Impact of Uncertainty and Risk Aversion on Price and Order Quantity in the Newsvendor Problem

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We consider a single-period inventory model in which a risk-averse retailer faces uncertain customer demand and makes a purchasing-order-quantity and a selling-price decision with the objective of maximizing expected utility. This problem is similar to the classic newsvendor problem, except: (a) the distribution of demand is a function of the selling price, which is determined by the retailer; and (b) the objective of the retailer is to maximize his/her expected utility. We consider two different ways in which price affects the distribution of demand. In the first model, we assume that a change in price affects the scale of the distribution. In the second model, a change in price only affects the location of the distribution. We present methodology by which this problem with two decision variables can be simplified by reducing it to a problem in a single variable. We show that in comparison to a risk-neutral retailer, a risk-averse retailer in the first model will charge a higher price and order less; whereas, in the second model a risk-averse retailer will charge a lower price. The implications of these findings for supply-chain strategy and channel design are discussed. Our research provides a better understanding of retailers' pricing behavior that could lead to improved price contracts and channel-management policies.
(*Pricing; Demand Uncertainty; Risk Aversion; Inventory*)

1. Introduction

We consider a single-period inventory model in which a risk-averse retailer faces uncertain customer demand and makes a purchasing-order-quantity and a selling-price decision with the objective of maximizing expected utility. In this model, the retailer purchases a certain quantity at a regular purchase price. If the realized demand is greater than the quantity ordered, then the retailer has the option to purchase the units that are short at an emergency purchase price that is higher than the regular price. If the demand is less than the order quantity, then the retailer has the option to return the leftover inventory at a salvage price that is lower than the regular price. This problem is similar to the classic newsvendor problem, except: (a) the distribution of demand is a function of the selling price, which is determined by the retailer; and (b) the objective of the retailer is to maximize his/her expected utility.

We use the newsvendor problem framework because of its increasing relevance due to shortening product life cycle. In addition, the newsvendor model forms the basic building block of many multiperiod dynamic inventory, capacity-planning, and contract-design problems. A recent summary and extensions of the results for the newsvendor problem that incorporates the pricing decision (but not risk aversion) is given in Petruzzi and Dada (1999). This problem was

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originally analyzed by Whitin (1955) and Karlin and Carr (1962). As mentioned by Petruzzi and Dada, the pricing decision plays a key role in managing the marketing-operations interface because it combines the study of operational efficiency issues (the order-quantity decision) with the study of marketing issues (the selling-price decision). It is of interest to note in the context of this special issue of *M&SOM,* that Petruzzi and Dada (1999, p. 183) remark, ". . . unlike the version of the newsvendor problem in which selling price is exogenous, this more strategic variant has received limited attention since the 1950's. This parallels in many ways the observation that since the 1950's, the practice of operations has emphasized functional efficiency at the expense of cross-functional effectiveness."

Pricing behavior of retailers is not only very difficult to understand, but is also of interest to manufacturers as well as to regulators. Manufacturers wish to see their product sold at the retail level at a specific price to ensure profitability, to maintain market share, and to gain brand equity. However, often to their dismay, the retailers either price higher or lower than the manufacturers' retail target price. (Legally, manufacturers can only suggest a retail price but not dictate it. The U.S. government permits manufacturers in certain situations to publicize the maximum retail price on the packaging of the product to prevent undue escalation of the retail price, see Stern and Eovaldi 1984, chapter 5.) In some cases, manufacturers force retailers to increase retail prices because discounted sales reduce the value of their brand and create channel conflict. In other instances, manufacturers attempt to keep retail prices reasonably low. For example, Levi-Strauss, the apparel giant, was accused of forcing its retailers not to *discount* prices (Emert 1999). In contrast to Levi's, manufacturers of candy products force retailers not to *increase* the retail price. We show that these seemingly disparate pricing behaviors of retailers can be attributed to risk aversion of small independent retailers.

Many consumer goods, including apparel, electronics, shoes, candy, food, and books, are sold through independent retailers. We expect such retailers to be more risk averse than larger retailers or integrated/chain retailers. Ironically, these industries also face short product life cycles and have high product variety. These factors result in high demand uncertainty. For example, Nahmias and Smith (1994) report from their study of the retail industry that it is common to observe a variance-to-mean ratio of demand that ranges from 3 to 300. (Eeckhoudt et al. 1995 give examples in which price-taking, risk-averse retailers will not stock an item due to high demand uncertainty.) Thus, we expect manufacturers in industries with short product life cycle and high product variety to be quite concerned about the compounding of the effects of risk aversion and high demand uncertainty. However, little is known about the pricing behavior of risk-averse retailers. In this paper we attempt to understand the pricing problem under uncertainty and aversion to risk.

Simply put, does risk aversion of retailers always lead to an increase or a decrease in prices? Equally persuasive arguments can be made for either possibility. A risk-averse retailer might choose to reduce the price in order to generate higher unit sales, and simultaneously achieve a smaller probability of unsold inventory. On the other hand, the same retailer might choose to increase the price, thereby generating lower demand, and consequently stock less of an item, thus obtaining a higher profit margin on each sale at lowered risk.

We are therefore interested in the effect of uncertainty on the *combined* price and order-quantity decisions of a risk-averse retailer. We wish to answer the following questions:

• How does uncertainty in demand influence the price and the order-quantity decisions of a risk-neutral retailer?

• Will the price charged by a risk-averse retailer increase or decrease with increasing aversion to risk?

• What happens to the order quantity when the retailer becomes more risk averse?

The order-quantity decision of a risk-averse retailer in the newsvendor-problem setting (with an exogenous fixed price) has been extensively studied in the literature. It has been shown that: (a) a risk-averse retailer's optimal order quantity (i.e., the one that maximizes the retailer's expected utility) will be smaller than the order quantity that maximizes expected profit (see Horowitz 1970, Baron 1973, Eeckhoudt et al. 1995);

(b) this optimal order quantity decreases with increasing risk aversion (see Bouakiz and Sobel 1992, Eeckhoudt et al. 1995, and Agrawal and Seshadri 2000 for structural properties).

As mentioned earlier, the pricing decision of a risk-neutral retailer in the newsvendor-problem setting has been studied in depth (Petruzzi and Dada 1999). Kalyanam (1996) models the effect of risk aversion on price using a power utility function (without considering the order-quantity decision) and uses a Bayesian demand-estimation procedure. He shows that the optimal price of a risk-averse retailer is lower than that of a risk-neutral retailer, and suggests that more insight is needed to understand the divergence between risk-neutral and risk-averse retailers. Our work is the first to combine the risk aversion of retailers (Eeckhoudt et al. 1995) and the joint ordering and pricing decision of the retailers (Petruzzi and Dada 1999).

