When can changes in expectations cause business cycle fluctuations in neo-classical settings?

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Abstract

It is often argued that changes in expectation are an important driving force of the business cycle. However, it is well known that changes in expectations cannot generate positive co-movement between consumption, investment and employment in the most standard neo-classical business cycle models. This gives rise to the question of whether changes in expectation can cause business cycle fluctuations in any neo-classical setting or whether such a phenomenon is inherently related to market imperfections. This paper offers a systematic exploration of this issue. Our finding is that expectation driven business cycle fluctuations can arise in neo-classical models when one allows for a sufficiently rich description of the production technology; however, such a structure is rarely allowed or explored in macro-models. In particular, we identify a multi-sector setting and a setting with a costly distribution system in which expectation driven business cycles can arise. © 2006 Published by Elsevier Inc.

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1. Introduction

A common perception among economic observers is that macroeconomic fluctuations are not driven only by current developments in the economy but are often influenced by perceptions of future developments, that is, they may be driven by changes in expectations about fundamentals as opposed to current changes in opportunities or preferences. In effect, to most business economists,
this is an undisputable fact. The 1999–2001 boom-bust cycle in the US is one example which may fit this idea. For many, the 1999–2001 period was one where agents’ rosy expectations of future productivity growth contributed significantly to the high growth rates of 1999 and 2000, while a revision of these expectations caused the downturn of 2001. Similar stories are given for the booms and busts observed in the late 1990s in Asia. Given the plausibility that at least some business cycle episodes are driven by expectations, it is relevant to circumscribe the classes of models which are capable of generating such phenomena.

The object of this paper is to examine whether, and if so under what conditions, expectation driven business cycles can arise in simple neo-classical settings. That is, we will examine whether changes in expectations alone could cause booms or busts—defined as positive co-movement in consumption, investment and employment—in a setting with constant returns to scale technology and perfect markets. It should immediately be noted that we are not searching to identify conditions under which an economy exhibits indeterminacy and therefore can support sunspot shocks (for a discussion of such models, see [5]). Instead, we are interested in asking whether an exogenous change in expectations about future fundamentals can cause positive co-movement in consumption, investment and employment. For example, consider the case where agents receive signals at time $t$ about productivity growth that will arise only at time $t + n$. In this case, the signal will change the agent’s expectation of the path of future productivity. The question we want to ask is whether such an exogenous change in expectation can cause business cycle co-movements in a perfect market setting.

We choose to examine this question within a constant returns to scale and perfect market setting for three reasons. First, a substantial body of empirical literature (see [6,1,2]) supports the assumption of constant returns to scale. Second, perfect competition appears to us as the good benchmark to begin a systematic exploration of this issue. In particular, by adopting this focus we can learn whether or not expectation driven business cycles are inherently related to market imperfections or whether they can arise in perfect market setting. Third, it is well known in the literature that the simplest one-sector neo-classical model is incapable of supporting expectation driven booms and bust since, in the absence of a current change in technology, consumption and employment on the one side and consumption and investment on the other side, always exhibit negative co-movement. Hence, part of our aim is to understand the generality of this last observation. In order to favor tractability and transparency, we limit our analysis to neo-classical models where the capital stock is the only state variable (this is what we mean by the term simple neo-classical models).

The two main results of the paper are: (1) expectation driven business cycles are possible in simple neo-classical settings, that is, we show that strictly positive co-movement between consumption, investment and employment can arise in simple perfect market settings as the result of an exogenous change in expectation; and (2) most commonly used macro models restrict the production possibility set in a manner that precisely rules out the possibility of expectation driven business cycles in the presence of market clearing. The main technological features we identify as being intimately linked with the possibility of expectation driven business cycles is that of a multi-sector setting where firms exhibit cost complementarity when supplying goods to different sectors of the economy. However, most commonly used macro models do not allow for rich inter-sectorial production technologies, or if they do, they impose linear production possibility frontiers which rule out any cost complementarities. Therefore these models cannot support expectation

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1 Such a consumption-investment correlation pattern can also be found in response to a sunspot shock, as discussed in [5].
driven booms and busts; instead in these models expectational change always lead to negative co-movement between consumption and investment. Hence, our results suggest that expectation driven business cycles can be explained in a perfect market explanation if one is ready to entertain the possibility of multi-product firms with internal cost complementarities between the production of different goods. Let us stress the necessary technological structure is not very exotic, nor absent from the literature, as it is implicitly implied by models which adopt multi-sectorial investment adjustment costs as in [13,14,9–11]. Whether or not real economies exhibit the cost structure needed to support the type of expectational driven fluctuations examined here remains an empirical issue, which we leave for future research. However, we believe that our analysis nicely isolates technological features which helps explain what is often coined as Keynesian type phenomena, that is expectation driven business cycles, without the need to invoke any market imperfections.

Before outlining the structure of the paper, let us stress that the aim of our analysis is to examine whether expectation driven business cycles can arise when all current spot markets are required to be in equilibrium. In the main part of our analysis, we will not need to specify whether the underlying change in expectation is the result of a properly forecasted change in future fundamentals or whether it is based on foolish perceptions. Instead we adopt a temporary equilibrium approach which allows us to directly focus on whether positive co-movements between consumption, investment and employment can arise as the result of some expectational change, whether it be rational or irrational. In order to understand our approach, it is useful to consider the phase diagram associated with a standard one sector neo-classical model as presented in Fig. 1. In this figure, we have traced the stable saddle path as well as some of the unstable paths. It is well known that starting from an initial steady state, an anticipated change in a fundamental will cause the economy to jump and evolve along one of the unstable paths until the anticipated change is realized. The nature of the dynamics in this case implies that a jump due to an anticipated change in fundamentals will always involve a negative co-movement between consumption and investment. Specifically, such a jump will either cause an increase in consumption and a decrease in investment or the converse, regardless of the source of the expectational change. Hence, the issue we want to explore is whether simple neo-classical models inherently imply a dynamic structure in which an anticipated change in fundamentals (whether it be due to signals about future changes in productivity, taxes or preferences) always lead to negative co-movement between consumption and investment; or alternatively, if it is possible for neo-classical models to exhibit the dynamic structure depicted in Fig. 2. In particular, in Fig. 2 we have illustrated a dynamic system which is saddle path stable and which has the property that a jump from the steady state to an unstable path involves a positive co-movement between consumption and investment. Therefore, our main question can be restated as examining the conditions under which (if any) a neo-classical model can generate a dynamic system similar to that depicted in Fig. 2. Note that the phase diagram in Fig. 2 satisfies saddle path stability and therefore does not exhibit the type of equilibrium indeterminacy studied by [5].

