionally lognormal, we can write this formula as

\[ p_t^k = E_t m_{t+1} + E_t p_{t+1}^{k-1} + \frac{1}{2} Var_t (m_{t+1} + p_{t+1}^{k-1}). \]

Hence, the expected excess return on a \( k \) period bond is given by

\[ E_{t^{r_{xt=1}} + \frac{1}{2} Var_t (m_{t+1} + p_{t+1}^{k-1})}, \]

or, equivalently

\[ (23) \quad E_t r_{xt+1} = -\frac{1}{2} Var_t (p_{t+1}^{k-1}) - Cov_t (m_{t+1}, p_{t+1}^{k-1}) \]

Thus, we see that expected excess returns, which we have termed as risk, are determined by a combination of movements in the conditional variance of the log of the pricing kernel, the conditional variance of bond prices, and the covariance between the log of the pricing kernel and the log of bond prices.

Using our solutions for bond prices in the formula for excess returns (??) gives the following proposition

**Proposition 2.** The expected excess returns on holding a long term bonds are given by

\[ (24) \quad E_t r_{xt+1}^k = D_k + F_k z_{2t} \]

where \( D_1 = F_1 = 0 \) and

\[ D_k = -B_{k-1} \left[ \frac{1}{2} B_{k-1} + 1 \right] \sigma_1^2 \] and \( F_k = \sigma_2 C_{k-1} \left[ \lambda - \frac{1}{2} C_{k-1} \sigma_2 \right] \) for \( k > 1 \).

Note from (??) that movements in expected excess returns on long bonds are a function only of movements in the business cycle component of interest rates \( z_{2t} \). Hence, a regression of excess returns on the yield spread of the form (??) in our model has a slope coefficients

\[ (25) \quad \beta^k = \frac{F_k}{C_k'/k + 1}. \]

We refer to these slope coefficients as the Fama-Bliss coefficients.
Foreign Currency Denominated Bonds  The expected excess return on a foreign currency denominated bond is given by

\[ \mathbb{E}_t r_{xt+1}^* = i_t^* + E_t e_{t+1} - e_t - i_t \]

where \( i_t^* \) denotes the log of the foreign short rate and \( e_t \) denotes the log of the exchange rate. To model these expected excess returns we also model the foreign pricing kernel \( m_{t+1}^* \). This foreign kernel prices foreign currency denominated assets and thus can be used to derive foreign bond prices in a manner similar to what we have done above for domestic bond prices. In particular, for the foreign currency denominated bond

\[ (26) \quad i_t^* = -E_t m_{t+1}^* - \frac{1}{2} \text{var}_t m_{t+1}^*. \]

The lack of arbitrage in complete financial markets implies that

\[ (27) \quad e_{t+1} - e_t = m_{t+1}^* - m_{t+1}. \]

so that taking conditional expectations gives

\[ (28) \quad E_t e_{t+1} - e_t = E_t \left[ m_{t+1}^* - m_{t+1} \right]. \]

Using (??), (??) and (??) gives that

\[ (29) \quad E_t r_{xt+1}^* = \frac{1}{2} \left[ \text{var}_t m_{t+1} - \text{var}_t m_{t+1}^* \right]. \]

We model the foreign pricing kernel in a symmetric fashion as the domestic pricing kernel as in (??), (??), and (??) and impose that the parameters in the two countries are identical. We also impose that secular component of interest rates is common to both countries in that \( z_{1t} = z_{1t}^* \). Under these assumptions

\[ (30) \quad E_t e_{t+1} - e_t = (1 - \frac{\lambda^2}{2}) (z_{2t}^* - z_{2t}) \]

and

\[ (31) \quad E_t r_{xt+1}^* = \frac{\lambda^2}{2} (z_{2t}^* - z_{2t}) = \frac{\lambda^2}{2} (i_t^* - i_t) \]

Note that with \( \lambda = \sqrt{2} \), the expected change in the exchange rate in our model is constant and hence exchange rates are a random walk. With this choice of \( \lambda \), the expected excess return to a foreign currency bond is simply \( E_t r_{xt+1}^* = z_{2t}^* - z_{2t} = i_t^* - i_t \).
Calibration

We have derived our model’s implications for the key features of the data on the dynamics of interest rates and risk that motivate our study. We use this model to decompose the observed postwar history of interest rates into a secular and a business cycle component and to measure the comovements of these components of the short rate with the conditional mean and the conditional variance of the pricing kernel.