A particular challenge in modeling the pricing decision under uncertainty is to specify how the distribution of demand changes as a function of the price. We model the effect of price on demand in two different ways. In our approach, we borrow two simple ideas from statistics, namely that the *scale* and the *location* of a distribution are two important quantities that are necessary to describe a distribution. In our first model, we assume that changing the price results in the rescaling of the demand distribution by a function $\alpha(P)$ of the price P. Thus, all moments are appropriately scaled. In particular, the mean and the standard deviation are scaled by $\alpha(P)$. In our second model, we assume that the location of the distribution changes by a function of the price. Therefore, only the mean demand changes with the price. These two models were labeled by Karlin and Carr (1962) (also see Petruzzi and Dada 1999) the "multiplicative demand model" and the "additive demand model," respectively. (In §5 we discuss the product and market characteristics that support scaling or a location shift of the distribution of demand when prices are changed.) The second challenge in analyzing this problem arises because there are two decision variables in the optimization problem. We develop new techniques for solving these types of problems in §§3 and 4.

We show that our two models capture two extreme pricing behaviors. In the first model, as risk aversion increases the price increases and the order quantity falls (§3); whereas in the second model the price falls and the effect on the order quantity depends on the relative effects of risk aversion and the sensitivity of the demand to price (§4). In particular, if the second effect (i.e., due to price) dominates, then the order quantity will also increase with increasing aversion to risk. The implications of these models to crafting channel strategies are discussed in §5.

2. Model and Notation

We consider a single-period problem in which a risk-averse retailer has to decide the quantity to order as well as the selling price of a single item. The retailer chooses a price P. The distribution of the demand is parameterized by the selling price, and is denoted by $F_P(x)$ with mean equal to $\alpha(P)$. We assume that the mean demand is downward sloping and concave with respect to the price, i.e., that $d\alpha(P)/dP \le 0$ and $d^2\alpha(P)/dP^2 \le 0$. The retailer has two modes of purchase: regular and emergency. Before the demand is realized the retailer purchases S units of the item at a "regular purchase price" of c. The quantity S will be referred to as the order quantity. Once the demand is realized, if it is greater than S then the retailer has the option to purchase the units that are short at an emergency purchase price of $e (e \ge c)$. If the demand is less than S, then the retailer has the option to return the leftover inventory at a "salvage" price of $s \le c$. Thus, $e \ge c \ge s$. The retailer's utility function is denoted by $U(\cdot)$, a function that is assumed to be increasing and concave in wealth, that is $U'(\cdot) \ge 0$ and $U''(\cdot) \le 0$. The retailer's objective is to determine the order quantity S and the selling price P that maximize his/her expected utility. Denote the random demand given a price P as $X(P)$. Let the retailer's profit be $\Pi(P, S, X(P))$, where the first two parameters are the decision variables of the retailer.

The retailer's expected profit is given by

$$\begin{aligned} E[\Pi(P, S, X(P)) &= E[PX(P) - cS + s[S - X(P)]^+ \\ &\quad - e[X(P) - S]^+] \\ &= P\alpha(P) - cS + sE[S - X(P)]^+ \\ &\quad - eE[X(P) - S]^+. \end{aligned} \tag{2.1}$$

Similarly, the expected utility to the retailer is given by

$$E[U(\Pi(P, S, X(P)))] = E[U(PX(P) - cS + s[S - X(P)]^{+} - e[X(P) - S]^{+})]. \quad (2.2)$$

Define

P^* equals Optimal price of a risk-neutral retailer under stochastic demand.

P_d^* equals Optimal price when demand is deterministic and is given by $\alpha(P)$.

P_u^* equals Optimal price of a risk-averse retailer.

$S^*(P)$ equals Optimal order quantity for a risk-neutral retailer given price P.

S^* equals Optimal order quantity for the risk-neutral retailer, i.e., $S^*(P^*)$

$S_u^*(P)$ equals Optimal order quantity of a risk-averse retailer for a given price P.

S_u^* equals Optimal order quantity of a risk-averse retailer, i.e., $S_u^*(P_u^*)$.

Formally,

$$(P^*, S^*) = \arg\max_{(P,S)} E[\Pi(P, S, X(P)).$$

$$(P_u^*, S_u^*) = \arg\max_{(P,S)} E[U(\Pi(P, S, X(P)))].$$

$$P_d^* = \arg\max_P ((P - c)\alpha(P)).$$

2.1. General Model of Price and Demand

An important feature of modeling this problem is to capture the effect of the selling price on the distribution of demand (i.e., to characterize the demand given a price P as $X(P)$). It is convenient to define a random variable X with distribution $F(x)$. Consider a general demand distribution that is parameterized by the price in one of two ways through two concave-decreasing functions $\alpha_1(P)$ and $\alpha_2(P)$ such that

$$X(P) = \alpha_1(P)x + \alpha_2(P). \quad (2.3)$$

In formulating (2.3) (see Young 1978), we borrow two simple ideas from statistics, namely that the *scale* and the *location* of a distribution are two important quantities used to describe a distribution. By scale we refer to the fact that we may scale an entire distribution to create a new distribution. By location we refer to the shifting the mean of a distribution by adding a constant. The *scale* effect of price on the demand distribution is represented by $\alpha_1(P)$ and the *location* effect by $\alpha_2(P)$. We focus on two extreme representations of Equation (2.3), where either $\alpha_1(P)$ or $\alpha_2(P)$ dominate so that the impact of the scale and location effects can be isolated. Therefore, in the first model $\alpha_2(P)$ is set to zero (§3), and in the second $\alpha_1(P)$ is set equal to one (§4).

The effect of price on the distribution of demand has been modeled in three different ways: (i) by assuming a specific distribution whose parameters are affected by the price (such as a Poisson process whose intensity is a function of price as in Bitran and Mondschein 1996, Feng and Gallego 1995, Gallego and Van Ryzin 1994, or a Weiner process as in Raman and Chatterjee 1995), (ii) by using a regression-based approach in which the random error term could be either independent of or dependent on the price (see Kalyanam 1996 and Lau and Lau 1988), and (iii) by assuming what has been termed an additive or a multiplicative model by Karlin and Carr (1962) and Petruzzi and Dada (1999).

In the context of our paper we note that: (i) We use the price models of Karlin and Carr, but refer to them as affecting the scale and location parameters because these terms directly refer to the effect of price on the distribution of demand. (ii) In Karlin and Carr (1962) and Petruzzi and Dada (1999), the penalty of lost sales is equated to the selling price minus the regular purchase price (p-c) plus a penalty cost, whereas we model the cost of understocking by introducing an emergency purchase price, similar to Eeckhoudt et al. (1995).