The remaining sections of the paper are as follows. In Section 2, we present the class of economies to be considered, and define our notion of expectation driven business cycles. In Section 3, we examine several standard macroeconomic models to see whether they can possibly support expectation driven business cycles. The models examined include standard one and two-

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Kim [11] shows that, in the absence of variable labor supply, multi-sectorial investment costs are observationally equivalent to intertemporal investment costs. However, as we will show, this equivalence does not hold with variable labor supply. In particular, intertemporal adjustment costs model cannot support positive co-movement between consumption, investment and employment due to changes in expectations, while models with multi-sector investment costs can.
sector models, models with adjustment cost and models with variable capacity utilization. In Section 4, we examine multi-sector models and show that expectation driven business cycles can arise in such settings if there are multi-product firms which supply intermediate goods to both the consumption and investment sectors, and exhibit cost complementarity in doing so. We also provide an interpretation of these cost complementarities in terms of the properties of a distribution system. Finally, we present a fully specified example where expectation driven business cycles arise under rational expectations about future tax changes. Section 5 offers concluding comments.

2. Structure

Here we present the economic environment and define the main concepts we use. Assumptions made for preferences and technology are standard. The only difference is in the notations we use for technology, that allows to treat comprehensively different production structures. Given those fundamentals, we define competitive allocations and expectation driven business cycles.
2.1. Preferences and technology

Let us consider an environment with a representative agent whose preferences over consumption and leisure are ordered by

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, 1 - L_t), \]

where \( U(\cdot, \cdot) \) is a twice continuously differentiable and quasi-concave function, \( C_t \) is consumption, \( 1 - L_t \) is leisure time (i.e. \( L_t \) is labor supply) with total per-period time endowment normalized to 1, and \( \beta \in [0, 1[. \) The one additional assumption we place on the utility function is that consumption and leisure are normal goods. More precisely, it is assumed that \( UC > 0, UL < 0, -UCC/UC > UCL/UL \) and \( ULL/UL > -UCL/UC \).

At a point in time \( t \), the production opportunities available in the economy are assumed to be represented by

\[ C_t = G(K_t, L_t, K_{t+1}; \psi_t), \]

Here \( K_t \) denotes today’s capital stock, \( K_{t+1} \) represents next period’s capital stock and \( \psi_t \) represents the state of technology. For a given \( \psi_t \), we assume that the function \( G \) is homogenous of degree one, that \( G_{K_t} > 0, G_{L_t} > 0, G_{K_{t+1}} < 0 \) and that the set \( \{C_t, K_t, L_t, K_{t+1}\} \) defined by \( C_t - G(K_t, L_t, K_{t+1}; \psi_t) \leq 0 \) is convex.

Remark. Our formulation of the technological opportunities encompasses many of the those used in the macro literature. For example, it encompasses the standard one-sector model since in this case the function \( G(\cdot) \) can be written as

\[ G(K_t, L_t, I_t; \psi_t) = F(K_t, L_t; \psi_t) - K_{t+1} + (1 - \delta)K_t, \]

where \( F(\cdot) \) is the one-sector concave production function and \( \delta \in [0, 1] \) is the rate of depreciation. One-sector models with convex capital adjustment cost are also encompassed, since in this case the function \( G(\cdot) \) can be written as

\[ G(K_t, L_t, K_{t+1}; \psi_t) = F(K_t, L_t; \psi_t) - K_{t+1} + (1 - \delta)K_t - \phi \left( \frac{I_t}{K_t} \right) \times K_t, \]

where \( F(\cdot) \) is the one-sector concave production function, \( \phi(\cdot) \) is a convex cost of adjustment function with \( I_t = K_{t+1} - (1 - \delta)K_t \).

This formulation also covers standard two-sector model since the function \( G(\cdot) \) can be seen as a value function defined by

\[ G(K_t, L_t, K_{t+1}; \psi_t) = \max_{K_t^C, K_t^I, L_t^C, L_t^I} F^C(K_t^C, L_t^C; \psi_t^C) \]

s.t.

\[ \begin{aligned}
F^I(K_t^I, L_t^I; \psi_t^I) &\geq K_{t+1} - (1 - \delta)K_t, \\
K_t^C + K_t^I &\leq K_t, \\
L_t^C + L_t^I &\leq L_t,
\end{aligned} \]

where \( F^C(K_t^C, L_t^C; \psi_t^C) \) is the production function for consumption goods, \( F^I(K_t^I, L_t^I; \psi_t^I) \) the production function for investment goods and \( \psi_t = (\psi_t^I, \psi_t^C) \).
Models with variable capacity utilization are also encompassed in this formulation, since \( G(\cdot) \) can be seen as the value function of the following problem:

\[
G(K_t, L_t, K_{t+1}; \psi_t) = \max_{v_t} F(v_t K_t, L_t; \psi_t) - K_{t+1} + (1 - \delta(v_t)) K_t,
\]

where \( v \) is the rate of capacity utilization and \( \frac{\delta(v_t)}{v_t} > 0 \). Finally, the function \( G(\cdot) \) can also accommodate models with “generalized intertemporal adjustment costs”, that is models where the accumulation equation is of the form:

\[
K_{t+1} = \left[ I_t^\rho + ((1 - \delta) K_t)^\frac{1}{\rho} \right]^\frac{1}{\rho}, \quad \rho < 1.
\]