To do so, we must choose parameter values for our model. We set the time period to be a month. We choose parameter values so that our model is quantitatively consistent with the four facts that motivate our analysis. Since we demean the data we need only choose parameters that affect our model’s implications of how interest rates and risk move as the secular and business cycle components move. Thus we need only set the parameters that determine $B_k$ and $C_k$ and the expected excess returns on long term bonds and foreign bonds. These parameters are $\lambda$, which determines how the conditional variance of the pricing kernel moves with the business cycle component of interest rates, and parameters $\varphi$ and $\sigma^2$ that govern the persistence of the business cycle component and how the conditional variance of the business cycle component moves with its level. We set these parameters to be $\lambda = \sqrt{2}$, $\varphi = .99$, and $\sigma^2 = .017$.

We now show that at these values the model reproduces the four regularities on interest rates and risk discussed above.

1. That the secular component of interest rates $z_{1t}$ in the model is a random walk that acts like a level factor on the yield curve is built in to the specification. It turns out that this level factor in our model corresponds closely to the first principal component of interest rates that we discussed above. We demonstrate this result in Figure 4 where we plot the loadings on the first principal component from the data for bonds of maturities 3 months and $k = 1, 2, \ldots, 13$ years, together with the coefficients $B_k/k$ (the “loadings” on $z_{1t}$) for the same maturities from our model.

2. That the business cycle component of interest rates $z_{2t}$ in the model acts like a slope factor is built into the specification. With our chosen parameters this slope factor in our model corresponds closely to the second principal component of interest rates that we discussed above. We demonstrate this in Figure 4 where we plot the loadings on
the second principal component from the data for bonds of maturities 3 months and $k = 1, 2, \ldots, 13$ years, together with the coefficients $C_k/k$ (the “loadings” on $z_{2t}$) for the same maturities from our model.

3. That movements in the yield spread are associated with movements in the expected excess returns on long bonds of similar magnitude follows from our parameter choices. Specifically, at these parameter values it follows from (??) that the Fama-Bliss coefficient for a 5 year bond is 1.

4. That movements in the short rate relative to foreign-currency short rates are associated with movements in the expected excess returns to holding foreign-currency bonds of a similar magnitude also follows from our parameter choices. Specifically, since $\lambda = \sqrt{2}$, (??) and (??) implies that exchange rates are a random walk and hence expected excess returns on foreign bonds move exactly one-for-one with the interest differential.

In summary, we have a quantitative pricing kernel that captures very well the dynamics of interest rates and is consistent with empirical evidence on how risk moves with interest rates.

3. The decomposition of interest rates

We now turn to our decomposition of the observed postwar history of interest rates. We set $z_{1t}$ and $z_{2t}$ equal to the observed history of the first and second principal components after scaling these components appropriately$^4$. As we have seen in Figure 4 the coefficients on $z_{1t}$ and $z_{2t}$ in the model correspond closely to the factor loadings on the first and second principal components. Hence, in our decomposition the constructed interest rates capture the dynamics of yield curve just as well as the first two principal components do in the data. Recall that these two components account for over 99% of the both the variance of the short rate and the overall variance of the yield curve. In this sense our decomposition captures the dynamics of interest rates extremely well.

With this definition of $z_{1t}$ and $z_{2t}$ we obtain the same decomposition of the short rate into secular and business cycle components shown in Figure 1. We now use our model of the

$^4$Movements in the principal components are determined only up to a scale factor. Motivated by (??) we set the scale factor on these components so that the response rate of the short rate to the first principal component is 1 and the response of the short rate to the second principal component is -1.
pricing kernel to derive some economic implications of this decomposition.

A. Expectations of Future Policy

Our model gives a simple interpretation of the decomposition in Figure 1. Movements in $z_{1t}$ in the figure represent movements in expectations of where the short rate will be in the long run. Under this interpretation in the postwar period over 90% of the variance in the Fed’s policy instrument, namely the short rate, is associated with movements in agents’ expectations of where the Fed will be setting its policy instrument in the distant future.