Petruzzi and Dada derive expressions for optimal order quantity and price for a risk-neutral newsvendor using a demand function characterized as $\alpha(P) = a - bP$ for the location case, and $\alpha(P) = aP^{-b}$ for the scale case. They show that the optimal price for a risk-neutral retailer under stochastic demand is higher in the scale case and lower for the location case when compared to the deterministic demand scenario. Because we use a different lost-sale penalty cost, our results for a risk-neutral retailer are slightly different, especially in the location case, where we show that the optimal prices in the stochastic and the deterministic cases are the same. This difference is explained in the next two sections and also in §5. Petruzzi and Dada (1999) also consider the general demand model and show that the optimal price is contingent upon the behavior of demand variance with respect to the mean when price is changed. They show that when demand variance decreases in P, and the coefficient of variation is nonincreasing in P (this condition is satisfied in the scale case), then $P^* \geq P_d^*$. On the other hand, when both the variance and the coefficient of variation increase in P (this condition is satisfied in the location case), then $P^* \leq P_d^*$. The optimal pricing behavior for a risk-neutral retailer when the variance is decreasing in P while the coefficient of variation is increasing in P is not known.

In this paper our primary objective is to study the impact of risk aversion on prices and order quantities, and the results of the risk-neutral retailer are presented so that we can make a consistent comparison due to the difference in our cost structure with the earlier literature. Moreover, since the pricing behavior of a risk-neutral retailer is itself difficult to analyze for the general model (unless it corresponds to the location or the scale case), we restrict our analysis to the scale case in §3 and the location case in §4.

3. Price Affects the Scale of the Demand Distribution

In the first model we assume that distribution of demand is *scaled* by a function of the price and that the expectation of X is equal to one. Hence,

$$X(P) = \alpha_1(P)X = \alpha(P)X \tag{3.1}$$

and

$$E[X(P)] = \alpha(P)E[X] = \alpha(P).$$

We shall first consider the decision-making problem for a risk-neutral retailer. We derive properties of its solution and use these to determine the optimal decision for a risk-averse retailer. For the risk-neutral retailer, $E[\Pi(P, S, X(P))]$ shown in (2.1) gives the expected profit for a given price P and stocking level S. Define the expected cost for a given P and S as $E[C(P, S, X(P))]$, thus

$$E[C(P, S, X(P))] = E[cS - s[S - X(P)]^+ + e[X(P) - S]^+]. \tag{3.2}$$

Define the average cost for a given P and S as $C_a(P, S)$, where

$$C_a(P, S) = \frac{E[C(P, S, X(P))]}{\alpha(P)}. \tag{3.3}$$

From Equation (2.1)

$$\begin{aligned} E[(\Pi(P, S, X(P))] &= E[PX(P)] - E[C(P, S, X(P))]) \\ &= E[PX(P)] - E\left[\frac{C(P, S, X(P))}{\alpha(P)}\alpha(P)\right] \\ &= P\alpha(P) - C_a(P, S)\alpha(P). \end{aligned} \tag{3.4}$$

Define:

- $C_a(P) = C_a(P, S^*(P)) =$ average cost given P and the optimal order quantity $S^*(P)$.
- $\zeta^* = F^{-1}(e - c/e - s)$.
- $c_a^* = E[c\zeta^* - s[\zeta^* - X]^+ + e[X - \zeta^*]^+]$ (3.5)

LEMMA 3.1.
(a) $S^*(P) = \alpha(P)\,\zeta^*$,
(b) $C_a(P) = C_a^*$.

PROOF.
(a) $S^*(P) = F_P^{-1}(y)$, where $y = e - c/e - s$ is constant. However, by definition $F_P(\alpha(P)X) = F(X)$. This implies that $S^*(P) = \alpha(P)\,\zeta^*$.
(b) From Equation (3.2),

$$\begin{aligned} E[C(P, \alpha(P)\zeta^*, X(P))] &= E[c\alpha(P)\zeta^* - s[\alpha(P)\zeta^* - X(P)]^+ \\ &\quad + e[X(P) - \alpha(P)\zeta^*]^+] \\ &= E[c\alpha(P)\zeta^* - s[\alpha(P)\zeta^* - \alpha(P)X]^+ \\ &\quad + e[\alpha(P)X - \alpha(P)\zeta^*]^+] \\ &= \alpha(P)E[c\zeta^* - s[\zeta^* - X]^+ \\ &\quad + e[X - \zeta^*]^+] \\ &= \alpha(P)C_a^*, \end{aligned} \tag{3.6}$$

where we have substituted $X(P) = \alpha(P)X$. Thus, by Definition (3.3), $C_a(P) = C_a^*$. □

REMARK. The constant C_a^* is the lowest average cost.

Lemma 3.1 can be used to determine the optimal price and order quantity for a risk-neutral retailer. However, characterizing the optimal decision is not as straightforward in the case of a risk-averse retailer. A major hurdle in the analysis of the risk-averse case is the fact that the expected utility depends on two decision parameters. A key contribution of this paper is to show how the analysis of problems of this type that have two decision variables can be simplified by reducing the analysis to a single decision variable. In order to effect this simplification we use the concept of an *S curve* which is defined as follows.

Select a value ζ. Define an *S curve* as the plot of the price versus the order quantity, when the quantity ordered at price P is given by $S(P, \zeta) = \alpha(P)\zeta$. The idea of defining the S curve is to exploit several properties of the optimal order quantity, the average cost, as well as the distribution of the profit that can be obtained when the price is changed *along* a given S curve. These properties are used to make the analysis of the risk-averse newsvendor more tractable. Our use of ζ is similar to the use of the "stocking factor, z," by Petruzzi and Dada (1999) to analyze the risk-neutral newsvendor's pricing problem. The necessary properties are developed in the next four lemmas and then used to prove the main theorem in this section.

COROLLARY 1. *The average cost along an S curve, $C_a(S(P, \zeta))$ is a constant and is greater than or equal to C_a^*.*

PROOF. Similar to the second part of Lemma 3.1. □

DEFINITION. Because of Corollary 1, we can denote the average cost along an *S curve* as $C_a(\zeta)$.

LEMMA 3.2. *The profit function $E[\Pi(P, S(P, \zeta), X(P))]$ is concave in P (that is, the expected profit is concave along an S curve).*

PROOF. From (3.4)

$$\frac{d^2E[(\Pi(P, S(P, \zeta), X(P))]}{dP^2} = (P - C_a(P, S(P, \zeta)))\alpha''(P) + \alpha'(P) \le 0,$$

where the final inequality follows from our assumptions that the mean demand is concave and decreasing in the price. □

Thus, the risk-neutral retailer's problem from Equation (3.4) and Lemma 3.1 can be restated as

$$\underset{P,S}{Max}\{E[\Pi(P, S, X(P))]\} = \underset{P}{Max}\{(P - C_a^*)\alpha(P)\}. \quad (3.7)$$

Similarly, the risk-averse retailer's problem is

$$\underset{P,S}{Max}\{E[U(PX(P) - cS + s[S - X(P)]^+ - e[X(P) - S]^+)]\}. \quad (3.8)$$

It is of interest to examine the effect of uncertainty on price even for a risk-neutral retailer under this model. For the purpose of making a comparison, let the demand be a known function of price, $\alpha(P)$.