In this case, the function \( G(\cdot) \) is given by

\[
G(K_t, L_t, K_{t+1}; \psi_t) = F(K_t, L_t; \psi_t) - \left[ K_{t+1}^\rho - ((1 - \delta) K_t)^\rho \right]^\frac{1}{\rho}.
\]

### 2.2. Equilibrium allocations

We consider a structure in which firms are owned by households. Since the Walrasian equilibrium for this economy is efficient, equilibrium quantities of consumption, employment, capital and investment are solutions of the following social planner problem:

\[
\max_{C_t, L_t, K_{t+1}} E_0^{t=0} \sum_{t=0}^\infty \beta^t U(C_t, 1 - L_t) - U_L(C_t, 1 - L_t) = U_C(C_t, 1 - L_t) G_L(K_t, L_t, K_{t+1}; \psi_t),
\]

s.t. \( C_t \leq G(K_t, L_t, K_{t+1}; \psi_t) \)

and therefore need to solve

\[
-C_L = U_C(C_t, 1 - L_t) G_L(K_t, L_t, K_{t+1}; \psi_t), \quad \text{(3)}
\]

\[
C_t = G(K_t, L_t, K_{t+1}; \psi_t), \quad \text{(4)}
\]

\[
U_C(C_t, 1 - L_t) = \beta E_t \left[ U_C(C_{t+1}, 1 - L_{t+1}) \left( \frac{-G_{K_{t+1}}(K_{t+1}, L_{t+1}, K_{t+2}; \psi_{t+1})}{G_{K_{t+1}}(K_t, L_t, K_{t+1}; \psi_{t})} \right) \right]. \quad \text{(5)}
\]

Eq. (3) describes the intratemporal choice between consumption and leisure and Eq. (5) describes the intertemporal choice between consumption today and consumption tomorrow. The other equation is simply the resource constraint in period \( t \).

### 2.3. Expectation driven business cycles

The question we now want to examine is under what conditions can changes in expectations lead to positive co-movement between consumption, investment and employment, holding current technology, \( \psi_t \), and preferences fixed. We will refer to such phenomena as expectation driven business cycles, as made explicit in the definition below. To answer this question it is useful to focus on the two equations (3) and (4). These two equations can be seen as defining combinations of consumption, investment and employment which are consistent with period \( t \) market clearing in both the goods market and the labor market. In other words, Eqs. (3) and (4) define a surface which represents the set of all possible temporary equilibrium quantities, where a temporary equilibrium
is defined as the set of current equilibrium quantity combinations that can arise for some expectation about the future. Hence, if changes in expectations are to lead to positive co-movement between consumption, investment (or equivalently next period capital) and employment, then it must be the case that the surface defined by Eqs. (3) and (4) has the property that \( \frac{\partial C_t}{\partial K_{t+1}} > 0 \) and \( \frac{\partial L_t}{\partial K_{t+1}} > 0 \). This leads us to the following statement:

**Definition 1.** Expectation driven business cycles represents a positive co-movement between consumption, investment and employment induced by a change in expectations holding current technology and preferences constant.

Given this definition, the following statement immediately follows:

**Lemma 1.** Expectation driven business cycles can arise in a Walrasian equilibrium only if the surface defined by Eqs. (3) and (4) has the property that \( \frac{\partial C_t}{\partial K_{t+1}} > 0 \) and \( \frac{\partial L_t}{\partial K_{t+1}} > 0 \).

The main objective of the following two sections will be to examine different production structures and preferences to see whether expectation driven business cycles could potentially arise in these environments. In particular, this will allow us to highlight the role of multi-product firms in allowing for expectation driven fluctuations.

3. **Can expectation driven business cycles arise in the Walrasian equilibrium of one- or two-sector macro model?**

We first state a general necessary condition for the existence of expectation driven business cycles. Then, we prove two negative results: expectation driven business cycles cannot arise in most one and two-sector models used in the macroeconomic literature. As technology \( \psi_t \) will be kept constant in the rest of the paper, we omit it as an argument of the production function.

3.1. **A necessary condition for the existence of expectation driven business cycles**

In this setup, we can prove the following proposition:

**Proposition 2.** An economy can exhibit expectation driven business cycles only if \( G_{L_tK_{t+1}} > 0 \).

**Proof of Proposition 2.** By fully differentiating (3) (with \( dK_t = d\psi = 0 \)), we obtain

\[
\frac{d L_t}{d K_t} = \kappa_1 (-\kappa_2 dC_t + \kappa_3 dK_{t+1})
\]

with

\[
\begin{align*}
\kappa_1 &= -(G_{L_t}U_{C_tL_t} + U_{C_tG_{L_tL_t}} + U_{L_tL_t})^{-1} > 0, \\
\kappa_2 &= G_{L_t}U_{C_tC_t} + U_{L_tC_t} < 0, \\
\kappa_3 &= U_{C_tG_{L_tK_{t+1}}} \geq 0.
\end{align*}
\]

\[
\text{In [3] we illustrated a decreasing returns to scale environment where the temporary equilibrium had the property that } \frac{\partial C_t}{\partial K_{t+1}} = 0 \text{ and } \frac{\partial L_t}{\partial K_{t+1}} > 0, \text{ and hence does not satisfy the more stringent conditions for expectation driven business cycles we are examining here.}
Taking now the full differentiation of (4) and using (6), we get
\[
\frac{dC_t}{dK_{t+1}} = \frac{G_{K_{t+1}} + G_{L_t}K_1K_3}{1 - G_{L_t}K_1K_2}.
\]
(7)

If \( G_{L_t}K_{t+1} < 0 \), then \( \kappa_3 < 0 \) and \( \frac{dC_t}{dK_{t+1}} < 0 \). □

### 3.2. One-sector models with different forms of capital accumulation

Proposition 2 allows us to prove the three following corollaries.

**Corollary 1.** Expectation driven business cycles cannot arise in one-sector models, even with convex cost of adjusting capital.

