*IIE Transactions* (2000) **32**, 819–831

# Risk intermediation in supply chains

VIPUL AGRAWAL and SRIDHAR SESHADRI

*Operations Management Department, Stern School of Business, New York University, New York, NY 10012, USA*
*E-mail: vagrawal@stern.nyu.edu* or *sseshadr@stern.nyu.edu*

Received August 1998 and accepted November 1999

This paper demonstrates that an important role of intermediaries in supply chains is to reduce the financial risk faced by retailers. It is well known that risk averse retailers when faced by the classical single-period inventory (newsvendor) problem will order less than the expected value maximizing (newsboy) quantity. We show that in such situations a risk neutral distributor can offer a menu of mutually beneficial contracts to the retailers. We show that a menu can be designed to simultaneously: (i) induce every risk averse retailer to select a unique contract from it; (ii) maximize the distributor's expected profit; and (iii) raise the order quantity of the retailers to the expected value maximizing quantity. Thus inefficiency created due to risk aversion on part of the retailers can be avoided. We also investigate the influence of product/market characteristics on the offered menu of contracts.

## 1. Introduction

We consider a single-period model in which multiple risk averse retailers purchase a single product from a common vendor. We assume that the retailers operate in identical and independent markets and face uncertain customer demand. The retailers are assumed to be price takers and to sell the product at the same fixed price. They accordingly make their purchase order quantity decision in order to maximize their expected utility. The vendor has to offer the same supply contract to each retailer. The terms of the contract offered to the retailers are similar to the ones found in the classical newsvendor problem. Under this contract, each retailer purchases a certain quantity at a regular purchase price. If the realized demand is greater than the quantity ordered then the retailer has the option to purchase the units that are short at an emergency purchase price that is higher than the regular price. If the demand is less than the order quantity then the retailer has the option to return the leftover inventory at a salvage price that is lower than the regular price. (This contract will be referred to as the Original Newsvendor Contract (ONC) in the sequel).

It is well known that the risk averse retailer's order quantity (i.e., the one that maximizes his expected utility) will be smaller than the order quantity that maximizes the retailer's expected profit (Baron, 1973; Eeckhoudt *et al.*, 1995; Horowitz, 1970). Eeckhoudt *et al.* (1995) give examples in which risk averse retailers will order nothing due to high demand uncertainty. Obviously, the reduction in the order quantity of the retailer leads to lower expected profit (for the retailer) compared to the expected profit obtained under the profit maximizing order quantity. Therefore, risk aversion of the retailers has been portrayed in the literature as leading to the loss of efficiency in supply chains. (We use the term "efficiency" to refer to the combined expected profit of the seller and the retailer. In general, this term refers to the total expected profit of all participants in a supply chain).

In this paper we show not only that this loss of efficiency can be eliminated through risk reducing pricing contracts but also that any risk neutral intermediary will find it beneficial to offer such risk reducing contracts to the retailers. In our model, the intermediary is referred to as the distributor[1] and purchases the goods as per the terms of the ONC from the vendor (see Fig. 1). In turn the distributor offers the goods to the retailers on contract terms that are less risky from the retailers' viewpoint. The distributor is a financial non-stocking intermediary such as a *buying group* or a *sourcing agent*, whose sole role in this paper is to "buy" goods at the ONC and "sell" them to retailers through less risky contracts without changing the flow of the products. This is done in order to isolate the risk intermediation function of the distributor. In Section 3 we show that even if there are economies associated with a stocking distributor, such as risk pooling and transportation consolidation, the nature of contracts offered is similar. We propose that, as opposed to the

[1] The distributor can be an independent firm, or the vendor, or one of the retailers. For the sake of clarity we will refer to the intermediary as the distributor, and the risk averse players facing uncertain demand as the retailers. The analysis, though, is valid for any two levels in a vertical marketing channel, where the lower level facing uncertain demand is risk averse and the upper level is risk neutral (or less risk averse).

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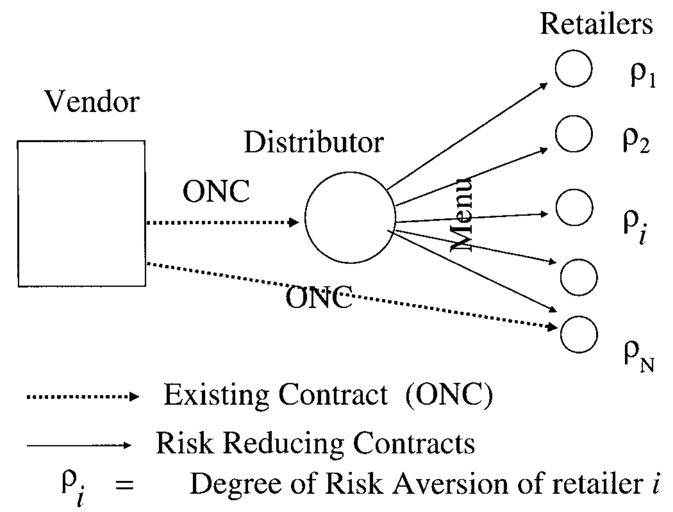


**Fig. 1.** Supply chain contract structure.

ONC, under the risk reducing contracts offered by the distributor to the retailers, the emergency purchase and the salvage prices should be set equal to the regular purchase price, and that in addition a fixed payment should be made by the distributor to the retailer (see Fig. 2).

Therefore, a retailer's payoff consists of a fixed component (independent of the demand) and a variable component that increases linearly with the realized demand. Consequently, as the retailer's payoff depends only upon the demand, the retailer is indifferent to the order quantity decision and is content to relegate the responsibility of determining an order quantity to the distributor. The distributor makes the order quantity decision fully aware that he has to satisfy all the demand faced by the retailer. The distributor bears the cost if necessary of buying the product at the emergency purchase cost and also if necessary of disposing of any unsold product at the salvage price. We prove that by performing this function of "(demand) risk intermediation" the distributor raises the retailers' order quantities such that the maximum efficiency is obtained. Our key contribution in this paper is to establish that the contracts offered to the retailer not only maximize the efficiency in the supply chain but are also optimal from the distributor's viewpoint. This paper is the first to show how supply contracts can play a crucial role in the reduction of financial risk resulting from demand uncertainty. This paper models the role of incentives (embodied in pricing contracts) in supply chains where uncertainty leads to inefficient decision making. Instead of entirely focusing on the risk averse retailers' order quantity decisions (as done in the entire literature dealing with the risk averse newsvendor problem) we are the first to focus on *optimal* mechanisms that can be used to influence the decisions of the retailers.

