Long-Run Inflation Risk and the
Postwar Term Premium*

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Abstract

Long-term bond yields in the U.S. steadily rose during the 1960s and 1970s and then retreated over the next two decades. This rise and fall is difficult to explain using only changes in long-term inflation expectations and real interest rates; instead, an additional role for changes in the term premium—the risk premium on long-term bonds—appears to be required. We explain the behavior of long-term bond yields with a New Keynesian DSGE model with nominal rigidities, Epstein-Zin-Weil preferences, and long-run inflation risk. We show that this model—unlike many others—is able to generate an empirically plausible level of the term premium without compromising the model’s ability to fit key macroeconomic variables. Moreover, by taking into account changes in the Federal Reserve’s perceived commitment to a low long-term U.S. inflation rate, the model’s predictions for inflation expectations and the term premium are able to explain the behavior of U.S. long-term bond yields in the postwar period.

*The views expressed in this paper are those of the authors and do not necessarily reflect the views of other individuals within the Federal Reserve System.
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1 Introduction

From 1955 to 1981, the nominal yield on the 10-year U.S. Treasury bond more than quintupled, rising steadily from 2.6 percent at the beginning of this period to over 15.3 percent by the end. From 1982 to 2008, the 10-year bond yield then reversed course, falling all the way back to 3.7 percent (see Figure 1). It is difficult to explain these dramatic changes in long-term bond yields solely as a result of changes in long-run inflation expectations or real interest rates. For example, Figure 2 plots survey data on long-term inflation expectations starting in 1980 (the date this question was first included in the survey). This measure of long-run inflation expectations peaked at about 8.25 percent in late 1980, and then declined steadily to about 2.5 percent. Although this fall is substantial, it can account for only about half of the decline in the long-term nominal bond yield over the same period. Similarly, it appears unlikely that elevated long-term real interest rates could account for the high nominal rates of the 1970s and 1980s: indeed, given the well-known period of slow productivity growth from the mid-1970s through the mid-1990s, it seems likely that equilibrium real interest rates were, if anything, lower during this period rather than higher. Finally, there is a large finance literature that argues that risk premia on long-term bonds are substantial and vary significantly over time (e.g., Fama and Bliss, 1987, Campbell and Shiller, 1991, Cochrane and Piazzesi, 2005). Thus, one is naturally led to wonder whether and to what extent variation in the term premium over the postwar period may have been responsible for the rise and fall of long-term bond yields.

In the present paper, we develop a dynamic structural general equilibrium (DSGE) model of the U.S. economy that is able to account for the dynamics of macroeconomic variables, changes in long-term inflation expectations, and changes in the risk premium on long-term bonds. To do this, our model requires three essential ingredients. First, as in Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003), and others, we require an important role for nominal rigidities in order to match the behavior of inflation and other nominal quantities. Without nominal rigidities, we would have little hope of matching the variation in the returns on nominal bonds.
Figure 1: Constant-maturity 10-year U.S. Treasury note yield. Source: Federal Reserve Board, H.15 release, monthly average yield.

Figure 2: Long-term inflation expectations of survey respondents (inflation expectations from 5 to 10 years ahead). Source: Blue Chip, semiannual frequency. Survey question first asked in 1980.
Second, in order to generate a nontrivial term premium, we depart from expected utility preferences and turn instead to the class of generalized recursive preferences proposed by Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1989). These preferences have been shown by a number of authors (Piazzesi and Schneider, 2006, Bansal and Shaliastovich, 2008) to be able to explain the term premium in an endowment economy. In addition, Epstein-Zin-Weil preferences separate the coefficient of relative risk aversion from the intertemporal elasticity of substitution, which increases the ability of the model to match asset prices even when intertemporal substitution possibilities are present in the model in the form of variable labor supply and investment. This contrasts sharply with the habit-based asset pricing specification of Campbell and Cochrane (1999) and Wachter (2005), which previous studies have had problems carrying over from an endowment economy to a DSGE setting because of households’ strong incentive to substitute intertemporally as a way of insuring themselves against risk (Jermann, 1998, Lettau and Uhlig, 2000, Rudebusch and Swanson, 2007).

The third and final key ingredient in the model is a mechanism that creates a substantial long-run nominal risk. Without a long-run risk, even though households in the model are risk averse, the economy is not risky enough for there to be an appreciable term premium on long-term U.S. Treasury bonds. We follow Gürkaynak, Sack, and Swanson (2005) and propose that the effective long-run inflation rate in the Federal Reserve’s monetary policy rule has varied over time in a manner that depends on the recent history of inflation. Gürkaynak et al. showed that a small degree of inflation “pass-through” of this form is necessary to explain the "excess sensitivity" of U.S. long-term bond yields to macroeconomic news.

Together, these three ingredients—nominal rigidities, Epstein-Zin-Weil preferences, and long-run inflation risk—allow our model to replicate the level and variability of the term premium without compromising the model’s ability to fit macroeconomic variables. The importance of having a model that can simultaneously explain both macroeconomic quantities and asset prices is sometimes underappreciated. Indeed, the standard approach in the economics literature up to the present has been to use DSGE models to explain macroeconomic
variables and reduced-form, latent-factor finance models to fit asset prices. However, this dichotomous modeling approach has two very serious shortcomings. First, as a theoretical matter, asset prices and the macroeconomy are inextricably linked, so a failure of the standard DSGE framework to explain asset prices suggests flaws in the model. As emphasized by Cochrane (2007), asset markets are the mechanism by which consumption and investment are allocated across time and states of nature, so asset prices, which equate marginal rates of substitution and transformation, are at the very foundation of the dynamics of macroeconomic quantities. If a DSGE model can match the data on macroeconomic quantities but not asset prices, then how does the model propose that marginal rates of substitution and transformation are being equated? Surely such behavior is a sign that the model itself is flawed or at least incomplete. Second, from a practical point of view, policymakers and others are often very interested in the interaction between macroeconomic variables and asset prices—both the effects of asset prices on macro variables and the effects of interest rates and other macro variables on asset prices. For example, how does a low term premium—the bond yield “conundrum”—affect the economy and how should monetary policy respond? As Rudebusch, Sack, and Swanson (2007) discuss in detail, answering this question requires a structural macro-finance model; it cannot be addressed with a dichotomous macroeconomic and financial modeling approach.

