A METHOD FOR STRATEGIC ASSET-LIABILITY MANAGEMENT WITH AN APPLICATION TO THE FEDERAL HOME LOAN BANK OF NEW YORK

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Strategic asset-liability management is a primary concern in today's banking environment. In this paper, we present a methodology to assist in the process of asset-liability selection in a stochastic interest rate environment. In our approach, a quadratic optimizer is embedded in a simulation model and used to generate patterns of dividends, market value and duration of capital, for randomly generated interest rate scenarios. This approach can be used to formulate, test, and refine asset-liability strategies. We present results of applying this methodology to data from the Federal Home Loan Bank of New York.

In this paper, we describe a methodology for the strategic asset-liability management (SALM) process and apply it to data from the Federal Home Loan Bank of New York (FHLBNY). "The primary goal of asset-liability management is to maximize earnings subject to acceptable levels of interest rate and liquidity risk as defined by management, (Bank Administration Institute, 1987)." To attain this goal, the SALM process must provide financial institutions the means to:

- Uncover asset-liability mismatches that could expose the bank to interest rate risk,
- Forecast earnings under multiple interest rate scenarios, and
- Assess the relative impact on income from alternative investment and funding strategies.

With these expectations in mind, we propose an integrated multiperiod planning model to assist in the SALM process. The model comprises several modules. For each period these modules provide inputs, such as the demand for funds, interest rates, and rate-sensitive cash flows, to an optimizer. The optimizer's outputs are used to update the balance sheet and to compute performance measures needed for SALM. These performance measures include dividends, net profit, and the market value of capital, computed across simulated interest rate paths. The background for the study, definitions, formulation of the problem, and analysis of numerical results are presented.

1. ENVIRONMENT

The Federal Home Loan Bank of New York is a member of the Federal Home Loan Bank (FHLB) System. The FHLB System was created in 1932 by an act of Congress to provide savings institutions with a specialized source of wholesale funds to facilitate home financing. The FHLB System comprises 12 district banks, their member institutions, and the Federal Housing Finance Board (FHFB). The 12 district Federal Home Loan Banks are both supervised and regulated by the FHFB, which is an independent federal agency in the executive branch of the United States Government. The member banks of the FHLB System are required to purchase capital stock in their district banks and in return receive dividends and access to a low-cost source of credit. The loans are provided to the member banks at a *spread* above the FHLB's cost of funds. Typically, this spread ranges from 10 to 30 basis points (one basis point is 0.01%). In more than 60 years of existence, the FHLBNY has never experienced a loan default, as evident from the Bank's AAA rating from Standard and Poor's (S&P).

The FHLBNY provides an array of financial products and correspondent services to its member banks (mostly thrifts), who in turn make mortgage funds available to consumers at reasonable rates. In addition, the FHLBNY is required by the government to subsidize Community Investment and Affordable Housing Programs that offer

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both specially priced advances for affordable housing and technical assistance to customers. The major financial products offered by the FHLBNY are loans, *interest rate swaps*, *letters of credit*, and deposits. Of these products, advances are by far the most important, comprising more than 60% of the balance sheet. Loans are available to qualified customers in maturities of up to 10 years and, upon request, for longer terms. The rates charged on loans are either fixed or variable and are governed by FHFB guidelines. The 12 district banks together issue debt securities to fund their activities and are jointly and severally liable for the debt. Traditional forms of financing include public monthly security sales, issuing securities with fixed rates, and fixed maturities through a group of designated dealers. Recently, the FHLB System has increasingly raised funds through *structured debt securities* placed via negotiated, directly placed, or competitively bid transactions. Selecting the mix of liabilities is the primary means by which the Bank controls its interest rate risk and profitability as explained below.

Selecting the mix of liabilities to match the demand for financial products is an ongoing process in any bank. For example, day-to-day decisions are made about structuring new products, funding new and existing loan demands, and pricing competitively. These decisions are normally constrained by policy guidelines obtained from the SALM process. An example of such a guideline would be to finance loans with maturity between one and two years using a specific mix of liabilities. Such a decision impacts the bank's performance not only in the short term via the cost of financial products (which is a function of the mix of liabilities used), but also in several other ways in the medium term. First, the rates for the liabilities and the assets need not change at the same time. This change in rates is also called repricing. For example, consider a \$100, 8% customer loan with a maturity of two years. Assume that the loan has been funded by borrowing an equal amount for a year at a rate of 6%. If after 12 months the cost of borrowing changes to 7%, then the profit margin on this loan will decline from 2% to 1%. This is an example of interest rate risk. The decision to finance loans using funds borrowed for a shorter maturity can also be taken consciously in anticipation of interest rates falling. The SALM process would then evaluate the risks associated with this decision for a class of financial products under different interest rate scenarios. Second, the bank could be faced with the problem of having too little cash either to meet its own obligations for maturing liabilities, or commitments made to customers (Saunders 1994, p. 294). This is an example of liquidity risk. This paper is mainly concerned with interest rate risk, and liquidity requirements are taken as policy inputs. More complex effects due to changes in interest rates are seen in the case of Mortgage-Backed Securities (MBS), and Collaterized Mortgage Obligations (CMOs), as described in Section 5.

Mix selection directly impacts profitability and risk, as shown above. It has other consequences, too. As in the example above, by matching a customer loan having a maturity of two years to liabilities with maturity of one year, the bank capitalizes on the upward sloping shape of the yield curve.[1] Thus by taking on short-term liabilities to fund long-term loans, the bank normally can realize a profit margin larger than just the spread that can be obtained by exactly matching the maturity of the loan to the maturity of the liability. However, by doing the latter the bank may not enjoy the profitability required to attract member banks to invest in equity. This leads to two related issues: (i) the impact of SALM on the market value of capital,[2] and (ii) regulatory restrictions as well as the interest rate risk associated with the size of the balance sheet.

Market Value of Capital. The market value or the net present value (NPV) of capital is the difference between the NPV of assets and the NPV of liabilities. One of the goals of the SALM process is to maintain or increase the NPV of capital over time, while paying out reasonable dividends to its shareholders. A change in interest rates could affect the NPV of assets differently compared to the NPV of liabilities because of the mismatch in funding. In the example given earlier, the cash flows from the \$100 customer loan and the matching liability of \$100 occur at different points in time. Suppose interest rates change a week after this transaction. Then the discounted value of these cash flows will change differently depending on how the two-year and one-year rates change—thus affecting the NPV of capital.

Size of the Balance Sheet. The size of the balance sheet is given by the ratio of assets to capital. The greater this ratio, the larger is the scope in the short term for making profits because with a larger size of the balance sheet, the bank can accommodate a larger amount of customer loans in its asset portfolio, thus earning more revenue while keeping the value of capital constant (an illustration is given in the appendix). Such an approach to earning higher profits can sometimes be disastrous, because a small adverse change in the present value of assets relative to liabilities can drive the NPV of capital below zero. Adverse changes are always probable in practice because assets and liabilities are rarely perfectly matched. Further illustration of the impact of size on performance in the short term and long term are given in Sections 4 and 5.

The complexity and uncertainty in the SALM process stem not just from interest rate changes. In recent years, the FHLBNY has encountered numerous challenges due to financial and structural changes, such as:

- The radical contraction of its primary membership base, namely the savings and loan industry;
- The encroachment of Wall Street firms who have made competitive derivative financial products available to member banks;
- Increased volatility of interest rates;

- The growth of the *secondary mortgage market*, which has provided member banks with an alternative source of liquidity; and
- The *FIRREA Act of 1989*.

The combined effect of these changes has been to lower income and consequently to underscore the importance of asset-liability management at the FHLBNY. SALM has become a primary concern in the financial services industry, as highlighted in the study by CSF Boston (1994).

2. DEFINITIONS AND LITERATURE REVIEW

In this paper we focus on a specific SALM problem: *to maximize earnings over a finite time horizon, given a collection of assets, liabilities, investment prospects, a forecast of future demand for funds, subject to acceptable levels of risk, government regulations and policy directives from the bank's board*. Different aspects of this problem have been studied and reported in the literature. Two central ideas found in all these studies are the notion of duration and the use of optimal portfolio theory (Markowitz 1952, Sharpe 1964). We make extensive use of these concepts and hence begin by giving brief descriptions of these general ideas.