B. The Short Rate and the Pricing Kernel

Consider next what the decomposition implies for the comovements of the short rate with the conditional mean and variance of the pricing kernel. Recall that

$$i_t = -E_t [m_{t+1}] - \frac{1}{2} \text{var}_t [m_{t+1}].$$

As we have discussed above standard monetary analyses impose that the conditional variances are constant so that

$$i_t = -E_t m_{t+1} + \text{constant}. \tag{33}$$

In our model (??) and (??) imply that when $\lambda = \sqrt{2}$

$$-E_t m_{t+1} = \text{constant} + z_{1t} \tag{34}$$

and

$$-\frac{1}{2} \text{var}_t (m_{t+1}) = \text{constant} - z_{2t}. \tag{35}$$

This result gives a very stark economic interpretation of the decomposition of the short rate shown in Figure 1: movements in the secular component of the short rate are movements in the conditional mean of the pricing kernel and movements in the business cycle component are movements in the conditional variance of the pricing kernel.

These results thus imply that, at least for business cycle analysis, existing monetary models completely miss the link between the short rate and the economy. In these models movements in the short rate are associated solely with movements in the conditional mean of the pricing kernel. Our quantitative model implies that in the data movements in the short rate are associated solely with movements in the conditional variance of the pricing kernel.
4. Towards a New View of Monetary Policy

We now put forward a simple economic model that delivers our pricing kernel. Using our pricing kernel we have made two points about the postwar history of the Fed’s policy instrument. First, most of the movements in this policy instrument are permanent. Second, the business cycle movements in this policy instrument are associated with movements in risk. In our economic model we interpret the secular movements in the Fed’s policy instrument as arising from permanent movements in the Fed’s inflation target. We interpret the business cycle movements in the Fed’s policy instrument as arising from the Fed’s endogenous policy response to exogenous changes in real risk in the economy.

Our model economy is a pure exchange economy with exogenous time-varying risk. Since the early contribution by McCallum (1994) a large literature has studied interest rates in such economies. Examples include Wachter (2006), Bansal and Shaliastovich (2007), Gallmeyer, Hollifield, Palomino, and Zin (2007), Piazzesi and Schneider (2007).

A. An Economic Interpretation of the Model

We interpret the secular component of interest rates in our model as corresponding to the Fed’s long run inflation target \( \pi_t^* = z_{1t} \) that follows a random walk. We interpret the shock \( \varepsilon_{3t+1} \) in the pricing kernel as the deviation of realized inflation \( \pi_{t+1} \) from the inflation target \( \pi_{t+1}^* \). Given this interpretation, realized inflation in our model is the sum of a random walk component and an i.i.d. component

\[
\pi_{t+1} = z_{1t+1} + \varepsilon_{3t+1}
\]

as in the model of inflation studied by Stock and Watson (2007).

We interpret the business cycle component of nominal interest rates in our model \( (z_{2t}) \) as corresponding to the real pricing kernel derived from the growth of the marginal utility of the representative agent in our economy as follows. Assume that the representative consumer has expected utility with external habit of the form

\[
E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} (C_t - X_t)^{1-\gamma}
\]
Since habit is external, the representative consumer’s marginal utility is given by

$$(C_t - X_t)^{-\gamma}$$

Following Campbell and Cochrane (1999), we define

$$S_t = \frac{C_t - X_t}{C_t}$$

Using lower case letters for logarithms of variables, we write the pricing kernel as

$$m_{t+1} = \log \beta - \gamma (c_{t+1} - c_t + s_{t+1} - s_t)$$

We assume that the logarithm of consumption growth is i.i.d. with

$$c_{t+1} - c_t = \delta_c + \sigma_c \epsilon_{2t+1}$$

Note that in this representative agent framework, $c_t$ is also aggregate consumption. We assume that the external habit level $X_t$ is a non-linear function of lagged values of consumption, habit, and a preference shock $z_{2t}$ given implicitly by

$$s_{t+1} = s_t + \eta(z_{2t}) \epsilon_{2t+1}$$

where $z_{2t}$ evolves according to

$$z_{2t+1} = (1 - \varphi) \theta + \varphi z_{2t} + \sigma_z z_{2t}^{1/2} \epsilon_{2t+1}$$

With

$$\eta(z_2) = \frac{\sqrt{2}}{\gamma} z_2^{1/2} - \sigma_c$$

and $\epsilon_{2t+1}$ independent of $\epsilon_{1t+1}$, the pricing kernel in this economy is given by (??), (??), and (??) with $\lambda = \sqrt{2}$.

