Examining the Bond Premium Puzzle with a DSGE Model

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Abstract

The basic inability of standard theoretical models to generate a sufficiently large and variable nominal bond risk premium has been termed the “bond premium puzzle.” We show that the term premium on long-term bonds is far too small and stable relative to the data in the canonical dynamic stochastic general equilibrium (DSGE) model used in macroeconomics. We find that introducing long-memory habits in consumption as well as labor market frictions can help fit the term premium, but only by seriously distorting the DSGE model’s ability to fit other macroeconomic variables, such as the real wage; therefore, the bond premium puzzle remains.

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1 Introduction

During the past few years, as long- and short-term interest rates moved in different directions, the risk premium on long-term nominal bonds has attracted widespread attention. Indeed, as Federal Reserve Chairman Alan Greenspan pointedly remarked to Congress in July 2005, “a significant portion of the sharp decline in the ten-year forward one-year rate over the past year appears to have resulted from a fall in term premiums,” a situation that Greenspan had previously described as a “conundrum.” Of course, assessing whether a particular historical configuration of interest rates and term premiums truly represents a puzzle requires a theoretical framework that accounts for their underlying macroeconomic and financial determinants. In this paper, we investigate the nature of bond risk premiums in a dynamic stochastic general equilibrium (DSGE) model that can completely characterize the relationship between asset prices and the economy.

Our analysis does not focus on the latest conundrum episode; instead, we attempt to account for the bond premium’s relatively large average size and volatility over a longer period. A similar exercise was conducted by Backus, Gregory, and Zin (1989) with a consumption-based asset pricing model in an endowment economy. They found that “the representative agent model with additively separable preferences fails to account for the sign or the magnitude of risk premiums” and “cannot account for the variability of risk premiums” (p. 397). This basic inability of a standard theoretical model to generate a sufficiently large and variable nominal bond risk premium has been termed the “bond premium puzzle.”

Since this early work, however, the “standard” theoretical model in macroeconomics has undergone dramatic changes and now includes a prominent role for nominal rigidities, such as staggered Taylor (1980) or Calvo (1983) price contracts. Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003) have shown that DSGE models with these
features can match the impulse responses of the economy both to nominal monetary policy shocks and real-side technology shocks. We investigate whether these models’ ability to fit the behavior of nominal variables has also allowed them to better match the prices of nominal assets and the risk premium on long-term nominal bonds. Unfortunately, we find that the bond premium puzzle remains—that is, canonical macroeconomic DSGE models are unable to account for the size and variability of the term premium.

Of course, one might question the importance of the bond premium puzzle or, more generally, the point of trying to match asset prices at all in a DSGE framework. Indeed, a common course of action in the literature has been to use DSGE models to explain macroeconomic variables and latent-factor finance models to fit asset prices. However, this dichotomous modeling approach suffers from two very serious shortcomings. First, as a theoretical matter, asset prices and macroeconomic dynamics are inextricably linked. As emphasized by Cochrane (2007), asset markets are the mechanism by which consumption and investment are allocated across time and states of nature, so asset prices, which equate marginal rates of substitution and transformation, are at the very foundation of the dynamics of macroeconomic quantities. If a DSGE model can match the data on macroeconomic quantities but not prices, then how does the model propose that marginal rates of substitution and transformation in the economy are being equated? Surely such behavior is a sign that the model itself is seriously flawed or at least incomplete. Second, from a practical point of view, policymakers and others are very interested in the interaction between macro variables and asset prices—both the effects of asset prices on macroeconomic variables and the effects of interest rates and other macroeconomic variables on asset prices. For example, how does a low term premium—the bond yield “conundrum”—affect the economy and how should monetary policy respond? As Rudebusch, Sack, and Swanson (2007) discuss in detail, answering this question requires a structural macro-finance model; it cannot be addressed with a dichotomous macroeconomic and financial modeling approach.

Turning to the literature, there have been a number of recent attempts to solve the bond

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4 These two shortcomings have fostered a new emphasis in the literature on the importance of macro-finance linkages, for example, Ang and Piazzesi (2003) and Diebold, Piazzesi, and Rudebusch (2005).
premium puzzle. In particular, Wachter (2006) and Piazzesi and Schneider (2006) have had notable success with the consumption-based asset pricing model in an endowment economy by using preferences that have been modified to include either an important role for habit, as in Campbell and Cochrane (1999), or “recursive utility,” as in Epstein and Zin (1989). While these successes in an endowment economy are encouraging, they are somewhat unsatisfying from a macroeconomist’s perspective because the lack of a structural relationship among the macroeconomic variables precludes many questions of interest. Macroeconomists have thus been naturally interested in extending the endowment economy results to more fully-specified DSGE models. Wu (2006), Bekaert, Cho, and Moreno (2005), Hördahl, Tristani, and Vestin (2006b), and Doh (2006) all use the stochastic discount factor from a standard DSGE model to study the term premium, but in order to solve the model, these authors have all essentially assumed that the term premium is constant over time—that is, they have essentially assumed the expectations hypothesis. Since we are interested in the variability as well as the level of the term premium, and in the relationship between the term premium and the macroeconomy, a higher-order approximate solution method or a global nonlinear method is required, as in Ravenna and Seppälä (2006), Rudebusch, Sack, and Swanson (2007), and Gallmeyer, Hollifield, and Zin (2005). Still, as we discuss in detail below, these last authors have mixed success in solving the bond premium puzzle, and in particular, it remains unclear whether the size and volatility of the bond premium can be replicated in a

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5 Hördahl, Tristani, and Vestin (2006a), Rudebusch and Wu (2007), and other macro-finance researchers have examined term premiums with a affine no-arbitrage structure and a log-linearized version of a DSGE model. However, these models employ an exogenous stochastic pricing kernel that does not enforce a consistency between the asset pricing structure and the utility function underlying the macro structure.

6 Wu (2006) and Bekaert, Cho, and Moreno (2005) use a log-linear, log-normal approximation to solve the model, which allows some second- and higher-order terms from the log-normal distribution to remain in these models, although the implied term premium is a constant. An additional drawback of this approach is that it treats some second-order terms as important while dropping other terms of similar magnitude. In contrast, Hördahl, Tristani, and Vestin (2006b), compute a full second-order approximate solution to the model, which treats all second-order terms equally; however, the term premium is also a constant in this approach, as we discuss below. Doh (2006) combines a full second-order solution with an ARCH process on one of the shocks; again, this method treats some third- and higher-order terms as being important while dropping other terms of similar magnitude.

7 Ravenna and Seppälä (2006) and Rudebusch, Sack, and Swanson (2007) use a third-order approximate solution to the model, which allows for a time-varying term premium. Gallmeyer, Hollifield, and Zin (2005) are able to compute a closed-form solution for bond prices for a very special monetary policy reaction function, but their method does not apply to more general monetary policy reaction functions such as the Taylor policy rule we consider below.
DSGE model without distorting its macroeconomic fit and stochastic moments. Our analysis sheds light on this issue.

Since Mehra and Prescott (1985), there have been a number of analyses of the equity premium puzzle in a DSGE framework, including Jermann (1998), Boldrin, Christiano, and Fisher (2000), and Uhlig (2007). While the equity premium puzzle has generally received more attention in the literature than the bond premium puzzle, the latter may be even more interesting from a DSGE modeling point of view for two reasons. First, unlike the equity premium, the bond premium involves the valuation of nominal assets and inflation dynamics; therefore, matching the term premium can be a useful metric in evaluating the canonical DSGE model’s nominal rigidities. Second, the term premium is arguably more important from a practical point of view than the equity premium, since the dollar volume of bonds outstanding in the U.S. is many times larger than the dollar volume of equities.

In the next section, we introduce our benchmark model, which is a simple DSGE model that is set well within the broad range of such models. Section 3 derives the term premium for this simple, canonical model and shows that is counterfactually small and stable. Section 4 explores whether long-memory habits, as in Campbell and Cochrane (1999), can help to explain the term premium in the model, as suggested by the endowment economy analysis of Wachter (2006). We find that long-memory habits by themselves are not sufficient to explain the term premium, so Section 5 considers whether adding labor market frictions to the model might improve its fit. Section 6 concludes.