Duration. Duration, in its simplest form (Macaulay 1938) is defined as follows. Let X_i be the cash flow from a security, in period i, $i = 1, 2, \ldots, N$. Let r be the current interest rate for all values of maturity, i.e., assume that the yield curve is flat. Then the duration of the security is given by

$$D = \left(\sum_{i=1}^{N} \frac{iX_i}{(1+r)^i} \right) \Big/ \left(\sum_{i=1}^{N} \frac{X_i}{(1+r)^i} \right).$$

In this definition, duration is the weighted average life (hence the name) of the cash flows from the security, in which the weight assigned to the life of i of the cash flow X_i is proportional to the NPV of X_i divided by the NPV of the entire cash flow stream. However, yield curves are not flat and normally are found to be upward sloping to reflect, among other things, the higher risk associated with the increasing term of the loan. The Macaulay measure of duration was therefore generalized by Fisher and Weil (1971) to account for a nonflat yield curve, by defining

$$D = \left(\sum_{i=1}^{N} \frac{iX_i}{(1+r_i)^i} \right) \Big/ \left(\sum_{i=1}^{N} \frac{X_i}{(1+r_i)^i} \right),$$

where r_i, $i = 1, 2, \ldots, N$, is the rate for the ith period. For this measure, the negative of duration ($-D$) gives the percentage change in the NPV for a *parallel* shift in the yield curve.

It has been shown that by correctly choosing the durations of assets and liabilities, called duration matching, the elasticity of the NPV of the portfolio to *parallel* shifts in the yield curve becomes zero (see Bierwag et al. 1983, Ho 1992, and Saunders 1994). On the regulatory side, the Office of Thrifts Supervision (OTS), which is the primary regulatory agency for most federal and state chartered savings associations and federal savings banks, requires analysis of risk exposure using parallel shocks to the yield curve.[3]

Apart from the criticism that duration is a sensitivity measure to parallel shifts of the yield curve, two shortcomings of duration matching are that (1) duration is a static measure because portfolios once immunized using duration matching need not remain balanced as time progresses, and (2) duration matching does not trade off risk versus return, as discussed below. Additionally, duration gives just the first-order impact of a change in interest rates and ignores second-order effects, and for certain instruments with embedded options the cash flows themselves could change as a result of a change in rates. More recent innovations address some of these problems by using *effective* duration, which is defined as the average change in the NPV of the security computed using rate-dependent cash flows along multiple interest-rate paths (OTS 1989 and Mulvey and Zenios 1994).

Despite these limitations, duration remains a simple, easily computed, powerful concept and is widely understood in the industry. The analysis of the SALM process given in this paper was partly motivated by some of these shortcomings of duration matching.

Portfolio Optimization. Duration matching does not explicitly make the tradeoff between risk and return, except under restrictive assumptions (see Grove 1974 and Prisman and Tian 1993). Researchers have used the notion of an efficient risk-versus-return frontier (Markowitz 1952 and Tobin 1958) to portray the tradeoffs. The tradeoff frontier is typically developed by solving a single-period portfolio optimization problem in which the objective is to minimize the variance of the change in value of the portfolio when subjected to a random shock (Elton and Gruber 1991).

In the multiperiod approach to portfolio optimization, interest-rate paths (or scenarios) are given and the objective is to maximize the expected return under the different scenarios (see Booth et al. 1994, Carino et al. 1994, and Mulvey 1994). This version of the portfolio selection problem can be solved using stochastic programming. Examples of successful applications can be found in Berger and Mulvey (1998), Boender (1997), Dempster (1998), Mulvey (1989, 1996), Wilkie (1995), Ziemba and Mulvey (1998), and Zenios (1993). The size of the problem was the major hurdle to using a stochastic program in our case; therefore, we used a framework similar to the one described in Mulvey and Zenios (1994).

Mulvey and Zenios (1994) suggested a framework for managing a portfolio of fixed income securities. They recommend a two-step approach: Step 1 obtains the correlation of prices or rates of return, and Step 2 uses an optimization model to select a portfolio. Two of the key conclusions reached by Mulvey and Zenios are that (1) correlation of the change in prices of different securities is

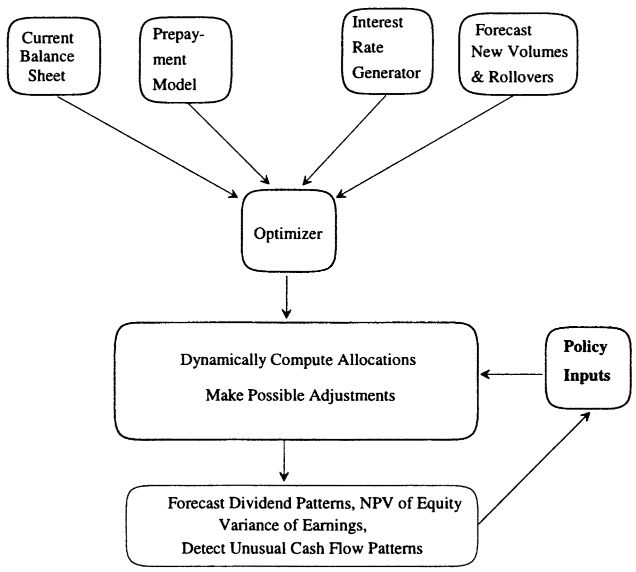


Figure 1. Integrated strategic asset-liability planning.

important, and (2) correlation changes over time and must be updated. They illustrate the use of this framework in a multiperiod setting by solving the single-period problem in each period and then stepping forward in time. While this approach reduces the computational burden, there are pros and cons to using this method as discussed in Mulvey and Zenios's paper. Our methods have many features in common with the approach of Mulvey and Zenios.

Other Techniques. Although a large number of techniques for portfolio optimization exist and are being used in bond portfolio management (Fabozzi 1993), the use of such methods for asset-liability management is not as common. However, risk management has received considerable attention in recent years. For example, Risk Metrics (1994) provides users with a unified way to measure and report risks associated with movements in currency, interest rates and equity markets. There are many applications of simulation to asset-liability management as well (e.g., see 1987 and CSF Boston 1994). In these applications, interest rate scenarios are given as inputs, and the performance across different scenarios compared.

3. INTEGRATED STRATEGIC ASSET-LIABILITY PLANNING

The SALM process can be conceptualized as shown in Figure 1. We have used the ideas discussed in Section 2 in developing this process model for SALM. A brief description of our overall approach called "Integrated Strategic Asset Liability Planning" is given below. Details of model formulation and some of the modules are given in Sections 4 and 5.

Figure 1 depicts five key modules as well as management policy as providing the inputs required to update the balance sheet each period. The existing balance sheet of the bank contains the details of the assets and liabilities. By delving into the security level information that makes up the balance sheet, we obtain the future cash flows and obligations arising from all assets and liabilities, excepting MBSs. For mortgage-backed securities, the cash flows will depend on the interest rates because the rate of mortgage prepayments will be different under different interest-rate scenarios.[4] The Prepayment module provides the cash flow forecasts for MBS. The Interest-Rate Generator produces the interest-rate paths, along which the dynamic behavior of the balance sheet and earnings are studied and compared. The Forecast module provides the demand forecast for new loans as well as the forecast of how the existing loans will behave (or roll over) when they mature. The policy inputs in our case include the maximum size of the balance sheet, limits and restrictions on the volume as well as features of MBS, limits on the duration of the bank's capital, dividend payout policy, desired level of earnings, and cash requirements. Policy inputs can be changed based on outputs from the model, and the model can be rerun to evaluate alternate strategies.

We visualize the SALM process as taking place in discrete time, with each period being two to four weeks in length. Consider the problem faced at the beginning of each period by a decision maker. During the previous period some assets have left the balance sheet because of loan repayments and maturing liabilities. Some liabilities have become due and leave the balance sheet. Interest income will be received, and interest has to be paid out by the bank.

After these adjustments have been made, new customer loans have to be made, i.e., new assets (created either as a result of new loans or maturing loans rolling over) are added to the balance sheet. Once these accounting adjustments have been made, the decision maker must decide upon the mix of liabilities and investments to fill the gaps created in the balance sheet as a result of these changes. The optimizer selects the mix of liabilities and investments to add to the balance sheet according to some decision criteria. Once the mix has been selected, time can be indexed forward and the whole sequence repeated. At the end of the planning horizon, results are summarized and analyzed over several interest-rate paths.

The optimizer's outputs also provide insights into the current structure of the balance sheet. Therefore, both for ease of explaining the single-period versus the multiperiod formulations and for describing the different insights provided by each, we discuss the two formulations separately in Sections 4 and 5. The single-period model is to assist in uncovering asset-liability mismatches, carrying out limited simulations of interest rate movements, and identifying worst case scenarios. Multiperiod simulations are to carry out comprehensive multiperiod scenario analysis, to suggest investment and liability selections, evaluate interest rate risk, and uncover key operating variables.

4. THE SINGLE-PERIOD MODEL

4.1. Formulation

We are given a balance sheet consisting of a set of assets and liabilities that are already committed at time t, a target value for the return, and a set of policy inputs such as limits on the size of the balance sheet and the duration of capital. The problem is to choose investments and the mix of liabilities to match the assets. Based on discussions with the bank's managers, we chose the objective of minimizing the variance in the change in market value of capital. This objective is identical to utility maximization if (1) the distribution of the random market value is Normal, or (2) utility functions are quadratic, or (3) the uncertainty in market value is of a sufficiently small order of magnitude such that third and higher moments can be neglected (see Elton and Gruber 1991, p. 212). The variance in the market value obtained in our numerical results, even under extremely volatile scenarios, is small (see Figure 4) and additionally justifies the choice of the minimum variance criterion.