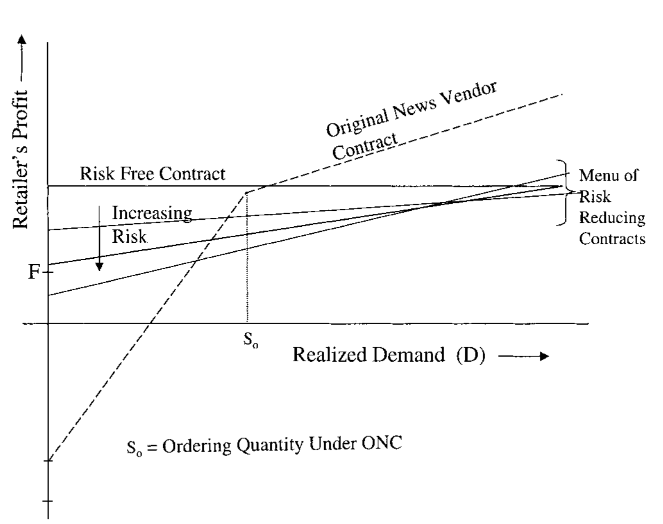


**Fig. 2.** Original contract versus suggested menu.

The problem of designing pricing contracts under demand uncertainty and risk aversion is complicated because of the following reasons:

(i) We allow the retailers to differ with regard to their aversion to risk. Therefore, different retailers may derive different expected utility from the same contract.
(ii) The distributor does not know how risk averse any particular retailer is and only knows the *distribution* of risk aversion among the retailers.
(iii) The laws against price discrimination prohibit the distributor from offering different contracts selectively to different retailers, i.e., any contract that the distributor decides to offer to a specific retailer, has to be offered to all the retailers.

Contracts similar to the ones proposed in this paper are being adopted within the context of Vendor Managed Inventory (VMI) programs. In many VMI programs the vendors make the inventory decisions on behalf of the retailers and also bear the risks and costs associated with these decisions (Andel, 1996). In addition to the contracts found in VMI programs, we have observed several supply contracts, for example in the publishing (Carvajal, 1998), cosmetics (Moses and Seshadri, 2000), personal computers (Kirkpatrick, 1997), apparel and grocery industries (Bird and Bounds, 1997; Lucas, 1996), that transfer the demand risk from the buyer to the vendor. We also see a trend in industry wherein simple price discounting contracts that were previously offered by the vendors (to induce the retailers to purchase a larger quantity) are being substituted by relatively sophisticated contracts that are designed to transfer the demand risk from the retailers to their vendors. As discussed in an extensive survey of pricing contracts by Tsay *et al.* (1999), channel coordination and risk sharing are important reasons for such contracts. Therefore, economic justification for such contracts is not complete unless, as done in this paper, the impact of risk reduction for risk averse retailers is carefully accounted for, and the optimal form of incentives (i.e., contracts) is established taking into account the nature of the risk and the retailer's aversion to risk.

In Section 2, we begin our analysis by showing that in general it is not in the distributor's best interest to offer the *same* contract to all retailers. In other words, the distributor does not in general maximize either his own expected profit or the efficiency, by simply buying and reselling the product to all retailers on the *same* terms. In Section 3, we derive the *menu* of contracts that maximizes the expected profit of the distributor. This menu rather interestingly also maximizes the efficiency. Every contract in the menu has two parameters: (i) a fixed payment that the distributor makes to the retailer; and (ii) a unit price that the distributor charges the retailer for every unit sold by the retailer. (The choice of this form of each contract in the menu is neither random nor accidental. We prove that these contract terms create stochastic payoffs for the risk averse retailer that dominate the payoffs under the terms of the ONC in a strong sense). We emphasize that the menu does not depend on the distributor's knowledge of the degree of risk aversion of each and every retailer, but only upon the knowledge of the distribution of risk aversion over the ensemble of retailers. The menu of contracts is either continuously or discretely parameterized by the fixed side payment and the selling price depending upon whether the distribution of risk aversion is a discrete one or a continuous one. Each retailer selects a contract from the menu, choosing the one that provides him the highest expected utility. The menu of contracts derived by us has the following special properties:

- Every (risk averse) retailer selects a unique contract from the menu.
- The menu always contains a risk free (fixed payment) contract. This contract is selected by a subset of retailers that are the most risk averse.
- Retailers who are less risk averse, prefer the contracts with higher expected profit and utility (and consequently bear higher risk).
- The retailer is willing to relegate the ordering decision to the distributor. Furthermore, the order quantity stipulated by the distributor is the Expected Value (EV) maximizing solution to the newsvendor problem. Therefore, the menu of contracts is also instrumental in maximizing efficiency.
- The menu of contracts maximizes the distributor's expected profit.

### 1.1. *Relevant literature*

#### 1.1.1. *Supply chain coordination*

A comprehensive review of recent research on contracts in supply chains is given in Tsay *et al.* (1999). In contrast to these papers, the inclusion of risk aversion in supply chain coordination, and optimal design within a class of contracts are the novel features of our work. Moreover, our research is aimed at understanding how contracts, and in particular risk sharing plays a role in determining the distribution channel structure.

Traditional quantity discounts while widely used can not achieve complete coordination between the buyers and the seller for two reasons: (i) the (risk sharing) benefits as described by us can not be attained by simply varying the price as a function of the order quantity; and (ii) franchise fees are often required in addition to quantity discounts for achieving maximal channel profits (Oren *et al.*, 1983; Weng, 1995). It is now well understood that such failures in coordination can arise as a result of using improperly constructed price schedules. The theory of non-linear pricing (Wilson, 1993) can be used to determine the price schedule in such situations. Quantity discounts and franchise fees are also examples of nonlinear pricing. The menu of contracts necessary for inducing the retailers' participation in the distributor's network is yet another example of using a nonlinear price schedule to separate out retailers with different attitudes towards risk.

Our work is also related to research on insurance markets (Rothschild and Stiglitz, 1976; Stiglitz, 1977). The menu offered to the retailers takes away some or all of the demand risk faced by them; and in return for this insurance the distributor extracts payment by reducing the expected value of the contract.

## 2. Model, assumptions, and analysis

We consider the classical single-period inventory problem. In this problem, the distribution of the demand as well as the retail price, $p$, are given. The retailer's problem is to choose the order quantity, $S$. In the contract, items are supplied at an initial unit price of $c$. If the demand in the period exceeds the quantity ordered, $S$, then the retailer obtains emergency shipments to cover any excess demand at a unit cost of $e$. If the demand is less than $S$, the retailer sends the unsold items back to the seller, and obtains a credit of $s$ per unit returned. The framework has been extended by us to incorporate risk aversion as well as to model multiple retailers as follows.

*Model assumptions*

1. The retailers are alike in terms of demand distribution and cost parameters, and differ only with regard to their aversion to risk. We will discuss in the conclusions how the optimal contracts derived by us can be extended when retailers of different size are present.
2. The retailer demands are independent and identically distributed random variables. The retail price

($p$) and the distribution of demand are unaffected by the contracts offered to the retailers.

3. Every contract has to be offered to every retailer. The retailer in turn selects a contract from the menu offered. This condition prevents direct (illegal) price discrimination by the distributor.
4. The retailers are not resellers, but purchase only to satisfy their own demand (Wilson, 1993).
5. The distributor is risk-neutral (the distributor can be the manufacturer itself or the least risk averse retailer).
6. We use utility functions for money. Until Section 3 we make no additional assumption about the utility functions of retailers, except that the utility functions are concave and nondecreasing in the amount of wealth. From Section 3 onwards, the small gambles framework is adopted in our analysis (Pratt, 1964).
7. Retailers are Expected Utility (EU) maximizer, where, EU = Expected Value (EV) − the risk premium.
8. We assume that the general form of contracts that are made available to the retailers are given by $\mathscr{C}(F, c, s, e)$, where,

   $F$ = fixed side payment to the retailer;
   $c$ = regular purchase price/unit;
   $s$ = salvage value/unit of the unsold retailer stock;
   $e$ = emergency purchase price/unit;

   and, $p \geq e \geq c \geq s$. The ONC the retailer can be written using this notation as $\mathscr{C}(0, c, s, e)$.
9. If two contracts provide the same EU to a retailer, the retailer will choose the contract that has the larger fixed payment. If the distributor offers a contract that provides the same EU as the ONC to the retailer, then the retailer will choose the contract offered by the distributor.