2 A Benchmark DSGE Model

We begin our investigation of the term premium using a simple benchmark DSGE model with nominal rigidities. The economy contains a continuum of households with a total mass of unity. Households are representative and seek to maximize utility over consumption and labor streams given by:

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left( \frac{(c_t - bh_t)^{1-\gamma}}{1-\gamma} - \chi \frac{l_t^{1+\chi}}{1+\chi} \right),$$

(1)
where $\beta$ denotes the household’s discount factor, $c_t$ denotes consumption in period $t$, $l_t$ denotes labor, $h_t$ a predetermined stock of consumption habits, and $\gamma$, $\chi$, $\chi_0$, and $b$ are parameters. In our baseline specification, we will set $h_t = C_{t-1}$, the level of aggregate consumption in the previous period (so the habit stock is external to the household), although we will consider alternative formulations such as long-memory habits later. The household’s nominal stochastic discount factor from period $t$ to $t + j$ in this model thus satisfies:

$$m_{t,t+j} \equiv \beta^j \frac{(c_{t+j} - bC_{t+j-1})^{-\gamma} P_t}{(c_t - bC_{t-1})^{-\gamma} P_{t+j}}.$$  

The economy also contains a continuum of monopolistically competitive intermediate goods firms indexed by $f \in [0, 1]$ that set prices according to Calvo contracts and hire labor competitively from households. In the baseline version of the model, firms have fixed, firm-specific capital stocks. Firms have Cobb-Douglas production functions:

$$y_t(f) = A_t \bar{k}^{1-\alpha} l_t(f)^\alpha,$$  \hspace{1cm} (2)

where $\bar{k}$ is a fixed, firm-specific capital stock (identical across firms) and where $A_t$ denotes an aggregate technology shock that affects all firms.\(^8\) The level of aggregate technology follows an exogenous AR(1) process:

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A,$$  \hspace{1cm} (3)

where $\varepsilon_t^A$ denotes an i.i.d. aggregate technology shock with mean zero and variance $\sigma_A^2$.

Intermediate goods are purchased by a perfectly competitive final goods sector that produces the final good with a CES production technology:

$$Y_t = \left[ \int_0^1 y_t(f)^{1/(1+\theta)} df \right]^{1+\theta}.$$  \hspace{1cm} (4)

Each intermediate goods firm $f$ thus faces a downward-sloping demand curve for its product given by:

$$y_t(f) = \left( \frac{p_t(f)}{P_t} \right)^{-(1+\theta)/\theta} Y_t,$$  \hspace{1cm} (5)

\(^8\) Several authors, such as Woodford (2003) and Altig, Christiano, Eichenbaum, and Linde (2004), have emphasized the importance of firm-specific fixed factors for generating a level of inflation persistence that is consistent with the data.
where
\[ P_t \equiv \left[ \int_0^1 p_t(f)^{-1/\theta} \, df \right]^{-\theta} \]  \hspace{1cm} (6)

is the CES aggregate price of a unit of the final good.

Each firm sets its price \( p_t(f) \) according to a Calvo contract that expires with probability \( 1 - \xi \) each period. There is no indexation, so the price \( p_t(f) \) is fixed over the life of the contract. When a contract expires, the firm is free to reset its price as it chooses. In each period \( t \), firms must supply whatever output is demanded at the posted price \( p_t(f) \). Firms hire labor \( l_t(f) \) from households in a competitive labor market, paying the nominal market wage \( w_t \). Marginal cost for firm \( f \) at time \( t \) is thus given by:
\[ mc_t(f) = \frac{w_t l_t(f)}{\alpha y_t(f)}. \]  \hspace{1cm} (7)

Firms are collectively owned by households and distribute profits and losses back to the households. When a firm’s price contract expires and it is able to set a new contract price, the firm maximizes the expected present discounted value of profits over the lifetime of the contract:
\[ E_t \sum_{j=0}^{\infty} \xi^j m_{t,t+j}[p_t(f)y_{t+j}(f) - w_{t+j}l_{t+j}(f)], \]  \hspace{1cm} (8)

where \( m_{t,t+j} \) is the representative household’s stochastic discount factor from period \( t \) to \( t + j \). The firm’s optimal contract price \( p_t^*(f) \) thus satisfies:
\[ p_t^*(f) = \frac{(1 + \theta)E_t \sum_{j=0}^{\infty} \xi^j m_{t,t+j}mc_{t+j}(f)y_{t+j}(f)}{E_t \sum_{j=0}^{\infty} \xi^j m_{t,t+j}y_{t+j}(f)}. \]  \hspace{1cm} (9)

To aggregate up from firm-level variables to aggregate variables, it is useful to define the cross-sectional price dispersion \( \Delta_t^\alpha \):
\[ \Delta_t^{1/\alpha} \equiv (1 - \xi) \sum_{j=0}^{\infty} \xi^j p_t^*(f)^{-(1+\theta)/(\theta \alpha)}, \]  \hspace{1cm} (10)

where the parameter \( \alpha \) in the exponent arises from the firm-specificity of capital. We can then write:
\[ Y_t = \Delta_t^{-1} A_t K_0^{1-\alpha} L_t^\alpha, \]  \hspace{1cm} (11)
where $\bar{K} = \bar{k}$ and
\[ L_t \equiv \int_0^1 l_t(f)df, \]  
(12)
and where equilibrium in the labor market requires $L_t = l_t$ (where the latter is the labor supplied by households).

In the baseline model, agents cannot invest in physical capital, although we do assume that an amount $\delta \bar{K}$ of output each period is devoted to maintaining the fixed capital stock. However, agents can buy and sell one-period risk-free nominal bonds, subject to an individual borrowing constraint that is not binding but rules out Ponzi schemes.

Optimizing behavior by households gives rise to the intratemporal condition:
\[ \frac{w_t}{P_t} = \frac{\lambda_0 l_t^K}{(c_t - bC_{t-1})^{-\gamma}}, \]  
(13)
and the intertemporal Euler equation:
\[ (c_t - bC_{t-1})^{-\gamma} = \beta \exp(i_t)E_t(c_{t+1} - bC_t)^{-\gamma}P_t/P_{t+1}, \]  
(14)
where $i_t$ denotes the continuously-compounded interest rate on the riskless one-period nominal bond.

The government levies lump-sum taxes $G_t$ on households and destroys the resources it collects. The aggregate resource constraint implies that:
\[ Y_t = C_t + \delta \bar{K} + G_t, \]  
(15)
where $C_t = c_t$, the consumption of the representative household. Government consumption follows an exogenous AR(1) process:
\[ \log G_t = \rho_G \log G_{t-1} + \varepsilon^G_t, \]  
(16)
where $\varepsilon^G_t$ denotes an i.i.d. government consumption shock with mean zero and variance $\sigma_G^2$.

Finally, there is a monetary authority in the economy which sets the one-period nominal interest rate $i_t$ according to a Taylor-type policy rule:
\[ i_t = \rho_i i_{t-1} + (1 - \rho_i) \left[ 1/\beta + \pi_t + g_y (Y_t - \bar{Y})/\bar{Y} + g_\pi (\pi_t - \pi^*) \right] + \varepsilon^i_t, \]  
(17)
where $1/\beta$ is the steady-state real interest rate in the model, $\pi_t$ denotes the four-period trailing average inflation rate (equal to $\log(P_t/P_{t-4})$), $\bar{Y}$ denotes the steady-state level of output, $\pi^*$ denotes the steady-state rate of inflation, $\epsilon^*_t$ denotes an $i.i.d.$ stochastic monetary policy shock with mean zero and variance $\sigma^2$, and $\rho_i$, $g_y$, and $g_\pi$ are parameters.

3 The Term Premium in a DSGE Model

We now derive the term premium in the benchmark DSGE model and calibrate the parameters of the model to quarterly U.S. data in order to compare the implications of the model to the data.