**Proof of Corollary 1.** In a one-sector model and in a one-sector model with convex adjustment costs, \( G_{L_t}K_{t+1} = 0 \) and hence by Proposition 2 such economies cannot support expectation driven business cycles. □

**Corollary 2.** Expectation driven business cycles cannot arise in one-sector models where variable capacity utilization generates production possibilities given by
\[
\max_y F(v_tK_t, L_t) - K_{t+1} + (1 - \delta(v_t))K_t, \quad \frac{\partial \delta}{\partial v} > 0.
\]

**Proof of Corollary 2.** By the envelop theorem, \( G_{K_{t+1}} = -1 \) and hence \( G_{K_{t+1}}L_t = G_{L_t}K_{t+1} = 0 \). Therefore by Proposition 2, such an economy cannot exhibit expectation driven business cycles. □

**Corollary 3.** Expectation driven business cycles cannot arise in one-sector models where capital accumulation is of the form
\[
K_{t+1} = I_t^\rho + ((1 - \delta)K_t)^\rho, \quad \rho < 1.
\]

**Proof of Corollary 3.** In this case the function is given by \( G = F(K_t, L_t) - [(K_{t+1})^\rho + ((1 - \delta)K_t)^\rho]^{\frac{1}{\rho}} \). Hence \( G_{K_{t+1}}L_t = G_{L_t}K_{t+1} = 0 \), and once again, by Proposition 2, expectation driven business cycles cannot arise. □

The above results indicate that expectation driven business cycles cannot arise in most one-sector models used in the macro literature.  

4 We now turn to examining whether expectation driven business cycles could arise in two-sector models. One reason why it may be interesting to look into two-sector models is that Proposition 2 cannot be used to rule out expectation driven business cycles, as the following example makes clear. This example also shows that the necessary condition on the function \( G \) is not an exotic one.

Suppose the production function for consumption goods is \( C = K^{1-\gamma}(L^c)^{\gamma} \), that it is \( I_t = (L^I)^{\gamma} \) for investment goods, that capital accumulates according to \( K_{t+1} = (1 - \delta)K_t + I_t \) and that \( L^C +
$L^I = L$. Then the $G(\cdot)$ for this economy is $G(K_t, L_t, K_{t+1}) = K^{1-a}(L - (K_{t+1} - (1 - \delta)K_t)^\gamma)^a$, which has $G_{L_t}K_{t+1} > 0$. Note that the condition $G_{L_t}K_{t+1} > 0$ is only a necessary condition, and we prove in the next section that it is not always sufficient. Typically, our results for two-sectors models show that one cannot obtain expectation driven business cycles in this specific example.

3.3. Two-sector models

The interest in examining two-sector models is that potentially they satisfy the necessary condition given in Proposition 2.

Let us define a two-sector model as a model where the production function for consumption goods and the production function for investment goods are distinct. To simplify matters, let us concentrate on the case where capital accumulation is given by $I_t = K_{t+1} - (1 - \delta)K_t$, with $I_t$ being the level of investment. In this case, the aggregate production possibility is given by

$$G(K_t, L_t, K_{t+1}) = \max_{K_t^C, K_t^I, L_t^C, L_t^I} \left\{ \begin{array}{l} F^C(K_t^C, L_t^C) \\
F^I(K_t^I, L_t^I) \geq I_t = K_{t+1} - (1 - \delta)K_t, \\
K_t^C + K_t^I \leq K_t, \\
L_t^C + L_t^I \leq L_t, \end{array} \right. \tag{12}$$

where $F^C(K_t^C, L_t^C)$ and $F^I(K_t^I, L_t^I)$ are constant returns to scale and concave production functions.

**Proposition 3.** Expectation driven business cycles cannot arise in two-sector models.

**Proof of Proposition 3.** To prove this proposition, it is helpful to work with the dual form of the temporary equilibrium. Accordingly, let $\Omega^C(w_t, r_t)$ represent the unit cost function for the consumption good sector (where $r$ is the rental rate of capital and $w$ in the wage rate) and let $\Omega^I(w_t, r_t)$ represent the unit cost function for the investment good sector. A temporary equilibrium, with the consumption good being the numeraire and with $q$ being the price of investment, must then satisfy the following set of five conditions:

$$\Omega^C(w_t, r_t) = 1, \tag{8}$$

$$\Omega^I(w_t, r_t) = q_t, \tag{9}$$

$$\Omega^C_C(w_t, r_t)C_t + \Omega^I_w(w_t, r_t)I_t = L_t, \tag{10}$$

$$\Omega^C_r(w_t, r_t)C_t + \Omega^I_r(w_t, r_t)I_t = K_t, \tag{11}$$

$$-U_L(C_t, 1 - L_t) = w_tU_C(C_t, 1 - L_t). \tag{12}$$

Note that as before, taking $K_t$ as given, this system implicitly defines a set of values for $C_t, I_t$ and $L_t$ (as well as values for $r_t, w_t$ and $q_t$). The claim is that with this set, an increase in $K_{t+1}$ (through its effect on $I_t$) is necessarily associated with a decrease in either $C_t$ or $L_t$ or both. Let us prove this to be a contradiction. If we assume that $C_t, I_t$ and $L_t$ increase, then Eq. (12) implies (by normality of leisure and consumption) that $w_t$ increases. Then if $w_t$ increases, Eq. (8) implies...
that \( r_t \) decreases, but then it is impossible to satisfy Eq. (11) given that concavity of the production function and constant returns to scale imply that \( \Omega_{r,t}^C < 0, \Omega_{w,r}^C > 0, \Omega_{r,t}^I < 0 \) and \( \Omega_{w,r}^I > 0 \).  

4. Models that can support expectation driven business cycles

In the previous two sections, we have show that most of the production structures used in the macroeconomic literature are inconsistent with expectation driven business cycles. In this section we present alternative specifications of technology which can support expectation driven business cycles. The first specification involves a multi-sector environment where intermediate good producing firms supply inputs to both the consumption and investment goods sector. As we shall show, if there are cost complementarities in supplying goods to these different sectors, then expectation driven business cycles can arise in this setting. Since this multi-sector production structure may appear exotic, we also present an alternative specification which simply adds a distribution system to an otherwise standard one sector model. The distribution system is assumed to use resources to allocate the goods to their final use. We show that expectation driven business cycles can arise in such an environment under very mild assumptions on the distribution system. Since the model with a distribution system has a similar reduced form to that of the multi-sector model, the two models are best seen as different interpretations of the same sufficient conditions that given rise to expectation driven business cycles.