These modeling assumptions hold if: (i) the product by itself contributes a small portion to the retailer's wealth (for Assumption 6); and (ii) retailers serve their local markets, and these markets have little or no overlap (for Assumption 2). Assumption 9 is standard in the analysis of such problems, for example see the discussion in Kreps (1990). The assumption allows us to work with weak inequalities in Section 3. In Section 4, we discuss how Assumption 1 can be relaxed.

### 2.1. *A single retailer*

In this section we consider the case of a single risk averse retailer, and show that a risk neutral intermediary has an incentive to offer a risk sharing contract to the retailer. This has the effect of raising the order quantity to the EV maximizing value. The single period demand will be denoted as $D$, its distribution, mean and standard deviation as $\mathscr{F}_D(\cdot)$, $\mu$, and $\sigma$. The retailer's utility function for payoffs is $U(\cdot)$, a concave non-decreasing function. Let $E[\cdot]$ stand for the expected value, and $[A]^+$ for the positive part of $A$. The random pay-off given the ordering quantity $S$, can be written as $\Pi(S, F, c, s, e) = F + pD - cS + s[S-D]^+ - e[D-S]^+$. The EU and EV maximizing order quantities, denoted as $S^U_{\text{opt}}$ and $S^{EV}_{\text{opt}}$, are given by

$$S^U_{\text{opt}}(F, c, s, e) = \arg\max_S E[U(pD - cS + s[S-D]^+ - e[D-S]^+)], \quad (1)$$

$$S^{EV}_{\text{opt}}(F, c, s, e) = \arg\max_S E[pD - cS + s[S-D]^+ - e[D-S]^+], \quad (2)$$

$$= \mathscr{F}_D^{-1}\left(\frac{e-c}{e-s}\right). \quad (3)$$

It is well known that: (i) $S^U_{\text{opt}}(0, c, s, e) \leq S^{EV}_{\text{opt}}(0, c, s, e)$; and (ii) $E[\Pi(S^U_{\text{opt}}, 0, c, s, e)] \leq E[\Pi(S^{EV}_{\text{opt}}, 0, c, s, e)]$. The latter fact can be exploited by a risk neutral intermediary to act as a distributor as follows. Assume that the distributor takes the ONC, and in turn offers the contract $\mathscr{C}(F, c', c', c')$, where $p \geq c' \geq e$. In this contract the distributor offers to pay a fixed fee $F$ to the retailer, and in addition charges a unit price $c'$ for every unit sold. Under this arrangement the distributor decides the order quantity, $S$, and bears the costs of emergency shipment and of salvage.

Lemma 2.1 gives the conditions under which a class of contracts of the form $\mathscr{C}(F, c', c', c')$ will be accepted by the retailer, and Lemma 2.3 shows that the distributor will use the EV maximizing value for the order quantity $S$.

**Lemma 2.1.** *Contract $\mathscr{C}(F, c', c', c')$ is preferred by the retailer to the contract $\mathscr{C}(0, c, s, e)$ if*: (i) $E[\Pi(S^{EV}_{\text{opt}}, F, c', c', c')] \geq E[\Pi(S^U_{\text{opt}}, 0, c, s, e)]$; *and* (ii) $p \geq c' \geq e \geq s$.

**Proof.** Set $F$ equal to $E[\Pi(S^U_{\text{opt}}, 0, c, s, e)]$ and $c' = p$. This contract's EV equals $E[\Pi(S^U_{\text{opt}}, 0, c, s, e)]$. Thus by construction we see that there exist contracts such that $E[\Pi(S^{EV}_{\text{opt}}, F, c', c', c')] \geq E[\Pi(S^U_{\text{opt}}, 0, c, s, e)]$, with $p \geq c' \geq e \geq s$.

The proof of the lemma is based on establishing that the distribution function of the retailer's profit under the new contract will cross the distribution function of profit under the ONC from above and at most at a single point. The profit to the retailer under the ONC can be written as a function of the demand ($D$) as follows

$$\Pi(S^U_{\text{opt}}, 0, c, s, e) = \begin{cases} (s-c)S^U_{\text{opt}} + (p-s)D & 0 \leq D \leq S^U_{\text{opt}}, \\ (e-c)S^U_{\text{opt}} + (p-e)D & S^U_{\text{opt}} \leq D. \end{cases} \quad (4)$$

Similarly, the profit to the retailer under the new contract is given by,

$$\Pi(S^{EV}_{\text{opt}}, F, c', c', c') = F + (p - c')D. \quad (5)$$

From Equations (4) and (5), and the condition, $p \geq c' \geq e \geq s$, the slope of profit as a function of $D$ under the new contract is not larger than the slope under the ONC, see Fig. 3. It follows from this observation and the condition $E[\Pi(S_{\text{opt}}^{EV}, F, c', c', c')] \geq E[\Pi(S_{\text{opt}}^{U}, 0, c, s, e)]$ that $\Pi(S_{\text{opt}}^{EV}, F, c', c', c')$ intersects $[\Pi(S_{\text{opt}}^{U}, 0, c, s, e)]$ at most once and from above. It can be verified that the intersection will take place if $S_{\text{opt}}^{U} \geq F/(c'-c)$, and it takes place at the point

$$D = \frac{F + (c-s)S_{\text{opt}}^{U}}{c' - s}.$$

From Theorem 3.A.1 of Shaked and Shanthikumar (1994) or from the work of Rothschild and Stiglitz (1970, 1971, 1972), if the utility function of the retailer is increasing and concave in wealth then, $E[U(\Pi(S_{\text{opt}}^{EV}, F, c', c', c'))] \geq E[U(\Pi(S_{\text{opt}}^{U}, 0, c, s, e))]$. ■

This result can be strengthened as follows. Define "$v$ to be more risk averse than $u$" when $v(\cdot) = k(u(\cdot))$, where $k(\cdot)$ is an increasing concave function, see for example Pratt (1964) and Arrow (1974).