3.1 Measuring the Term Premium

The price of any asset in the model economy must satisfy the standard stochastic discounting relationship in which the stochastic discount factor $(m_{t+1} \equiv m_{t,t+1})$ is used to value the state-contingent payoffs of the asset in period $t+1$. For example, the price of a default-free $n$-period zero-coupon bond that pays one dollar at maturity satisfies:

$$p_t^{(n)} = \mathbb{E}_t[m_{t+1}p_{t+1}^{(n-1)}],$$

where $p_t^{(n)}$ denotes the price of the bond at time $t$ and $p_t^{(0)} \equiv 1$, i.e., the time-$t$ price of one dollar delivered at time $t$ is one dollar.\(^9\)

In the U.S. data, the benchmark long-term bond is the 10-year Treasury note. Thus, we wish to model the term premium on a bond with a duration of about 10 years. For computational reasons, it turns out to be inconvenient to work with a zero-coupon bond that has more than a few periods to maturity; instead, it is much easier to work with an infinitely-lived consol-style bond that has a time-invariant or time-symmetric structure. Thus, we assume that households in the model can buy and sell a long-term default-free nominal consol which pays a geometrically declining coupon in every period in perpetuity.

\(^9\) Cochrane (2001) provides a comprehensive treatment of this asset pricing framework.
The nominal consol’s price per $1 of coupon in period $t$, which we denote by $p_t^{(\infty)}$, then satisfies:

$$p_t^{(\infty)} = 1 + \delta_c E_t m_{t+1} p_{t+1}^{(\infty)},$$

(19)

where $\delta_c$ is the rate of decay of the coupon on the consol. By setting an appropriate value for $\delta_c$, which determines the Macauley duration of the bond, we can model prices of a bond of any desired maturity.

The continuously-compounded per-period yield to maturity on the consol is given by:

$$\log \left( \frac{\delta_c p_t^{(\infty)}}{p_t^{(\infty)} - 1} \right),$$

(20)

We define the risk-neutral consol price $p_t^{(\infty)rn}$ to be the deterministically discounted price of the bond:

$$p_t^{(\infty)rn} = 1 + \delta_c \exp(-i_t) E_t p_{t+1}^{(\infty)rn},$$

(21)

and the implied term premium on the bond is then given by:

$$\psi_t \equiv \log \left( \frac{\delta_c p_t^{(\infty)}}{p_t^{(\infty)} - 1} \right) - \log \left( \frac{\delta_c p_t^{(\infty)rn}}{p_t^{(\infty)rn} - 1} \right),$$

(22)

which is the difference between the observed yield to maturity on the bond and the risk-neutral yield to maturity. For a given set of structural parameters of the model, we will choose $\delta_c$ so that the bond has a Macauley duration of 10 years, and we will multiply equation (22) by 40,000 in order to report the term premium in units of annualized basis points rather than logs.

A technical issue that arises in solving the model above is the relatively large number of state variables it includes—eleven, including $C_{t-1}$, $A_{t-1}$, $G_{t-1}$, $i_{t-1}$, $\Delta_{t-1}$, the three lags of inflation underlying $\pi_t$, and the three shocks, $\varepsilon^A_t$, $\varepsilon^G_t$, $\varepsilon^\pi_t$. Because of this dauntingly high level of dimensionality, value-function iteration-based methods such as projection methods (or, even worse, discretization methods) are computationally completely intractable. Thus, the number of state variables can be reduced a bit by noting that $G_t$ and $A_t$ are sufficient to incorporate all of the information from $G_{t-1}$, $A_{t-1}$, $\varepsilon^G_t$, and $\varepsilon^\pi_t$, but the basic point remains valid, namely that the number of state variables in the model is large from a computational point of view.
we solve the model using the standard macroeconomic technique of approximation around the nonstochastic steady state—so-called perturbation methods. However, a first-order approximation of the model (i.e., a linearization or log-linearization) eliminates the term premium entirely, because equations (19) and (21) are identical to first order, a manifestation of the well-known property of certainty equivalence in linearized models. A second-order approximation to the solution of the model produces a term premium that is nonzero but constant (a weighted sum of the variances $\sigma_A^2$, $\sigma_G^2$, and $\sigma_i^2$). Since our interest in this paper is not just in the level of the term premium but also in its volatility and its variation over time, we must compute a third-order approximate solution of the model around the nonstochastic steady state. We do so using the $n$th-order perturbationAIM algorithm of Swanson, Anderson, and Levin (2006), which automatically and quickly computes $n$th-order approximate solutions do dynamic discrete-time rational expectations models of this type. For the baseline model above with 11 state variables, a third-order accurate solution can be computed in about 45 minutes on a standard laptop computer. Additional details of this solution method and the computation of impulse responses and unconditional standard deviations for the variables in the model are provided in the Appendix.

3.2 The Term Premium in the Benchmark DSGE Model

The baseline parameter values for our benchmark model are reported in the first column of Table 1 and are fairly standard in the literature (see, e.g., Levin, Onatski, Williams, and Williams, 2005). We set the household’s discount factor to .99 per quarter (implying a steady-state real interest rate of 4.02 percent per year), firms’ output elasticity with respect to labor to .7, firms’ steady-state markup to .2 (implying a price-elasticity of demand of 6), and the average price contract duration to 4 quarters. The importance of habits in the household’s utility is set to .66, consistent with typical estimates in the macro literature. We set the utility curvature parameter $\gamma$ to 2, which is a little on the high side of standard macroeconomic estimates so as to give the model a better chance of generating an appreciable term premium. We set the utility curvature parameter on labor $\chi$ to 1.5 (implying a Frisch
Table 1

Average Term Premiums in Alternative Parameterizations of Baseline Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline case value</th>
<th>Low case mean[\bar{\psi}_t]</th>
<th>High case mean[\bar{\psi}_t]</th>
</tr>
</thead>
<tbody>
<tr>
<td>\alpha</td>
<td>.7</td>
<td>.5</td>
<td>1.9</td>
</tr>
<tr>
<td>\beta</td>
<td>.99</td>
<td>.97</td>
<td>2.0</td>
</tr>
<tr>
<td>\theta</td>
<td>.2</td>
<td>.05</td>
<td>1.1</td>
</tr>
<tr>
<td>\xi</td>
<td>.75</td>
<td>.5</td>
<td>3.9</td>
</tr>
<tr>
<td>\gamma</td>
<td>2</td>
<td>.5</td>
<td>-1.4</td>
</tr>
<tr>
<td>\chi</td>
<td>1.5</td>
<td>0</td>
<td>.8</td>
</tr>
<tr>
<td>b</td>
<td>.66</td>
<td>0</td>
<td>1.6</td>
</tr>
<tr>
<td>\rho_A</td>
<td>.9</td>
<td>.7</td>
<td>.4</td>
</tr>
<tr>
<td>\rho_G</td>
<td>.9</td>
<td>.7</td>
<td>2.0</td>
</tr>
<tr>
<td>\sigma_A^2</td>
<td>.01^2</td>
<td>.005^2</td>
<td>.7</td>
</tr>
<tr>
<td>\sigma_G^2</td>
<td>.004^2</td>
<td>.002^2</td>
<td>2.0</td>
</tr>
<tr>
<td>\rho_i</td>
<td>.73</td>
<td>0</td>
<td>5.5</td>
</tr>
<tr>
<td>g_\pi</td>
<td>.53</td>
<td>.05</td>
<td>-3.7</td>
</tr>
<tr>
<td>g_y</td>
<td>.93</td>
<td>0</td>
<td>4.0</td>
</tr>
<tr>
<td>\sigma_i^2</td>
<td>.004^2</td>
<td>.002^2</td>
<td>1.8</td>
</tr>
<tr>
<td>K/(4Y)</td>
<td>2.5</td>
<td>1.25</td>
<td>1.6</td>
</tr>
<tr>
<td>C/Y</td>
<td>.17</td>
<td>.1</td>
<td>1.7</td>
</tr>
<tr>
<td>\pi^*</td>
<td>0</td>
<td>0</td>
<td>–</td>
</tr>
</tbody>
</table>

memo:
\chi_0        | 4.74                |
\delta_c      | .9848               |