LEMMA 3.3. *The optimal price under uncertainty for a risk-neutral retailer is higher than the optimal price under deterministic demand, i.e., $P^* \ge P_d^*$.*

PROOF. The optimal order quantity in the deterministic case is $S^*(P) = \alpha(P)$ and the optimal price is given by solving for P to satisfy the first-order necessary condition for optimality:

$$\frac{d\Pi(P, S)}{dP} = \alpha(P) + (P - c)\alpha'(P) = 0. \quad (3.9)$$

Similarly, the optimal price for risk-neutral retailer in the presence of demand uncertainty is given by solving for P in

$$\frac{dE[\Pi(P, S, X(P))]}{dP} = \alpha(P) + (P - C_a^*)\alpha'(P) = 0. \quad (3.10)$$

From Equation (3.9)

$$\frac{d^2\Pi(P, S]}{dcdP} = -c\alpha'(P) \ge 0.$$

Because the normal purchase price is less than the average cost to the retailer, i.e., $c \le C_a^*$, therefore

$$\left[\frac{dE[\Pi(P, S, X(P))]}{dP}\right]_{P=P_d^*} \ge 0.$$

Therefore, as $E[\Pi(P, S, X(P))]$ is a concave function of price (by Lemma 3.2), $P^* \ge P_d^*$. □

REMARKS. When price affects scale, then: (1) When the distribution of demand becomes larger in convex stochastic order, it leads to an increase in the average cost and therefore an increase in the price. (A random variable X is said to be larger than another random variable Y in the convex stochastic order if $E[f(X)]$ is larger than $E[f(Y)]$ for all convex functions; see Shaked and Shanthikumar 1994.) The reader is referred to Ridder et al. (1998) and Gerchak and Mossman (1992) who provide conditions under which the average cost C increases (thus the price increases) with increasing *variability* of demand. (2) Karlin and Carr (1962) showed that the same result, i.e., $P^* \ge P_d^*$, is obtained when the cost of understocking is modeled as $(p - c)$ (instead of e as in our paper). The difference in the relationship between P^* and P_d^* between our model and Karlin and Carr's arises in the second model.

We now prove the main theorem of this section, which is used to establish the relationship between the optimal decisions of the risk-neutral and risk-averse

Table 1 Conditions for Higher Prices of Risk-Averse Retailer (Based on Theorem 3.1) for Some Common Demand Distributions

Demand Distribution		Condition 1: High Risk Aversion — Optimal Fill Rate of a Risk-Averse Retailer is less than	Condition 2: Low $\frac{e-c}{e-s}$ — Optimal Fill Rate of a Risk-Neutral Retailer is less than
Uniform [0, 1]		100%	100%
Triangular [0, 2]		50%	84.75%
Normal			
Mean	SD		
1.0	3.0	96.4%	98.2%
1.0	2.0	91.9%	97.1%
1.0	1.0	75%	91.9%
1.0	0.5	46%	81.6%
1.0	0.1	13%	61.8%

retailers. We need some definitions and preliminary lemmas to establish this theorem.

DEFINITIONS.

- Let $P^*(\zeta)$ be the optimal price for a risk-neutral retailer along an *S curve*, i.e., $P^*(\zeta) = \arg\max_P\{E[\Pi(P, S(P, \zeta), X(P))]$.
- Let $\zeta_u^*(P) = \arg\max_\zeta\{E[U(\Pi(P, S(P, \zeta), X(P)))]$, i.e., the optimal value of ζ, for a risk-averse retailer given P.
- Let ζ_u^* be the optimal value of ζ for a risk-averse retailer, when the price is P_u^*.

LEMMA 3.4. *If ζ is less than ζ^*, then $P^*(\zeta)$ is greater than P^*, i.e., the optimal price for a risk-neutral retailer is higher when the order quantity is smaller than the optimal order quantity.*

PROOF. Please see Appendix.

LEMMA 3.5. *The optimal value of ζ for a risk-averse retailer, i.e., ζ_u^*, will be less than or equal to the corresponding quantity for the risk-neutral retailer; in other words, $\zeta_u^* \leq \zeta^*$.*

PROOF. It is well known (see Agrawal and Seshadri 2000 or Eeckhoudt et al. 1995) that for a given price the risk-averse retailer's quantity is smaller than that of a risk-neutral retailer. This implies $\zeta_u^* \leq \zeta^*$. This does not automatically imply that the order quantity for the risk-averse retailer will be smaller than the order quantity for the risk-neutral retailer, because that involves comparing the quantities, $\alpha(P_u^*)\zeta_u^*$ and $\alpha(P^*)\zeta^*$. These quantities are compared in Theorem 3.2. □

THEOREM 3.1.

(i) *If $E[X \mid X < y]/E[X \mid X \geq y]$ is increasing in y then $P_u^* \geq P^*$, i.e., the optimal price for a risk-averse retailer is not lower than the optimal price of a risk-neutral retailer.*

(ii) *If $E[X \mid X \leq \zeta_u^*]/E[X \mid X \geq \zeta_u^*]$ is less than $E[X \mid X < \zeta^*]/E[X \mid X \geq \zeta^*]$ (in particular if the retailer is sufficiently risk averse) then $P_u^* \geq P^*$.*

(iii) *If the emergency price e equals the regular price c then $P_u^* = P^*$.*

(iv) *The risk-averse retailer's optimal price is greater than the optimal price of a risk-neutral retailer under deterministic demand, i.e., $P_u^* \geq P_d^*$.*

PROOF. See Appendix. □

Theorem 3.1 suggests that there are two conditions under which the risk-averse retailer will set a higher price: (1) The optimal fill rate of a risk-averse retailer is low, i.e., the risk-averse retailer orders a small quantity; or (2) the optimal fill rate of a risk-neutral retailer is low, i.e., risk-neutral retailer orders a small quantity. Condition 1 holds if the order quantity of the risk-averse retailer is small or, equivalently, when the retailer is sufficiently risk averse. Condition 2 holds if either the demand uncertainty is high, or if the salvage value is low compared to purchase price and emergency price. In Table 1 we provide the threshold values of the fill rates for Conditions 1 and 2 for some common distributions. Interestingly, for the uniform distribution the price of a risk-averse retailer is always higher.