Although the difficulty of modeling the term premium has received far less attention in the literature than Mehra and Prescott’s (1985) equity premium puzzle, it is every bit as interesting and important. First, the term premium provides a very different theoretical perspective on model performance than does the equity premium. For example, Boldrin, Christiano, and Fisher (2001) can explain the equity premium puzzle in a two-sector DSGE model because the immobility of capital across sectors greatly increases the variance of the price of capital (and thus stock prices) as well as its covariance with consumption. However, this mechanism will not explain the term premium, which involves the pricing of a default-free bond with a fixed nominal coupon. Second, successfully modeling the term premium can be a very useful metric for testing the nominal rigidities of the model, and in particular whether
they are able to match the dynamic behavior of inflation and other nominal quantities. As mentioned above, these nominal rigidities are a key feature of models that are currently in use in macroeconomics. Third, as a practical matter, understanding the term premium is perhaps of greater interest than understanding the equity premium because the value of long-term bonds outstanding in the U.S. greatly exceeds the value of equities.

The remainder of the paper proceeds as follows. Section 2 lays out a stylized version of the model with the three key ingredients discussed above. Section 3 presents results for the stylized version of the model and shows how it is able to match the term premium without compromising the model’s fit to macroeconomic variables. Section 4 extends our basic specification to a less stylized DSGE models and applies the model to the explanation of the rise and fall of long-term bond yields in the postwar U.S. Section 5 provides discussion of additional issues related to the model and its parameterization. Section 6 concludes.

2 A Simple New Keynesian Model with Epstein-Zin-Weil Preferences

2.1 Epstein-Zin-Weil Preferences

In the macroeconomics literature, it is typically assumed that a representative household chooses state-contingent plans for consumption and labor so as to maximize the expected present discounted value of a utility kernel $u(c_t, l_t)$:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t),$$  \hspace{1cm} (1)

subject to an asset accumulation equation, where $\beta \in (0, 1)$ is the household’s discount factor and where $u(c_t, l_t)$ is twice-differentiable, concave, increasing in $c$, and decreasing in $l$. The maximand in equation (1) can be expressed in first-order recursive form as:

$$V_t \equiv u(c_t, l_t) + \beta E_t V_{t+1},$$  \hspace{1cm} (2)

where the household’s state-contingent plans are chosen so as to maximize $V_0$. 5
In this paper, we follow the finance literature and generalize (2) to an Epstein-Zin (1989)-
Weil (1989) specification:

\[ V_t \equiv u(c_t, l_t) + \beta \left( E_t V_{t+1}^{\alpha} \right)^{1/\alpha}. \] (3)

If \( u \leq 0 \) everywhere, then it is natural to let \( V \leq 0 \) and take equation (3) to mean:

\[ V_t \equiv u(c_t, l_t) - \beta \left[ E_t (-V_{t+1})^{\alpha} \right]^{1/\alpha}. \] (4)

In this paper, we will exclude the case where \( u \) is sometimes positive and sometimes negative,
but it can be handled by requiring that \( \alpha \) be an even integer (or \( \alpha = 1 \)), which ensures that
(3) is a well-defined real-valued function. If \( u \geq 0 \) everywhere or \( u \leq 0 \) everywhere, as we
will assume in this paper, then \( \alpha \) is unrestricted and can be any real number.\(^1\)

The case \( \alpha = 1 \) clearly corresponds to the standard expected utility framework. When
\( u \geq 0 \) everywhere, lower values of \( \alpha \) correspond to greater degrees of risk aversion, and when
\( u \leq 0 \) everywhere, higher values of \( \alpha \) correspond to greater degrees of risk aversion. Note
that, traditionally, Epstein-Zin-Weil preferences over consumption streams have been written
as:

\[ \tilde{V}_t \equiv \left[ c_t^\rho + \beta \left( E_t \tilde{V}_{t+1}^{\alpha} \right)^{\rho/\alpha} \right]^{1/\rho}, \] (5)

but by setting \( V_t = \tilde{V}_t^\rho \) and \( \alpha = \tilde{\alpha}/\rho \), this can be seen to correspond to (3).

The advantage of using (3) over (2) for household preferences is that (3) breaks the
connection between the household’s intertemporal elasticity of substitution and coefficient of
relative risk aversion that is present in (2). In (3), the intertemporal elasticity of substitution
over determinstic consumption paths is exactly the same as in (2), but now the household’s
risk aversion to uncertain lotteries over \( V_{t+1} \) can be amplified by the additional parameter
\( \alpha \).\(^2\)

We now turn to the utility kernel \( u \). It is common in the DSGE literature to specify:

\[ u(c_t, l_t) \equiv \frac{c_t^{1-\gamma}}{1 - \gamma} - \frac{\chi_0}{1 + \chi} + \frac{l_t^{1+\chi}}{1 + \chi}, \] (6)

\(^1\) The case \( \alpha = 0 \) corresponds to \( V_t = u(c_t, l_t) + \beta \exp(E_t \log V_{t+1}) \). Negative values for \( \alpha \) are also
permissible.