**B. The New View**

This economic interpretation of our model leads to a new interpretation of the history of monetary policy in the postwar period. Under this new interpretation, the business cycle movements in the Fed’s policy instrument, the short rate, arise as a result of the Fed’s need to compensate for exogenous business cycle fluctuations in risk as it aims for its inflation target. Specifically, under this interpretation of our model, expected growth of consumption
is always constant and the Fed is always hitting its inflation target, at least in expectation. In a standard model, with constant risk, the movements in the short rate would then correspond only to movements in the Fed's inflation target, that is

\[ i_t = \text{constant} + \pi_t^*. \]

In this model, however, risk is time varying because of exogenous shifts in habit, so that the short rate has a business cycle component that is driven by these business cycle fluctuations in risk

\[ i_t = \text{constant} + \pi_t^* - \frac{1}{2} \text{Var}_t m_{t+1} = \text{constant} + \pi_t^* - z_{2t}. \]

These business cycle fluctuations in the Fed's policy instrument are required to ensure that inflation stays on target and correspond in the data to fluctuations in the slope of the yield curve.

A simple way to summarize our view about what the Fed does over the business cycle is that it is simply responding to exogenous changes in the real risk, specifically to exogenous changes in the conditional variance of the real pricing kernel with the aim of maintaining inflation close to a target level. Consider how different this view is from the standard view of what the Fed does over the business cycle espoused both in the academic literature and within the Fed itself. In our experience as Fed staff members the typical policy meeting at the Fed consists of detailed discussions of forecasts of economic variables that enter the mean of the pricing kernel, such as expected real growth and expected inflation. These discussions are often summarized by a Taylor rule for policy. Our interpretation of the historical record is that over the business cycle what the Fed actually did has little to do with these forecasts about changes in conditional means of growth and inflation. Instead, policy mainly responds to exogenous changes in real risk.

5. A Research Agenda

We take away from our interpretation of the decomposition of the short rate in Figure 1 two important questions for monetary policy analysis.

The first question regards the secular movements in the Fed's policy instrument. We interpret these as arising from random walk movements in the Fed's inflation target. We view
this interpretation as a purely mechanical accounting of these secular movements. The central question here is, Why did the Fed choose these secular movements in its policy instrument?

The second question regards the business cycle comovements between the Fed's policy instrument and the macroeconomy as captured in the standard Euler equation. Canzoneri, Cumby, and Diba (2007) have documented that, in practice, standard monetary models miss this link. The central question here is, How to fix our models so that they capture this link?

A. Why did the Fed choose the secular movements in policy?

The literature has offered two basic approaches to modeling the secular movements in the short rate in postwar U.S. data. The first approach mechanically describes aspects of Fed policy over this period that led to these movements. The economic model that we have proposed is representative of this approach. The second approach explicitly models the Fed’s objectives and information that led to its behavior.

In our economic model we have followed the first approach that mechanically describes the secular movements in Fed policy as arising from a random walk inflation target. As we have documented the random walk component in policy is large in that it accounts for over 90% of the variance in the short rate over the postwar period. This model seems adequate as a purely statistical description of Fed policy but seems useless for answering fundamental questions at any more than a superficial level: Why did the great inflation of the 1970s occur? Why did it end? Is it likely to occur again? and How can we change institutions to reduce that likelihood?

A few papers in the existing literature have begun wrestling with these questions. For example, Orphanides (2002) argues that the Fed’s difficulties in interpreting real time economic data in the 1970s played a key role in shaping the Fed’s choice of the short rate during that time. It is unclear, however, what mechanism in this framework would lead to a large random walk component in policy. Thus, we do not see how an explanation of this sort would be able to account for secular component of Fed policy.

Sargent, Williams, and Zha (2005) and Primiceri (2006) represent the most ambitious attempts to reconcile the observed secular movements in Fed policy with optimizing behavior by the Fed. In these papers the Fed uses a misspecified model to choose policy and continually
revises that model in light of the data. This approach is clearly aimed at fundamental questions in analysis of monetary policy in the post World War II period. Unfortunately, however, data on the secular movements in Fed policy pose a formidable challenge to models of this type. The basic problem is these models have a very difficult time generating volatile long run expectations of policy simply from learning dynamics.