In summary, Conditions 1 and 2 state that retailers price higher due to a combination of two effects: (1) the economics of stocking the product, risk aversion, and demand uncertainty force them to provide a *relatively low service level* to customers, (2) due to the scaling of the demand distribution, an increase in price results in a *decrease* in the relative variability of demand (i.e., the variance to mean ratio, also see Nahmias and Smith

1994). The second effect is evident in the normal-distribution example in which, if the standard deviation is high, the threshold values for Conditions 1 and 2 are higher.

THEOREM 3.2. *Under the conditions stated in Theorem 3.1, the optimal order quantity for a risk-averse retailer is smaller than the optimal order quantity of a risk-neutral retailer, i.e.,* $S_u^*(P_u^*) \le S^*(P^*)$.

PROOF. From Theorem 3.1 (i) or (ii), $P_u^* \ge P^*$, therefore $\alpha(P_u^*) \le \alpha(P^*)$. It then follows from Lemma 3.1 that

$$S^*(P_u^*) \le S^*(P^*). \tag{3.11}$$

It is well known (see Agrawal and Seshadri 2000 or Eeckhoudt et al. 1995) that for a given price the risk-averse retailer's quantity is smaller than that of a risk-neutral retailer. Therefore,

$$S_u^*(P_u^*) \le S^*(P_u^*). \tag{3.12}$$

The proof follows from Inequalities (3.11) and (3.12). □

4. Price Affects the Location of the Demand Distribution

In our second model we assume that only the location of the demand distribution is affected by price. Formally, define as before a random variable X with distribution $F(.)$. Assume without loss of generality that the expectation of X, $E[X]$, equals zero. Let the distribution of demand be given by

$$X(P) = X + \alpha_2(P) = X + \alpha(P) \tag{4.1}$$

Similar to the first model, given a price P, the mean demand is equal to $\alpha(P)$. Instead of the *S curve* defined in the previous section, define an *S curve* to be such that, $S(P, \zeta) = \zeta + \alpha(P)$. It is then straightforward to write,

$$\begin{aligned} \Pi(P, S(P, \zeta), x) &= P(x + \alpha(P)) - c(\zeta + \alpha(P)) \\ &\quad + s[\zeta + \alpha(P) - (x + \alpha(P))]^+ \\ &\quad - e[(x + \alpha(P)) - (\zeta + \alpha(P))]^+ \\ &= P\alpha(P) + Px - c(\zeta + \alpha(P)) \\ &\quad - s[\zeta - x]^+ - e[x - \zeta]^+. \end{aligned} \tag{4.2}$$

This model is relatively easy to analyze because, as established below, the expected profit, as well as the expected utility, is a concave function of the price.

LEMMA 4.1.
(1) *$E[\Pi(P, S(P, \zeta), x)]$ is concave in P.*
(2) *$E[U(\Pi(P, S(P, \zeta), x))]$ is concave in P.*

PROOF. See Appendix. □

Define:
- $\zeta^* = F^{-1}(e - c/e - s)$
- $C^* = E[c\zeta^* + s[\zeta^* - x]^+ - e[x - \zeta^*]^+]$

Note that ζ^* is the expected profit-maximizing order quantity and C^* is the optimal expected cost of a newsvendor problem for a product whose demand distribution is $F(.)$ (as defined earlier).

LEMMA 4.2.
(1) $S^*(P) = \alpha(P) + \zeta^*$,
(2) $E[C(P, S^*(P), x)] = C^* + c\alpha(P)$.

PROOF.
(1) From the solution to the newsvendor problem, $F_P(S^*(P)) = (e - c/e - s)$. However, by definition $F_P(S^*(P)) = F(S^*(P) - \alpha(P))$. Therefore,

$$S^*(P) - \alpha(P) = F^{-1}\left(\frac{e - c}{e - s}\right) = \zeta^*.$$

(2) Using $S^*(P) = \alpha(P) + \zeta^*$, we get,

$$\begin{aligned} E[C(P, S^*(P), x)] &= E[\Pi(P, S^*(P), x)] - P\alpha(P) \\ &= E[c(\zeta^* + \alpha(P)) + s[\zeta^* - x]^+ \\ &\quad - e[x - \zeta^*]^+] \\ &= c\alpha(P) + E[c\zeta^* + s[\zeta^* - x]^+ \\ &\quad - e[x - \zeta^*]^+]. \quad \square \end{aligned}$$

LEMMA 4.3. *The optimal price for a risk-neural retailer is the same as the price under deterministic demand, i.e.,* $P^* = P_d^*$.

PROOF. Using Lemma 4.2,

$$E[\Pi(P, S^*(P), x)] = (P - c)\alpha(P) + C^*,$$

where C^* does not depend on price. Therefore, the functions optimized under stochastic and deterministic demand differ by a constant. □

It is interesting to note that in contrast to the previous model the price is independent of the uncertainty of demand for a risk-neutral retailer. Our result is also different when compared to the results of Karlin and Carr (1962) and Petruzzi and Dada (1999), who show that under the additive model of demand the risk-neutral retailer's price will be lower compared to the price under deterministic demand. Therefore, by having recourse to emergency shipment (as in our model) instead of foregoing profit on a lost sale, a risk-neural retailer's optimal price becomes independent of demand uncertainty.

THEOREM 4.1. *The derivative of the expected profit with respect to the price along an S curve is less than or equal to the derivative of the expected utility with respect to the price along the same S curve.*

PROOF. This theorem is established by writing the derivative of the expected utility as the integral of the product of two quantities, namely, the derivative of the utility with respect to the profit and the derivative of the profit with respect to the realized demand. It turns out that the first of these quantities is decreasing while the second one is increasing in the profit. This leads to the desired result. We formalize these arguments now. From Equation (4.2) we get

$$\frac{d\Pi(P, S(P, \zeta), x)}{dP} = (P - c)\alpha'(P) + \alpha(P) + x, \quad (4.3)$$

where x is the observed demand. From Equation (4.3) we observe that $[d\Pi(P, S(P, \zeta), x)]/dP$ is increasing in x. From Equation (4.2), $\Pi(P, S(P, \zeta), x)$ is increasing in x. Therefore, because $U(.)$ is concave, $[\partial U(\Pi(P, S(P, \zeta), x))]/\partial\Pi$ is decreasing in x. We can write

$$\frac{d}{dP}\int_{x=0}^{\infty} U(\Pi(P, S(P, \zeta), x))dF(x) = \int_{x=0}^{\infty} \frac{\partial U(\Pi(P, S(P, \zeta), x))}{\partial \Pi}\frac{d\Pi(P, S(P, \zeta), x)}{dP}\, dF(x). \quad (4.4)$$