**Theorem 2.1.** *Given $\beta \geq 0$ and a retailer "x" who has an increasing utility function, $u(\cdot)$, such that this retailer is indifferent between the contracts $\mathscr{C}(F-\beta, c', c', c')$ and the ONC, then every retailer with an increasing utility function $v(\cdot)$, who is more risk averse than retailer "x" will prefer $\mathscr{C}(F-\beta, c', c', c')$ to the ONC.*

**Proof.** Follows from Lemma 2.2 and the theorem due to Landsberger and Meilijson (1994) given below. ■

**Lemma 2.2.** *$\Pi(S_{\text{opt}}^{EV}, F, c', c', c')$ is smaller than $\Pi(S_{\text{opt}}^{U}, 0, c, s, e)$ in the dispersive order.*

**Proof.** Let the distribution functions of the profit under the ONC and the contract $\mathscr{C}(F, c', c', c')$ be $G_{\text{ONC}}$ and $G_{\text{NEW}}$. From Fig. 1, we observe that $G_{\text{NEW}}^{-1}(\alpha) - G_{\text{ONC}}^{-1}(\alpha)$

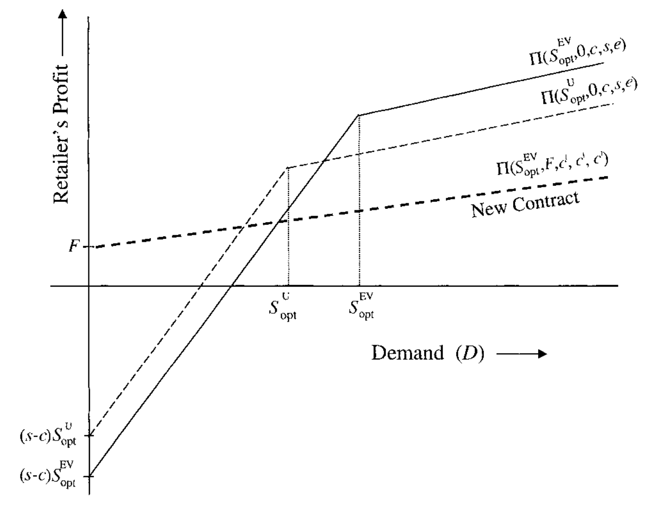


**Fig. 3.** Retailer's profit under different contracts.

is increasing in $\alpha \in (0, 1)$. From the discussion given in Section, 2.B.1 of Shaked and Shanthikumar (1994), this is a sufficient condition for $\Pi(S_{\text{opt}}^{EV}, F, c', c', c')$ to be smaller than $\Pi(S_{\text{opt}}^{U}, 0, c, s, e)$ in the dispersive order. (For a definition of dispersive ordering see Bickel and Lehmann (1979) or Shaked and Shanthikumar (1994)). ■

Landsberger and Meilijson (1994) have proved the following theorem:
*The following two relations between distribution functions F and G are equivalent*: (i) *F is less dispersed than G*; (ii) *for every non-decreasing function U and non-decreasing and concave function h such that $U(x)$ and $h(U(x)) = V(x)$ are integrable under G*,

$$\int U(x-c)\mathrm{d}F(x) \geq \int U(x)\mathrm{d}G(x) \Rightarrow \int V(x-c)\mathrm{d}F(x) \geq \int V(x)\mathrm{d}G(x). \tag{6}$$

Lemma 2.1 thus gets sharpened by Theorem 2.1 to include utility functions that are increasing in wealth. We also know that a risk neutral retailer will either prefer or be indifferent to the new contract. Therefore the contract offered by the distributor will of course be preferred by any retailer who is more risk averse than the risk neutral retailer.

Given that the distributor can induce the retailer to take a "risk sharing" contract, the order quantity has to be decided upon by the distributor. We now prove that the optimal order quantity that the distributor will stipulate to the retailer is the EV maximizing solution to the original newsvendor problem.

**Lemma 2.3.** *The optimal ordering quantity for the distributor is $S_{\text{opt}}^{EV}(0, c, s, e)$.*

**Proof.** If the distributor offers the contract $\mathscr{C}(F, c', c', c')$ then the distributor's expected profit as a function of $S$ can be written as

$$\begin{aligned} &E(-F + c'D - cS + s[S-D]^+ - e[D-S]^+) \\ &\quad = E(-F + c'D - pD + pD - cS \\ &\qquad + s[S-D]^+ - e[D-S]^+), \\ &\quad = E[\Pi(S, 0, c, s, e)] - (F + (p-c')\mu). \end{aligned} \tag{7}$$ ■

## 2.2. *Multiple retailers*

In this section we consider the case when there are multiple retailers. Assume that the distributor offers the same contract $\mathscr{C}(F, c', c', c')$ to all the retailers. We restrict attention to practical contracts in which $c' \leq p$. This class of contracts will be called $\mathscr{C}_{\text{eq}}$. The contract $\mathscr{C}(F, p, p, p)$ will be called a "risk free" contract because the retailer is paid a fixed fee $F$ regardless of the quantity sold.

**Theorem 2.2.** *If the distributor offers a single contract to the whole population of retailers, and there are no diseconomies of scale in distribution, then the contract offered will be a risk free contract of the form $\mathscr{C}(F,p,p,p)$. Moreover the contract that maximizes the distributor's expected profit need not be selected by the entire population of retailers.*

**Proof.** Let the distributor offer a contract $\mathscr{C}(F,c',c',c')$. Let the set of retailers who accept this contract be $\mathscr{M}$. The total number of retailers that accept this contract is given by $m = \text{card}[\mathscr{M}]$. Let $c' < p$. Consider the alternate contract, $\mathscr{C}(F_1,p,p,p)$, such that $F_1 = F + (p - c')\mu$. Contract $\mathscr{C}(F_1,p,p,p)$ gives the same expected profit to the distributor as contract $\mathscr{C}(F,c',c',c')$. From Lemma 2.1 the new contract will be taken by all the retailers in $\mathscr{M}$. Moreover, because of Assumption 9, retailers who are not in $\mathscr{M}$ may also take this contract. The proof of the first part now follows from the assumption that there are no diseconomies of scale in distribution.

For the second part, we observe that the fixed side payment will have to increase in order to attract more retailers to accept the risk free contract. Therefore counter examples can be constructed to show that a contract of the form $\mathscr{C}(F,p,p,p)$ that maximizes distributor's expected profit need not be selected by all the retailers. ■

Risk pooling in supply chains has been studied by several researchers, see for example Eppen and Schrage (1981), Federgruen and Zipkin (1984), Schwartz *et al.* (1985), Jackson and Muckstadt (1989) and Schwartz (1989). Tsay *et al.* (1999) discuss pricing contracts that exploit these economies for risk neutral retailers. As shown by their work, risk pooling leads to economies of scale in distribution under fairly general conditions. Therefore, from the viewpoint of minimizing the total cost of holding inventory, cost of emergency shipment and salvage cost, the distributor prefers to *add* more retailers to the distribution network. On the other hand, in order to attract more retailers, the distributor has to make increasingly attractive offers to *all* the retailers. Thus after attracting several of the most risk averse retailers to take a contract, the marginal profit to the distributor from inducing an extra retailer to accept the contract can become negative. The risk pooling and risk sharing effects therefore work in opposite directions. We have carried out numerical investigations to understand just how the risk pooling and risk sharing effects interact when the distributor is constrained to offer a single contract of the form $\mathscr{C}(F,c',c',c')$ to all the retailers. Details can be obtained from the authors.