To illustrate this point we graph in Figure 5 the time series for long run averages of expected inflation over horizons of 20 and 30 years from the model of Sargent, Williams, and Zha (2005) together with the secular component of Fed policy from our quantitative model. Clearly, this model fails to account for the secular patterns in postwar behavior of policy.

In sum, existing approaches to the forces driving the secular component of policy have not been successful. Thus a new approach is needed.

In thinking about a new approach it is worth noting that the secular component of interest rates has always been so volatile. In fact, the postwar period stands out from the historical record as a period with exceptionally high volatility of the secular component of interest rates. To illustrate this point in Figure 6A we graph a short rate and a long rate for the United States from 1835 through the present. For the short rate we use the US 3-month Commercial Paper rate and for the long rate we use the yield on a 10 year U.S. Treasury bond (available at www.globalfinancialdata.com.). Clearly, in the pre-World War II period, fluctuations in the long rate (which we associate with the secular component of interest rates) are a much smaller fraction of overall fluctuations in the short rate than they are in the post-World War II period.

This difference in pre- and post-war behavior of long and short rates is also evident in the data for of a large number of countries, including the United Kingdom (Figure 6B), France (Figure 6C), Germany (Figure 6D), and the Netherlands (Figure 6E).

A central question in the analysis of monetary policy at the secular level then is what institutional changes led to this pattern. To answer this question at a mechanical level, we see that the Gold Standard was the main institution governing monetary policy in the pre-war era and that after the war most countries switched to a fiat standard governed for part of

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5Tao Zha kindly provided us with these long run expectations of inflation from the Sargent, Williams, and Zha model
the time by the Bretton Woods agreement. But this answer is, at best, a superficial one. In
the pre war era, countries chose to be on the Gold Standard for the majority of the time and
chose to leave it when it suited their purposes. Thus, the relevant question is what forces at a
deeper level led agents to have confidence that their governments would choose stable policy
over the long term, what forces led them to lose this confidence after World War II.. Only
if we can quantitatively account for this history can we give advice on how to avoid another
Great Inflation.

B. The Short Rate and the Euler Equation

As we have discussed in modern monetary models the policy instrument enters the
economy through the Euler equation that links the short rate to expectations of growth
in the marginal utility of consumption and inflation. Canzoneri, Cumby, and Diba (2007)
document that this Euler equation in standard models does a miserable job of capturing this
link between policy and the economy at business cycle frequencies. We give some intuition
for why this is so here. We then argue that existing attempts to fix this Euler equation are
misguided and we propose a new direction.

Consider first what aspects of the comovements of the short rate and macroeconomic
aggregates that we miss in the Euler equation of standard monetary models. The basic
problem with the simplest of these models is that the terms

\[-E_t \log \frac{U_{ct+1}}{U_{ct}} + E_t \log \pi_{t+1}\]

are too smooth relative to the short rate at business cycle frequencies so they account for
virtually none of the fluctuations in the policy variable, the short rate, at these frequencies. To
illustrate this point, we\(^6\) have estimated a version of the Smets Wouters (2007) model where
we have replaced their habit preferences with standard CRRA preferences and computed the
errors in the consumption Euler equation, where the error is computed as

\[
error_t = i_t - \left[-E_t \log \frac{U_{ct+1}}{U_{ct}} + E_t \log \pi_{t+1}\right].
\]

\(^6\)Actually, we asked Ellen McGrattan to reestimate the model using codes kindly provided by Smets and
Wouters and she kindly obliged. A similar remark applies later to the computations underlying Figures 10
and 11.
In Figure 7, we plot the HP filtered short rate (the Fed Funds Rate) and the HP filtered error in the Euler equation. (We HP filter both $i_t$ and error$_t$ so as to focus on business cycle frequencies.) We find this figure striking. As we have explained, in theory the standard monetary models imply that movements in the short rate are associated one-for-one with the sum of the movements in the expected growth of the log of marginal utility for the representative consumer and expected inflation. Figure 7 shows that, in practice in a standard monetary model, movements in the short rate are associated almost one-for-one with Euler equation error and the model captures essentially none of the link between the short rate and the macroeconomy. Since this Euler equation is the fundamental link between monetary policy and the macroeconomy, this version of the model can hardly be said to be useful for analyzing monetary policy at business cycle frequencies if the observed movements in the monetary policy instrument at these frequencies correspond simply to the unexplained error in this equation.