\(^2\) Indeed, the linearization or log-linearization of (3) is exactly the same as that of (2), which turns out to
be very useful for matching the model to macroeconomic variables, since models with (2) are already known
to be able to fit macroeconomic quantities reasonably well. We will return to this point in Section 3, below.
because having additive separability between consumption and labor makes incorporating nominal wage rigidities into the model tractable. When we turn to the larger-scale Smets-Wouters model in Section 4, below, we will thus want a utility kernel of the form (6), so we will also use (6) in the simple stylized New Keynesian model in this section. We will ensure that \( u \leq 0 \) everywhere by setting \( \gamma > 1 \). However, one can also consider the case \( \gamma \leq 1 \) by defining:

\[
u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi} + \frac{\chi l_t}{1+\chi}, \tag{7}\]

where \( \bar{l} \) denotes the household’s time endowment, which ensures that \( u \geq 0 \) everywhere.\(^3\)

It is important to note, however, that utility kernel (7) is observationally the same as (6) only in the case of expected utility (see the household’s first-order conditions below). Thus, adding a constant term to the utility kernel when \( \alpha \neq 1 \) is not innocuous and alters the household’s attitudes towards risk.

### 2.2 The Household’s Optimization Problem

Now we turn to the representative household’s optimization problem under the preference specification (3). Households are representative and choose state-contingent consumption and labor plans so as to maximize (3). Households have access to an asset that pays a possibly stochastic nominal rate of return \( r \) each period. Households are free to buy and sell the asset each period subject to a constraint that its asset holdings \( a_t \) are always greater than some lower bound \( a_0 \geq 0 \), which does not bind in equilibrium but rules out Ponzi schemes. Households face a price per unit of consumption of \( P_t \), and supply labor in a competitive market that pays nominal wage \( w_t \). Households also own stock in firms and receive a per-period lump-sum transfer from firms in the amount \( d_t \).

The household’s flow budget constraint is thus:

\[
a_{t+1} = (1 + r_{t+1})(a_t + w_t l_t + d_t - P_t c_t). \tag{8}\]

\(^3\) There are, of course, other ways of doing this that also maintain additive separability between consumption and labor.
The household’s optimization problem is perhaps easiest to solve when formulated as a Lagrange problem with the states of nature explicitly specified. To that end, we let \( s^0 \in S_0 \) denote the initial state of the economy at time 0, we let \( s_t \in S \) denote the realizations of the shocks that hit the economy in period \( t \), and we let \( s^t \equiv \{s^{t-1}, s_t\} \in S_0 \times S^t \) denote the initial state and history of all shocks up through time \( t \). The household’s optimization problem is then to choose a sequence of vector-valued functions, \([c_t(s^t), l_t(s^t), a_t(s^t)]\): \( S_0 \times S^t \rightarrow \mathbb{R}_+ \times [0, 1] \times \mathbb{R} \) so as to maximize (3) subject to the sequence of budget constraints (8) and the lower bound \( a_t \geq a \). For clarity, in this section we will assume that \( s^0 \) and \( s_t \) can take on only a finite number of possible values (i.e., \( S_0 \) and \( S \) have finite support), and we will explicitly index each variable by the (finitely-valued) state of the economy \( s^t \) as well as time \( t \). We let \( \pi_{s^t|s^t}, \tau \geq t \geq 0 \), denote the probability of realizing state \( s^\tau \) at time \( \tau \) conditional on being in state \( s^t \) at time \( t \). We also define \( s^t_{t-1} \) to be the projection of the history \( s^t \) onto its first \( t \) components; that is, \( s^t_{t-1} \) is the history \( s^t \) viewed at time \( t-1 \), before time-\( t \) shocks have been realized.

The household’s optimization problem can be formulated as a Lagrangean, where the household chooses state-contingent plans for consumption, labor, and asset holdings, \((c_{t,s^t}, l_{t,s^t}, a_{t,s^t})\), that maximize \( V_0 \) subject to the infinite sequence of state-contingent constraints (3) and (8), that is, maximize:

\[
\mathcal{L} \equiv V_{0,s^0} - \sum_{t=0}^{\infty} \sum_{s^t} \mu_{t,s^t} \left\{ V_{t,s^t} - u(c_t(s^t), l_t(s^t)) - \beta \left( \sum_{s^{t+1}} \pi_{s^{t+1}|s^t} V_{t+1,s^{t+1}}^{\alpha} \right)^{1/\alpha} \right\} - \\
\sum_{t=0}^{\infty} \sum_{s^t} \sum_{s^{t+1} \geq s^t} \lambda_{t+1,s^{t+1}} \left\{ a_{t+1,s^{t+1}} - (1 + r_{t+1,s^{t+1}}) (a_t(s^t) + w_{t,s^t} l_t(s^t) + d_{t,s^t} - P_{t,s^t} c_{t,s^t}) \right\} \theta
\]
Making substitutions and defining the stationary Lagrange multipliers \( \widetilde{\lambda}_{t,s} \equiv \beta^{-1}\pi_{st|s_0}^{-1}\lambda_{t,s} \) and \( \widetilde{\mu}_{t,s} \equiv \beta^{-1}\pi_{st|s_0}^{-1}\mu_{t,s} \), these become:

\[
\begin{align*}
\frac{\partial \mathcal{L}}{\partial a_{t,s}} & : \quad \tilde{\lambda}_{t,s} = \beta E_{t,s}\tilde{\lambda}_{t+1,s+1}(1 + r_{t+1,s+1}) \\
\frac{\partial \mathcal{L}}{\partial c_{t,s}} & : \quad \tilde{\mu}_{t,s}u_1|_{(c_{t,s},\lambda_{t,s})} = P_{t,s} \sum_{s_{t+1} \supseteq s_t} \lambda_{t+1,s_{t+1}+1}(1 + r_{t+1,s_{t+1}+1}) \\
\frac{\partial \mathcal{L}}{\partial l_{t,s}} & : \quad -\tilde{\mu}_{t,s}u_2|_{(c_{t,s},\lambda_{t,s})} = w_{t,s} \sum_{s_{t+1} \supseteq s_t} \lambda_{t+1,s_{t+1}+1}(1 + r_{t+1,s_{t+1}+1}) \\
\frac{\partial \mathcal{L}}{\partial \nu_{t,s}} & : \quad \tilde{\mu}_{t,s} = \beta \pi_{st|s_{t-1}}^{-1}\mu_{t-1,s_{t-1}} \left( \sum_{s_{t-1} \supseteq s_{t-1}} \pi_{s_{t-1} | s_{t-1}} \nu_{t,s_{t-1}} \right)^{(1-\alpha)/\alpha} V_{t,s_{t-1}}^{\alpha-1}, \quad \mu_{0,s_0} = 1 \\
\end{align*}
\]