## 3. Multiple contracts

The question that naturally follows from the second part of Theorem 2.1 is whether there is an incentive (at all) for the distributor to offer contracts to all the retailers. In order to answer this question, we adopt the small gambles framework, see Assumption 6. It is quite difficult to obtain closed form results without this simplification. However, the insights we obtain into the optimal menu of contracts, in our view, adequately compensates for the simplification. In this framework, the "local" aversion to risk determines the relative preference of a decision maker between two gambles, see for example Pratt (1964). Define the coefficient of risk aversion $\rho$ at a wealth level of $w$ to be equal to $-U''(w)/U'(w)$, where $U(\cdot)$ is the utility function of the decision maker. Consider a small gamble, $Z$, that has mean $E[Z]$ and variance, $\text{Var}[Z]$. Then, $E[U(w+Z)] \approx U(w) + E[Z]U'(w) + U''(w)\text{Var}[Z]/2$. Thus, the preference for small gambles is determined by the function (under suitable conditions as discussed in Pratt) $E[Z] - \rho\,\text{Var}[Z]/2$. Thus noting that utility functions are unique up to the addition of a constant and/or scaling by a positive constant, without loss of generality the expected utility can be expressed as $E[Z] - \rho\,\text{Var}[Z]/2$. Moreover we can now define the reservation utility of a retailer to be $r = E[\Pi(S^U_{\text{opt}},0,c,s,e)] - \rho\,\text{Var}[\Pi(S^U_{\text{opt}},0,c,s,e)2]$. Now consider a set of $n$ retailers, denoted by $\mathscr{N}$. The retailers are indexed by $i$ and ordered in the decreasing order of risk aversion. Let,

$\rho_i$ = co-efficient of risk aversion of retailer $i$; where $\rho_i \geq \rho_{i+1}$;

$r_i$ = reservation utility for retailer $i$, which is defined as the expected utility derived by retailer $i$ under the ONC, $\mathscr{C}(0,c,s,e)$.

We assume that every retailer in $\mathscr{N}$ has a strictly positive order quantity under the ONC, and therefore has a strictly positive reservation utility.

We shall consider contracts from the class $\mathscr{C}_{\text{eq}}$. To simplify the notation, we denote a contract from this class as $\mathscr{C}(F,c')$. The expected utility to retailer $i$ from the contract $\mathscr{C}(F,c')$ is given by

$$E[U(\Pi(S,F,c',c',c'))] = F + (p-c')\mu - \rho_i(p-c')^2\sigma^2/2. \quad (8)$$

The distributor offers the same menu of contracts to all the retailers in $\mathscr{N}$ (Assumption 3). The menu will be written as $\mathscr{Q} = \{(F_i,c_i)\}$. The set of retailers who accept a contract from this set, $\mathscr{Q}$, will be denoted as $\mathscr{M}(\mathscr{Q})$. To keep the notation simple, we also denote the contract *accepted* by retailer $i$ from the set $\mathscr{Q}$ as $(F_i,c_i)$. To focus on the risk sharing role played by the distributor, we have chosen not to model any scale economies obtained from risk pooling or from transportation. Therefore we assume that the distributor does not carry any inventory and trans-shipments between retailers are not allowed. There is no loss of generality in making this assumption as long as there are no scale diseconomies in distribution, see Section 4. The distributor's profit maximization problem is shown below.

$$(\mathbf{P}):\ \max_{\mathcal{Q}} \sum_{i\in\mathcal{M}(\mathcal{Q})} (E[\Pi(S_{\text{opt}}^{EV}, 0, c, s, e)] - (F_i + (p - c_i)\mu)), \tag{9}$$

Subject to

$$\text{If } i \in \mathcal{M}(\mathcal{Q}) \Rightarrow F_i + (p - c_i)\mu - \rho_i(p - c_i)^2\sigma^2/2 \geq r_i, \tag{10}$$

$$F_i + (p - c_i)\mu - \rho_i(p - c_i)^2\sigma^2/2 \geq F_j + (p - c_j)\mu - \rho_i(p - c_j)^2\sigma^2/2, \quad \forall i \in \mathcal{M}(\mathcal{Q}), j \in \mathcal{Q}. \tag{11}$$

The distributor's objective is to offer a menu that will maximize her expected profit, see Equation (7). The first set of constraints, (10), defines the set $\mathcal{M}(\mathcal{Q})$ – a retailer will accept some contract from the menu of the contracts, only if his expected utility from the contract is at least as great as his reservation utility (Assumption 9). The second set of constraints given in (11) requires that the retailer will pick that contract from the menu which gives him the highest EU. We assume that $\mathcal{Q}$ consists of undominated contracts, that is, there is no contract in the menu which is strictly preferred to another by all retailers. We need the following properties of the retailers' reservation utilities for characterizing the set $\mathcal{Q}$.

**Lemma 3.1.** (i) *The reservation utilities, $r_i$'s, are non-decreasing in $i$; and* (ii) *if $\rho_i > \rho_{i+1}$ then $r_i < r_{i+1}$.*

**Proof.** For the sake of convenience we use, $g(S)$ for $E[\Pi(S, 0, c, s, e)]$ and $h(S)$ for $\text{Var}[\Pi(S, 0, c, s, e)]/2$. By our definition of reservation utilities,

$$r_i = \max_S [E[\Pi(S, 0, c, s, e)] - \rho_i \text{Var}[\Pi(S, 0, c, s, e)]/2] = \max_S [g(S) - \rho_i h(S)].$$

Let $S_i = S_i(0, c, s, e)$ be the optimal ordering quantity under the ONC which gives the retailer $i$ his reservation utility $r_i$. Assume that $r_i > r_{i+1}$, i.e.,

$$g(S_i) - \rho_i h(S_i) > g(S_{i+1}) - \rho_{i+1} h(S_{i+1}). \tag{12}$$

Since, $\rho_i \geq \rho_{i+1}$,

$$g(S_i) - \rho_{i+1} h(S_i) \geq g(S_i) - \rho_i h(S_i). \tag{13}$$

Equations (12) and (13) contradict the optimality of $S_{i+1}$ for retailer $(i + 1)$. The proof of part (ii) is similar. ■

**Lemma 3.2.** (i) *$r$ is a convex function of $\rho$, i.e., for $\rho_{i-1} > \rho_i > \rho_{i+1}$,*

$$\frac{r_{i+1} - r_i}{\rho_i - \rho_{i+1}} \geq \frac{r_i - r_{i-1}}{\rho_{i-1} - \rho_i}.$$

(ii) *If $e > c > s$, $\sigma > 0$, and the demand distribution is continuous, then $r$ is a strictly convex function of $\rho$.*

**Proof.** (i) Using $S_i$ as the ordering quantity for both retailers $i$ and $i - 1$, we get,

$$r_i = g(S_i) - \rho_i h(S_i), \text{ and, } r_{i-1} \geq g(S_i) - \rho_{i-1} h(S_i), \tag{14}$$

$$\Rightarrow r_i - r_{i-1} \leq (\rho_{i-1} - \rho_i) h(S_i) \Rightarrow h(S_i) \geq \frac{r_i - r_{i-1}}{\rho_{i-1} - \rho_i}. \tag{15}$$

Similarly using $S_i$ as the ordering quantity for retailers $i$ and $i + 1$, we get,

$$r_{i+1} - r_i \geq (\rho_i - \rho_{i+1}) h(S_i) \Rightarrow h(S_i) \leq \frac{r_{i+1} - r_i}{\rho_i - \rho_{i+1}}. \tag{16}$$

Part (i) of the lemma follows from combining inequalities (15) and (16).
(ii) If the salvage value $s$, is strictly smaller than the unit price $c$, then for all $i$ the retailers' optimal order quantity under the ONC, $S_i$, is less than some large number $M$. By assumption $S_i > 0$. We know that $f(S) - \rho g(S)$ is a continuously differentiable function of $S$. Therefore, the first order condition for optimality should hold at $S_i$ for every retailer $i$, i.e.,

$$\frac{\mathrm{d}}{\mathrm{d}S}[g(S) - \rho_i h(S)]_{S=S_i} = 0.$$

Moreover, if $S > 0$ and $\sigma > 0$, then $h(S) > 0$. Hence, if $\rho_j \neq \rho_i$, then

$$\frac{\mathrm{d}}{\mathrm{d}S}[g(S) - \rho_j h(S)]_{S=S_i} \neq 0.$$

This implies that $g(S_i) - \rho_j h(S_i) < g(S_j) - \rho_j h(S_j)$, and goes to prove that inequalities (15) and (16) are strict. ■

We now prove in Theorem 3.1 that every retailer will select a contract from the menu in the optimal solution to the distributor's problem. After showing this result, the precise characterization of the optimal menu of contracts will be given in Theorem 3.2.