How should we fix this problem? To address this question consider the Euler equation that allows for movements in conditional variances

$$i_t = -E_t \log \frac{U_{ct+1}}{U_{ct}} + E_t \log \pi_{t+1} - \frac{1}{2} \text{var}_t \left[ \log \frac{U_{ct+1}}{U_{ct}} \frac{1}{\pi_{t+1}} \right].$$

(36)

The approach taken in most of the literature to date has been to use more exotic preferences, such as preferences with habit persistence, but to continue to log-linearize the model and assume constant conditional variances. Mechanically, this approach amounts to making the conditional means of marginal utility growth ($E_t \log U_{ct+1}/U_{ct}$) more volatile while assuming that the conditional variances are still constant.

That this approach is a failure is well-documented by Canzoneri, Cumby, and Diba (2007). For example, consider what happens when we repeat the experiment Figure 7 using the Smets-Wouters model as specified with habit persistence. In Figure 8 we plot the HP filtered short rate and the HP filtered Euler equation error from the model. Clearly adding habit is not improving matters.

Our decomposition suggests the approach being taken in the literature to fixing the Euler equation is wrong-headed. Our decomposition indicates we should not be trying to make the conditional mean more volatile at business cycle frequencies—we find that at these
frequencies it is approximately constant. Instead we should be looking for a framework that delivers smooth conditional means and volatile conditional variances of the pricing kernel at business cycle frequencies.

Note that the economic model we proposed, while useful in helping us interpret the data, is probably not the full answer to this problem. In that model we made special assumptions that guaranteed that the conditional mean of the pricing kernel is constant. (We made consumption growth i.i.d. and engineered the habit process appropriately.) If one believes, as Canzoneri, Cumby, and Diba (2007) do that there is time variation in expected consumption growth then one would guess that our model would have similar problems to the ones that they document for other models. The reason is that when there is time variation in expected consumption growth the conditional mean of the pricing kernel in our model would likely to become volatile.

6. Conclusion

We have used a simple model of the pricing kernel to interpret the postwar U.S. data on the dynamics of interest rates and risk and to draw out implications from these data for new research directions in monetary policy analysis.

We also see our paper as pointing to new directions in empirical work on the dynamics of interest rates and risk. We have used a very simple model of the pricing kernel and have shown that, given the data, it yields a very sharp characterization of the dynamics of the short rate, the conditional mean of the pricing kernel, and its conditional variance. The short rate has a random walk component that accounts for the vast bulk of its movements in sample. The conditional mean of the pricing kernel closely tracks this random walk component. The short rate also has stationary component that accounts for almost all of the remainder of its movements. The conditional variance of the pricing kernel closely tracks this stationary component. We think that it would be useful if empirical researchers could refine our simple characterization. Specifically, there is a huge literature that uses a wide variety of different affine models of the pricing kernel to model the dynamics of interest rates and risk. Dai and Singleton (2002) and Cochrane and Piazzesi (2008) are prominent recent examples. We believe it would be useful if researchers would take the most promising of these models and
decompose movements in the short rate into movements in the conditional mean and the conditional variance of the pricing kernel.

Here we have put forth a view of monetary policy under which the Fed must continually adjust the short term nominal interest rate in response to exogenous time variation in risk even if the sole objective of the central bank is to maintain a constant level of expected inflation. We term this the exogenous risk approach. An alternative approach, termed the *endogenous risk* approach reverses the direction of causality. In this alternative approach the Fed is an active player in generating time-varying risk. Alvarez, Atkeson, and Kehoe (2002 and 2007) propose such an approach. Clearly, before progress can be made in modeling monetary policy it is essential to sort out which way the causality runs: from risk to the Fed or from the Fed to risk.
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Data Appendix

This appendix refers to the data in figures 4A-4E. All the data is available at www.globalfinancialdata.com.