The alternative objective of maximizing utility had two drawbacks in our setting: (1) We needed to develop a utility function of a group of managers who act together in the SALM process, and (2) maximizing utility for the current period does not guarantee that the next period's performance will be better due to the decisions in this period. In contrast, by reducing the variability we find that the path of the balance sheet is kept within a narrow band of preferred values, as required by the bank (see Section 5 and Mulvey 1994, §3.1). Recent developments by Bell (1995) and Jia and Deyer (1995) have made it easier to construct a Von Neumann Morgenstern (VM) utility function. Moreover, the VM utility function handles skewed returns in a "natural" fashion, whereas the mean/variance approach assumes that the distribution of returns is close to normal. The distribution of the returns is not close to normal in our case.

The proposed approach for solving the single-period problem is similar to the approach used in standard portfolio theory. The single-period model is repeatedly solved to determine the selection of assets and liabilities for each period in the multiperiod problem; accordingly, all variables and constants in the single-period model are indexed by time t. We introduce the following notation:

T = Life of balance sheet (number of time periods).

n_a = Number of different asset types in which the bank can invest.

n_1 = Number of liabilities types used by the bank to raise funds.

$ac_j(t)$ = Cash inflow in period $(t + j)$ from assets already committed by period t, $j = 1, 2, \ldots, T$.

$lc_j(t)$ = Cash outflow in period $(t + j)$ generated from liabilities already committed by period t.

$cf^a_{k,j}(t)$ = Cash inflow in period $(t + j)$ from one dollar invested in asset type k at time t, $k = 1, 2, \ldots, n_a$, $j = 1, 2, \ldots, T$.

$cf^l_{k,j}(t)$ = Cash outflow in period $(t + j)$ from one dollar raised using liability type k at time t, $k = 1, 2, \ldots, n_1$, $j = 1, 2, \ldots, T$.

$\bar{y}^{ac}(t)$ = Average yield from assets already committed at period t.

$\bar{y}^{lc}(t)$ = Average yield from liabilities committed at period t.

$f^a_k(t)$ = Amount invested in asset type k at time t, $k = 1, 2, \ldots, n_a$.

$f^l_k(t)$ = Amount raised using liability type k at time t, $k = 1, 2, \ldots, n_1$.

tc^a_k = Transactions cost per dollar invested in asset type k, $k = 1, 2, \ldots, n_a$.

tc^l_k = Transactions cost per dollar raised using liability type k, $k = 1, 2, \ldots, n_1$.

$K(t)$ = Bank's total capital at time t, including all reserves and undistributed dividends.

$R(t)$ = Target return desired by bank at time t.

$r^*_j(t)$ = Treasury spot rate at time t for a loan of j periods, $j = 1, 2, \ldots, T$.

$sp^a_j(t)$ = Spread on Treasury spot rate $r^*_j(t)$ that the bank charges at time t to customers for a loan of maturity j, $j = 1, 2, \ldots, T$.

$sp^l_j(t)$ = Spread on Treasury spot rate $r^*_j(t)$ that the bank has to pay at time t to raise a loan of maturity j, $j = 1, 2, \ldots, T$.

$r_j(t) = r^*_j(t) + sp^l_j(t)$ = cost of funds for the bank at time t for a maturity of j periods, $j = 1, 2, \ldots, T$. (The collection of $r_j(t)$'s is the yield curve.)

$y^a_k(t) = r^*_j(t) + sp^a_j(t)$ = average yield from investment in asset type k, $k = 1, 2, \ldots, n_a$, where j is the maturity of asset type k.

$\Delta r_j(t) = r_j(t + 1) - r_j(t)$ = random change in cost of funds for period j between times t and $(t + 1)$.

ρ_{ij} = Covariance of $\Delta r_i(t)/(1 + r_i(t))$ and $\Delta r_j(t)/(1 + r_j(t))$ (assumed to be stationary).

$\mathrm{npv}^{ac}(t)$ = Net present value of committed assets at time t.

$\mathrm{npv}^{lc}(t)$ = Net present value of committed liabilities at time t.

$d^a(t)$ = Duration of all assets at time t (derived in the quadratic program).

$d^l(t)$ = Duration of all liabilities at time t (derived in the quadratic program).

$E(t)$ = Market value of capital at time t (derived in the quadratic program).

$\Delta E(t)$ = Change in market value of capital due to a change in rates between periods t and $(t + 1)$.

BS = Maximum size of balance sheet expressed as a multiple of the $K(t)$.

$a\text{gap}(t)$ = Gap in assets at time t to be filled using investments in asset types, $k = 1, 2, \ldots, n_a$.

$l\text{gap}(t)$ = Gap in liabilities to be filled by raising funds using liabilities of type, $k = 1, 2, \ldots, n_1$.

$A(t), L(t)$ = Book value of total committed assets and liabilities at time t.

The following relations help in simplifying the statement of the single-period problem:

$$\text{npv}^{ac}(t) = \sum_{j=1}^{T} \frac{ac_j(t)}{(1 + r_j(t))^j}, \tag{1}$$

$$\text{npv}^{lc}(t) = \sum_{j=1}^{T} \frac{lc_j(t)}{(1 + r_j(t))^j}, \tag{2}$$

$$a\text{gap}(t) = [BS \times K(t) - A(t)]^+, \tag{3}$$

$$l\text{gap}(t) = A(t) + a\text{gap}(t) - L(t) - K(t), \tag{4}$$

$$a_j(t) = \left(ac_j(t) + \sum_{k=1}^{n_a} f_k^a(t) cf_{k,j}^a - lc_j(t) - \sum_{k=1}^{n_a} f_k^l(t) cf_{k,j}^l\right) \Big/ (1 + r_j(t))^j, \tag{5}$$

$$E(t) = \sum_{j=1}^{T} a_j(t), \tag{6}$$

$$\Delta E(t) \approx -\sum_{j=1}^{T} j a_j(t) \frac{\Delta r_j(t)}{(1 + r_j(t))}. \tag{7}$$

In Equation (5), $a_j(t)$ is the NPV of the cash flow in period j. We assume that the change in the interest rate in a period will be small and therefore will not affect the cash flow patterns from mortgage-backed securities. In the multiperiod simulations, this assumption is no longer valid, and a prepayment model is necessary. A simple example in the appendix illustrates yield curve, spread, balance sheet size, return on equity, and calculation of the duration of a portfolio. The single-period problem, denoted as **P**, can now be formally stated as:

$$\mathbf{P}: \quad \min \sum_{i=1}^{T} \sum_{j=1}^{T} ij a_i(t) a_j(t) \rho_{ij}$$

subject to

$$\sum_{k=1}^{n_a} f_k^a(t) = a\text{gap}(t), \tag{8}$$

$$\sum_{k=1}^{n_l} f_k^l(t) = l\text{gap}(t), \tag{9}$$

$$\text{npv}^{ac}(t)\bar{y}^a(t) - \text{npv}^{lc}(t)\bar{y}^l(t) + \sum_{k=1}^{n_a} f_k^a(t) y_k^a(t) - f_k^l(t) y_k^l(t) \geq R \times K(t),$$

$$f_k^a(t), f_k^l(t) \geq 0. \tag{10}$$

The objective is to minimize the variance in the market value of capital, Var$[E(t)]$. The decision variables, $f_k^a(t)$s and $f_k^l(t)$s, are the amounts of assets of n_a types and liabilities of n_1 types to add to the balance sheet. The values of these variables must sum up to fill the gaps in the balance sheet; see Equations (8) and (9). Constraint (10) requires the *average* yield to exceed a desired rate of return, R (also see assumptions below). In practical applications, other types of constraints must also be addressed. Our model allows for such constraints as detailed below.

- **Constraints on Distribution:** The bank could restrict the allocations to different asset and liability types as follows:

$$f_k^a(t) \leq U_k^a a\text{gap}(t), \quad k = 1, 2, \ldots, n_a,$$
$$f_k^l(t) \leq U_k^l l\text{gap}(t), \quad k = 1, 2, \ldots, n_l, \tag{11}$$

where U_k^a, U_k^l are the permitted proportions of different asset and liability types. This constraint is useful in the multiperiod context, because it eliminates opportunistic behavior, smoothes out the cash flows, and diversifies the portfolio.