These first-order conditions are very similar to the standard expected utility case except for the introduction of the additional Lagrange multipliers \( \tilde{\mu}_{t,s} \), which translate utils at time \( t \) into utils at time 0, allowing for the “twisting” of the value function by \( \alpha \) that takes place at each time 1, 2, \ldots, \( t \). Note that in the expected utility case (\( \alpha = 1 \)), \( \tilde{\mu}_{t,s} = 1 \) for every \( t \) and \( s_t \), and equations (10) through (13) reduce to the standard optimality conditions.

Substituting out for \( \tilde{\lambda}_{t,s} \) and \( \tilde{\mu}_{t,s} \) in (10) through (13), we get the household’s intratemporal and intertemporal (Euler) optimality conditions:

\[
\begin{align*}
-\frac{u_2|_{(c_{t,s},\lambda_{t,s})}}{u_1|_{(c_{t,s},\lambda_{t,s})}} & = \frac{w_{t,s}}{P_{t,s}} \\
u_1|_{(c_{t,s},\lambda_{t,s})} & = \beta E_{t,s}(E_{t,s} V_{t+1,s+1}^{\alpha})^{(1-\alpha)/\alpha} V_{t,s}^{\alpha-1} u_1|_{(c_{t+1,s+1},\lambda_{t+1,s+1})}(1 + r_{t+1,s+1}) P_{t,s}/P_{t+1,s+1} \\
\end{align*}
\]

Finally, let \( p_{t,s}, t \leq \tau \), denote the price at time \( t \) in state \( s_t \) of a state-contingent bond that pays one dollar at time \( \tau \) in state \( s_\tau \) and 0 otherwise. If we insert this state-contingent
security into the household’s optimization problem, we see that, for \( t < \tau \):

\[
p_{t,s}^{r_t} = \beta E_{t,s}(E_{t,s}V_{t+1,s}^{\alpha})^{(1-\alpha)/\alpha} \frac{u_1 |(c_{t+1,s+1}, l_{t+1,s+1})|}{u_1 |(c_{t,s}, l_{t,s})|} \frac{P_{t,s}}{P_{t+1,s+1}} p_{t+1,s+1}^{r_{t+1}}. \tag{14}
\]

That is, the household’s (nominal) stochastic discount factor at time \( t \) in state \( s_t \) for stochastic payoffs at time \( t + 1 \) is given by:

\[
m_{t,s,t+1} = \left( \frac{V_{t+1,s+1}}{(E_{t,s}V_{t+1,s+1})^{1/\alpha}} \right)^{1-\alpha} \beta u_1 |(c_{t+1,s+1}, l_{t+1,s+1})| \frac{P_{t,s}}{P_{t+1,s+1}} \tag{15}
\]

Despite the twisting of the value function by \( \alpha \), the price \( p_{t,s}^{r_t} \) satisfies the standard relationship:

\[
p_{t,s}^{r_t} = E_{t,s} m_{t,s,t+1} m_{t+1,s+1,t+2} m_{t+2,s+2} \cdots P_{t+\tau,s+\tau}^{r_{t+\tau}} \tag{16}
\]

and the asset pricing equation (14) is linear in the future state-contingent payoffs, so that we can price any compound security by summing over the prices of its individual constituent state-contingent payoffs.

### 2.3 The Firm’s Optimization Problem

As is standard in DSGE models with nominal rigidities, the economy contains a continuum of monopolistically competitive intermediate goods firms indexed by \( f \in [0, 1] \) that set prices according to Calvo contracts and hire labor from households in a competitive labor market. Firms have identical Cobb-Douglas production functions:

\[
y_{t,f} = A_t \kappa^{1-\eta} l_{t,f}^\eta, \tag{16}
\]

where \( \kappa \) is the fixed, firm-specific capital stock,\(^4\) and where \( A_t \) denotes an aggregate technology shock that affects all firms in the economy. Note that we have suppressed the explicit

\(^4\) Several authors, such as Woodford (2003) and Altit, Christiano, Eichenbaum, and Linde (2004), have emphasized the importance of firm-specific fixed factors for generating a level of inflation persistence that is consistent with the data. With firm-specific capital stocks, the term premium is higher as well as inflation being more persistent.
state-dependence of the variables in this equation and in the remainder of the paper to ease the notational burden. The technology shock \( A_t \) follows an exogenous AR(1) process:

\[
\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A, \tag{17}
\]

where \( \varepsilon_t^A \) denotes an \( i.i.d. \) aggregate technology shock with mean zero and variance \( \sigma_A^2 \).

Firms set prices according to Calvo contracts that expire with probability \( 1 - \xi \) each period. When the Calvo contract expires, the firm is free to reset its price as it chooses, and we denote this price that firm \( f \) sets in period \( t \) by \( p_t(f) \). There is no indexation, so the price \( p_t(f) \) that the firm sets is fixed over the life of the contract. In each period \( \tau \geq t \) that the contract remains in effect, the firm must supply whatever output is demanded at the contract price \( p_t(f) \), hiring labor \( l_{\tau}(f) \) from households at the nominal market wage \( w_{\tau} \).