Thus, in the integral on the right-hand side of (4.4), $[\partial U(\Pi(P, S(P, \zeta), x))]/\partial\Pi$ puts smaller weight on larger values of $[d\Pi(P, S(P, \zeta), x)]/dP$. The statement of the theorem follows by observing that if we replace $[\partial U(\Pi(P, S(P, \zeta), x))]/\partial\Pi$ by the expected value of $[\partial U(\Pi(P, S(P, \zeta), x)]/\partial\Pi$ we obtain a (positive) multiple of the derivative of the expected profit that is larger than the expected utility. (This statement can be formalized by use of the mean value theorem, see, for example, Widder 1989.) □

THEOREM 4.2. $P_u^* \leq P^*$, *i.e., the optimal price for a risk-averse retailer is not higher than the optimal price of a risk-neutral retailer.*

PROOF. The derivative of the expected profit with respect to price (see (4.5)) does not depend on S. Moreover, the expected profit is concave in price. Thus, if the derivative of the expected profit is less than or equal to zero at some price P, then regardless of the value of S, we know that P is less than P^*. The theorem then follows from Theorem 4.1 and concavity of the expected utility in price (Part Two of Lemma 4.1). □

REMARK. Theorem 4.1 does not depend on the assumption that $\alpha(P)$ is concave in price. However, the proof of Theorem 4.2 uses the strong unimodality of the function $(p - c)\alpha(P)$. We may thus relax the assumption of concavity on $\alpha(P)$ and only require $(p - c)\alpha(P)$ be strongly quasi-concave.

LEMMA 4.4. *Given two retailers i and j, if retailer i is more risk averse in the Arrow-Pratt sense than retailer j, then the optimal price for retailer i is lower.*

PROOF. The proof follows from replicating the steps used to prove Theorem 4.1. □

Effect of risk aversion on the order quantity. In this case, we cannot in general predict the effect of risk aversion on the order quantity. An increase in risk aversion will lead to a lower price, and therefore a higher demand, thus to a desire to order more. On the other hand, given a price an increase in risk aversion will result in a reduction of the order quantity. Therefore, the combined effect of the two factors can be either an increase or a decrease in the order quantity.

5. Discussion of Results

Table 2 summarizes the main results of §§3 and 4. It is interesting to observe that risk aversion of retailers distorts their pricing decision in different ways depending on the impact of price on the distribution of demand. In the first model (the scale assumption) as risk aversion increases, the price increases and the order quantity decreases. Therefore, risk aversion compounds the effect of uncertainty in reducing a manufacturer's expected profit. In the second model as risk aversion increases the price falls, and the effect on the

Table 2 Summary of Results

Distribution Assumption	Decision	Risk-Neutral Retailer	Risk-Averse Retailer
Scale	Price	Greater than deterministic price	(a) Greater than deterministic price (b) Greater than risk-neutral price for high risk aversion and high variability
	Order Quantity	(Deterministic quantity) $\times$ (a factor independent of price)	Lower than risk-neutral
Location	Price	Equal to deterministic price	Lower than deterministic price
	Order Quantity	(Deterministic quantity) $+$ (a factor independent of price)	Lower than risk-neutral if price elasticity is low and risk aversion is high, and vice versa.

order quantity depends on the relative effects of risk aversion and the sensitivity of the demand to price. In particular, if the second effect (i.e., due to price) dominates, then the order quantity will also increase with increasing aversion to risk. Hence, in the second model risk aversion lowers expected profit and channel conflict between larger retailers and smaller retailers. These inefficiencies in the supply chain caused by price and order-quantity distortion due to risk aversion should be considered in the channel-design (integrated vs. independent) and contract-design decisions.

Manufacturers must investigate whether the distribution of demand for their product primarily exhibits a location shift or a scale effect due to price change. We now present some conjectures supporting the shift and scale effects—systematic empirical research is necessary to verify these claims.

• For branded products (such as Levi's jeans) that exhibit substantial customer demand, the reduction in price should result in higher customer traffic to the retailer, thus resulting in a predictable increase in sales. We can argue that the uncertainty in demand in this situation is mainly due to a random forecast error that is independent of price. Therefore, a price change in this case is expected to affect only the location of the demand distribution. This provides a possible explanation for the lower prices charged by an independent Levi's jeans retailer, as mentioned in the introduction.

There can be other reasons why a small independent retailer offers a lower price: (1) It is possible that emergency replenishment is not available to the retailer because of long lead times or lack of support from the manufacturer. In this case even a risk-neutral retailer may price lower than the deterministic price (from the newsvendor model without emergency supply as discussed in Karlin and Carr 1962, Petruzzi and Dada 1999, and Van Meigham and Dada 1999) in order to take advantage of reduced relative demand variability (location effect). (2) It may be the case that small independent retailer outlets are owned by entrepreneurs who are less risk averse than their corporate counterpart. In such a case, if the demand distribution exhibits the scale effect the less risk-averse retailers will post lower prices.

• For a retailer whose main channel function is geographical convenience, such as a corner-store deli, reduction in the price does not necessarily result in increased customer traffic for that specific product (because the demand is generated locally). However, a "regular" customer might purchase a larger quantity of the product when the price is lower. This could result in the scaling of the demand distribution. The higher prices of candy in corner-store delis can be explained based on this observation and Theorem 3.1.

• For high-fashion products or new products, the product may be a *hit* or a *miss* in the marketplace. For a product that becomes a *hit,* the average sales will be higher if the price is lower. On the other hand, a *miss* product may have dismal sales irrespective of the price. So the difference between the *hit* demand and the *miss* demand is magnified if prices are reduced. This is again similar to the scaling of the demand distribution.

• The same product can exhibit scale or location effects depending on the intensity of demand. Consider the Poisson process. When the demand intensity is low, for example a rate of 1, and a reduction in price increases the intensity to, let us say, 2 or 3, the variability of the demand also increases considerably. This effect is closer to the scaling of the demand distribution. On the other hand, if the intensity of the demand increases from 100 to 110 due to a price decrease, the effect on demand variability is much less, thus exhibiting a location effect.

To summarize, our results suggest that in small independent stores: prices of new, low demand, or low brand recognition products will be higher; while the prices of mature, high demand, or branded products will be lower, when compared to prices in chain stores.

An open question is whether manufacturers can reduce the inefficiencies and price distortions due to risk aversion of retailers by offering suitable pricing contracts to their retailers. For risk-neutral retailers significant research has been done in designing pricing contracts to mitigate the loss in efficiency attributed to double marginalization and the misalignment of incentives (see Tsay et al. 1998 and Moses and Seshadri 2000 for a review). In the newsvendor framework, price contracts are discussed by Pasternack (1985), Weng (1997), and by Lariviere and Porteus (1999) for a risk-neutral price-taking retailer.

For the newsvendor model under risk aversion, Agrawal and Seshadri (2000) show that the decrease in total expected profits of a supply chain due to reduced order quantities of risk-averse retailers can be eliminated by a risk-neutral intermediary offering a suitably designed menu of pricing contracts to the retailers. Such a menu reduces the demand risk of the retailers and restores the retailers' order quantities to the one that maximizes the retailers' expected value. Our initial analysis shows that it may not be possible to create similar price contracts (that achieve the first best) when the risk-averse retailers are also price setters. Further research is necessary to determine effective contracts that reduce the price and order-quantity distortions due to risk aversion.