United States

For the short rate the series used is the US 3 month Commercial Paper which consists of short-term, unsecured promissary notes issued primarily by corporations and it is derived from data supplied by the Depository Trust Company. The sources for this series are: Walter B. Smith and Arthur H. Cole, Fluctuations in American Business, Cambridge: Harvard Univ. Press, 1935, Federal Reserve Bank, National Monetary Statistics, New York: FRB, 1941, 1970 (annually thereafter). For the long rate we use the US Long-Term Bond Yield. This series is a combination of several indices. From February 1861 until December 1877, the 6% U.S. Government bonds of 1881 are used. From January 1878 until January 1895, the 4% U.S. Government Bonds of 1907 are used, and from February 1895 until December 1918, the 4% U.S. Government Bonds of 1925 were used. Where no trades were recorded during a given month, the previous month’s yield was used. The source for this data is William B. Dana Co., The Financial Review, New York: William B. Dana Co. (1872-1921) which reprinted data published by The Commercial and Financial Chronicle. Beginning in 1919, the Federal Reserve Board’s 10-15 year Treasury Bond index is used. This is used through 1975. In 1976, the 20-year Bond is used, and beginning on February 26, 1977, the 30-year bond is used. Beginning on February 19, 2002, the 30-year Bond series includes all bonds of 25 or more years. The sources for these series are: Sydney Homer, A History of Interest Rates, Princeton: Rutgers, 1963 from Joseph G. Martin, Martin’s Boston Stock Market, Boston: 1886 (1800-1862), Hunt’s Merchants Magazine (1843-1853), The Economist (1854-1861), The Financial Review (1862-1918), Federal Reserve Bank, National Monetary Statistics, New York: FRB, 1941, 1970 (annually thereafter); and Salomon Brothers, Analytical Record of Yields and Yield Spreads, New York: Salomon Brothers, 1995

United Kingdom

The short rate for the UK is the UK Private Discount Rate. Data are for the beginning of the month from 1867 until 1917. Data for 1824-1857 are for first class bills at undetermined periods. Thereafter, the data are for three-month Banker’s bills whenever given, or the nearest item to this type of paper or the closest period. The sources for this data is Sydney Homer, A History of Interest Rates, Sydney: Princeton University Press, 1967. (1800-1823) (NBER) Parliamentary papers, 1857, X, Pt. I; Report from the Select Committee on Bank Activity, pp. 463-464 (1824-May 1857); The Economist and Investor’s Monthly Manual (1867-1939), Central Statistical Office, Annual Abstract of Statistics, London: CSO (1919-). The long rate is the UK is the 2.5% Consol Yield. The British consol paid 3% from August 1753 until December 1888, 2 3/4% from 1889 through 1906, and 2 1/2% beginning in 1907. The actual price for the annuities/consols is provided in IGGBRCPM. Series for notes and bonds are also included. A series for 4-5 year notes issued by the British government
is quarterly from 1937 through III/1947 and monthly thereafter. This series used the 5% Conversion Loan, 1944-64 from 1935 to 1938; 2.5% National War Bonds 1952-54 from 1947 to 1949; and Exchequer stock and Treasury stock of 4-5 years maturity thereafter. A series for 10-year bonds is also included begins in 1958. The series for 10-year bonds uses the 3.5% War Loan of 1932 (callable in 1952) from 1933 through 1946, 3% Savings Bonds 1960-70 in 1947; 2.5% Savings Bonds 1964-67 in 1948 to 1950; 3% Savings Bonds 1965-75 from 1951 to 1958, and 3.5% Treasury Stock 1979-1981 from 1959. The sources for this data is Larry Neal, The Rise of Financial Capitalism, Cambridge: Cambridge University Press, 1990 for data between 1698 and 1823, The Times of London for data from 1823 until 1844, The Economist and The Banker’s Magazine from data from 1844 onwards.

France

Germany
For the short rate we used the Berlin SE Discount Rate from 1860 to 1914. The sources for this data are: The Economist and Investor’s Monthly Manual (1860-1894), Statistisches Reichsamt (1895-1945). From 1953 to 2007 we used the Germany 3-month Treasury Bill Yield from the Deutsche Bundesbank, Monthly Report. The long series used is the Germany 10-year Benchmark Bond available from the Bundesbank. The benchmark bond is used for this series. The benchmark bond is the bond that is closest to the stated maturity without exceeding it. When the government issues a new bond of the stated maturity, it replaces the bond used for the index to keep the maturity as close to the stated time period as possible.