- **Constraints on the Duration of Capital:** Constraints can be added on the duration of capital, such as:

$$\sum_{j=1}^{T} j a_j(t) \leq D_{\max} \sum_{j=1}^{T} a_j(t), \tag{12}$$

where $D_{\max}$ is the maximum allowed duration. The addition of inequalities (11) and (12) helps in controlling interest-rate risk, and conforms to accepted asset-liability management policies at the bank. (Also see discussion on regulatory aspects related to interest rate risk measurement in Saunders 1994, p. 143). If it were desired to model the sensitivity of the duration to changes in key rates (Ho 1992), inequalities of the following form could be added:

$$|j a_j(t)| \leq D_{\max}^j \quad \text{for selected values of } j.$$

- **Constraint on Convexity:** Equation (7) captures just the first-order approximation of the change in the market value of capital (i.e., the first term in a Taylor series expansion). For relatively large changes in the rates, a second-order approximation to the change in NPV is preferred because in contrast to first-order effects, which nearly cancel out when durations of assets and liabilities are very close to one another, second-order effects could add up. To specify the convexity constraint, it is useful to assume a worst-case scenario of changes in rates, such as a 200 basis point shock. This can be used to impose a maximum allowed change in the market value of capital as follows:

$$-\Delta E^{\max} \leq \sum_{j=1}^{T} a_j(t) \left(\frac{j \Delta r_j^{\max}}{(1 + r_j(t))} - \frac{1}{2} \frac{j(j+1)(\Delta r_j^{\max})^2}{(1 + r_j(t))^2} \right) \leq \Delta E^{\max}, \tag{13}$$

where $\Delta r_j^{\max}$, $j = 1, 2, \ldots, T$ are the maximum expected changes in rates[5] and $\Delta E^{\max}$ is the ceiling on the change in the market value of capital.

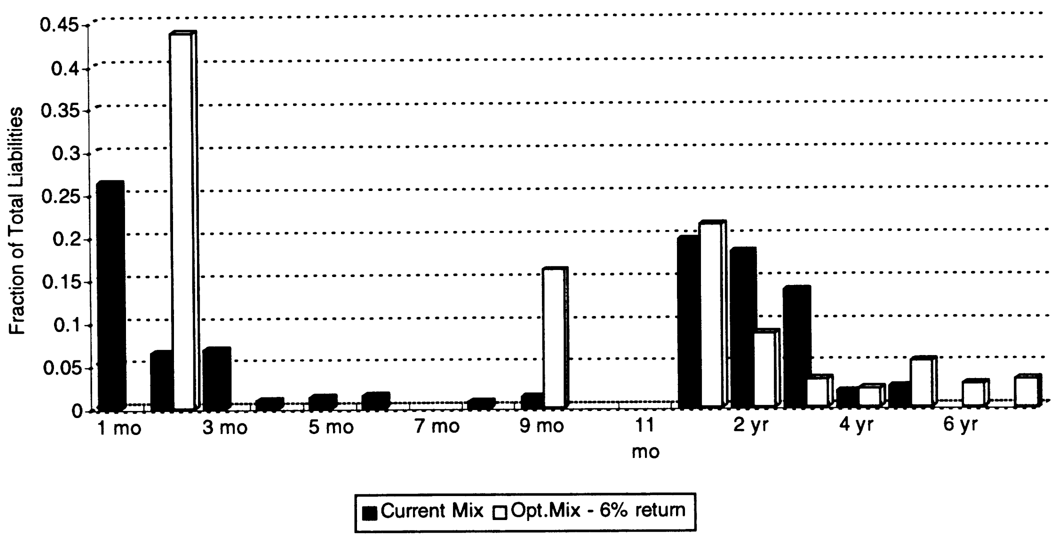


Figure 2. Current versus optimal mix.

- **Transaction Costs:** Restructuring the portfolio will incur transaction costs. This effect can be captured by modifying (10) as follows:

$$\mathrm{npv}^{ac}(t)\bar{y}^{a}(t) - \mathrm{npv}^{lc}(t)\bar{y}^{l}(t) + \sum_{k=1}^{n_a} f_k^a(t)(y_k^a(t) - tr_k^a)$$

$$- \sum_{k=1}^{n_l} f_k^l(t)(y_k^l(t) + tr_k^l) \geqslant RK(t). \tag{14}$$

The assumptions underlying the formulation of **P** are as follows:

Assumption 1. *The covariance matrix ρ_{ij} is assumed to be stationary; see Mulvey and Zenios* (1994) *and the next section for a related discussion.*

Assumption 2. *The objective function ignores second-order effects of rate changes. This has been found to be reasonable when the change in rates is small, say up to* 50 *basis points. Constraint* (13) *may be necessary for larger changes in the rates.*

Assumption 3. *Yields are averaged for ease of formulation. As a result, the actual return may differ from the target rate of return. Numerical results given below indicate that this assumption does not substantially affect the average rate of return in practice.*

It may be verified that the above three assumptions are valid if the planning period is sufficiently small.

The data necessary to solve **P** were provided by the FHLBNY, and include the balance sheet, the cash flows from various securities, the spot rates and spreads, and policy variables such as the required rate of return, size of the balance sheet, maximum duration, and the permitted distribution (allocation) of assets and liabilities. The ρ_{ij}s were computed using the assumption of stationarity and daily data on spot rates for 30 weeks.

4.2. Computational Results

We implemented our own algorithm for solving the quadratic program **P**. This algorithm was coded in FORTRAN 77 and tested on an IBM-compatible PC. Three experiments were performed to study quality of the solution obtained using the single-period model. In each experiment, it is assumed that *all* the existing liabilities can be reconfigured (i.e., there are no committed liabilities) and that all assets, with the exception of investments, are committed. Four different mixes were obtained from solving **P**. Three of these mixes correspond to the current size of the balance sheet and target rate of returns of 6%, 7.5%, and 10%. The fourth mix, denoted as BIG10%, was obtained using a larger balance sheet with a 10% target rate of return. In the first experiment, the four optimal mixes were compared with each other and with the present balance sheet. Remarkably, the optimal mix for different values of the target return were found to be similar. The solutions from **P** indicate that the target return can be achieved using a smoother allocation (i.e., a more evenly spread allocation of liabilities across all maturities) compared to the distribution of liabilities in the existing balance sheet; see Figure 2. On viewing these results, the risk managers at the bank concurred that solving **P** helps in identifying mismatches between assets and liabilities, a primary task of SALM.

In the second experiment, the solutions were tested for sensitivity to interest rate changes using standard techniques. The balance sheet was subjected to three types of random rate changes:

- 1000 random parallel shifts of the spot curve with the magnitude of the shift distributed uniformly in the interval [−50, +50] basis points,
- a single random shock of +50 basis points,
- a skew of −100 basis points in the first 64 months and +100 basis points in months 65 to 360.

Table I
Single-Period Model—Sensitivity Analysis

	Target Return				
	6%	7.5%	10%	BIG 10%	Current
Duration of liabilities (months)	13.26	12.95	11.69	13.15	12.44
Average change in market value* (xxx dollars)	−0.01	−0.01	−0.02	−0.01	−0.01
Std. deviation of change in market value* (xxx dollars)	6.7	8.9	17.45	6.1	12.5
Change in market value due to a shock of +50 basis points	−5.8	−7.7	−15.1	−5.3	−10.1
Change in market value due to a skew of −100 basis points in months 1 through 64, and +100 points for remaining	−4.67	−11.35	13.28	−8.66	−28.21

*1000 simulation runs of a random parallel shift uniformly distributed over [−50, +50] basis points.

The results of the experiment are summarized in Table I. As expected under random and parallel shifts, the average change in the market value of capital was close to zero (because the expected value of the shift is zero). The standard deviation of the change in value of capital is more significant and of greater interest.

Table I shows that the standard deviation increases with the rate of return, indicating a risk versus return tradeoff. The BIG10% solution yields the lowest standard deviation. Apparently financial leverage counters interest-rate risk for our data. This phenomenon occurs in our case for two reasons: (1) a larger balance sheet size allows greater scope for matching the durations of liabilities to the duration of assets (portfolio diversification), and (2) a larger balance sheet allows us to mismatch assets and liabilities just slightly and obtain the desired return while keeping the duration of the portfolio small (see appendix, and a similar result in the multiperiod case). Table I also shows the effects of a parallel shift and a skew in the spot curve. Compared to the existing balance sheet, the change in the market value of capital for the "optimal" portfolios is smaller for both types of shifts, indicating that the "optimal" mixes of liabilities match assets better than the bank's current portfolio of liabilities.

In the third experiment, we studied whether the method for estimating the covariance affects the results. The covariance matrix, (ρ_{ij}), was calculated in two different ways: (1) by using the daily, and (2) the weekly changes in the spot curve. The different estimates were then used for solving **P**. The results obtained were identical regardless of the estimation method.