Firms are collectively owned by households and distribute profits and losses back to the households each period. When a firm’s price contract expires, the firm chooses the new contract price \( p_t(f) \) to maximize the value to shareholders of the firm’s cash flows over the lifetime of the contract (equivalently, the firm chooses a state-contingent plan for prices that maximizes the value of the firm to shareholders). That is, the firm maximizes:

\[
E_t \sum_{j=0}^{\infty} \xi^j m_{t,t+j}[p_t(f)y_{t+j}(f) - w_{t+j}l_{t+j}(f)], \tag{18}
\]

where \( m_{t,t+j} \) is the representative household’s stochastic discount factor from period \( t \) to \( t+j \).

Output of each intermediate firm \( f \) is purchased by a perfectly competitive final goods sector that aggregates the continuum of intermediate goods into a single final good using a CES production technology:

\[
Y_t = \left[ \int_0^1 y_t(f)^{1/(1+\theta)} df \right]^{1+\theta}. \tag{19}
\]

Each intermediate firm \( f \) thus faces a downward-sloping demand curve for its product given by

\[
y_t(f) = \left( \frac{p_t(f)}{P_t} \right)^{-(1+\theta)/\theta} Y_t, \tag{20}
\]
where $P_t$ is the CES aggregate price per unit of the final good:

\[
P_t \equiv \left[ \int_0^1 p_t(f)^{-1/\theta} df \right]^{-\theta}.
\]  

(21)

Differentiating (18) with respect to $p_t(f)$ yields the standard optimality condition for the firm’s price:

\[
p_t(f) = \frac{(1 + \theta) E_t \sum_{j=0}^\infty \xi^j m_{t,t+j} m_{t+j}(f)y_{t+j}(f)}{E_t \sum_{j=0}^\infty \xi^j m_{t,t+j} y_{t+j}(f)}.
\]

(22)

where $mc_t(f)$ denotes the marginal cost for firm $f$ at time $t$:

\[
m_c(f) \equiv \frac{w_t l_t(f)}{\eta y_t(f)}.
\]

(23)

2.4 Aggregate Resource Constraints

To aggregate up from firm-level variables to aggregate quantities, it is useful to define the cross-sectional price dispersion $\Delta_t$:

\[
\Delta_t^{1/\eta} \equiv (1 - \xi) \sum_{j=0}^\infty \xi^j p_{t-j}(f)^{(1+\theta)/(\theta \eta)},
\]

(24)

where the parameter $\eta$ in the exponent arises from the firm-specificity of capital. We define $L_t$, the aggregate quantity of labor demanded by firms, by:

\[
L_t \equiv \int_0^1 l_t(f) df.
\]

(25)

Then $L_t$ satisfies:

\[
Y_t = \Delta_t^{-1} A_t \bar{K}^{1-\eta} L_t^\eta,
\]

(26)

where $\bar{K} = \bar{k}$ is the fixed capital stock. Equilibrium in the labor market requires that $L_t = l_t$, where the latter denotes the aggregate labor supplied by the representative households.

In order to study the effect of fiscal shocks, we assume that there is a government in the economy that levies lump-sum taxes $G_t$ on households and destroys the resources it collects. Government consumption follows an exogenous AR(1) process:

\[
\log G_t = \rho G \log G_{t-1} + \varepsilon_t^G.
\]

(27)
where \( \varepsilon_t^G \) denotes an \( i.i.d. \) government consumption shock with mean zero and variance \( \sigma^2_G \).

Although agents cannot invest in physical capital, we do assume that an amount \( \delta \bar{K} \) of output each period is devoted to maintaining the fixed capital stock. Thus, the aggregate resource constraint implies that

\[
Y_t = C_t + \delta \bar{K} + G_t, \tag{28}
\]

where \( C_t = c_t \), the consumption of the representative household.

### 2.5 The Monetary Authority and Long-Run Inflation Risk

Finally, there is a monetary authority in the economy which sets the one-period nominal interest rate \( i_t \) according to a Taylor-type policy rule:

\[
i_t = \rho_i i_{t-1} + (1 - \rho_i) \left[ 1/\beta + \pi_t + g_y (Y_t - \bar{Y})/\bar{Y} + g_{\pi} (\pi_t - \pi^*_t) \right] + \varepsilon^i_t, \tag{29}
\]

where \( 1/\beta \) is the steady-state real interest rate in the model, \( \pi_t \) denotes the four-period trailing average inflation rate (equal to \( \log(P_t/P_{t-4}) \)), \( \bar{Y} \) denotes the steady-state level of output, \( \pi^*_t \) denotes the monetary authority’s target rate of inflation, \( \varepsilon^i_t \) denotes an \( i.i.d. \) stochastic monetary policy shock with mean zero and variance \( \sigma^2_i \), and \( \rho_i, g_y, \) and \( g_{\pi} \) are parameters.\(^5\)

As mentioned previously, we assume that the monetary authority’s target rate of inflation \( \pi^*_t \) may vary over time. We do not take a stand on why this might be so, but note only that financial market perceptions of the long-run rate of inflation in the U.S. seem to have varied considerably over the past 50 years, as can be seen in Figure 2 and as Gürkaynak, Sack, and Swanson (2005) found in the “excess sensitivity” of long-term bond yields to macroeconomic and monetary policy announcements.