4.1. A multi-sector environment

Let us consider a multi-sector environment with \( N > 0 \) intermediate sectors. The setup will slightly generalize standard multi-sector models by treating intermediate goods firms as multi-product producers that sell potentially different inputs to the consumption or investment sector. The total output of consumption good \( C_t \) is given by an aggregator function \( C_t = J^C(X_1, \ldots, X_N) \) where \( X_i \) denotes the quantity of an intermediate good produced by sector \( i \) for the consumption good sector.

Similarly, the investment good is produced by an aggregate of intermediate inputs, that is, \( I_t = J^I(Z_1, \ldots, Z_N) \) where \( Z_i \) denotes the quantity intermediate inputs produced by sector \( i \) for the investment good sector. \( J^C \) and \( J^I \) are increasing, concave, and symmetric functions of their argument (i.e. they are invariant under permutation of their argument) and that they are twice continuously differentiable and homogeneous of degree one. Again, we focus on the case where the investment good is used to increase the capital stock according to the accumulation equation \( K_{t+1} = (1 - \delta)K_t + I_t \).

Each intermediate good sector \( i \) can produce both \( X_i \) and \( Z_i \) according to the following multi-product production function:

\[
H(X_i, Z_i) = F(K_i, L_i),
\]

where \( F_i(\cdot) \) is a constant returns to scale (CRS) and concave function, \( H(\cdot) \) is a CRS convex function, and \( K_i \) and \( L_i \) represent the amount of labor and capital used in sector \( i \).

In most multi-sector models used in macro, the function \( H(X_i, Z_i) \) is simply the linear function \( X_i + Z_i \). For this case, we have the following proposition.

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5 This proposition can be easily generalized to models with adjustment costs to capital and variable rates of capital utilization.
Proposition 4. Expectation driven business cycles cannot arise in a multi-sector setting if \( H(\cdot) \) is linear.

Proof of Proposition 4. In this case, because of symmetry, the model collapses to a two-sector model and the proof of Proposition 3 applies. □

The above results suggest that expectation driven business cycles may not be possible in a neo-classical setting. However, this is not the case. The following Proposition gives conditions under which expectation driven business cycles can arise and we provide an interpretation of these conditions in terms of the cost function of multi-product firms.

Proposition 5. Expectation driven business cycles can arise in a multi-sector setting if the following condition is satisfied:

\[

g L F L L - L \left( \frac{F_L}{F} \right) \left( X \frac{H_X Z}{H_Z} \right) \left( \frac{H}{X H_X} \right) > L \frac{U_L L}{U_L} - L \frac{U_C L}{U_L},
\]

when \( X = C \) and \( Z = I \).

Proof of Proposition 5. In this case, because of symmetry, the temporary equilibrium can be reduced to the following two conditions:

\[

\begin{align*}
H(C_t, K_{t+1} - (1 - \delta) K_t) &= F(K_t, L_t), \\
-U_L(C_t, 1 - L_t) &= F_L(K_t, L_t), \\
U_C(C_t, 1 - L_t) &= H_C(C_t, I_t).
\end{align*}
\]

Taking the total differential of this system to obtain \( \frac{dC}{dK_{t+1}} \), and setting this greater than zero gives the above condition after manipulation. □

The above condition is rather hard to interpret, especially the left side of the inequality. The right side of the inequality has an easy interpretation as the inverse of the inter-temporal elasticity of labor supply (that is, the elasticity of the consumption constant labor supply). Hence a high labor supply elasticity makes this condition less stringent. Close inspection of the condition given in Proposition 5 reveals that a necessary condition for expectation driven business cycles in this multi-product firm setting is that \( H_{X, Z} \) be negative. Interestingly, if \( H(\cdot) \) is not linear, then \( H_{X, Z} \) is always negative since \( H(\cdot) \) is assumed to satisfy CRS and convexity. From this observation, we can see that expectation driven business cycles can arise in a non-trivial multi-product firm setting (that is, when \( H(\cdot) \) is not linear), if the inter-temporal elasticity of labor supply is sufficiently high and if \( F(K, L) \) does not exhibit strong decreasing returns to labor. In such a case, the term \( L \frac{F_{LL}}{F_L} \) is close to zero, the term \( \left( \frac{F_L}{F} \right) \left( \frac{H}{X H_X} \right) \) is close to one, and the inverse of inter-temporal elasticity of labor supply is close to zero, and hence in such limit case the condition of Proposition 5 is satisfied by the simple fact that \( H_{X, Z} \) is negative. In order to gain further insight into the interpretation of

Another way of stating this condition is in terms of the economy’s production possibility set. If the economy’s production possibility set between consumption and investment is strictly convex and is homothetic with respect to increases in employment, then such an economy can exhibit expectation driven business cycles if the inter-temporal elasticity of labor supply is sufficiently high and if the economy does not exhibit strong decreasing returns to labor.
the right side of the condition, it is useful to expresses it in terms of properties of the multi-product firm’s short run cost function.

4.1.1. Interpretation in terms of cost complementarity

Let us define an intermediate goods firm short run cost function, \( \Omega(X, Z, K, w) \), as follows:

\[
\begin{align*}
\Omega(X, Z, K, w) &= \min_{L} wL, \\
s.t. \quad H(X, Z) &= F(K, L).
\end{align*}
\]

(Q)

Proposition 6. The intermediate firm’s production function and its associated cost function satisfy the following relationship:

\[
L \frac{F_{LL}}{F_L} - \left( \frac{F_L}{F} \right) \left( X \frac{H_{X,Z}}{H_Z} + \frac{H}{X H_X} \right) = - \left( \frac{X \frac{\Omega_{X,Z}}{\Omega_Z}}{\frac{\Omega_{X,Z}}{\Omega_Z}} \right).
\]

Proof of Proposition 6. Let us set up the Lagrangian associated with the cost minimization problem (Q) as \( wL + \lambda (H(X, Z) - F(K, L)) \). Then by the envelop theorem, \( \Omega_X = \lambda H_X(X, Z) \) and \( \Omega_{X,Z} = \frac{\partial \lambda H_X(X, Z)}{\partial Z} \). By totally differentiating the first order conditions associated with the minimization problem to get \( \frac{\partial \lambda H_X(X, Z)}{\partial Z} \), we can rearrange terms to get the above expression.

