Asset Prices in Business Cycle Analysis

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Pictures: leads and lags in US data

Equations: the usual suspects + bells & whistles

Computations: loglinear approximation

More pictures: leads and lags in the model

Extensions
Leads and lags in US data

Cross-correlation functions of GDP with

- Stock price indexes
- Interest rates and spreads
- Consumption and employment

US data, quarterly, 1960 to present

Quarterly growth rates \((\log x_t - \log x_{t-1})\) except

- Interest rates and spreads (used as is)
- Occasional year-on-year comparisons \((\log x_{t+2} - \log x_{t-2})\)
Stock prices and GDP

Leads and lags in data

Backus, Routledge, & Zin (NYU & CMU)
Leads and lags in data

Stock prices and GDP (year-on-year)

S&P 500 (yoy)
Stock prices and GDP

Leads and lags in data

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Interest rates and GDP

Leads, lags, and logs

Backus, Routledge, & Zin (NYU & CMU)
Consumption and GDP

- Consumption
- Services
- Nondurables
- Durables

Backus, Routledge, & Zin (NYU & CMU)
Investment and GDP

- Investment
- Structures
- Equipment and Software
- Residential
Employment and GDP

Leads and lags in data
Lead/lag summary

Things that lead GDP

- Stock prices
- Yield curve and short rate
- Maybe consumption (a little)

Things that lag GDP

- Maybe employment (a little)

Why?
(Almost) the usual equations

Streamlined Kydland-Prescott except

- Recursive preferences (Kreps-Porteus/Epstein-Zin-Weil)
- CES production
- Adjustment costs
- Unit root in productivity
- Predictable component in productivity growth
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- **Predictable component in productivity growth**
Preferences

Equations

\[
U_t = V[u_t, \mu_t(U_{t+1})]
\]

\[
u_t = c_t(1 - n_t)^\lambda
\]

\[
V(u_t, \mu_t) = [(1 - \beta)u_t^\rho + \beta\mu_t^\rho]^{1/\rho}
\]

\[
\mu_t(U_{t+1}) = (E_t U_{t+1}^\alpha)^{1/\alpha}
\]

Interpretation

\[
IES = 1/(1 - \rho)
\]

\[
CRRA = 1 - \alpha
\]

\[
\alpha = \rho \implies \text{additive preferences}
\]
Technology: production

Equations

\[ y_t = f(k_t, z_t n_t) \]
\[ = [\omega k_t^\nu + (1 - \omega)(z_t n_t)^\nu]^{1/\nu} \]
\[ y_t = c_t + i_t \]

Interpretation

Elast of Subst \( = 1/(1 - \nu) \)
Capital Share \( = \omega(y/k)^{-\nu} \)
Technology: capital accumulation

Equations

\[ k_{t+1} = g(i_t, k_t) \]

\[ = (1 - \delta)k_t + k_t[(i_t/k_t)^\eta(i/k)^{1-\eta} - (1 - \eta)(i/k)]/\eta \]

Interpretation

No adjustment costs if \( \eta = 1 \)
Productivity

Equations

\[
\log x_{t+1} = (I - A) \log x + A \log x_t + B w_{t+1} \\
\{w_t\} \sim \text{NID}(0, I) \\
\log z_{t+1} - \log z_t = \log x_{1t+1} \text{ (first element)}
\]

Interpretation

\[
A = [0] \implies \text{no predictable component}
\]
Computation overview

Scaling
- Recast as stationary problem in “scaled” variables

Loglinear approximation
- Loglinearize value function (not log-quadratic)
- Loglinearize necessary conditions
- With constant variances, recursive preferences irrelevant to quantities (but not asset prices)
Scaling the Bellman equation

Key input: \((V, \mu, f, g)\) are hd1

Natural version

\[
J(k_t, x_t, z_t) = \max_{c_t, n_t} V \left\{ c_t (1 - n_t)^\lambda, \mu [J(k_{t+1}, x_{t+1}, z_{t+1})] \right\}
\]

subject to:
\[
k_{t+1} = g[f(k_t, z_t n_t) - c_t, k_t]
\]

plus productivity process & initial conditions

Scaled version \([\tilde{k}_t = k_t/z_t, \tilde{c}_t = c_t/z_t]\)

\[
J(\tilde{k}_t, x_t, 1) = \max_{\tilde{c}_t, n_t} V \left\{ \tilde{c}_t (1 - n_t)^\lambda, \mu [x_{1t+1} J(\tilde{k}_{t+1}, x_{t+1}, 1)] \right\}
\]

subject to:
\[
\tilde{k}_{t+1} = g[f(\tilde{k}_t, n_t) - \tilde{c}_t, \tilde{k}_t]/x_{1t+1}
\]

plus productivity process & initial conditions
Loglinear approximation

Objective: loglinear decision rules \([\hat{k}_t \equiv \log \tilde{k}_t - \log \tilde{k}, \text{ etc}]\)

\[
\begin{align*}
\hat{c}_t &= h_{ck} \hat{k}_t + h_{cx}^\top \hat{x}_t \\
\hat{n}_t &= h_{nk} \hat{k}_t + h_{nx}^\top \hat{x}_t
\end{align*}
\]

Key input:

\[
\log J(\tilde{k}_t, x_t) = p_0 + p_k \log \tilde{k}_t + p_x^\top \log x_t
\]