**Theorem 3.1.** *Every retailer will be included in the set $\mathcal{M}(\mathcal{Q})$ in the optimal solution to problem* (**P**).

**Proof.** The theorem follows from Lemmas 3.3 to 3.5 given below. Lemmas 3.3 and 3.5 show that the distributor's profit maximizing menu of contracts is such that the most and the least risk averse retailers in $\mathcal{N}$ select a contract from it. Lemma 3.4 shows that every other retailer will also select a contract from the menu. ■

Define the expected utility to retailer $i$, from the contract $\mathscr{C}(F_i, c_i)$, as $\hat{r}_i(\mathscr{C}(F_i, c_i))$. Where there is no scope for confusion, we shall omit the dependence on $\mathscr{C}(F_i, c_i)$, and write $\hat{r}_i$ for $\hat{r}_i(\mathscr{C}(F_i, c_i))$. We now state properties of the optimal solution to problem (**P**) that are used to prove Lemmas 3.3 to 3.5. The proofs of the properties as well as the lemmas are given in Appendix A.

**Property 1.** *If $i < j$, and there exists a $k$, $i < k < j$, such that retailer $k \notin \mathcal{M}(\mathcal{Q})$, then $c_i \neq c_j$.*

**Property 2.** *If $\rho_i > \rho_j$, then $c_i \geq c_j$.*

**Property 3.** *If $\rho_i > \rho_j$, and $c_i \geq c_j$ $(c_i > c_j)$, then $F_i \geq F_j (F_i > F_j)$.*

**Property 4.** *If $c_i > c_j$, $\sigma > 0$, and retailer j prefers the contract $(F_j, c_j)$ over $(F_i, c_i)$, then all retailers $k \geq j$ will also prefer (out of the two contracts) $(F_j, c_j)$ to $(F_i, c_i)$.*

**Property 5.** *If $c_i > c_j$, $\sigma > 0$, and retailer i prefers the contract $(F_i, c_i)$ over $(F_j, c_j)$, then all retailers $k \leq i$ will also prefer (out of the two contracts) $(F_i, c_i)$ to $(F_j, c_j)$.*

**Property 6.** *If $i < j$, and the two retailers take the contracts $(F_i, c_i)$ and $(F_j, c_j)$, then retailer j gets at least the same EV from his contract as retailer i from the contract $(F_i, c_i)$.*

**Property 7.** *If the EU to a retailer is the same from two contracts $(F_i, c_i)$ and $(F_j, c_j)$, and $c_i \geq c_j$, then the EV to the retailer from the contract $(F_j, c_j)$ is greater than the EV from the contract $(F_i, c_i)$.*

**Property 8.** *If a risk averse retailer i obtains the same EU from the contract $(F_i, c_i)$ as from the ONC, then the distributor stands to profit from offering the contract $(F_i, c_i)$ to this retailer.*

Recall that indices 1 and $n$ denote the most and the least risk averse retailers in $\mathcal{N}$. Let $i_{\text{MIN}}$ and $i_{\text{MAX}}$ stand for the most and least averse retailers in $\mathcal{M}(\mathcal{Q})$.

**Lemma 3.3.** *If $i_{\text{MIN}} > 1$, then the distributor should also include a contract in the menu that will be selected by any retailer "a", whose coefficient of risk aversion $\rho_a > \rho_{i_{\text{MIN}}}$ (or equivalently, $a < i_{\text{MIN}}$). In words, if there is a more risk averse retailer in $\mathcal{N}$, who is currently not included in $\mathcal{M}(\mathcal{Q})$, then it is in the distributor's interest to offer a contract that is attractive to such a retailer.*

**Lemma 3.4.** *If the distributor's menu contains contracts that are taken by retailers i and j, $(j > i, \rho_i > \rho_j)$, there is no retailer $a \in \mathcal{M}(\mathcal{Q})$ such that $i < a < j$, but $\exists\, a \in \mathcal{N}$, $i < a < j$, then the distributor should also include a contract which is attractive to retailer "a". In words, the distributor will not skip a retailer in offering contracts.*

**Lemma 3.5.** *If $i_{\text{MAX}} < n$, then the distributor's menu should also contain a contract that will be taken by retailer "a", whose coefficient of risk aversion $\rho_a < \rho_{i_{\text{MAX}}}$. In words, if there is a less risk averse retailer in $\mathcal{N}$ who is not included in $\mathcal{M}(\mathcal{Q})$, then it is in the distributor's interest to include such a retailer by offering a suitable contract.*

We have shown that it is in the distributor's interest to offer a menu such that every retailer selects a contract from it. Now we shall investigate the structure of the optimal menu of contracts.

**Lemma 3.6.** *In the optimal (and undominated) menu of contracts, there will be (exactly) one risk free contract.*

**Proof.** If the most risk averse retailer, namely retailer 1, is not offered a risk free contract in the optimal menu, then consider offering this retailer the contract, $F_1 = r_1, c_1 = p$. This contract has a lower EV (by Property 7), will be taken by the most risk averse retailer (and any others who have the same reservation utility), and will not affect the decisions of other retailers who have a larger reservation utility. This proves the lemma. ■

It should be noted that the risk free contract is a less expensive contract for the distributor to offer because the distributor does not have to pay any risk premium. However, the distributor has to offer all retailers the same menu of contracts. To entice all retailers and maximize profits simultaneously she may perforce have to the offer "riskier" contracts, with $(c_i < p)$. On the other hand, the distributor always has the option of designing the risk free contract to attract more than just the most risk averse retailer. Let $k$ be the number of retailers that take the risk free contract in the optimal menu. Before determining the optimal value of $k$ we require the optimal structure of the menu of contracts offered to retailers, $k+1, k+2, \ldots, n$. To obtain this characterization we make an additional assumption, namely that the reservation utility, $r_i$ is an increasing and strictly convex function of $\rho_i$. This assumption holds good when $e > c > s$, $\sigma > 0, S_i > 0$, and the demand distribution is continuous, see Lemma 3.2.

**Theorem 3.2.** *For a given value of k (i.e., retailers $1, 2, \ldots, k$ accept the risk free contract, $F_k = r_k, c_k = p$), the distributor's profit is maximized by offering the contract $(F_i, c_i), i \geq k+1$ given by,*

$$c_i = p - \left( \frac{2(r_i - r_{i-1})}{(\rho_{i-1} - \rho_i)\sigma^2} \right)^{0.5}, \tag{17}$$

$$F_i + (p - c_i)\mu - \rho_i (p - c_i)^2 \sigma^2 / 2 = r_i. \tag{18}$$

**Proof.** The theorem will be proved by induction on the value of $n$. Throughout the proof, we assume that $i \geq k+1$. Without loss of generality, we shall also assume $\rho_i > \rho_{i+1}$. Before we prove the theorem, we need some additional properties of the contracts, $(F_i, c_i)$ given in Equations (17) and (18).