Note: Term premium means are measured in basis points.

elasticity of about 0.7), which is again a little higher than typical macro estimates (but in line with estimates from the labor literature) to give the model a better chance of matching the term premium facts. The shock persistances \rho_A and \rho_G are 0.9, as is common, and the shock variances \sigma_A^2 and \sigma_G^2 are set to .01^2 and .004^2, respectively, consistent with typical estimates in the literature. The monetary policy rule coefficients are taken from Rudebusch
Table 2

Unconditional Moments of Four Empirical Measures of the Term Premium

<table>
<thead>
<tr>
<th>Measures</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bernanke, Reinhart, and Sack</td>
<td>180</td>
<td>69</td>
</tr>
<tr>
<td>Rudebusch and Wu</td>
<td>211</td>
<td>17</td>
</tr>
<tr>
<td>Kim and Wright</td>
<td>106</td>
<td>54</td>
</tr>
<tr>
<td>Vector autoregression</td>
<td>150</td>
<td>67</td>
</tr>
<tr>
<td>Average</td>
<td>162</td>
<td>52</td>
</tr>
</tbody>
</table>

Note: Term premium means and standard deviations are measured in basis points.

(2002) and are also typical of those in the literature. We assume the steady-state capital-output ratio is 2.5, which is close to what is found in the data, and steady-state government spending is about 17 percent of output. As is usual, we set the baseline steady-state inflation rate in the model to 0 percent per year. The parameter $\chi_0$ is chosen so as to normalize the steady-state quantity of labor to unity and, as discussed above, the parameter $\delta_c$ is chosen to set the Macaulay duration of the consol in the model to 10 years.

For the benchmark model with these standard parameter values, the average term premium, denoted mean[$\psi_t$], is just 2.0 basis points, and the unconditional standard deviation of the term premium is about 0.1 basis point. To anyone who has ever observed that long-term interest rates are higher than short-term interest rates on average and can be quite volatile, these magnitudes appear ludicrously small. To quantify this observation, Table 2 provides summary statistics for four empirical measures of the term premium on 10-year zero-coupon nominal U.S. Treasury securities calculated from 1994 through 2006. Three of the measures are taken from estimated no-arbitrage finance or macro-finance models in the literature, specifically, Bernanke, Reinhart, and Sack (2005), Rudebusch and Wu (2004, 2007), and Kim and Wright (2005). The fourth measure is obtained from a simple, standard, three-variable macroeconomic vector autoregression. The approaches used to obtain these four empirical measures of the term premium, which are described in detail in Rudebusch, Sack, and Swanson (2007), differ considerably in terms of the variables included and the theoretical restrictions imposed. Nevertheless, the measures have broadly similar uncondi-
tional moments, and we take their average mean of 162 basis points and standard deviation of 52 basis points (shown in the bottom row of Table 2), as indicative of the approximate magnitudes that a model should deliver.\textsuperscript{11} By this metric, the DSGE model term premium moments are too small by two orders of magnitude.\textsuperscript{12}

Although the benchmark DSGE model that we have used to conduct this experiment is fairly simple, we have found similar results in other more complicated DSGE models in the literature. For example, in the moderately-sized DSGE model of Christiano, Eichenbaum, and Evans (2005), the mean term premium is just 1.0 basis points—even smaller than in our benchmark model.\textsuperscript{13} We will give some intuition for this result later, after presenting results from several variations of our benchmark model.

From the point of view of a second- or third-order approximation to a macroeconomic model, these results should not be too surprising. The shocks in the benchmark model have a standard deviation of only about 1 percent, so first-order terms in the model have a magnitude that is roughly proportional to (.01), second-order terms have a magnitude that is roughly proportional to (.01)\textsuperscript{2}, where the constant of proportionality is related to the curvature of the model, and third-order terms have a magnitude that is roughly proportional to (.01)\textsuperscript{3}. Because the shocks in a macro model like our benchmark model are typically so small, the second-order terms should be expected to be roughly 100 times smaller than the first-order terms for a relatively flat model, and the third-order terms should be expected to be roughly 10,000 times smaller. Only for an extremely curved model, or for much larger shock standard deviations, could we reasonably expect the second- or third-order terms to matter very much.

\textsuperscript{11} Although the term premium itself is not directly observable, one can think of this approach as indirect inference. That is, the observable long-term bond yield data have generated a historical pattern of ex post realized excess returns that we are attempting to match. One way to match the historical ex post realized excess returns on long-term bonds is by replicating the term premium estimates that they imply in a standard finance or macro-finance model. This is the approach we have taken here.

\textsuperscript{12} The lack of variation in the baseline model’s estimate of the term premium is also illustrated by its impulse response to economic shocks. For example, the term premium moves less than five one-hundredths of one basis point on impact to a 1 percent technology shock and decays thereafter. See Rudebusch, Sack, and Swanson (2007) for further discussion.

\textsuperscript{13} In contrast to Christiano, Eichenbaum, and Evans (2006), we assume that the central bank follows a Taylor-type reaction function for the short-term nominal interest rate (equation (22)) rather than a money growth rule. This modification to the model is standard practice in the large-scale DSGE models being put into practice at central banks and the IMF, among others.
This basic intuition is supported by the remaining columns in Table 1, in which we vary each of the parameters over a wide range, broadly covering the various empirical estimates of each parameter in the literature, to see if the mean term premium changes significantly. The middle columns of Table 1 report the mean term premiums for the “low” parameter values, while the columns on the right are based on the “high” values. (As each parameter is altered, other parameters are fixed at their baseline values.) For example, setting \( \alpha = .5 \) instead of \( .7 \) reduces the mean term premium to 1.9 basis points, while setting \( \alpha = .85 \) increases it to 2.2 basis points. Across all of these parameter variations, the mean term premium is always at least an order of magnitude too small relative to the data. Still, in line with the intuition above, some parameters are more important than others. In particular, the mean term premium appears most sensitive to the variance and persistence of the technology shock \( (\sigma_A^2 \text{ and } \rho_A) \) and to the curvature of the utility function \( (\gamma \text{ and } \chi) \). These results foreshadow the two key approaches to increase risk premia in DSGE models that we consider below.

### 3.3 Increased Shock Volatility

Two recent papers in the literature, Hördahl, Tristani, and Vestin (2006b) and Ravenna and Seppälä (2006), denoted HTV and RS hereafter, generate a large mean term premium that is roughly in line with the data in a DSGE model that is very similar to our benchmark model. However, both of these analyses appear to accomplish this feat by introducing extremely large shocks into the model. For example, HTV assume that the technology shock has a quarterly standard deviation of 2.4 percent and a persistence of .986, compared with our baseline values of \( \sigma_A = 1 \) percent and \( \rho_A = .9 \). Adopting the two HTV parameter values in our model increases the mean term premium in our model from 2.0 to 100.3 basis points, which is now within the range of empirical estimates in Table 2 and confirms HTV’s results.