From the above two propositions, we can see that a necessary condition for the multi-sector economy to possibly exhibit expectation driven business cycles is that \( \Omega_{X,Z} \) be negative, that is, that the marginal cost of producing \( X \) decreases with the production of \( Z \). The property that a cost function for a multi-product good firm has a negative cross derivative is generally referred to as a cost complementarity property. Let us emphasize that this cost complementarity condition does not violate convexity of the cost function as long as \( (\Omega_{X,Z})^2 < (\Omega_{X,X})(\Omega_{Z,Z}) \). With this result in mind, we have the following corollary on the possibility of expectation driven business cycles:

Corollary 4. If \( U(C, I - L) = \log C + \theta(1 - L) \) (i.e., Hansen–Rogerson preferences), then a multi-sector economy can exhibit expectation driven fluctuations if the cost function of intermediate good firms exhibits cost complementarity defined as \( \Omega_{X,Z} < 0 \).

Proof of Corollary 4. This follows directly from Propositions 5 and 6 when noticing that right side of the condition in Proposition 5 is zero with these preferences.

In the more general case where preferences do not satisfy Hansen–Rogerson, then the condition is that the cost complementarity be sufficiently large relative to the slope of the consumption-constant labor supply curve.

The fact that cost complementarities can allow an economy to exhibit expectation driven business cycles is rather intuitive. Consider a change in expectation that favors increasing investment now. This most standard model, this leads to an increase in the cost of labor, and therefore reduces the incentives to produce more consumption goods. However, with cost complementarities, the economy becomes more efficient at producing consumption goods when the production of investment goods goes up. Hence, it is possible to simultaneously have an increase in investment, real wages and consumption.
4.1.2. An example

An example of a cost function exhibiting cost complementarity is given by

$$\Omega(X, Z, K, w) = wK^{1-\frac{1}{\sigma}} ([X^\sigma + Z^\sigma]^{\frac{1}{\sigma}}), \quad 0 < \sigma < 1, \quad \sigma > \frac{1}{z}. $$

The associated production function is

$$[X^\sigma + Z^\sigma]^{\frac{1}{\sigma}} = K^{1-\frac{1}{\sigma}}L^\sigma. $$

This structure has been used in the literature by [13,14,9–11], and is interpreted as a production function with multi-sectorial adjustment costs.7

If one assumes that preferences are Hansen–Rogerson preferences ($U(C, I - L) = \log C + \theta(1 - L)$), then, using Corollary 4, it is straightforward to prove that economy exhibits expectation driven business cycles if $\sigma \geq \frac{1}{z}$. This result on the possibility of expectation driven business cycles can be extended to the presence of variable capacity utilization. In such a case, the amount of curvature in the $H(\cdot)$ function needed to support expectation driven business cycles can be arbitrarily small (or one can assume an arbitrarily small multi-sectorial adjustment costs). To see this, assume the production function is

$$[X_i^\sigma + Z_i^\sigma]^{\frac{1}{\sigma}} = (v_i K_i)^{1-\frac{1}{\sigma}}L_i^\sigma$$

and the accumulation equation is

$$K_{t+1} = I_t + \left(1 - \delta_0 - \frac{\nu_i^{1+\gamma}}{1+\gamma}\right)K_t, \quad \gamma \geq 0$$

then the condition for expectation driven business cycles is

$$\sigma(\frac{1+\gamma}{\gamma}) - 1 > \frac{1}{z}. $$

As $\gamma$ approaches zero (high elasticity of capacity utilization), $\sigma$ can approach 1.8

4.2. An environment with costly distribution

The above result indicates that expectation driven business cycles can arise in an environment where intermediate good producing firms sell input to both the consumption good and investment good sectors. In this section we present a closely related environment where, instead of introducing intermediate good producers, we augment a standard one sector model by including a distribution system. This distribution system can be interpreted as reflecting the wholesale, retail and transportation industries. To this end, let the function $\Phi(C, I)$ represent the resource costs associated with distributing an amount of consumption good $C$ and investment good $I$ to its final use, $\Phi_C \geq 0, \Phi_I \geq 0$. Given our focus on models where the competitive equilibrium is optimal and can support balanced growth, the function $\Phi(C_t, I_t)$ is assumed to be convex and satisfy

---

7 It should be noted that such an aggregate production function is also found in [5], but as the outcome of an equilibrium with sector-specific externalities.

8 This result can be related to [15]. In a different context, he shows that high elasticity of capacity utilization facilitates the occurrence of indeterminacy in the [4] model.
 CRS. Again, we assume that capital goods accumulate according \( K_{t+1} = (1 - \delta)K_{t} + I_{t} \). The economy’s resource constraint can then be specified as:
\[
C_{t} + I_{t} = F(K_{t}, L_{t}) - \Phi(C_{t}, I_{t})
\]
In this environment, there are two possible scenarios: either \( \Phi(\cdot) \) is linear and \( \Phi_{CC} = \Phi_{II} = \Phi_{CI} = 0 \), or \( \Phi(\cdot) \) is not linear. In the latter case, convexity implies that \( \Phi_{CC} > 0, \Phi_{II} > 0 \) and \( \Phi_{CI} < \Phi_{CC}\Phi_{II} \). Also imposing CRS put the extra restriction \( \Phi_{CI} < 0 \). In the first case when \( \Phi(\cdot) \) is linear, it is easy to see that the economy will not satisfy the necessary condition for expectation driven business cycles identified in Proposition 2. In contrast, if \( \Phi(\cdot) \) is not linear, the economy will possibly exhibit expectation driven business cycles as indicated in the following corollary.