As our experiments show, the solutions obtained using the single-period model help identify mismatches in the portfolio, provide immunization under different types of movements in the spot curve, and are insensitive to the method used for estimating (ρ_{ij}). We conclude that the performance of the single-period model is sufficiently robust to be used in the multiperiod context. The model can also be used on a stand-alone basis for carrying out rapid "what-if" analysis. Once the data for **P** has been set up, it is easy to carry out simulations in a spreadsheet; the entire exercise as described above can then be completed within two hours.

5. THE MULTIPERIOD SIMULATIONS

We project the balance sheet forward in time using the simulation/optimization algorithm shown in Figure 3. The model has five input modules, and additional inputs are required in the form of policy guidelines. In each period, the single-period model is used to determine the asset and liability selections. These selections are then employed to update the balance sheet, collect statistics, and update future cash flows. We describe the modules below and then describe the results.

Step 0. Read balance sheet and rate information, policy and other inputs, length of simulation (NSIM) and number of replications (NREP).

Step 1. Do NREP times:
Simulate for NSIM time periods. In each time period:
- 1.1 Get New Spot Rates
- 1.2 Update levels of existing assets and liabilities (including rollovers). Add *new* volumes of assets and capital stock. Reprice assets and liabilities, compute prepayment flows. Compute maximum MBS allowed based on total capital and add more MBS if needed. Compute profit, and calculate dividends on quarterly basis if any.
- 1.3 Compute the inputs to the single-period model, namely the gaps in assets and liabilities, the durations and the cash flows for existing and new assets and liabilities. Compute other inputs such as yields that are used to generate the constraints.
- 1.4 Solve for the optimal mix to fill the gaps using the quadratic optimizer.
- 1.5 Add the assets and liabilities produced as output from the optimizer to the balance sheet, and collect statistics.

Figure 3. Algorithm for the multiperiod simulations.

5.1. Description of Modules

The Interest-Rate Generator. The rate generator provides the Treasury spot curve for each period. A variety of interest-rate models are available in the literature (see Hull 1993 for a review). We employ two different procedures for generating interest rates, namely an empirical approach and the Heath-Jarrow-Morton (HJM) arbitrage-free two-factor rate generator (Heath et al. 1992, which we shall abbreviate as HJM 1992). The simulations using these two approaches are labeled EMP and HJM, respectively. We now describe the two approaches.

EMP. Let $\hat{r}_j(t)$ be the spot rate generated in period t, $j = 1, 2, \ldots, T$. Then the spot rate for period $(t + 1)$ is given by

$$\hat{r}_j(t+1) = \hat{r}_j(t) + \text{volatility} \times (r_j^*(M+1) - r_j^*(M)),$$
$$j = 1, 2, \ldots, T,$$

where volatility is a constant and M is a randomly chosen integer between 1 and 360. We have used the empirical data on spot rates for the previous 360 weeks for the values of the r_j^*s. The use of such an empirical approach is not uncommon (see for example OTS 1989 and Litterman and Scheinkman 1991). The empirical approach is appealing because in some sense we are testing the model using historically observed data. There are other reasons, too, for using this method. Theoretical interest rate generators need not duplicate shocks, twists, or increased curvature of the yield curve observed in practice. The immunization of a portfolio to such shocks is often critical in practice.

HJM. We have used a two-factor version of the Heath-Jarrow-Morton rate generator (HJM 1992). Definitions of some of the technical terms are given in the glossary; for space considerations we concentrate on the algorithmic details. Let $f(t, T)$ be the instantaneous forward rate, as seen at time t, for the period $[T, T + dT]$. Informally, the forward rate is the cost of borrowing or lending funds at time t for the period $[T, T + dT]$. Let W_1 and W_2 be two standard independent Brownian motions (BMs), and let the volatility associated with these BMs be σ_1 and $\sigma_2 e^{-\lambda(T-t)}$. The drift factor (see Equation (16)) is denoted as $\alpha(t, T)$, and it is calculated in the model. The movement of the forward rates is governed by

$$df(t, T) = \alpha(t, T)dt + \sigma_1 dW_1(t) + \sigma_2 e^{-\lambda(T-t)} dW_2(t), \tag{15}$$

and

$$\alpha(T, t) = \sigma_1^2(T - t) + \sigma_2^2 e^{-\lambda(T-t)}(1 - e^{-\lambda(T-t)})/\lambda. \tag{16}$$

The drift correction given in (16) is the sufficient condition to guarantee that the rates generated are arbitrage free, i.e., an investor trading interest rate securities cannot realize nonzero profits under all interest rate scenarios (HJM 1992). When the HJM methodology is used to price derivative securities, the parameters σ_1, σ_2, and λ are chosen to fit observed interest rate option prices (Hull 1993, Jarrow and Turnbull 1996). We used historically observed values to derive a range for these parameters, choosing the range of [0.01, 0.04] for σ_1, and σ_2 and selecting λ in the range of [0.2, 0.4]. The spot rates are related to the forward rates as follows:

$$\hat{r}_j(t) = \frac{\left(\int_t^{t+j} f(t, u)\, du\right)}{j}. \tag{17}$$

The Prepayment Module. Mortgage prepayments are affected by rate changes. Prepayment models used for pricing MBS are usually multifactor models that have been adapted to specific lending environments. Factors such as economic indices, demographic indices, interest rates, and the terms and conditions of the particular instrument are used to predict the prepayment rates along simulated interest rate paths. Such models can be extremely complex and are useful when working with security-level data. For our purpose, some degree of aggregation was necessary due to the time and cost involved in developing a detailed prepayment model.

We chose to use the OTS methodology for estimating the value of mortgages and mortgage-backed bonds and securities. We use this model for several reasons: (1) The model deals with weighted average balances of MBS instead of security-level data; (2) MBS constitute only about 10–15% of the bank's portfolio; (3) for strategic planning purposes we can vary the parameters of the prepayment model (see below) and carry out sensitivity analysis if necessary; and (4) the OTS methodology is used to assess risk in bank portfolios across the United States. Figlewski et al. (1992) depict the prepayment forecasts made by 10 different banks for the same security. While there is some variance across these forecasts, the *shape* of prepayments over time is similar for all 10 forecasts. This similarity in shape supports the aggregation approach used in the OTS model.

The OTS model classifies the MBS into subsets, depending on the type, coupon rate, and average remaining life. The type of mortgage includes details such as whether the mortgage is for 15 or 30 years, and whether the mortgage rate is fixed or floating. The coupon rate is the rate paid on the mortgage, and average life denotes the weighted average remaining life to maturity (OTS 1989). The prepayment at time t for many of the mortgage types described in the OTS model is characterized by

$$\text{Prepayment rate}(t),\ \text{cpr}(t) = \text{seasoning}(t) \times \text{seasonality}(t) \times \text{refinance}(t), \tag{18}$$

$$\text{seasoning}(t) = 0.0333 \text{ for a new mortgage and increases linearly to } \textit{one} \text{ at month 30}, \tag{19}$$

$$\text{seasonality}(t) = 1 + 0.2 \sin\left(1.571\left(\frac{\text{month} + t - 3}{3}\right) - 1\right),$$

$$\text{refinance}(t) = a - b \arctan \left[c \left(d - \frac{\text{coupon rate}}{\begin{array}{c}\text{5 year simulated refinancing}\\ \text{rate lagged by 3 months}\end{array}} \right) \right], \tag{20}$$

Monthly prepayment rate,

$$\text{month}(t) = 1 - (1 - \text{cpr}(t))^{1/12}. \tag{21}$$

There are four parameters in this model, all of which are found in Equation (20). These parameters lie in the range $a \in [0.1173, 0.3135]$, $b \in [0.0705, 0.1831]$, $c \in [3.130, 6.818]$, and $d \in [1.041, 1.093]$. Given the variety of securities covered in the OTS model, the variation in parameters is indeed small. Based on the data obtained from the bank, we used just two different types of MBS, namely 30-year fixed-rate MBS with a weighted average remaining life of 55 months, and floating-rate MBS that reprice every month, to broadly represent the MBS portfolio. We assume that (1) fixed MBS will always be available for purchase such that we can maintain the average remaining life of MBS to be 55 months, and (2) we can purchase new MBS at a fixed spread above the cost of borrowing. The weighted average coupon (WAC) rate is recomputed using the opening balances and new volumes at each step of the simulation.[6] Equations (18)–(21) give the cash flows arising from prepayments based on the index rates prevailing three periods before. These cash flows and earnings from MBS are then input to the single-period model. This module gives us a starting point for analysis (as is typical in developing any aggregate plan).