---

\(^5\) Note that in equation (29) (and equation (29) only), we will express \( i_t, \pi_t, \) and \( 1/\beta \) in annualized terms, so that the coefficients \( g_{\pi} \) and \( g_y \) correspond directly to the estimates in the empirical literature. We also follow the literature by assuming an “inertial” policy rule with \( i.i.d. \) policy shocks; however, there are a variety of reasons to be dissatisfied with the assumption of AR(1) processes for all stochastic disturbances except the one associated with short-term interest rates. Indeed, Rudebusch (2002, 2006) and Carrillo, Fève, and Matheron (2007) provide strong evidence that an alternative policy specification with serially correlated shocks and little gradual adjustment is more consistent with the dynamic behavior of nominal interest rates.
Following Gürkaynak et al., we assume that $\pi_t^*$ loads to some extent on the recent history of inflation:

$$
\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + (1 - \rho_{\pi^*}) \theta_{\pi^*} (\bar{\pi}_t - \pi_t^*) + \varepsilon_t^{*}. 
$$

(30)

There are two main advantages of using specification (30) rather than a simple random walk or AR(1) specification in which $\theta_{\pi^*} = 0$. First, (30) allows long-term inflation expectations to respond to current news about inflation and economic activity in a manner that is consistent with the bond market responses documented by Gürkaynak et al. Thus, $\theta_{\pi^*} > 0$ seems to be consistent with the data (Gürkaynak et al. find that a value of $\theta_{\pi^*} = .02$ is roughly consistent with the bond market data).

Second, if $\theta_{\pi^*} = 0$, then even though $\pi_t^*$ varies over time, it does not do so systematically with output or consumption. As a result, long-term bonds are not particularly risky, in the sense that their returns are not very correlated with the household’s stochastic discount factor. Long-term bonds even have some elements of insurance in this case, because a negative shock $\varepsilon_t^{*}$ leads the monetary authority to raise interest rates and depress output at the same time that it causes long-term bond yields to fall and bond prices to rise, so long-term bonds carry a negative risk premium because of their insurance properties. By contrast, if $\theta_{\pi^*} > 0$, then a negative technology shock today raises inflation and long-term inflation expectations and depresses bond prices at exactly the same time that it depresses output, which makes holding long-term bonds risky. Thus, if we want the model to generate a term premium that is positive on average, we will require $\theta_{\pi^*} > 0$.

### 2.6 The Term Premium in the Model

The price of any asset in the model economy satisfies the standard stochastic discounting relationship in which the stochastic discount factor ($m_{t+1} \equiv m_{t,t+1}$) is used to value the state-contingent payoffs of the asset in period $t + 1$. For example, the price of a default-free $n$-period zero-coupon bond that pays one dollar at maturity satisfies:

$$
p_t^{(n)} = \mathbb{E}_t[m_{t+1} p_{t+1}^{(n-1)}],
$$

(31)
where \( p_t^{(n)} \) denotes the price of the bond at time \( t \) and \( p_t^{(0)} \equiv 1 \), i.e., the time-\( t \) price of one dollar delivered at time \( t \) is one dollar.

In the U.S. data, the benchmark long-term bond is the ten-year Treasury note. Thus, we wish to model the term premium on a bond with a duration of about ten years. For computational reasons, it turns out to be inconvenient to work with a zero-coupon bond that has more than a few periods to maturity; instead, it is much easier to work with an infinitely lived consol-style bond that has a time-invariant or time-symmetric structure. Thus, we assume that households in the model can buy and sell a long-term default-free nominal consol which pays a geometrically declining coupon in every period in perpetuity. The nominal consol’s price per dollar of coupon in period \( t \), which we denote by \( p_t^{(\infty)} \), then satisfies

\[
p_t^{(\infty)} = 1 + \delta_c E_t m_{t+1} p_{t+1}^{(\infty)},
\]

where \( \delta_c \) is the rate of decay of the coupon on the consol. By setting an appropriate value for \( \delta_c \), which determines the Macauley duration of the bond, we can model prices of a bond of any desired maturity. The continuously compounded per-period yield to maturity on the consol is given by

\[
\log \left( \frac{\delta_c p_t^{(\infty)}}{p_t^{(\infty)} - 1} \right).
\]

In the literature, the term premium is typically defined to be the difference between the yield on a bond and the (unobserved) risk-neutral yield for that same bond. To define the term premium in our model, then, we first define the risk-neutral price of the consol, \( p_t^{(\infty)rn} \):

\[
p_t^{(\infty)rn} \equiv E_t \sum_{j=0}^{\infty} e^{-i_{t,t+j} \delta_c^j},
\]

where \( i_{t,t+j} \equiv \sum_{n=0}^{j} i_n \). Equation (34) is the expected present discounted value of the coupons of the consol, where the discounting is performed using the risk-free rate rather than the household’s stochastic discount factor.\(^6\) Equivalently, equation (34) can be expressed in

\(^6\) In computing the term premium, some authors take the expectation over yields rather than over prices (with the difference between the two approaches being a convexity term). Equation (34) follows the no-arbitrage finance and macro-finance literatures (e.g., Ang and Piazzesi, 2003), which compute risk-neutral bond prices by setting the prices of risk to zero.
first-order recursive form as:

\[ p_t^{(\infty)rn} = 1 + \delta_c e^{-it} E_t p_{t+1}^{(\infty)rn}, \]  

which directly parallels equation (32). The implied term premium on the consol is then given by:

\[ \psi_t \equiv \log \left( \frac{\delta_c p_t^{(\infty)}}{p_t^{(\infty)} - 1} \right) - \log \left( \frac{\delta_c p_t^{(\infty)rn}}{p_t^{(\infty)rn} - 1} \right), \]  

which is the difference between the observed yield to maturity on the consol and the risk-neutral yield to maturity. For a given set of structural parameters of the model, we will choose \( \delta_c \) so that the bond has a Macauley duration of ten years, and we will multiply equation (36) by 40,000 in order to report the term premium in units of annualized basis points rather than logs.

### 2.7 Solving the Model

The model outlined above is complicated enough that closed-form solutions for the household’s decision rules do not exist in general. As a result, we must solve the model numerically.