**Corollary 5.** Expectation driven business cycles will arise in a one-sector models augmented with a costly distribution system, if the following condition is met.
\[
L \left( \frac{F_{LL}}{F_{L}} + \left( \frac{L}{F_{L}} \right) \left( \frac{C \Phi_{CI}}{(1 + \Phi_{I})} \right) \left( \frac{C + I + \Phi}{C(1 + \Phi_{C})} \right) \right) > L \left( \frac{U_{L,L}}{U_{L}} - \frac{U_{C,L}}{U_{L}} \right).
\]

**Proof of Corollary 5.** This corollary follows directly from Propositions 5. To see this, first note that the resource constraint can be rewritten \( C_{t} + I_{t} + \Phi(C_{t}, I_{t}) = F(K_{t}, L_{t}) \), or therefore we can define the function \( H(C, I) \) as \( H(C_{t}, I_{t}) = C_{t} + I_{t} + \Phi(C_{t}, I_{t}) \), which allows us to invoke Proposition 5.

Corollary 5 indicates that if the inter-temporal elasticity of labor supply is sufficiently high and if \( F(K, L) \) does not exhibit strong decreasing returns to labor, then an economy with a costly distribution system can exhibit expectation driven business cycles since the condition identified in Corollary 5 can be met. In particular, if either \( \Phi_{II} \) or \( \Phi_{CC} \) is sufficiently large then this condition will be met. Note that \( \Phi_{CI} \) being negative, which is implied by a convex and CRS cost function, has a nice interpretation: the condition for expectation driven business cycles to arise in the presence of a distribution system reduces to the presence of cost complementarities in the distribution of \( C \) and \( I \). The idea of cost complementarities in a distribution system appears is rather plausible. For example, if there are some joint inputs in the delivery of goods, such as reception and shipping staff, or if transportation line are sometimes used in one direction for investment goods and another direction for consumption goods, then the distribution system will exhibit cost complementarities.

**4.3. A complete example of expectation driven business cycles under rational expectations**

Up to now, we have identified a set of properties that a model must satisfy if changes in expectations are to cause positive co-movement between consumption, investment and employment. In this section we present a fully developed example whereby expectation driven business cycles arise under rational expectations. For this to be possible, two conditions must be met. One the one hand, there must be a driving force which is at least partially predictable in order to have agent’s expectations change without current changes in fundamentals. Second, the environment must satisfy the conditions outlined above, which implies the presence of more than one sector. One possibility would be to consider the effect of expected changes in technology. However, given we are in a multi-sector environment, modelling an expected technological change is complicated by the fact that we must specify which sector is affected and that the results are sensitive
Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Degree of productive complementarity</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Share of labor income</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Disutility of leisure parameter</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Psychological discount factor</td>
</tr>
</tbody>
</table>

to such specification. We choose to focus on expected changes in taxes in order to bypass this difficulty and to allow a better understanding of the mechanisms. We will assume that agents receive information about a tax change before it is actually implemented. Hence, agents will react to this information, which will be the underlying fundamental driving force behind expectations. On the production side, we will adopt the multi-sector (or intratemporal adjustment cost) framework presented in the example in Section 4.3. We will compare the behavior of the model under the assumption that $\sigma = 1$ (one-sector or no intratemporal adjustment cost) and the case where $\sigma > 1$, for both the cases when the tax change is anticipated and when it is not. As we will see, the models’ implications are almost identical when the tax change is an entirely unanticipated shock, but that the two models (i.e., models having different $\sigma$) differ dramatically in their responses to news about future tax changes.

The non-stochastic aspects of the model are standard in that we assume that instantaneous utility is given by $\log C_t + \eta(1 - L_t)$. The only non-standard feature of the model is the specification of the aggregate production function, which is given by $(C_t^\sigma + I_t^\sigma)^{\frac{1}{\sigma}} = A(K_t)^{1-\sigma}(L_t)^{\sigma}$. Note that $\sigma = 1$ corresponds to the standard one-sector model and that capital accumulation is given by $K_{t+1} = (1 - \delta)K_t + I_t$. As explained above, $\sigma > 1$ can be interpreted as reflecting the presence of multi-product firms with exhibit cost complementarities when supply intermediate goods to different sectors.

We introduce into this economy a capital income tax $\tau_k$ and a labor income tax $\tau_l$, and we assume that the proceeds of those taxes are redistributed to households in a lump sum way. Hence, these taxes simply create distortions in the economy and have no other role.

The parameters values we use are presented in Table 1. The length of a period is set at a quarter. The parameters ($\alpha$, $\delta$, $\eta$, $\beta$) are set to values commonly accepted in the literature. In the case with productive complementarity, we assume that $\sigma = 1.65$, which is slightly above the limit value $1/\alpha (=1.5)$ that we have computed in the previous section. Using the same functional form of intratemporal adjustment costs, [14] calibrates $\sigma$ to match the estimated responses of investment to three shocks: a preference shock, a technological shock and a shock to intratemporal adjustment costs. He obtains a value $\sigma = 1.8$. A specification similar to our is also used in [13], where it is assumed there, without justification, that $\sigma = 3$.

We assume an initial 20% uniform income taxation, so that $\tau_k = \tau_k = 20\%$. The shock is a permanent four hundred basis points cut in both tax rate (from 20% to 16%). This shock is either a surprise in period one; or is announced in period one and implemented five periods later. Of course, a more complex shock structure would be needed to reproduce fluctuations, allowing also for technology shocks, noisy signals and gradual resolution of uncertainty (see [3] for a richer

---

9 For example, expected technological change in the capital goods producing sector generally causes a recession, while expected technological change in the consumption good sector can generate a boom.
modeling of the informational structure in a setup with news about future productivity growth). Here, we are only interested in conditional responses to news, we think it is more illustrative to keep a simple shock structure.

With these parameters, consumption represents 75% of output, capital is 9.18 times quarterly output and the labor share is two-thirds.