Unfortunately, the increased shock volatility in the model also increases the volatility of output and other macroeconomic and financial variables in the model. In Table 3, we report the mean and unconditional standard deviation of the term premium \( \text{mean}[\psi_t] \text{ and sd}[\psi] \) and the standard deviations of seven other macroeconomic and financial variables of interest:
Table 3
Unconditional Moments in Three Parameterizations of Benchmark DSGE Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameterizations of DSGE Model</th>
<th>Baseline</th>
<th>HTV</th>
<th>RS</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean measured in basis points</td>
<td>2.0</td>
<td>100.3</td>
<td>29.8</td>
<td></td>
</tr>
<tr>
<td>standard deviations measured in percent</td>
<td>1.24</td>
<td>12.7</td>
<td>5.23</td>
<td></td>
</tr>
<tr>
<td>sd[C]</td>
<td>0.79</td>
<td>7.98</td>
<td>3.30</td>
<td></td>
</tr>
<tr>
<td>sd[Y]</td>
<td>2.40</td>
<td>9.64</td>
<td>5.16</td>
<td></td>
</tr>
<tr>
<td>sd[L]</td>
<td>1.89</td>
<td>12.6</td>
<td>10.1</td>
<td></td>
</tr>
<tr>
<td>sd[w]</td>
<td>2.20</td>
<td>15.5</td>
<td>7.84</td>
<td></td>
</tr>
<tr>
<td>standard deviations measured in basis points</td>
<td>209</td>
<td>1560</td>
<td>772</td>
<td></td>
</tr>
<tr>
<td>sd[i]</td>
<td>53</td>
<td>1049</td>
<td>290</td>
<td></td>
</tr>
<tr>
<td>sd[ytm]</td>
<td>0.1</td>
<td>228</td>
<td>13.7</td>
<td></td>
</tr>
</tbody>
</table>

consumption, output, labor, the real wage, the inflation rate, the one-period nominal interest rate, and the yield-to-maturity on the long-term bond (sd[C], sd[Y], sd[L], sd[w], sd[π], sd[i], and sd[ytm], respectively).

The first column in Table 3 provides the means and standard deviations of several variables, as implied by the baseline parameterization of our model. The volatilities are similar to those computed from the data, except for the long-term yield, which should have a volatility fairly close to the short rate, and, of course, the term premium, which has a much larger standard deviation (and mean) in the data. The HTV parameterization produces an unconditional standard deviation of the term premium of 228 basis points, a bit large but of the same order of magnitude as the empirical estimates. However, for every other variable, the model’s implications for volatility are an order of magnitude larger than in the baseline parameterization. That is, the HTV parameterization appears to solve the bond premium puzzle but substantially worsens the model fit along other dimensions.
The RS parameterization provides a similarly poor overall fit, as can be seen in the third column of Table 3. Instead of an unusually large technology shock, RS introduce a very large taste shock into their model, changing the households’ preferences from (1) to:

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left( \frac{(c_t - bh_t)^{1-\gamma}}{1-\gamma} - d_t \lambda_0 \frac{l_t^{1+\chi}}{1+\chi} \right), \quad (23)$$

where $d_t$ is an AR(1) marginal rate of substitution shock which RS assume has a quarterly standard deviation of 8 percent and a serial correlation of .95. Introducing a taste shock $d_t$ of this size into our model leads to an average term premium in our model of 29.8 basis points and an unconditional standard deviation of the term premium of 13.7 basis points, both of which are only a little lower than the empirical measures. However, Table 3 shows that, as for the HTV case, the price of this solution to the bond premium puzzle is unrealistically large volatilities for all of the other variables in the model.

Thus, in order to simultaneously fit both the volatility of the term premium and the volatilities of other major macroeconomic and financial variables, we need to look beyond simply large shocks.

### 4 Habit Formation and the Term Premium

The analysis in the preceding section shows that the standard macro DSGE model is unable to match the level and volatility of the term premium in the data. This bond premium puzzle is reminiscent of the famous “equity premium puzzle” of Mehra and Prescott (1985). To resolve the equity premium puzzle, the macro-finance literature has proposed several modifications, including long-memory habit formation in consumption (Campbell and Cochrane 1999), time-inseparable preferences (Epstein and Zin 1989), and heterogeneous agents (Constantinides and Duffie 1996, Alvarez and Jermann 2001). Since these modifications to the basic model have been relatively successful at producing substantial risk premiums in endowment model economies, it is natural to ask whether incorporating these modifications might help our DSGE model match the level and volatility of the term premium. In the
present paper, we focus on the long-memory habit specification of Campbell and Cochrane (1999), because the standard DSGE models in macroeconomics and our benchmark model above already include a prominent role for habit in consumption. Moving from the standard habit specification used in these macroeconomic models to the Campbell-Cochrane specification of habits thus represents the smallest deviation from the standard macroeconomic framework that might still be expected to fit the term premium. We thus hope to extend our benchmark DSGE model to fit the term premium without significantly degrading its fit to the macroeconomic data.

Campbell and Cochrane (1999) propose replacing the standard habit preferences (1) with a habit stock $h_t$ that has a much longer memory over past consumption, and a parameter $b$ that is much closer to unity, which increases the importance of habits in agents’ utility. Moreover, to prevent current consumption from ever falling below habits (which would cross a singularity of the utility function (1)), Campbell and Cochrane define habits implicitly through surplus consumption $s_t$, as follows.

First, define the household’s surplus consumption ratio:

$$s_t = \frac{c_t - bh_t}{c_t}.$$  \hfill (24)

The habit stock $h_t$ is assumed to be external to the household (“keeping up with Joneses” habits), so letting capital letters denote aggregate quantities as above, household habits $h_t$ are defined to equal the aggregate habit stock:

$$h_t \equiv \frac{C_t(1 - S_t)}{b},$$  \hfill (25)

which is in turn defined implicitly by a process on $S_t$:

$$\log S_t = (1 - \phi) \log \overline{S} + \phi \log S_{t-1} + \left(\sqrt{1 - 2\log(S_t/\overline{S})} - 1\right) \left[\log(C_t/C_{t-1}) - E_{t-1} \log(C_t/C_{t-1})\right],$$  \hfill (26)

where $\phi$ and $\overline{S}$ are parameters. The primary advantage of this complicated definition of habits is that it ensures household surplus consumption is always positive, which is important
when the habit stock is a large fraction of current consumption. Campbell and Cochrane (1999) discuss the parameterization of (26) in detail, but surplus consumption and the habit stock must be persistent ($\phi$ close to 1) to match the persistence of risk premia and $\overline{S}$ must be very low (the habit stock must be very large relative to consumption) to keep the risk-free rate stable.

We investigate whether these long-memory habits can potentially explain the term premium by replacing the definition of $h_t$ in our benchmark model with the definition of habits given by equations (24)–(26). In all other respects, we keep the benchmark model the same.\(^{14}\) From the point of view of a Taylor series approximation, it is clear how these Campbell-Cochrane preferences could help make second- or even third-order terms more important. By increasing the size of habits relative to consumption (making $\overline{S}$ small), this specification greatly increases the curvature of the household’s utility function with respect to consumption—from a value of $\gamma$ in the model with no habits or $\gamma/(1 - b)$ in our baseline DSGE model, to $\gamma/\overline{S}$ in the model with long-memory habits. When $\overline{S}$ is small, such as the value of .0588 calibrated by Campbell and Cochrane (1999), then the curvature of the utility function is magnified by a factor of more than 16 compared to a model with no habits and by a factor of more than 5 relative to our baseline model. Such a large increase in the curvature of the model should be expected to increase the importance of higher-order terms in the Taylor series expansion.

Perhaps surprisingly, even with Campbell-Cochrane habits, our benchmark DSGE model is still unable to match the level and volatility of the term premium. The mean term premium implied by this model rises to 3.7 basis points, which is bigger than the 2.0 basis point term premium in the baseline version of our model but still nowhere near the 60 basis points mean term premium estimated in the data. Moreover, this model does essentially no better at matching the term premium’s volatility, as the unconditional standard deviation of the term premium is still far less than 1 basis point.

\(^{14}\) As in the baseline model, we set $\chi_0$ to normalize the quantity of labor $L = 1$ in steady state. However, because the marginal utility of consumption is so much higher with long-memory habits, the marginal disutility of labor must also be higher to arrive at the same steady-state quantity of labor, which produces $\chi_0 = 158.5$, much larger than in the baseline version of the model.
In Table 4, we check how robust this finding is by varying all of the parameters of the model over a plausible range of values, as we did previously in Table 1 for the baseline version of the model. The results in Table 4 confirm that the finding is robust: the introduction of Campbell-Cochrane preferences into the model by itself does not have a substantial effect on the term premium, as the mean term premium in the model remains fairly small.