(a) By the assumption of strict convexity of $r_i$ and Equation (17), $c_i$ is strictly decreasing in $\rho_i$.
(b) From Equations (17) and (18), retailer $i$ gets the same EU from contracts $(F_i, c_i)$ and $(F_{i+1}, c_{i+1})$.

However, from Property 3, $F_i > F_{i+1}$. Therefore, from Assumption 9, retailer $i$ prefers contract $(F_i, c_i)$ to $(F_{i+1}, c_{i+1})$.

(c) Because, $c_i > c_{i+1}$, the difference in EU, $(F_i + (p - c_i)\mu - \rho(p - c_i)^2\sigma^2/2) - (F_{i+1} + (p - c_{i+1})\mu - \rho(p - c_{i+1})^2\sigma^2/2)$, is increasing in $\rho$. Thus, as retailer $i$ obtains the same EU from the two contracts, retailer $i+1$ strictly prefers the contract $(F_{i+1}, c_{i+1})$ to $(F_i, c_i)$.

(d) It follows from (b) and (c) above that given the menu of contracts in Equations (17) and (18), each retailer $i$ will choose the contract $(F_i, c_i)$.

(e) It also follows that, given: (i) retailers $i$ and $i+1$ obtain EU's of $r_i$ and $r_{i+1}$; and (ii) we wish retailer $i$ to continue taking his contract and not switch to the contract offered to retailer $i+1$; then the maximum price that the distributor can charge retailer $i+1$ is given by Equation (17).

**Proof.** We need, $r_i = (F_i + (p - c_i)\mu - \rho_i(p - c_i)^2 \sigma^2/2) \geq (F_{i+1} + (p - c_{i+1})\mu - \rho_i(p - c_{i+1})^2\sigma^2/2)$, and $r_{i+1} = (F_{i+1} + (p - c_{i+1})\mu - \rho_{i+1}(p - c_{i+1})^2 \sigma^2/2)$. This pair of conditions yields, $r_{i+1} - r_i \leq (\rho_i - \rho_{i+1})((p - c_{i+1})^2\sigma^2/2)$. The right-hand side of this inequality is decreasing in $c_{i+1}$, and equality is attained when

$$c_{i+1} = p - \left(\frac{2(r_{i+1} - r_i)}{(\rho_i - \rho_{i+1})\sigma^2}\right)^{0.5}. \quad \blacksquare$$

(f) Assume we are given that: (i) retailers $i$ and $i+1$ obtain EU's of $r_i$ and $r_{i+1}$; and that (ii) we wish that none of retailers $1, 2, \ldots, i$ should switch to the contract offered to retailer $i+1$. Then the value of $c_{i+1}$ in Equation (17) maximizes the distributor's profit. This conclusion follows from the maximality of $c_{i+1}$ shown in (e) above and Property 7.

Now we are in a position to prove the theorem. From (a)–(f), the theorem is true when $n = k+1$. Let the theorem be true for some $n > k$. We will prove that the theorem is true for $n+1$. Assume that we are given an alternate menu of contracts for the $n+1$ retailers, denoted by $\mathcal{Q}_{\text{alt}}$. By the induction hypothesis, the distributor's EV from the contracts offered to retailers $k+1, k+2, \ldots, n$ is maximized by using the menu given in Equations (17) and (18). Denote the contract offered to retailer $n+1$ in $\mathcal{M}(\mathcal{Q}_{\text{alt}})$, as $(F^{\text{alt}}_{n+1}, c^{\text{alt}}_{n+1})$. Replace the contracts offered to the retailers $k+1, k+2, \ldots, n$ in $\mathcal{M}(\mathcal{Q}_{\text{alt}})$, by the menu given in Equations (17) and (18). This switch will not increase the EVs of these contracts. If necessary, change the value of $F^{\text{alt}}_{n+1}$ and reduce the EU of retailer $n+1$ to $r_{n+1}$. This will not increase the contract's EV. Finally, for retailer $(n+1)$, change to the contract given in Equations (17) and (18). This will not increase its EV by (e) above. We have therefore shown that given any alternate menu of contracts (for retailers $(k+1), \ldots, n$), the menu given in the theorem has an equal or smaller EV. $\blacksquare$

We summarize the properties of the optimal menu of contracts.

1. From Property 8, the distributor makes a profit from all contracts.
2. The prices charged to the retailers are decreasing in $i$, i.e., $c_i > c_{i+1}, i \geq (k+1)$.
3. From Property 3, the fixed side payments made to the retailers are decreasing in $i$, i.e., $F_i > F_{i+1}, i \geq (k+1)$.
4. From Property 7 and the fact that retailer $i$ obtains the same EU from contracts $(F_i, c_i)$ and $(F_{i+1}, c_{i+1})$, the EV's of the contracts $(F_i, c_i)$ are increasing in $i \geq k$.
5. Retailers $1, 2, \ldots, k-1$ obtain EU's greater than their reservation utility. All other retailers get exactly their reservation utility.

We will now discuss how many retailers should get the fixed contract, i.e., the decision variable is now $k$. The distributor's profit maximization problem is (see problem (**P**)):

$$\max_k \Bigg( (E[\Pi(S^{EV}_{\text{opt}}, 0, c, s, e)] - r_k)k + \sum_{i>k} (E[\Pi(S^{EV}_{\text{opt}}, 0, c, s, e)] - (r_i + \rho_i(p - c_i)^2\sigma^2/2)) \Bigg), \tag{19}$$

where $F_i$ and $c_i$ are as defined in Equations (17) and (18). This problem can be solved numerically using a search technique.

## 4. Future research and conclusion

This paper demonstrates that an important role of an intermediary in distribution channels is to reduce the risk faced by retailers. The sharing of risk can be achieved by offering mutually beneficial risk sharing contracts which also raise the retailers' order quantity to the expected value maximizing quantity. Thus inefficiency created due to risk aversion on part of the retailers can be avoided.

We have shown that it is to the benefit of the distributor to offer a completely risk free contract to one (or more) of the most risk averse retailers. In practice this can be interpreted as an integrated channel in which the distributor owns the retail channel. We wish to emphasize that the distributor need not necessarily be risk neutral. The most efficient outcome is possible (i.e., the solution that yields the highest expected profit in the supply chain) only when the distributor is risk neutral. Otherwise, a person who is less risk averse than all the retailers will still find it beneficial to perform the role of risk intermediation.

We have also demonstrated an important decentralization result. The distributor is responsible for the ordering decision in our model, and the retailer is shown to be content with this arrangement. Therefore, even if there are economies of scale in distribution (for e.g., risk pooling), the distributor will offer the menu of contracts, and independently optimize with respect to her distribution costs. We can also extend our results to the case when the retailers are of different size (Assumption 1). In that case the retailer population can be partitioned into classes of retailers that are similar in size. A separate menu can then be derived for each size class.

Our analysis forms the basis for generalizations that have to be explored to get a more comprehensive insight into channel structure. Of particular interest is the case when there is competition between the distributors. As discussed in Rothschild and Stiglitz (1976) there may not be an equilibrium in this market. Another case of interest is when the price $p$ is a decision variable (i.e., the retailers are price setters). In this case the presence of the distributor may result in higher prices and lower consumption. In future work, we also plan to study the effects of different size of markets, correlation of retailer demands, and of effort aversion on incentives for risk sharing.