Given parameters values, we solve for the Walrasian equilibrium by log-linearizing the equilibrium conditions to obtain an approximate solution to the model. We then compute impulse response functions for the model with and without cost complementarities.

4.4. Responses to a tax cut surprise and to a tax cut news

Let us first consider responses to a tax surprise in the two cases $\sigma = 1$ (standard one-sector model) and $\sigma = 1.65$ (multi-sector model with cost complementarities). Four variables are plotted in Fig. 3: the income tax rate, worked hours, consumption and investment. Each variable is expressed in percentage deviation from the initial steady-state. In both versions of the model, an unanticipated permanent decrease in the income tax rate permanently increases consumption,
investment and hours. The qualitative response of all variables is similar in both versions of the model, as the tax cut surprise creates an aggregate boom. In the case with $\sigma = 1$, investment and hours overshoot in order to smooth consumption. Notice that in the multi-sector case, the productive complementarity dampens investment response and pushes up the response of consumption. This comes from the fact that there is less of a tradeoff here between investment and consumption.

We now turn to a tax cut announcement in period 1, that is implemented in period 6. Fig. 4 presents the responses of the four variables under consideration during the interim periods (the first five periods), i.e. between the announcement date and the implementation of the tax cut. The two models are now qualitatively very different. In the case of $\sigma = 1$, consumption increases on impact because of a wealth effect, stays above its initial steady state level but decreases after period 1 until the tax cut is effectively implemented. Simultaneously, investment and hours decrease on impact and during interim periods. Therefore, the announced reduction in distortionary taxes creates a recession in output, investment and hours. In contrast, the figure shows that in the multi-sector model all variables increase on impact and during the interim periods: hence the tax cut announcement creates an expectation driven aggregate expansion.
5. Conclusion

In this paper, we have shown that: (1) expectation driven business cycles are possible in simple neo-classical settings, meaning that strictly positive co-movement between consumption, investment and employment can arise in simple perfect market settings as the result of changes in expectation; and (2) most commonly used macro models restrict the production possibility set in a manner that precisely rules out the possibility of expectation driven business cycles in the presence of market clearing. The main technological features we identified as being necessary for expectation driven business cycles is that of a multi-sector setting where firms experience cost complementarities when supply goods to different sectors of the economy.

In closing, let us emphasize that a model which allows for expectation driven business fluctuations is a close cousin to a simple Keynesien cross model since in both cases an autonomous increase in investment leads to a more than one-for-one increase in output. Our results therefore can be thought as linking the possibility of Keynesian cross type phenomena with a technological feature that is close in spirit to those emphasized in the complementarity and coordination literature.\(^{10}\) In particular, our results identify technological conditions under which business cycles may arise as a purely demand driven phenomena, as in traditional Keynesian models, even in the absence of sticky prices, imperfect competition, increasing returns to scale or externalities. In this sense, our analysis highlights a new mechanism which may help explain why market economies may exhibit business cycle fluctuations driven by changes in expectations.

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Appendix A. A model with time to build

Here we examine a time to build model that possesses more than one state variable. The model can be described as follows. Preferences have the standard form

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, 1 - L_t). \]  \hspace{1cm} (A.1)

Output is produced according to

\[ Y_t = F(K_t, L_t). \]  \hspace{1cm} (A.2)

To model time to build, we use the formulation of \[12,7\]. Investment in period \(t\) is given by

\[ I_t = \sum_{j=1}^{n} \omega_j S_{j,t}, \]  \hspace{1cm} (A.3)

where \(S_{j,t}\) is the volume of projects \(j\)-periods away from completion at the beginning of period \(t\) and \(\omega_j\) is the resource cost associated with work on a project \(j\) periods away from completion.

\(^{10}\) See \[8\] for an exposition of that literature.
for $j = 1, \ldots, n$. Investment projects progress according to

$$S_{j,t+1} = S_{j+1,t} \quad \text{for } 1 < j \leq n$$

(A.4)

and starts during period $t$ are represented by $S_{n,t}$. Thus the capital stock evolves according to

$$K_{t+1} = (1 - \delta)K_t + S_{1,t}.$$  

(A.5)

In the following, we restrict ourselves to $n = 2$ without loss of generality for the result. The optimal allocation program can be reduced to

$$\max_{S_{2,t}, L_t} E_0 \sum_{t=0}^{\infty} \beta^t U \left( F[(1 - \delta)K_{t-1} + S_{2,t-2}, L_t] + (1 - \delta)((1 - \delta)K_{t-1} + S_{2,t-2}) 
\quad - \omega_1 S_{2,t-1} - \omega_2 S_{2,t} + 1 - L_t \right)$$

with $K_0$ and $S_{1,0}$ given. The first-order conditions associated with this program are

$$C_t + \omega_1 S_{1,t} + \omega_2 S_{2,t} = F(K_t, L_t), \quad (A.6)$$

$$U_1(C_t, 1 - L_t)F_2(K_t, L_t) = U_2(C_t, 1 - L_t), \quad (A.7)$$

$$U_1(C_t, 1 - L_t)\omega_2 + \beta E_t \left[ U_1(C_{t+1}, 1 - L_{t+1})\omega_1 \right] = \beta^2 E_t \left[ U_1(C_{t+2}, 1 - L_{t+2})(F_1(K_{t+2}, L_{t+2}) + 1 - \delta) \right]$$

(A.8)

and a transversality condition.

In a given period $t$, $K_t$ and $S_{1,t}$ are given. A temporary equilibrium of this economy is a triplet $(C_t, L_t, S_{2,t})$ that satisfies Eqs. (A.6) and (A.7). Here, $S_{2,t}$ stands for new investment. Let us define the $G$ function as

$$C_t = G(K_t, L_t, S_{1,t}, S_{2,t}) = F(K_t, L_t) - \omega_1 S_{1,t} - \omega_2 S_{2,t}.$$ 

As previously shown, a necessary condition for expectation driven business cycles is $G_{L,S_2} > 0$. In the time to build model $G_{L,S_2} = -\omega_2 < 0$, so that one cannot generate expectation driven business cycles.

References