This result stands in sharp contrast to Wachter (2006), who finds that Campbell-Cochrane habits can match the mean term premium in an endowment economy, where the exogenous process for consumption and inflation is estimated from the data. What can explain this dramatic difference in conclusions? The key, as also emphasized by Boldrin, Christiano, and Fisher (2000) and Jermann (1998), is that in a production-based model households can endogenously choose their labor-consumption tradeoff; in contrast, in an endowment-based economy, households must consume whatever the endowment turns out to be.\footnote{In Jermann (1998), households are unable to vary their labor supply but can vary investment instead, so the basic point is the same.} If households are hit by a negative shock in a production-based model, they can compensate for the shock by increasing their labor supply and working more hours. As a result, they have the ability to insure themselves to some extent from the effects of the shock on consumption by endogenously varying their labor supply in response. Households in an endowment economy do not have this opportunity, so the consumption cost of shocks in an endowment economy is correspondingly greater and risky assets in an endowment-based economy will tend to carry a larger risk premium. In the Campbell-Cochrane version of our benchmark model, this ability of households to self-insure appears to offset the large effects that long-memory habit preferences would otherwise have on the term premium.

This observation suggests that if labor in the model is not perfectly flexible or not completely within the household’s control, then the ability of households to self-insure themselves from shocks will be substantially diminished, and risk premia increase toward the higher levels in the endowment economy case. Boldrin, Christiano, and Fisher (2000) and Uhlig (2007) have also emphasized the importance of labor market frictions for matching the equity premium in a production economy. We explore this case in the next section.
Table 4
Average Term Premiums in Alternative Parameterizations of
the Campbell-Cochrane Version of Benchmark DSGE Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline case</th>
<th>Low case mean(ψ_t)</th>
<th>High case mean(ψ_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>.7</td>
<td>.5 3.5</td>
<td>.85 3.7</td>
</tr>
<tr>
<td>β</td>
<td>.99</td>
<td>.97 3.6</td>
<td>.995 3.7</td>
</tr>
<tr>
<td>θ</td>
<td>.2</td>
<td>.05 1.9</td>
<td>.4 4.5</td>
</tr>
<tr>
<td>ξ</td>
<td>.75</td>
<td>.5 8.3</td>
<td>.9 1.1</td>
</tr>
<tr>
<td>γ</td>
<td>2</td>
<td>.5 -1.0</td>
<td>6 6.9</td>
</tr>
<tr>
<td>χ</td>
<td>1.5</td>
<td>0 1.2</td>
<td>5 10.0</td>
</tr>
<tr>
<td>S</td>
<td>.0588</td>
<td>.03 3.8</td>
<td>.1 3.5</td>
</tr>
<tr>
<td>ϕ</td>
<td>.87</td>
<td>.7 2.5</td>
<td>.95 5.2</td>
</tr>
<tr>
<td>ρ_A</td>
<td>.9</td>
<td>.7 0.9</td>
<td>.95 9.0</td>
</tr>
<tr>
<td>ρ_G</td>
<td>.9</td>
<td>.7 3.6</td>
<td>.95 3.7</td>
</tr>
<tr>
<td>σ^2_A</td>
<td>.01^2</td>
<td>.005^2 1.3</td>
<td>.02^2 13.0</td>
</tr>
<tr>
<td>σ^2_G</td>
<td>.004^2</td>
<td>.002^2 3.6</td>
<td>.008^2 3.8</td>
</tr>
<tr>
<td>ρ_i</td>
<td>.73</td>
<td>0 10.0</td>
<td>.9 1.7</td>
</tr>
<tr>
<td>g_π</td>
<td>.53</td>
<td>.05 -3.7</td>
<td>1 6.8</td>
</tr>
<tr>
<td>g_y</td>
<td>.93</td>
<td>0 5.7</td>
<td>2 0.8</td>
</tr>
<tr>
<td>σ^2_i</td>
<td>.004^2</td>
<td>.002^2 3.3</td>
<td>.008^2 5.2</td>
</tr>
<tr>
<td>K/(4Y)</td>
<td>2.5</td>
<td>1.25 3.1</td>
<td>5 5.0</td>
</tr>
<tr>
<td>G/√Y</td>
<td>.17</td>
<td>.1 3.3</td>
<td>.3 4.6</td>
</tr>
<tr>
<td>π*</td>
<td>0</td>
<td>0 –</td>
<td>.01 4.3</td>
</tr>
<tr>
<td>memo:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>χ_0</td>
<td>158.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>δ_c</td>
<td>.9848</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Term premium means are measured in basis points.
5 Labor Market Frictions and the Term Premium

We find that the Campbell-Cochrane long-memory habit specification is unable to solve the bond premium puzzle in a DSGE model. However, limiting the ability of households to vary labor supply through labor market frictions should improve the model’s ability to fit the term premium. There are many ways to introduce labor market rigidities into our benchmark DSGE model, and we consider three such frictions below. We begin with the simplest form of labor market friction, a quadratic adjustment cost, then we consider two frictions with more institutional realism: real wage rigidities and staggered nominal wage contracting.

5.1 Quadratic Labor Adjustment Costs

We begin with the simplest form of labor market friction, namely, a quadratic adjustment cost on changes in the quantity of labor from one period to the next. Specifically, in each period, households must pay an adjustment cost \( \kappa (\log(l_t/l_{t-1}))^2 \), which is proportional to the squared log percentage change in labor from the previous quarter. This friction is particularly useful for gaining intuition because the size of the friction clearly varies with the size of the cost parameter \( \kappa \). As this cost parameter increases, it becomes more expensive for households to insure themselves against a shock by varying their labor supply, so the magnitude of the term premium should increase.

Figure 1 displays the relationship between the mean term premium and the labor adjustment cost parameter \( \kappa \). We trace out this relationship for two versions of our model with quadratic adjustment costs: one with Campbell-Cochrane habits and one without. The horizontal axis measures \( \kappa \) in units of output cost for a 1% change in labor—that is, if \( \kappa = 50\sqrt{Y} \), then a 1% change in labor from the previous quarter costs households 0.5% of quarterly steady-state output today. The unconditional standard deviation of labor in these models is about 2.5%, so a change in labor from one quarter to the next of about 1% is about the right order of magnitude for this representative-agent model.
Figure 1 shows that both a moderate level of adjustment costs and long-memory habits are necessary to generate a mean term premium that is roughly consistent with the data. Without such habits, even extreme levels of adjustment costs to labor do not have much of an effect on the term premium because variation in consumption is simply not that abhorrent to households. With Campbell-Cochrane habits, consumption variation is much more undesirable, so adjustment costs to labor quickly begin to generate a substantial aversion by households to risky assets. Indeed, with Campbell-Cochrane habits and adjustment costs of around 0.5% of output for a 1% change in labor \((\kappa = 50Y)\), the mean term premium in this model is 79.7 basis points, which is well within range of our earlier empirical estimates.

In Table 5, we consider varying each of the parameters of the model with Campbell-Cochrane habits and adjustment costs \(\kappa = 50Y\). As can be seen in the table, once we have introduced long-memory habits and some degree of labor adjustment costs into the model, then it suddenly becomes relatively easy for the model to match the average term premium in the data. Slightly smaller values of the price contract duration parameter \(\xi\), slightly higher steady-state inflation \(\pi^*\), or slightly larger technology shocks all increase the volatility of nominal variables and can help fit the term premium while reducing the model’s reliance on labor adjustment costs. A lower Frisch elasticity of labor supply (a higher \(\chi\)) also helps raise the term premium, which is not surprising given that a higher value of \(\chi\) is just another way of increasing the costs that households must pay to vary their labor supply in response to shocks.\(^{16}\)