A technical issue that arises in solving even this simple, stylized New Keynesian model above is the relatively large number of state variables it includes—11, including \( C_{t-1}, A_{t-1}, G_{t-1}, i_{t-1}, \Delta_{t-1} \), the three lags of inflation underlying \( \pi_t \), and the three shocks, \( \varepsilon_t^A, \varepsilon_t^G, \) and, \( \varepsilon_t^i \). Because of this high level of dimensionality, value-function iteration-based methods such as projection methods (or, even worse, discretization methods) are computationally completely intractable. We instead solve the model using the standard macroeconomic technique of approximation around the nonstochastic steady state—so-called perturbation methods.

However, a first-order approximation of the model (i.e., a linearization or log-linearization) eliminates the term premium entirely, because equations (32) and (35) are identical to first

---

7 The number of state variables can be reduced a bit by noting that \( G_t \) and \( A_t \) are sufficient to incorporate all of the information from \( G_{t-1}, A_{t-1}, \varepsilon_t^G, \) and \( \varepsilon_t^A \), but the basic point remains valid, namely that the number of state variables in the model is large from a computational point of view.
order, a manifestation of the well-known property of certainty equivalence in linearized models. A second-order approximation to the solution of the model produces a term premium that is nonzero but constant (a weighted sum of the variances $\sigma_A^2$, $\sigma_G^2$, and $\sigma_i^2$). Since our interest in this paper is not just in the level of the term premium but also in its volatility and variation over time, we must compute a third-order approximate solution to the model around the nonstochastic steady state. We do so using the $n$th-order perturbation AIM algorithm of Swanson, Anderson, and Levin (2006), which automatically and quickly computes $n$th-order approximate solutions to dynamic discrete-time rational expectations models of this type. For the baseline model above with 11 state variables, a third-order accurate solution can be computed in about 45 minutes on a standard laptop computer. Additional details of this solution method are provided in Swanson, Anderson, and Levin (2006) and Rudebusch, Sack and Swanson (2007).

2.8 Parameterization

The baseline parameter values for our simple New Keynesian model are reported in Table 1 and are fairly standard in the literature (see, e.g., Levin, Onatski, Williams, and Williams, 2005). We set the household’s discount factor, $\beta$, to .99 per quarter (implying a steady-state real interest rate of 4.02 percent per year), households’ utility curvature with respect to consumption, $\gamma$, to 1.5 (implying an intertemporal elasticity of substitution in consumption of about 2/3), and households’ utility curvature with respect to labor, $\chi$, to 1.5 (implying a Frisch elasticity of about 2/3), both of which are in line with estimates from the micro literature. The Epstein-Zin-Weil coefficient $\alpha$ is set to 30, which implies a coefficient of relative risk aversion over wealth gambles of about 16, similar to values used by Bansal and Yaron (2004) and Bansal and Shaliastovich (2008) to explain the equity premium, term premium, and foreign exchange premium in an endowment economy. Note that the coefficient of relative risk aversion over wealth gambles in (3), the way we have written it, is not $1 - \alpha$ but rather $\gamma + (1 - \gamma)(1 - \alpha)$.8

8 Technically, the coefficient of relative risk aversion over wealth gambles is only $\gamma + (1 - \gamma)(1 - \alpha)$ for the case where labor is held fixed and the model is homothetic in wealth. In this case, the household’s value
We set firms’ output elasticity with respect to labor, $\eta$, to .7, firms’ steady-state markup, $\theta$, to .2 (implying a price-elasticity of demand of 6), and the Calvo frequency of price adjustment, $\xi$, to .75 (implying an average price contract duration of four quarters).

The shock persistences $\rho_A$ and $\rho_G$ are set to .9, as is common, and the shock variances $\sigma^2_A$ and $\sigma^2_G$ are set to .01$^2$ and .004$^2$, respectively, consistent with typical estimates in the literature. The monetary policy rule coefficients are taken from Rudebusch (2002) and are also typical of those in the literature. We assume the steady-state capital-output ratio is 2.5, which is close to what is found in the data, and steady-state government spending is about 17 percent of output. The persistence of the monetary authority’s inflation target, $\rho_{\pi^*}$, is set equal to .995, as estimated by Levin et al. (2006), and the loading of the monetary authority’s inflation target on the recent history of inflation, $\vartheta_{\pi^*}$, is set to .02, the same value that Gürkaynak et al. found was necessary to explain the excess sensitivity of long-term bond yields in response to news.

Finally, the parameter $\chi_0$ is chosen to normalize the steady-state quantity of labor to unity and, as discussed above, and the parameter $\delta_c$ is chosen to set the Macauley duration of the consol in the model to ten years.

Table 1:
Parameter Values for the Simple New Keynesian Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>.99</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\chi$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>30</td>
</tr>
<tr>
<td>$\eta$</td>
<td>.7</td>
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<tr>
<td>$\theta$</td>
<td>.2</td>
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<tr>
<td>$\xi$</td>
<td>.75</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>.73</td>
</tr>
<tr>
<td>$g_\pi$</td>
<td>.53</td>
</tr>
<tr>
<td>$g_y$</td>
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</tr>
<tr>
<td>$\rho_{\pi^*}$</td>
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<tr>
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<td>.001$^2$</td>
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<tr>
<td>$\sigma^2_{\pi^*}$</td>
<td>.001$^2$</td>
</tr>
<tr>
<td>$\sigma^2_A$</td>
<td>.01$^2$</td>
</tr>
<tr>
<td>$\sigma^2_G$</td>
<td>.01$^2$</td>
</tr>
<tr>
<td>$K/(4Y)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$\delta K/Y$</td>
<td>.2</td>
</tr>
<tr>
<td>$\overline{G/Y}$</td>
<td>.17</td>
</tr>
</tbody>
</table>

memo:
CRRA 16
$\chi_0$ 4.74
$\delta_c$ .9848

function is essentially wealth$^{1-\gamma}$. We will discuss this issue in more detail in Section 5, below.
3 Results for the Simple Model

Table 2 reports the unconditional mean of the term premium ($\psi$) and the unconditional standard deviations of the term premium and a number of other macroeconomic and financial variables of interest as implied by the simple New Keynesian DSGE model developed in the preceding section.