The Balance Sheet. The size of the current balance sheet of the bank is over \$20 billion. We aggregated the line items into 41 classes of assets and 21 classes of liabilities. There are nine broad categories of assets: short-term advances, long-term advances, London Interbank Offered Rate (LIBOR) advances, variable advances, term Fed funds, MBS pools, MBS, DDA (overnight) loans, and commercial paper. Forecasts are provided for three of these categories, namely the short-term, long-term, and LIBOR advances. The levels of floating and fixed MBS are bounded above as fixed multiples of capital stock. MBS are the most profitable category of investments. As MBS run off they are replaced to the maximum extent possible by fresh MBS. Decisions on investment in term Fed funds are provided by the optimizer. The remaining asset categories are either kept constant or allowed to drop off the balance sheet as they mature. The categories of liabilities include overnight deposits, term deposits, discount notes, regular bonds, and repricing bonds. The mix of regular bonds and discount notes to be added to the balance sheet is determined using the optimizer. All other liabilities are either kept at a fixed level or allowed to drop off the balance sheet as they mature.

Forecast Module. This module contains the summary of forecast data on future demand for loans, pricing and cost of funds, and the projection of capital stock. The data are the result of internal forecasting exercises carried out within the bank and are used regularly by the bank for planning purposes. In the case of the FHLBNY, the loans are priced to be attractive to the member banks, and the amount of loan available to a member bank is a function of its contribution to equity. The period-by-period forecast of new capital stock is included in the data provided by the bank and is based on the projected volumes of loans. To update the balance sheet each period, we use the given data on (1) the volume and mix of new loans (attributable to new capital stock) and (2) information on how each class of maturing assets will roll over (or leave) into different asset categories, in each period. To see the need for the latter, observe that when a loan given to a customer for five years matures, she has the option to renew the loan either partially or completely, and may request funds for the same or a different value of maturity. The spreads on assets and liabilities are also fixed and are given as data for each class of assets and liabilities. Thus to obtain the rates for liabilities and assets we add the given spread to the value on the spot curve for the appropriate values of maturity. While we did not do so, standard forecasting and pricing routines can be incorporated into this module.

Optimizer. The optimizer is the single-period model described earlier. Linear constraints are imposed to incorporate factors such as the desired return on capital, upper and lower bounds on the duration of capital, the maximum investment permitted in fixed and floating MBS, a target fraction of available funds to be invested in term Fed funds, and the target fraction of funds that must be raised in the form of notes and bonds. When using the empirical approach, EMP, we do not update the covariance matrix in the objective function; but when HJM is used to generate the rates, the estimate of the covariance matrix (ρ_{ij}) is updated every six months, using the last 40 periods' data.

Policy Inputs. The policy inputs include the maximum size of the balance sheet, the maximum amount of MBS allowed, the proportion of floating to fixed MBS, the required rate of return, the duration of capital, the spreads on each category of assets and liabilities, and the dividend payout policy. Additional inputs are required to specify the volatility of interest rates and the bounds as expressed by constraint (11) in the single-period model.

5.2. Simulation Algorithm

The simulation algorithm is outlined in Figure 3. The simulation generates the interest rates, updates the balance sheet, formulates the quadratic program, solves the program, and adds assets and liabilities to the balance sheet. The major work involves accounting for the demand for funds, maturing assets and liabilities, prepayments, and the change of rates on assets and liabilities (called repricing).

In order to do the accounting (Step 1.2):

- We remove all maturing assets and liabilities. Then we add back that part of the assets which roll over into different asset categories, depending on the forecast. New volumes of assets, representing new customer loans, and new equity are also added.

- The added back assets will result in additional cash flow streams, created by both principal and interest payments. These cash flows are calculated on a monthly basis. The interest payments are calculated by adding the (given) spread on these assets to the rate given by the current yield curve.
- The MBS are updated and cash flows calculated as described under prepayment. We maintain the maximum permitted amount of MBS in the portfolio, because these are the most profitable investments and the optimizer always chooses the maximum allowable amount to invest in MBS.
- Some of the assets and liabilities will reprice, i.e., their rates will change because they are indexed to the prevailing interest rates. The change in rates will change interest income, and these are computed. For the assets that do not reprice, the previous period's cash-flow calculations are simply rolled back by one period.
- Minimum cash requirement is checked, average yield of the mix of assets and liabilities computed using book values and the rate of the security, and gaps in the asset and liability sides of the balance sheet are calculated using the size of the balance sheet.
- For each liability and investment that may be used to fill these gaps, we use the prevailing rates and given spreads to obtain the corresponding rate from the spot curve. The monthly cash flows for one dollar of each of these securities is calculated.
- A call is made to the quadratic optimizer, after adding constraints on duration, permissible distribution or allocation of funds, gaps, etc., as described in Section 4.
- If the quadratic optimizer finds a solution, we update the balance sheet, calculate performance indices, and go to the next period. Otherwise, we relax some of the constraints as described below and try again.

Several practical issues are involved in the implementation. The simulation cannot be placed on "auto pilot" without preliminary runs and familiarity with the dynamics of the balance sheet. The problem lies in ensuring that the quadratic program is feasible, while avoiding large fluctuations in any performance parameters such as dividends and the duration of capital. The following ideas helped considerably in this regard, especially when the empirical approach, EMP, was used to generate rates:

- Adding a penalty function to the objective to penalize deviations from the target duration of capital.
- Making the return target a soft constraint, i.e., computing a return, r_f (for feasible return), based on an assumed pattern of asset and liability selection and the size of the gaps in assets and liabilities, and imposing this return as the constraining factor if r_f is less than the desired average rate of return.
- Choosing appropriate scaling factors to preserve numerical accuracy.
- Adding constraints that not more than 30% of the gaps in assets or liabilities can be invested in any single type of asset or liability (see (11)).
- If the quadratic program turns out to be infeasible, predetermining a sequence in which some of the constraints will be relaxed.

We encountered two major difficulties during implementation: integrating the data from the modules into a single model and determining whether the model outputs were correct. The first difficulty was resolved by manually combining all the data first into a spreadsheet. The data were then exported in the required format to files. This was the most time-consuming step. The model outputs were verified for accuracy manually for each period and for several simulation traces. The outputs were also found to be similar to the outputs from a spreadsheet-based simulation model that was being used by the bank.

5.3. Results

The programs were coded in FORTRAN 77 and run on the network of SUN workstations at the School of Business at New York University. The simulation of one typical run took 1.5 to 2.0 minutes.

EMP. In the first set of simulations, rates were generated using EMP, the prepayment rates from MBS were kept fixed throughout the simulations, the target return was fixed at 15% and the size of the balance sheet constrained to be 16 times the total capital. The planning horizon (NSIM) was chosen as 36 months, and 100 replications were carried out under three interest scenarios, with each scenario having a different value of volatility. Figure 4 shows the average over the 100 runs of the net present value of capital and net profit for 36 months. The insights obtained from these simulations are as follows:

- Volatility: Volatility of rates affects the variance of returns, the rate of growth of the net present value of equity as well as the magnitude of swings in the duration of equity.
- Embedded Cash Flows: The effects of volatility in rates are less significant than the effects arising from embedded (committed) cash flows in the current balance sheet. For example, controlling the duration of capital in certain months proved to be difficult because a large amount of assets or liabilities were maturing in these periods, and therefore matching funds could not be found to replace them.
- Assumption of Stationary Covariance Matrix: We experimented with using a *subset* of the data to construct the covariance matrix. In these experiments, the original set of 360 weeks of data were used to generate the interest rates, whereas only 30 weeks of data were used