Solution

- Brute force loglinearization of necessary conditions
- Riccati equation separable: first \(p_k\), then \(p_x\)
- Lots of algebra, but separability allows you to do it by hand
Necessary conditions

First-order conditions

\[
(1 - \beta) \tilde{c}_t^{\rho - 1} (1 - n_t)^{\rho \lambda} = M_t g_{it}
\]

\[
\lambda (1 - \beta) \tilde{c}_t^\rho (1 - n_t)^{\rho \lambda - 1} = M_t g_{it} f_{nt}
\]

Envelope condition

\[
J_{kt} = J_{t}^{1-\rho} M_t (g_{it} f_{kt} + g_{kt})
\]

“Massive expression”

\[
M_t = \beta \mu_t (x_{1t+1} J_{t+1})^{\rho - \alpha} E_t[(x_{1t+1} J_{t+1})^{\alpha - 1} J_{kt+1}]
\]
Expectations and certainty equivalents

Example: let \( \log x \sim \mathcal{N}(\kappa_1, \kappa_2) \)

Expectations and certainty equivalents for lognormals

\[
\begin{align*}
E(x) &= \exp(\kappa_1 + \kappa_2/2) \\
E(x^\alpha) &= \exp(\alpha \kappa_1 + \alpha^2 \kappa_2/2) \\
\mu(x) &= \left[ E(x^\alpha) \right]^{1/\alpha} = \exp(\kappa_1 + \alpha \kappa_2/2).
\end{align*}
\]

Effect of risk is multiplicative (like \( \beta \))
Approximation: two flavors

Problem: find decision rule \( u_t = h(x_t) \) satisfying

\[
E_t F(x_t, u_t, w_{t+1}) = 1, \quad w_t \sim N(0, \kappa_2)
\]

Judd + many others

- Taylor series expansion of \( F \)
- \( n \)th moment shows up in \( n \)th-order term

Us + much of modern finance

- Taylor series expansion of \( f = \log F \) in

\[
E_t \exp[f(x_t, u_t, w_{t+1})] = 1
\]

- All moments show up even in linear approximation
Approximation: example

Linear “perturbation” method

- Linear approximation of $F$

$$F(x_t, u_t, w_{t+1}) = F + F_x(x_t - x) + F_u(u_t - u) + F_w w_{t+1}$$

$$E_t F = 1 \Rightarrow u_t - u = (1 - F)/F_u - (F_x/F_u)(x_t - x)$$

- Decision rule doesn’t depend on variance of $w$ (or higher moments)

“Affine” finance method

- Linear approximation of $f = \log F$

$$f(x_t, u_t, w_{t+1}) = f + f_x(x_t - x) + f_u(u_t - u) + f_w w_{t+1}$$

$$E_t \exp(f) = 1 \Rightarrow u_t - u = -(f + f_w \nu_2/2)/f_u - (f_x/f_u)(x_t - x)$$

- Note impact of variance $\nu$ (higher moments would show up, too)
Growth model: no labor or adjustment costs

Three processes for productivity growth

- Random walk \((A = 0)\)
- Two-period lead
- Small predictable component

The challenge

- Barro and King
Random walk: impulse responses
Random walk: cross correlations
Two-period lead: cross correlations
Predictable component: cross correlations
Summary and extensions

Summary

- Data: interest rates lead the cycle
- Model: ditto from predictable component in productivity growth

Extensions

- Labor dynamics: Gali’s result?
- Stochastic volatility
- Could this result from endogenous dynamics? Monetary policy?
Related work

Leads and lags in data
- Ang-Piazzesi-Wei, Beaudry-Portier, King-Watson, Stock-Watson

Predictable components in models
- Bansal-Yaron, Jaimovich-Rebelo

(Log)linear approximation
- Campbell, Hansen-Sargent, Lettau, Tallarini, Uhlig

Kreps-Porteus pricing kernels
- Hansen-Heaton-Li, Weil
Autocorrelations of quarterly growth rates

Autocorrelations of Growth Rates

Bartlett’s formula for MA(q) 95% confidence bands

GDP

Consumption

Investment

Government Purchases

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Random walk: autocorrelations

- GDP
- Consumption
- Investment
- Interest Rate

Backus, Routledge, & Zin (NYU & CMU) Leads, lags, and logs
Predictable component: autocorrelations
Predictable component: impulse responses

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**Graphs:**

- **Productivity**
  - Y-axis: 0 to 1
  - X-axis: Quarters after Shock (0 to 20)

- **Consumption**
  - Y-axis: 0 to 0.4
  - X-axis: Quarters after Shock (0 to 20)

- **Investment**
  - Y-axis: 0 to 5
  - X-axis: Quarters after Shock (0 to 20)

- **Interest Rate**
  - Y-axis: 0 to 10
  - X-axis: Quarters after Shock (0 to 20)
Theory and reality (circa 1964)

Realist MacLeod:

- Mundell’s article [ignores] complications associated with speculation in the forward market. It can only bring discredit on the economics profession to leave unchallenged his attempt to draw from the model policy conclusions that are applicable in the real world.

Theorist Mundell:

- Theory is the poetry of science. It is simplification, abstraction, the exaggeration of truth, a caricature of reality. Dr McLeod calls my assumptions unrealistic. I certainly hope he is right. I left out a million variables that made my caricature of reality unrealistic. At the same time, it enabled me to find fruitful, but refutable, empirical generalizations.
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