The first column of Table 2 reports results for the version of the model with expected utility preferences ($\alpha = 1$). The model does a reasonable job of matching the unconditional standard deviations of the standard macroeconomic variables such as output, labor, real wages, and inflation, and the short-term nominal interest rate the yield to maturity on the long-term bond, producing unconditional standard deviations of 1 or 2 percent, very similar to the U.S. data over the postwar period. However, the term premium implied by the expected utility version of the model is both small on average—about 2 basis points—and extremely stable over time, with a standard deviation of less than one-tenth of one basis point.

The second column of Table 2 reports results from the version of the model with Epstein-Zin-Weil preferences with a CRRA of about 16 ($\alpha = 30$). The model fits all of the macroeconomic variables essentially exactly as well as the expected utility version of the model. This is a straightforward implication of two features of the model: First, the linearization or log-linearization of Epstein-Zin-Weil preferences (3) is exactly the same as that of standard expected utility preferences (2), so to first order, these two utility specifications are the same. Second, the shocks that we consider here and which are standard in macroeconomics have standard deviations of only 1 percent or less, so a linear approximation to the model is typically extremely accurate. Only for models with enormous curvature (e.g., $\gamma \gg 1$ or $\chi \gg 1$), or for much larger shocks, would we expect second- or higher-order terms of the model to matter very much.

For asset prices, however, the implications of the Epstein-Zin-Weil and expected utility models are very different. Here, second- and higher-order terms are the whole story, since to first order the model is certainty equivalent and hence there are no first-order risk premium
terms. It turns out that even for very high values of $\alpha$, and hence very high levels of risk aversion in the model, the dynamics of the macroeconomic variables implied by the model are essentially unchanged, a finding that was also noted by Tallarini (2000) and Backus, Routledge, and Zin (2007). The fact that the models are first-order equivalent seems to dominate, for practical purposes, the additional curvature that is introduced by the parameter $\alpha$.

This is not true for models that rely on habits in consumption to increase households’ risk aversion. In contrast to the model with Epstein-Zin-Weil preferences, a New Keynesian DSGE model with habits is not equivalent to the standard expected utility model to first order and turns out to be unable to match both the term premium and macroeconomic facts. Column 3 of Table 2 reports results from Rudebusch and Swanson (2007) for a very similar New Keynesian model to the one developed above, except with habit-based expected utility as opposed to Epstein-Zin-Weil preferences. Even with the extreme degree of habits implied by Campbell and Cochrane’s (1999) specification, the DSGE model with habits is unable to produce a reasonably large or time-varying term premium. The intuition for this result, as also emphasized by Rudebusch and Swanson (2007), Boldrin, Christiano, and Fisher (2001), and Jermann (1998), is that in a production-based model households can endogenously choose their labor-consumption tradeoff. If households are hit by a negative shock in a production-based model, they can compensate for the shock by increasing their labor supply and working more hours. As a result, they have the ability to insure themselves to some extent from the effects of the shock on consumption by endogenously varying their labor supply in response. Households in an endowment economy do not have this opportunity, so the consumption cost of shocks in an endowment economy is correspondingly greater and risky assets in an endowment-based economy will tend to carry a larger risk premium. In the Campbell-Cochrane version of our New Keynesian model, this ability of households to self-insure appears to offset the large effects that those habit preferences would otherwise have on the term premium.

---

9 In Jermann (1998), households are unable to vary their labor supply but can vary investment instead, so the basic point is the same.
Even if we try to prevent households from self-insuring by adding labor adjustment costs to the model with Campell-Cochrane habits, we are still unable to match both the macroeconomic and financial moments in Table 2. While the Campbell-Cochrane version of our model without adjustment costs (in the third column) was unable to match the level and volatility of the term premium, that version of the model implied a volatility for the other macroeconomic and financial variables that was roughly consistent with the data. By contrast, in the model with C-C habits and quadratic labor adjustment costs (the last column of the table), the volatility of the term premium can be made much larger, but only at the cost of greatly increasing the unconditional standard deviations of real wages, inflation, and short-term nominal interest rates. The volatility of the real wage in particular is over 2.24 log percentage points.10

Thus, the flexibility of the Epstein-Zin-Weil specification allows us to use first-order terms to match the dynamic behavior of macroeconomic variables in the model while simultaneously allowing second- and higher-order terms to match the behavior of asset prices.

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10 In Rudebusch and Swanson (2007), we endeavored, without success, to find a parameterization that could deliver both a large term premium and plausible real wage volatility. See that paper for details.
### Table 2
Unconditional Moments in Four Versions of the Simple DSGE Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model with Expected Utility Preferences</th>
<th>Model with Epstein-Zin-Weil preferences</th>
<th>Model with Expected Utility, and Campbell-Cochrane Habits</th>
<th>Model with Expected Utility, C-C Habits, and Labor Adj costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean measured in basis points</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean[ψ]</td>
<td>0.4</td>
<td>24.7</td>
<td>3.7</td>
<td>79.7</td>
</tr>
<tr>
<td>standard deviations measured in percent</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sd[C]</td>
<td>2.17</td>
<td>2.20</td>
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</tr>
<tr>
<td>sd[Y]</td>
<td>1.36</td>
<td>1.38</td>
<td>0.64</td>
<td>0.70</td>
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<tr>
<td>sd[L]</td>
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</tr>
<tr>
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</tr>
<tr>
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<tr>
<td>sd[i]</td>
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<td>294</td>
<td>218</td>
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<tr>
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<td>21.3</td>
<td>0.2</td>
<td>12.7</td>
</tr>
</tbody>
</table>

4 **Long-Run Inflation Risk and the Postwar Term Premium**

(to be written)

5 **Discussion**

(to be written)

6 **Conclusions**

(to be written)
References