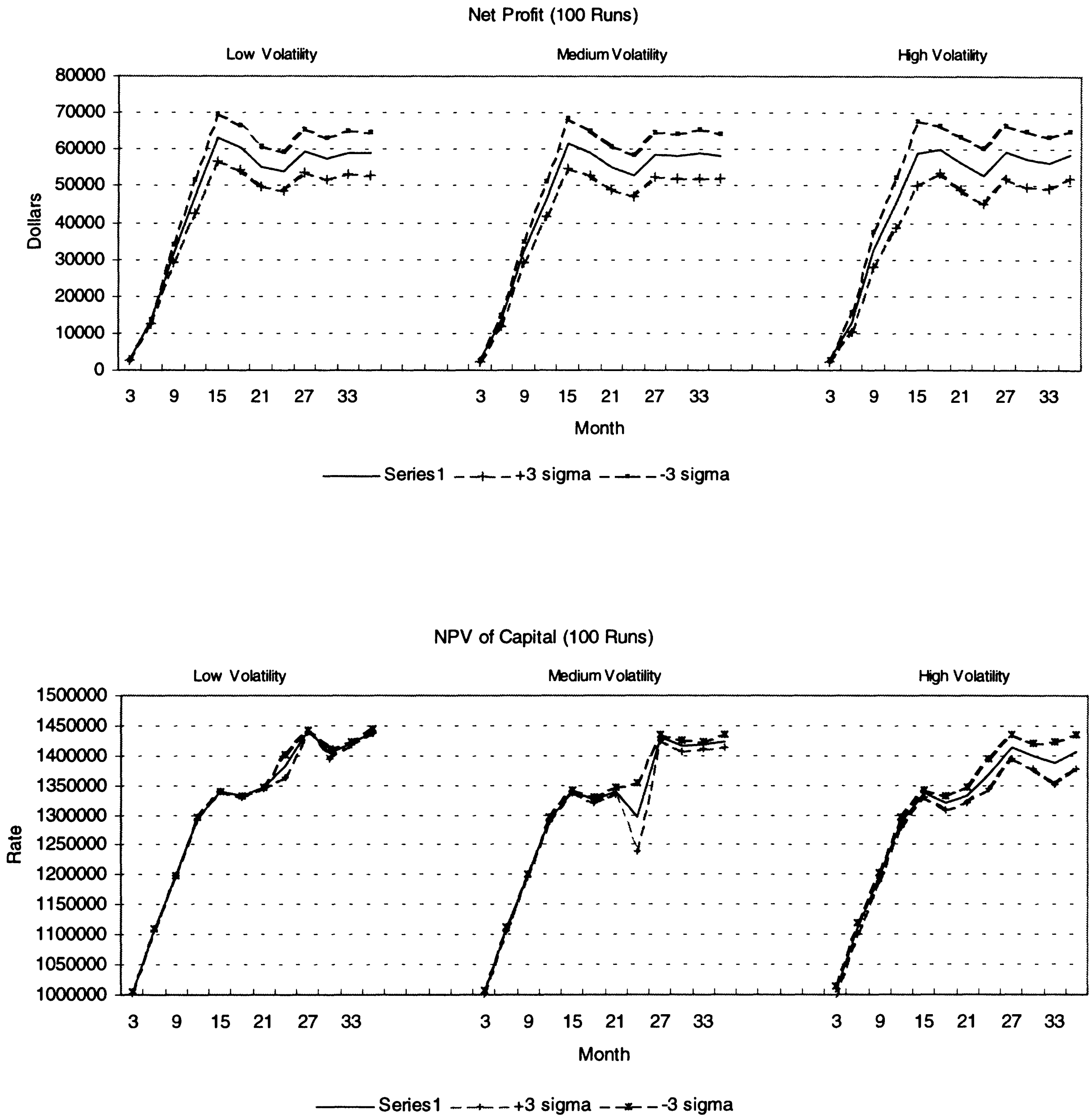
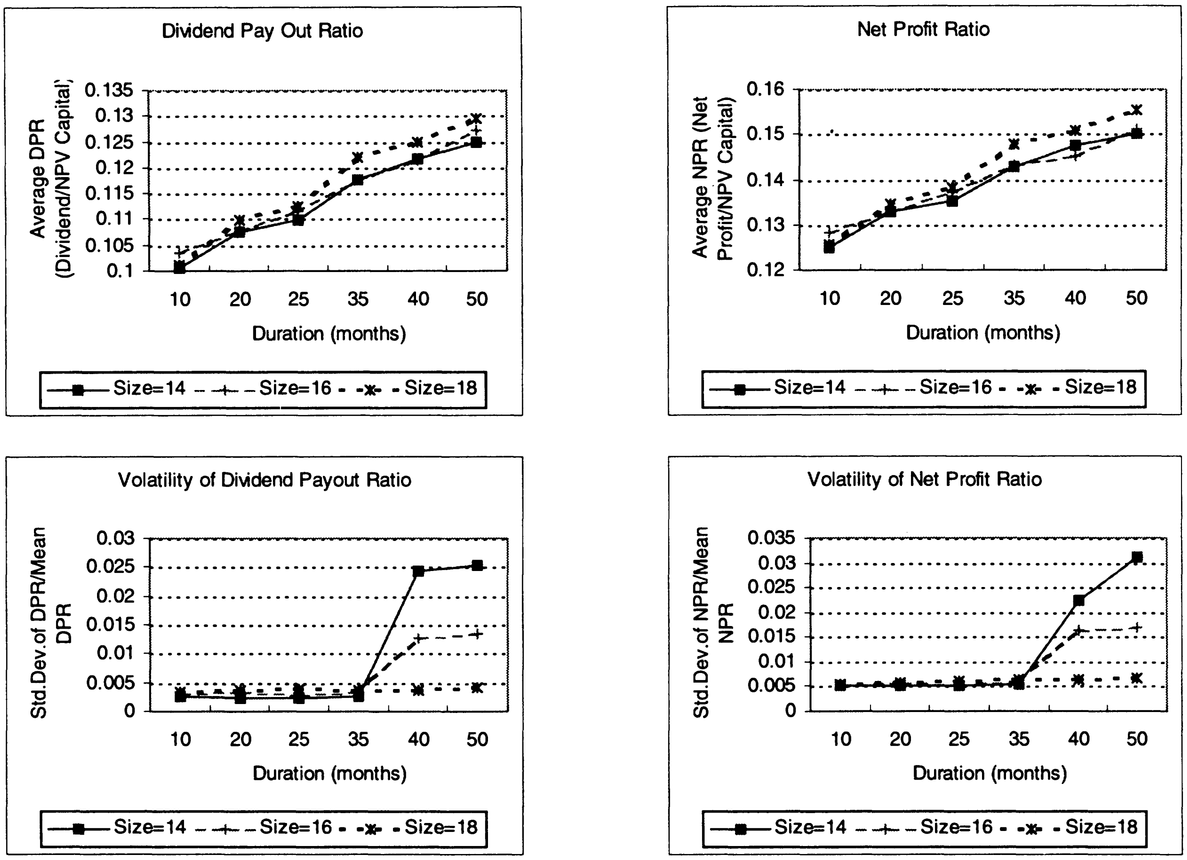


Figure 4. Net present value of capital and net profit (100 runs).

to estimate the covariance matrix. The results were similar to the ones produced using the full data set.

- Sensitivity of Demand for Funds: We created three scenarios: high, medium, and low interest rates. The mix of funds demanded under these three scenarios was assumed to be different—with more long-term funds being demanded when rates fell, and more short-term funds demanded when rates increased. For interim values of rates, we used linear interpolation as follows. Let the vector of funds of type i demanded in time period t under scenario j be $\{X^j_{it}\}$, where j can take the values 0.09, 0.06, or 0.03. If the 12-month rate is within the range [0.03, 0.09] then r is set equal to this rate. Otherwise (as was rarely the case), r is set equal to 0.03 or 0.09 if it falls below 0.03 or rises above 0.09, respectively. The forecast for funds of type i is determined by interpolating using the value of r. The performance was nearly unaffected by this change.
- Distribution of Profits: The kurtosis measure of the distribution of the profit in period 36 was 1.45, −0.248, and 3.43, respectively, under the three scenarios. The measures of skewness were −0.564, −0.416, and 1.513, respectively. We see that the deviation of the distribution of profit from the normal distribution can be quite significant. As discussed in Section 2, the mean-variance criterion is not adequate for handling such deviation and should be either replaced with or supplemented with a utility maximization criterion in future work.



Note: (i) DPR or Dividend Payout Ratio, is the ratio of dividends to NPV of Capital.
(ii) NPR is the ratio of Net Profits to the NPV of Capital.

Figure 5. Profit, duration, and size.

HJM. In the second set of simulations, we used the HJM model for generating rates, and added the prepayment module. In this case, the single-period model was always feasible, so we did not use many of the heuristic corrections needed under EMP. One experiment was carried out using this rate generator to understand the effect of the size of the balance sheet on risk and return; we show the results in Figure 5. We varied the duration of capital from 10 to 50 months, and selected three different balance sheet sizes, 14, 16 and 18. For ease of exposition, denote the ratio of dividend to NPV of capital as DPR (for Dividend Payout Ratio) and the ratio of net profit to capital as NPR (or Net Profit Ratio). In Figure 5, the averages of these values are plotted against the duration of capital for the three sizes of the balance sheet. We observe that profitability increases with both duration and size, but the rate of increase is smaller with respect to the size of the balance sheet. The increase in profitability with duration is due to an upward sloping yield curve. An inverted curve could lead to a different conclusion.

We depict the volatility of returns using two ratios, the standard deviation of (DPR) *divided* by the mean of (DPR), and the standard deviation of (NPR) *divided* by the mean of (NPR). When the duration is in the range of 10 to 35 months, a larger balance sheet leads to greater the volatility of returns, and moreover, for each size of the balance sheet, the volatility is almost constant. We interpret this component of volatility as being similar to systematic risk—i.e., the component of risk that cannot be diversified away (Elton and Gruber 1991). (Note that Figure 5 shows the ratio of the standard deviation to the mean return, and this ratio being constant does not imply that the risk has not increased. In fact, the variance of returns is growing faster than the mean.) Beyond 35 months, the smaller balance sheets exhibit much greater variability in returns, and the source of the variability is apparently nonsystematic. In contrast, the larger balance sheet shows steady performance over the entire range. This result is in accordance with the single-period results—the larger the size of the balance sheet, the easier it is to match durations of assets and liabilities, and obtain higher returns. However, a larger balance sheet also implies greater risk arising from adverse rate changes. The multiperiod approach allows us to portray this tradeoff and helps in choosing the size of the balance sheet.