\(^{16}\) Uhlig (2007) suggests introducing a leisure habit into the model, which accomplishes the same thing. While imposing adjustment costs on labor or reducing the Frisch elasticity of labor supply are very simple and effective ways of hindering households’ ability to vary labor, these methods also contrast with much of the real business cycle literature since the 1980s, which has generally sought to increase labor volatility and elasticity while reducing the variability of real wages. This should not be viewed as a major shortcoming of this approach, however, since a low household labor supply elasticity can be reconciled with a larger macroeconomic elasticity by appealing to extensive vs. intensive margins as in Hansen (1985).
Table 5
Average Term Premiums in Alternative Parameterizations of Campbell-Cochrane Version of Benchmark DSGE Model with Labor Adjustment Costs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline case</th>
<th>Low case</th>
<th>High case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>value</td>
<td>mean[$\psi_t$]</td>
<td>value</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>.7</td>
<td>.5</td>
<td>90.7</td>
</tr>
<tr>
<td>$\beta$</td>
<td>.99</td>
<td>.98</td>
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</tr>
<tr>
<td>$\xi$</td>
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<td>.5</td>
<td>40.4</td>
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<tr>
<td>$\chi$</td>
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<td>0</td>
<td>54.8</td>
</tr>
<tr>
<td>$\bar{S}$</td>
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</tr>
<tr>
<td>$\phi$</td>
<td>.87</td>
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<td>32.9</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>.9</td>
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<td>$\rho_G$</td>
<td>.9</td>
<td>.7</td>
<td>79.6</td>
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<td>.005$^2$</td>
<td>20.7</td>
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<td>$\sigma_G^2$</td>
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<tr>
<td>$g_y$</td>
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<td>0</td>
<td>81.3</td>
</tr>
<tr>
<td>$\sigma_i^2$</td>
<td>.004$^2$</td>
<td>.002$^2$</td>
<td>79.3</td>
</tr>
<tr>
<td>$K/(4Y)$</td>
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<td>$G/Y$</td>
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<td>1</td>
<td>.3</td>
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<tr>
<td>$\pi^*$</td>
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<td>0</td>
<td>–</td>
</tr>
</tbody>
</table>

memo:
$\lambda_0$ 158.5
$\delta_c$ .9848

Note: Term premium means are measured in basis points.
Unfortunately, adding labor adjustment costs to the model comes at a cost in terms of fitting macroeconomic quantities. In Table 6, we report the unconditional standard deviations for the term premium and seven other major macroeconomic and financial variables. While the Campbell-Cochrane version of our model without adjustment costs (the second column of Table 6) was unable to match the volatility of the term premium, that version of the model implied a volatility for the other macroeconomic and financial variables that was roughly consistent with the data. By contrast, in the model with C-C habits and quadratic labor adjustment costs of $\kappa = 50Y$ (the third column of the table), the volatility of the term premium is much larger, but so are the unconditional standard deviations of real wages, inflation, and short-term nominal interest rates. The volatility of the real wage in particular is over 224 log percentage points.

Intuitively, the presence of labor adjustment costs along with Campbell-Cochrane habits means that agents do not want to vary either labor or consumption in response to a shock. Yet when there is a shock, one or the other of these two quantities must give; as a result, the real wage must vary tremendously in order to achieve equilibrium. These large movements in the real wage in turn cause firms’ marginal costs to be extremely volatile, which passes through to prices. The Taylor-type policy rule implies that the movements in inflation pass through to the short-term interest rate and the long-term bond yield. Both the marginal utility of consumption and the bond price are much more volatile in this version of the model with adjustment costs, hence the term premium is much greater in magnitude.
### Table 6
Unconditional Moments in Four Versions of the Benchmark DSGE Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Baseline</th>
<th>Campbell-Cochrane</th>
<th>C-C with quadratic adj. costs to labor</th>
<th>C-C with quadratic and real wage rigidity</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean $\psi$</td>
<td>2.0</td>
<td>3.7</td>
<td>79.7</td>
<td>22.1</td>
</tr>
<tr>
<td>sd[C]</td>
<td>1.24</td>
<td>0.99</td>
<td>1.07</td>
<td>3.12</td>
</tr>
<tr>
<td>sd[Y]</td>
<td>0.79</td>
<td>0.64</td>
<td>0.70</td>
<td>1.97</td>
</tr>
<tr>
<td>sd[L]</td>
<td>2.40</td>
<td>2.57</td>
<td>3.72</td>
<td>2.07</td>
</tr>
<tr>
<td>sd[$w^r$]</td>
<td>1.89</td>
<td>1.80</td>
<td>224.2</td>
<td>2.63</td>
</tr>
<tr>
<td>sd[$\pi$]</td>
<td>2.20</td>
<td>2.14</td>
<td>19.7</td>
<td>5.35</td>
</tr>
<tr>
<td>sd[i]</td>
<td>209</td>
<td>218</td>
<td>907</td>
<td>556</td>
</tr>
<tr>
<td>sd[ytm]</td>
<td>53</td>
<td>56</td>
<td>134</td>
<td>193</td>
</tr>
<tr>
<td>sd[$\psi^*$]</td>
<td>0.1</td>
<td>0.2</td>
<td>12.7</td>
<td>8.7</td>
</tr>
</tbody>
</table>

### 5.2 Real Wage Rigidities

Adding simple quadratic labor adjustment costs with Campbell-Cochrane preferences solved the bond premium puzzle but at the cost of seriously distorting the model’s ability to fit the macroeconomic data. Therefore, it is interesting to consider whether other approaches to modeling labor market frictions might provide a better combination of macroeconomic and term premium fit. Real (and nominal) wage rigidities are perhaps a more realistic labor market friction than are quadratic adjustment costs and have been widely used in the macroeconomics literature, so they are a natural alternative labor market friction to consider. Moreover, wage rigidities of this type could perhaps be combined with quadratic adjustment costs to the quantity of labor to help dampen the enormous volatility of real wages that was implied by the quadratic adjustment cost version of the model in the preceding section.
Following Blanchard and Gali (2005), we introduce a wage bargaining friction into our benchmark model by assuming that the real wage follows the process:

$$\log w^r_t = \omega + \mu \log w^r_{t-1} + (1 - \mu)w^r_{t-1}^*, \quad (27)$$

where $w^r$ denotes the real wage, $w^r_*$ denotes the frictionless real wage that would obtain in the absence of the wage rigidity, $\omega$ denotes a steady-state wedge between the real wage and the households’ marginal rate of substitution, and $\mu$ denotes the sluggishness of wages in adjusting toward the frictionless real wage. Although equation (27) does not explicitly model Nash bargaining between workers and firms, Blanchard and Gali motivate it as a simple real wage friction that captures the essential features of such bargaining on real wage dynamics.

When we introduce equation (27) into our benchmark model, however, it turns out to have essentially no effect on the term premium, either in our baseline parameterization of the model or in our version with Campbell-Cochrane habits. In the version with C-C habits, the mean term premium in the model increases from 3.67 to just 3.75 basis points, and varying the parameters $\omega$ and $\mu$ over plausible ranges has little effect.

This contrasts sharply with our previous findings for adjustment costs on labor itself. Intuitively, equation (27) is a wage friction, but it fails to increase risk premia in the model because it does not place any restrictions upon households’ ability to self-insure. That is, equation (27) limits the adjustment of wages to shocks, but households in the model are still completely free to vary the number of hours that they work and thereby insure themselves from the effects of a shock. As a result, wage rigidities of the type in Blanchard and Gali (2005), Shimer (2005), and Hall (2005)—and in contrast to labor rigidities on the quantity of hours worked—appear to be unable to help macroeconomic models explain the term premium.

If real wage rigidities cannot help to explain the term premium by themselves, perhaps they could be combined with labor rigidities in the Campbell-Cochrane version of the model in order to damp down the excessive volatility of real wages that we documented in the previous section? Unfortunately, the answer here is also not promising. In the Campbell-Cochrane version of the benchmark model, the marginal rate of substitution is so volatile
that only with extreme degrees of wage rigidity ($\mu > .999$) can the variation in real wages and inflation be brought back down to reasonable levels. In the fourth column of Table 6, we report the mean term premium and unconditional standard deviations for this case, with quadratic adjustment costs to labor of 0.5% of output for a 1% change in labor and with a real wage rigidity parameter of $\mu = .999$. While this extreme degree of wage rigidity does bring the standard deviations of the real wages and other macroeconomic variables back toward more reasonable levels, it also reduces the term premium as well, both in mean and standard deviation, to a point that is still about five times smaller than in the data. Thus, not only does the required degree of real wage rigidity appear to be implausibly large, but even assuming such wage rigidity, we are unable to fit both the term premium and macroeconomic variables.