In this paper we augment the understanding of decision making under uncertainty at the interface of marketing and operations, i.e., of making ordering-quantity and pricing decisions. The particular problem that we analyze poses a technical challenge: namely, the joint optimization of two decision variables, one of which (price) impacts the demand distribution itself, and the risk aversion of the retailers that adds another layer of complexity. We have formalized a method for analyzing this problem by using S curves. This allows us to isolate the effect of one variable (the stocking level) and concentrate on the analysis of the price. Moreover, we have not made any assumptions about the utility function beyond the fact that it is concave and increasing, nor assumed a functional form for the $\alpha(P)$ except that it is concave (or strongly quasi-concave). In establishing the main result for the "price affects scale" case, we have optimized over a class of functions to obtain the bounds for the fill rate under which the statement of Theorem 3.1 is true. On the other hand, when the mean demand is not a concave function of price, we have only partial results for the location case. Extensions are also possible to the case when the penalty of stockout is as given by Karlin and Carr (1962) and Petruzzi and Dada (1999). Finally, analysis of the general model of demand presented in §2.1 should yield further insights into the problem. It is an open problem to extend the analysis in our paper to determine the most general conditions under which the price will increase or decrease with increase or decrease in risk aversion.

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Appendix

Proof of Lemma 3.4. The average cost $C_a(\zeta)$ is higher than $C_a(\zeta^*)$ by definition of ζ^*. Thus,

$$\frac{d}{dp}\alpha(P)(P - C_a(\zeta)) = \alpha'(P)(P - C_a(\zeta)) + \alpha(P) > \alpha'(P)(P - C_a(\zeta^*)) + \alpha(P).$$

We also know that $\alpha'(P)/\alpha(P)$ is decreasing in P, i.e., $\alpha''(P)/\alpha(P) - \alpha'(P)^2/\alpha(P)^2$; is less than or equal to zero. Therefore, $(\alpha'(P)/$

$\alpha(P))(P - C_a(\zeta^*)) + 1 \geq (\alpha'(P^*)/\alpha(P^*))(P^* - C_a(\zeta^*)) + 1 = 0$. Thus, $(d/dp)\alpha(P)(P - C_a(\zeta))$ is greater than or equal to zero when P is less than or equal to P^*. □

Proof of Theorem 3.1. Denote $U(\Pi)(P, S(P, \zeta), x))$ and $[dU(\Pi(P, S(P, \zeta), x))]/d\Pi$ as $U(x)$ and $U'(x)$ respectively. For the risk-averse retailer, the first-order condition for optimality with respect to ζ is given by

$$0 \equiv \frac{dE[U(x)]}{d\zeta} = \int_0^{\zeta} U'(x)\alpha(P)(-c + s)dF(x) + \int_{\zeta}^{\infty} U'(x)\alpha(P)(-c + e)dF(x). \tag{A.1}$$

Thus,

$$\frac{\int_{\zeta}^{\infty} U'(x)dF(x)}{\int_0^{\infty} U'(x)dF(x)} = \frac{c - s}{e - s} \tag{A.2}$$

Similarly, the first-order optimality condition with respect to P yields,

$$0 \equiv \frac{dE[U(x)]}{dP} = \int_0^{\zeta} U'(x)(\alpha'(P)(Px - c\zeta + s(\zeta - x)) + \alpha(P)x)dF(x) + \int_{\zeta}^{\infty} U'(x)(\alpha'(P)(Px - c\zeta - e(x - \zeta)) + \alpha(P)x)dF(x). \tag{A.3}$$

If both (A.1) and (A.3) have to hold simultaneously, then we can eliminate the terms involving ζ in (A.3) and simplify (A.3) to read

$$\int_0^{\zeta} U'(x)(R(P)x - sx)dF(x) + \int_{\zeta}^{\infty} U'(x)(R(P)x - ex)dF(x) = 0 \tag{A.4}$$

where $R(P) = P + (\alpha(P))/(\alpha'(P))$. Observe that

$$\frac{d}{dP}R(P) = 2 - \frac{\alpha(P)\alpha''(P)}{\alpha'(P)^2} > 0. \tag{A.5}$$

From (A.4), we obtain

$$\frac{\int_{\zeta}^{\infty} U'(x)xdF(x)}{\int_0^{\infty} U'(x)xdF(x)} = \frac{R(P) - s}{e - s}. \tag{A.6}$$

If we show that the expression on the left-hand side of (A.6) is greater than $R(P^*) - s/e - s$ for any choice of $\zeta \leq \zeta^*$ (see Lemma 3.5), then $P \geq P^*$ and the proof of the first two parts of the theorem will follow. The left-hand side of (A.6) can be rewritten and defined as

$$\left(1 + \frac{\int_0^{\zeta} U'(x)xdF(x)}{\int_{\zeta}^{\infty} U'(x)xdF(x)}\right)^{-1} \equiv \frac{1}{1 + G(\zeta)}. \tag{A.7}$$

We seek conditions under which $G(\zeta) \leq G(\zeta^*)$ for $\zeta \leq \zeta^*$. This is a nontrivial problem. However, we can readily prove Parts (iii) and (iv) of the theorem now. □

Proof of Part (iii). Observe from (3.10) that

$$R(P^*) = C_a^* \leq e. \tag{A.8}$$

Thus, if $e = c$, $\zeta_u^* = \zeta^* = 0$ (Lemmas 3.1(a) and 3.4) and from (A.6) $G(\zeta_u^*) = G(\zeta^*)$. Therefore, the optimal price coincides for both the risk-neutral as well as the risk-averse retailer. □

Proof of Part (iv). By the mean value theorem there exist quantities $\overline{X_\zeta}$ and $\overline{X_\infty}$ such that

$$\frac{\int_{\zeta}^{\infty} U'(x)xdF(x)}{\int_0^{\infty} U'(x)xdF(x)} = \frac{\overline{X_\zeta}\int_{\zeta}^{\infty} U'(x)dF(x)}{\overline{X_\infty}\int_0^{\infty} U'(x)dF(x)} = \frac{\overline{X_\zeta}}{\overline{X_\infty}}\frac{c - s}{e - s} \geq \frac{c - s}{e - s} \tag{A.9}$$

where, the second equality follows from (A.2) and the inequality from the fact that $\overline{X_\zeta} \geq \overline{X_\infty}$. Thus, combining this with (A.6) yields

$$R(P) \geq c \Rightarrow \alpha'(P)P + \alpha(P) \leq c\alpha'(P) \Rightarrow \alpha'(P)(P - c) + \alpha(P) \leq 0 \Rightarrow P_u^* \geq P_d^*$$

where we have used the definition of R(P) to obtain the first inequality and the analysis used in Lemma 3.3 to obtain the last inequality. □