6. CONCLUSIONS

Development of a fully integrated planning tool for a large bank involves an enormous commitment of time and resources. Our approach has been to start with a description of the SALM process, develop an integrated planning model broken up into modules, use aggregation techniques, carry out sensitivity analysis, and based on learning as well as feedback, target individual modules for further improvements. As part of this methodology, we have outlined a set of tools and techniques that may be used in the SALM process. The single-period model and the multiperiod simulations are seen to be two such important tools. The single-period model can be used to identify mismatches and to carry out limited but rapid what-if analysis. The risk managers at the bank concurred that solving the single-period problem helps in identifying mismatches between assets and liabilities, a primary task of SALM.

The multiperiod simulations permit extensive what-if analysis of key factors such as volatility of interest rates, demand for funds, and the size of the balance sheet. The experience gained from the simulations, in particular examining the dynamic nature of liability selection, the intertemporal nature of portfolio management, and the role of policy variables in controlling the evolution of the balance sheet over time, gave the bank a deeper understanding of its asset-liability management process and practices.

Based on the experience gained in modeling this problem, several directions of research suggest themselves. The first has to do with approximating multiperiod models with simpler models. Given the robust performance of the single-period model, we believe that the results are not likely to improve if a true multi-period model is used. However, determining bounds on the maximum deviation from optimal values, particularly with regard to the variance of profits and the variance of capital, is likely to be very useful. Research is required to understand what can be gained from embedding a two- or three-period model instead of a single-period model in the multi-period simulations. Second, the model remains to be tested in a highly leveraged situation. Extensive testing and use in such a situation as well as the development of guidelines for aggregating assets and liabilities into larger classes for planning purposes will increase the credibility and the acceptance of simulation/optimization methods for asset-liability management.

APPENDIX

EXAMPLE OF LIABILITY SELECTION AND THE EFFECT OF SIZE

Assume that the present value of assets is one dollar, and $\$L$ ($0 < L < 1$) of liabilities have to be selected. Let the yield curve be linear for maturity values between 1 and 2 years, with the borrowing rate (i.e., rate given by the yield curve) equal to 7.5% for 1 year and 8% for 2 years. Assume that the desired rate of return on equity of $(1 - L)$ is 10%. The size of the balance sheet is given by the ratio, assets/equity $= 1/(1 - L)$. Assume that the spread is 25 basis points, that assets have a maturity of 1.5 years, yielding a rate of

$$\begin{aligned}&(\text{rate on borrowed funds for a}\\ &\quad\text{maturity of 1.5 years + spread})\\ &\qquad = (0.075 + (0.08 - 0.075) \times (2 - 1.5)) + 0.0025\\ &\qquad = 0.08 \text{ or } 8\%.\end{aligned}$$

Assume that we choose liabilities of duration $t \in [1, 2]$ years. To obtain the return of 10% on equity, we will need

$$\begin{aligned}&(\text{return on assets} - \text{cost of } L \text{ amount}\\ &\quad\text{of liabilities of duration } t)/(\text{Equity})\\ &\qquad = (0.08 - (0.075 + 0.05(t - 1))L)/(1 - L) = 0.1\end{aligned}$$

or

$$t = (0.3L - 0.2)/(0.05L) \text{ years} = (6L - 4)/L \text{ years}.$$

Since $t \geq 1$, $L \geq 4/5$. The duration of equity for this choice of t will be given by

$$\begin{aligned}&(\text{value of assets} \times \text{duration of assets}\\ &\quad - \text{value of liabilities} \times \text{duration of liabilities})/(\text{Equity})\\ &\qquad = (\$1 \times 1.5 \text{ years} - \$L \times t)/(1 - L)\\ &\qquad = (1.5 - (6L - 4))/(1 - L) = 6 - 0.5/(1 - L).\end{aligned}$$

Thus the duration of equity decreases with the size of the balance sheet, $(1/(1 - L))$. This implies that in this example the sensitivity to interest rate risk *decreases* with the size of the balance sheet when $L \in [4/5, 11/12]$.

GLOSSARY OF TECHNICAL TERMS

CMO: Mortgage-backed bonds that separate mortgage pools into different maturity classes called tranches. This is accomplished by applying income (payments and prepayments of principal and interest) from mortgages in the pool in the order that the CMOs pay out.

Commercial Paper: Short-term unsecured debt issued by large corporations.

Convexity: The second derivative of the price yield relationship. Following Saunders (1994), let P be the price of a security, ΔR the change in the rate R, D the duration of the security, and CX the convexity of the security. Then

$$\frac{\Delta P}{P} \approx -D \frac{\Delta R}{(1 + R)} + \frac{1}{2} CX(\Delta R)^2.$$

Derivative Financial Products: Financial instruments whose value is determined by an underlying instrument. Stock options, futures contracts, and mortgage-backed securities are all examples of derivative securities.

Discount Notes: Short-term debt instruments that are issued at a discount and repaid at full face value.

FIRREA ACT, 1989: Legislation whose purpose was to resolve the crisis in the savings and loan industry and prevent such crises in the future. The Act established new regulatory structure as well as regulations pertaining to most federal savings banks and state or federal chartered savings associations. The new regulations include directives on risk-based capital requirement, leverage limit and tangible capital requirement.

Interest Rate Swaps: A contract between two parties to exchange a series of cash flows. In a plain vanilla interest rate swap, one party periodically pays flows determined by a fixed interest rate and receive cash flows determined by a floating interest rate.

Letters of Credit: Guarantee of sufficient funds.

LIBOR: (London Inter-Bank Offer Rate) The rate that the most creditworthy banks charge one another for loans in the London market.

MBS Pools: Group of mortgages sharing similar characteristics in terms of property, interest rate, and maturity.[7]

MBS: Securities backed by mortgages. They are the output of the process of securitizing mortgages on assets. Investors receive payments out of interest and principal on the underlying mortgages. The income passes from debtors to investors through intermediaries (like FNMA, FHLB) who take pooled debt obligations and repackage them as securities.[8]

OTS: The Office of Thrifts Supervision, set up by the FIRREA Act. This office is "under the umbrella of the Treasury department . . . (and is the) primary federal regulator for federal and state-chartered savings associations and S&L holding companies" (*Special Report FIRREA ACT* 1989).

Regular Bonds: Obligations of the FHLB System that pay a fixed rate of interest, have a stated maturity, and have semi-annual coupon payments.

Repricing Bonds: Obligations on which the rate paid changes or reprices according to an agreed-upon time interval and interest rate index.

Secondary Mortgage Market: Where existing pools of securitized mortgage loans are bought and sold.

Spread: The difference between the rate on a customer loan for a maturity of x years, and the cost to the bank of raising funds for the same maturity of x years.

Structured Debt Securities: Debt offerings that are tailored to meet the specific cash flow requirements of a particular investor or class of investors.

Term Fed Funds: Short-term fixed-rate investments of the money market.

Treasury Spot Rate: "The n year spot rate is the interest rate on an investment that is made for a period of time starting today and lasting for n years" (Hull 1993, p. 81).

Variable Rate Advances: Adjustable rate loans of the FHLB System. The rate paid on these loans changes or reprices according to an agreed upon time interval and interest rate index.

Yield Curve: "The yield curve gives the rate of return of an instrument for different values of maturity" (CSF Boston 1994, p. 17). In this paper the yield curve denotes the bank's cost of borrowing funds for different values of maturity.

ENDNOTES

1. "The yield curve gives the rate of return of an instrument for different values of maturity" (CSF Boston 1994, p. 17). In this paper we shall use yield curve to denote the Bank's cost of borrowing funds for different values of maturity.
2. We use the term *capital* to denote everything that is owned by the stockholders, including reserves and undistributed profits.
3. "These particular (parallel shifts) scenarios provide a convenient basis for analyzing risk exposure; they are not meant to be interest rate forecasts. It has become standard practice in financial analysis to examine the impact of potential changes of these magnitudes on the value of financial instruments and/or portfolios" (OTS 1989, §420, p. TB-13).
4. For example, as rates change, home owners may decide to refinance, thus affecting the level of prepayments.
5. The maximum change in E might occur at an interior rate point, i.e., $\Delta r_j < \Delta r_j^{\max}$. In order to guard against this, a few more constraints of the type (13) at selected values of Δr may be added.
6. WAC_{new} = (WAC_{old} × Opening Balance + Current Rate × New Volume)/(Opening Balance + New Volume).
7. Dictionary of Finance and Investment Terms, (3rd ed.), J. Downes and J. E. Goodman, Barrons, 1991.
8. Ibid.

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