5.3 Staggered Nominal Wage Contracting

As an alternative to real wage frictions, we also considered staggered nominal wage contracts as in Erceg, Henderson, and Levin (2000). These wage contracts have been used in many medium- and large-scale DSGE models. For example, Christiano, Eichenbaum, Evans (2005) and Smets and Wouters (2003) provide complete discussions of the details of these contracts. We provide a brief summary here. As in the Calvo price contracting specification in the benchmark model, each household is assumed to be a monopolistic supplier of a differentiated type of labor, which is bundled by a perfectly competitive labor market aggregator into the final labor input that is used by firms. A critical element that maintains tractability is the assumption of complete financial markets, which allows households to trade state-contingent securities and ensures that—despite the heterogeneity across households in the wage charged and in hours worked—all households have identical consumption in every period in equilibrium.

When we incorporate such Calvo staggered nominal wage contracts into the benchmark model, either under our baseline parameterization or with Campbell-Cochrane habits, there is again no significant effect on the term premium. In the model with Campbell-Cochrane
preferences, the term premium actually decreases from 3.67 to 1.26 basis points in the model with Calvo wage contracts, and varying the parameters of the model over plausible ranges has essentially no effect on this result.

Again, this is in sharp contrast to our results for labor rigidities, and is surprising at first glance because—unlike with the Blanchard-Gali specification—here the quantity of labor worked by households truly is demand-determined (for households that are unable to reset their wage this period) and is not under the household’s own control. However, the assumption of complete financial markets that is required for tractability in these models also has the side effect of allowing households to insure their consumption streams through the financial markets. Thus, even though most households in the model cannot self-insure against negative shocks by working more, they can purchase state-contingent claims that pay off in the event of a negative income shock and in the event that the household is unable to reset its wage. Because of this assumption, households are still largely able to insure their consumption streams from the consequences of negative shocks, and the term premium in the model is very small.

As a result, the staggered nominal wage contracting is going to be unable to help standard macroeconomic DSGE models fit the term premium data.

6 Conclusions

All in all, our results cast a pessimistic light on the ability of habit-based DSGE models to fit the term premium. Even in versions of the model with very strong, long-memory habits such as Campbell and Cochrane (2005), the ability of households to vary their labor supply and thereby self-insure themselves from consumption fluctuations led to term premia that were too small and far too stable relative to what we see in the data. When we tried to reduce households’ ability to self-insure by introducing standard labor market frictions into the model, we ended up dramatically increasing the volatility of households’ marginal rate of substitution and the real wage—and hence marginal costs, inflation, and the short-term...
nominal interest rate—to a point far in excess of the data. Thus, the success that Wachter (2006) reports in fitting the term premium in an endowment economy does not appear to generalize very well to the standard macroeconomic DSGE framework, where labor supply and demand are endogenous.

While our results are perhaps discouraging from the point of view of habit-based DSGE explanations for the term premium, there are many other approaches one might take to fit the term premium in a DSGE framework. First, more exotic habit-based DSGE models than the one we have considered here might still be able to fit the term premium where variations on our benchmark model have failed. For example, Boldrin, Christiano, and Fisher (2000) consider a two-sector model with immobile, sector-specific capital stocks and find that this feature of the model helps it to fit both the equity premium and the macro data; however, standard DSGE models in use today do not have this feature. Alternatively, Piazzesi and Schneider (2006) have recently reported success using generalized “recursive utility” preferences, as in Epstein and Zin (1989), to explain the term premium in an endowment economy, which suggests that this approach might also be very fruitful for explaining term premia in a DSGE setting. The methods used in this paper—such as second- and third-order approximations and a generalized consol to model the term premium—can likewise be applied to the case of recursive utility in a DSGE framework and make the solution of the term premium in such models computationally tractable. We view this as a promising direction for future research. Finally, models based on heterogeneous agents with incomplete insurance markets, as in Constanides and Duffie (1996) and Krebs (2007), might also be able to explain the term premium, although merging these frameworks into standard macroeconomic DSGE models appears challenging.
Appendix: Computational Algorithm

This appendix provides details regarding how we solve our benchmark DSGE models for the term premium and other variables.

As discussed in the text, the primary challenge in solving the model is the relatively large number of state variables, from as few as 11 in the baseline parameterization of the model to as many as 15 in the Campbell-Cochrane version of the model with labor adjustment costs and real wage rigidities. Because of this high degree of dimensionality of the model, value-function iteration-based methods such as projection methods (or, even worse, discretization methods) are computationally completely intractable. We instead solve the model above using the standard macroeconomic technique of approximation of the model’s solution around the nonstochastic steady state—so-called perturbation methods.

However, a first-order approximation (i.e., a linearization or log-linearization) of the model around the steady state eliminates the term premium entirely, since equations (19) and (21) are identical to first order. A second-order approximation produces a nonzero but constant term premium (a sum of the variances $\sigma_A^2$, $\sigma_G^2$, and $\sigma_i^2$). Since our interest in this paper is not just in the level of the term premium but also in its variation over time, we must compute a third-order approximation to the solution of the model around the nonstochastic steady state. We do so using the $n$th-order perturbation AIM algorithm of Swanson, Anderson, and Levin (2006), for additional details the reader is referred to that paper.

The perturbation AIM algorithm requires that the equations of the model be put into a recursive form, which for the model above is fairly standard—the most difficult equation is (9), which can be written in first-order recursive form as:

\[
\left( \frac{p_t^*(f)}{P_t} \right)^{1+\frac{1-\alpha}{\alpha} + \frac{1+\theta}{\sigma}} = \frac{z_{n,t}}{z_{d,t}} \tag{28}
\]

\[
z_{d,t} = Y_t(C_t - bC_{t-1})^{-\gamma} + \beta E_t \pi_{t+1}^{1/\theta} z_{d,t+1} \tag{29}
\]

\[
z_{n,t} = (1 + \theta) \frac{\lambda}{1 - \alpha} L_t^{1+\gamma} \Delta_{t}^{-1/\alpha} + \beta E_t \pi_{t+1}^{(1+\theta)/(\theta \alpha)} z_{n,t+1}. \tag{30}
\]
The computational time required to solve our benchmark model to third order ranges from about 45 minutes for the simplest, baseline version of the model up to about 30 hours for the most complicated specification.

Computing impulse responses for this model is actually simpler than the use of a third-order approximation might suggest. We are interested in the responses of output and the term premium to an exogenous shock to $\varepsilon_t^A$, $\varepsilon_t^G$, or $\varepsilon_t^i$. For macroeconomic variables such as output, we plot the standard first-order (i.e., log-linear) responses of output to each shock. For small shocks, such as those of the size we are considering here (1 percent), these responses are by far the dominant terms of the Taylor series expansion and thus are highly accurate. Of course, for the term premium, the first- and second-order responses of that variable to each shock would be identically zero, so we plot the third-order terms to trace out the impulse responses of that variable; again, these are the dominant terms of the Taylor series expansion for the term premium, so they should be highly accurate. These third-order terms all have the form $\sigma_Z^2 X$ where $Z \in \{A, G, i\}$ and $X$ is one of the state variables of the model, so once we substitute in the relevant values for $\sigma_A^2$, $\sigma_G^2$, and $\sigma_i^2$, these terms have a linear structure in the state variables of the model and can be easily plotted.

Similarly, when we compute unconditional standard deviations, we focus on the dominant terms of the Taylor series expansion to compute these impulse responses. For macroeconomic variables, this amounts to computing unconditional standard deviations using the first-order approximation to the solutions for those variables. For the term premium, we use the third-order approximate solution to compute the unconditional standard deviations, as for impulse responses above.

\[17\] In perturbation analysis, stochastic shocks of the model are given an auxiliary “scaling” parameter, so these shocks are third-order in a rigorous sense. See Swanson et al. (2006) for details.
References


Figure 1
Mean Term Premium in Campbell-Cochrane and Baseline Versions of Benchmark DSGE Model