Proof of Parts (i) and (ii). Given a utility function and the values of ζ and P that satisfy the first-order conditions for optimality for the risk-averse retailer denote and fix the quantities $\int_0^{\zeta} U'(x)xdF(x) = K_1$ and $\int_{\zeta}^{\infty} U'(x)xdF(x) = K_2$. By the mean value theorem, (A.2), (A.7), and Lemma 3.1 there exist quantities $\overline{X_1}$ and $\overline{X_2}$ such that

$$G(\zeta) = \frac{\int_0^{\zeta} U'(x)xdF(x)}{\int_{\zeta}^{\infty} U'(x)xdF(x)} = \frac{\overline{X_1}\int_0^{\zeta} U'(x)dF(x)}{\overline{X_2}\int_{\zeta}^{\infty} U'(x)dF(x)} = \frac{\overline{X_1}}{\overline{X_2}}\frac{c - s}{e - s} = \frac{\overline{X_1}}{\overline{X_2}}\frac{F(\zeta^*)}{F^C(\zeta^*)}. \tag{A.10}$$

We now determine a decreasing (we use decreasing to include nonincreasing) function $\Psi(x)$ such that Ψ is continuous, Ψ' exists and is continuous on $[0, \zeta)$ as well as on $[\zeta, \infty)$ such that the function maximizes $\overline{Y_1}/\overline{Y_2}$ subject to

$$\int_0^{\zeta} \Psi(x)xdF(x) = K_1 \text{ and } \int_0^{\zeta} \Psi(x)xdF(x) = \overline{Y_1}\int_0^{\zeta} \Psi(x)dF(x) \tag{A.11}$$

$$\int_{\zeta}^{\infty} \Psi(x)xdF(x) = K_2 \text{ and } \int_{\zeta}^{\infty} \Psi(x)xdF(x) = \overline{Y_2}\int_{\zeta}^{\infty} \Psi(x)dF(x) \tag{A.12}$$

$$\Psi(\zeta) \geq U'(\zeta) \tag{A.13}$$

$$\frac{\int_0^{\zeta} \Psi(x)dF(x)}{\int_{\zeta}^{\infty} \Psi(x)dF(x)} = \frac{F(\zeta^*)}{F^C(\zeta^*)}. \tag{A.14}$$

We desire that under the conditions of the theorem the maximum value of $\overline{Y_1}/\overline{Y_2}$ yielded by solving this problem should be less than $E[X \mid X < \zeta^*]/E[X \mid X \geq \zeta^*]$. This will imply that the conditions stated yield $G(\zeta) \leq G(\zeta^*)$ for $\zeta \leq \zeta^*$ as desired. We relax the above problem by dropping (A.14) from further analysis. Moreover, instead of maximizing the ratio $\overline{Y_1}/\overline{Y_2}$ we shall solve two problems: P1: maximize the numerator ($\overline{Y_1}$) subject to (A.11) and P2: minimize the denominator ($\overline{Y_2}$) subject to (A.12) and (A.13). It is clear that because Ψ is decreasing, P1 can be solved by letting Ψ constant, say l, on $[0, \zeta)$. Thus, $l \int_0^\zeta x dF(x) \geq \overline{Y_1} \int_0^\zeta l dF(x) = \overline{Y_1} l F(\zeta)$ or $\max \overline{Y_1} \leq E[X \mid X < \zeta]$. In order to solve P2, observe that due to (A.12), to minimize $\overline{Y_2}$ it is sufficient to maximize $\int_\zeta^\infty \Psi(x) dF(x)$ subject to (A.13). However, integrating by parts

$$\int_\zeta^\infty \Psi(x) dF(x) = \Psi(\zeta) F^C(\zeta) + \int_\zeta^\infty \Psi'(\eta) F^C(x) dx \leq \Psi(\zeta) F^C(\zeta)$$

because $\Psi(x)$ is decreasing. Thus, once again we can conclude that $\Psi(x)$ should be constant on $[\zeta, \infty)$. Thus, $\Psi(\zeta) \int_\zeta^\infty x dF(x) \leq \overline{Y_2} \Psi(\zeta) \int_\zeta^\infty dF(x) = \overline{Y_2} \Psi(\zeta) F^C(\zeta)$ or $\min \overline{Y_2} \geq E[X \mid X \geq \zeta]$. Putting the solution of P1 and P2 together, we obtain that:

$$\frac{\overline{X_1}}{\overline{X_2}} \leq \max \frac{\overline{Y_1}}{\overline{Y_2}} \leq E[X/X < \zeta]/E[X \mid X \geq \zeta]. \tag{A.15}$$

Combining (A.6), (A.7), (A.10), and (A.15) we get that the conditions stated in parts (i) and (ii) of the theorem are sufficient to guarantee that the risk-averse retailer will charge a higher price than the risk-neutral retailer. It is also possible to construct a utility function with derivatives shaped like Ψ such that the conditions are necessary for such a function. Thus, the conditions are necessary and sufficient if they have to hold for the entire *class* of concave functions. □

Proof of Lemma 4.1.

(1) $$\frac{d^2 E[(\Pi(P, S(P, \zeta), x)]}{dP^2} = \int_{x=0}^\infty ((P - c)\alpha''(P) + 2\alpha'(P)) dF(x) \leq 0$$

as $\frac{d\alpha(P)}{dP} \leq 0$ and $\frac{d^2\alpha(P)}{dP^2} \leq 0$.

(2) $$\begin{aligned} \frac{d^2 E[U(\Pi(P, S(P, \zeta), x))]}{dP^2} &= \frac{d}{dp} \int_{x=0}^\infty \frac{dU(\Pi(P, S(P, \zeta), x))}{d\Pi} \cdot \frac{d\Pi(P, S(P, \zeta), x)}{dP} dF(x) \\ &= \int_{x=0}^\infty \left(\frac{d^2 U(\Pi(P, S(P, \zeta), x))}{d\Pi^2} \cdot \left(\frac{d\Pi(P, S(P, \zeta), x)}{dP} \right)^2 + \frac{dU(\Pi(P, S(P, \zeta), x))}{d\Pi} \cdot \frac{d^2 \Pi(P, S(P, \zeta), x)}{dP^2} \right) dF(x). \end{aligned} \tag{A.16}$$

From Equation (4.2)

$$\frac{d^2 \Pi(P, S(P, \zeta), x)}{dP^2} = (P - c)\alpha''(P) + 2\alpha'(P) \leq 0. \tag{A.17}$$

The proof follows from Equations (A.16) and (A.17). □

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