Netherlands
The short rate consists of two series: Netherlands Private Discount Rate from 1860 to 1914 and Netherlands 3-month Treasury Bill Yield from 1946-2007. The source for the private discount rate is The Economist (1867-1914). The 3-month Treasury Bill yield consists of the 3-month T-bills through 1985 and three 3-month loans to local authorities are used beginning in 1986 because the issues of short-term government securities (Dutch Treasury Certificates) are insignificant as the total amount outstanding of short-term government securities is usually less than 5% of the total amount outstanding of government debt. For the long rate we use the 10-year Government Bond Yield. Data for the Dutch 3s are used from 1814 through June 1870, the 2.5% consol from July 1870 through July 1914 and the 3% consol from March 1907 through 1917. Data are also available on the Dutch 4s from April 1833 through June 1870. For the 1900s, the 2.5% consol is used from 1900 through July 1914, and the 3% consol from November 1915 through December 1917. The 2 1/2% consol is used from 1946 until 1954, the 3 1/4% issue of 1948 is used from 1955 until October 1964, and an index of the three or five longest running issues of the Dutch government begins in November 1964. The sources for this series are: The Economist and Banker’s Magazine (1844-1918), International Statistical Institute, International Abstract of Economic Statistics, London: International Conference of Economic Services (1919-1930) (published 1934) and 1931-1936 (published 1938), League of Nations, Statistical Yearbook, Geneva: League of Nations (1926-45), Central Bureau of Statistics, Maandschrift (1946-).
Figure 1: Short rate and the secular and business cycle components *

* The short rate is the 3 month T-Bill rate. The secular and business cycle component are the first two principal components derived from a decomposition of the covariance matrix of a vector of 14 yields: the 3 month rate and the imputed zero coupon yields for maturities k=1,...,13 years over 1946:12-2007:12. For the period 1946:12-1991:2 we use data from McCulloch and Kwon (1991) and for the period 1991:3-2007:12 we use data from Gurkaynak, Sack and Wright (2006).
Figure 2: Long rate and the secular component *

* The long rate is the imputed zero coupon yield for 13 years bonds over 1946:12-2007:12.
Figure 3: Yield spread and the business cycle component *

* The yield spread $y_{t-L,t}$ is defined as the difference between the the imputed zero coupon yield for 13 years bonds and the 3 month T-Bill rate. For the business cycle component see note to Figure 1.
Figure 4: Loadings on the secular and business cycle components data and model *

* The loadings on the secular and business cycle components in the data are the factor loadings in the principal components decomposition. The loadings are the secular components in the model are the coefficients $B_k/k$ and $C_k/k$ respectively.
Figure 5: Sargent-Williams-Zha expectations of 20 and 30 year average inflation and secular component of interest rates.
Figure 6A:
Long and Short Rates in the United States*

* The short rate is the 3 month commercial paper rate and the long rate is the yield of a long term bond. For detailed information see data appendix.
Figure 6B:
Long and Short Rates in the United Kingdom *

* The short rate is the private discount rate and the long rate is the 2.5% consol yield. For detailed information see data appendix.
Figure 6C:
Long and Short Rates in France *

* The short rate is the private discount rate for the period 1860-1914 and the 3 month T-Bill for 1960-2007. The long rate is the 10-year government bond yield. For detailed information see data appendix.
Figure 6D:
Long and Short Rates in Germany *

* The short rate is the Berlin Discount Rate for the period 1860-1914 and the 3 month T-Bill from 1953-2007. The long rate is the 10-year government bond yield. For detailed information see data appendix.
Figure 6E: 
Long and Short Rates in Netherlands *

* The short rate is the private discount rate for the period 1860-1914 and the 3 month T-Bill from 1946-2007. The long rate is the 10-year government bond yield. For detailed information see data appendix.
Figure 7: HP Filtered Fed Funds Rate And HP Filtered Euler Equation
Error CRRA utility
Figure 8: HP Filtered Fed Funds Rate And HP Filtered Euler Equation Error With Habit