## Acknowledgements

We thank Professor J.G. Shanthikumar of the University of California, Berkeley, for pointing out that Lemma 2.1 can be generalized by the use of the dispersive ordering. We are grateful to Professor Robert M. Solow of the Massachussets Institute of Technology for helpful discussions, comments and suggestions. We thank the Associate Editor and two anonymous reviewers for many helpful comments.

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## Appendices

### *Appendix A*

**Property 1.** *If $i < j$, and there exists a $k, i < k < j$, such that retailer $k \notin \mathcal{M}(\mathcal{Q})$, then $c_i \neq c_j$.*

**Proof.** Assume that $c_i = c_j$. Then, by definition of $\hat{r}_i$ and Assumption 8,

$$\hat{r}_i = F_i + (p - c_i)\mu - \rho_i(p - c_i)^2\sigma^2/2, \qquad \text{(A1)}$$

$$\hat{r}_j = F_i + (p - c_i)\mu - \rho_j (p - c_i)^2 \sigma^2/2, \quad \text{(A2)}$$

$$r_k > F_i + (p - c_i)\mu - \rho_k (p - c_i)^2 \sigma^2/2. \quad \text{(A3)}$$

Equations (A1) and (A3) imply,

$$\frac{r_k - \hat{r}_i}{\rho_i - \rho_k} > (p - c_i)^2 \sigma^2/2, \quad \text{(A4)}$$

whereas Equations (A2) and (A3) imply,

$$\frac{\hat{r}_j - r_k}{\rho_k - \rho_j} < (p - c_i)^2 \sigma^2/2. \quad \text{(A5)}$$

Equations (A4) and (A5) contradict Lemma 3.2. ■

**Property 2.** *If $\rho_i > \rho_j$, then $c_i \geq c_j$.*

**Proof.** By the fact that retailer $i(j)$ prefers the contract $F_i, c_i$ $(F_j, c_j)$,

$$F_i + (p - c_i)\mu - \rho_i (p - c_i)^2 \sigma^2/2 \geq F_j + (p - c_j)\mu - \rho_i (p - c_j)^2 \sigma^2/2, \quad \text{(A6)}$$

$$F_i + (p - c_i)\mu - \rho_j (p - c_i)^2 \sigma^2/2 \leq F_j + (p - c_j)\mu - \rho_j (p - c_j)^2 \sigma^2/2. \quad \text{(A7)}$$

Equations (A6) and (A7) imply that,

$$\rho_j((p - c_i)^2 - (p - c_j)^2) \geq \rho_i((p - c_i)^2 - (p - c_j)^2) \Rightarrow ((p - c_i)^2 - (p - c_j)^2) \leq 0.$$ ■

**Property 3.** *If $\rho_i > \rho_j$, and $c_i \geq c_j$ $(c_i > c_j)$, then $F_i \geq F_j$ $(F_i > F_j)$.*

**Proof.** If $F_i < F_j$, then the contract $(F_j, c_j)$ will dominate $(F_i, c_i)$. ■

**Property 4.** *If $c_i > c_j$, $\sigma > 0$, and retailer $j$ prefers the contract $(F_j, c_j)$ over $(F_i, c_i)$, then all retailers $k \geq j$ will also prefer (out of the two contracts) $(F_j, c_j)$ to $(F_i, c_i)$.*

**Proof.** The difference,

$$F_i + (p - c_i)\mu - \rho(p - c_i)^2 \sigma^2/2 - (F_j + (p - c_j)\mu - \rho(p - c_j)^2 \sigma^2/2) = F_i + (p - c_i)\mu - (F_j + (p - c_j)\mu) - \rho((p - c_i)^2 - (p - c_j)^2)\sigma^2/2,$$

is strictly increasing in $\rho$. Therefore retailers $k \geq j$, who have $\rho_k \leq \rho_j$, will prefer $(F_j, c_j)$ to $(F_i, c_i)$.

**Property 5.** *If $c_i > c_j$, $\sigma > 0$, and retailer $i$ prefers the contract $(F_i, c_i)$ over $(F_j, c_j)$, then all retailers $k \leq i$ will also prefer (out of the two contracts) $(F_i, c_i)$ to $(F_j, c_j)$.*

**Proof.** Similar to that for Property 4. ■

**Property 6.** *If $i < j$, and the two retailers take the contracts $(F_i, c_i)$ and $(F_j, c_j)$, then retailer $j$ gets at least the same EV from his contract as retailer $i$ from the contract $(F_i, c_i)$.*

**Proof.** From Property 2, we know that $c_i \geq c_j$. Therefore, as retailer $j$ prefers the contract $(F_j, c_j)$,

$$F_i + (p - c_i)\mu - \rho_j (p - c_i)^2 \sigma^2/2 \leq F_j + (p - c_j)\mu - \rho_j (p - c_j)^2 \sigma^2/2,$$

$$\Rightarrow (F_i + (p - c_i)\mu) - (F_j + (p - c_j)\mu) \leq \rho_j \sigma^2/2((p - c_i)^2 - (p - c_j)^2) \leq 0.$$ ■

**Property 7.** *If the EU to a retailer is the same from two contracts $(F_i, c_i)$ and $(F_j, c_j)$, and $c_i \geq c_j$, then the EV is higher to the retailer from the contract $(F_j, c_j)$ compared to the EV from the contract $(F_i, c_i)$.*

**Proof.** Similar to that for Property 6. ■

**Property 8.** *If a risk averse retailer $i$ obtains the same EU from the contract $(F_i, c_i)$ as from the ONC, then the distributor stands to profit from offering the contract $(F_i, c_i)$ to this retailer.*

**Proof.** The EV to the retailer from the contract $(F_i, c_i)$ is not larger than the EV to the retailer from the ONC from Lemma 2.1. The distributor's profit is the difference in the EV between the ONC and the contract $(F_i, c_i)$. ■

The next Lemma will be used to prove Lemmas 3.3–3.5.

**Lemma A.1.** *In a solution to problem* **(P)**, *let the contracts taken by retailers $i$ and $j$ $(i, j \in \mathscr{M}(\mathscr{Q}))$, be given by $\mathscr{C}(F_i, c_i)$ and $\mathscr{C}(F_j, c_j)$. Then, $\rho_i > \rho_j$, $\sigma > 0$, and $c_i \neq p$ imply that $\hat{r}_i(\mathscr{C}(F_i, c_i))$ is less than $\hat{r}_j(\mathscr{C}(F_j, c_j))$. In words, the retailer who is less risk averse obtains a higher expected utility by selecting a contract from the distributor's menu.*

**Proof of Lemma A.1.** Assume instead that $\hat{r}_i \geq \hat{r}_j$, i.e.,

$$F_i + (p - c_i)\mu - \rho_i (p - c_i)^2 \sigma^2/2 \geq F_j + (p - c_j)\mu - \rho_j (p - c_j)^2 \sigma^2/2.$$

Since $\rho_i > \rho_j, \sigma > 0$, and $c_i \neq p$, this would imply

$$F_i + (p - c_i)\mu - \rho_j (p - c_i)^2 \sigma^2/2 > F_j + (p - c_j)\mu - \rho_j (p - c_j)^2 \sigma^2/2.$$

But this would in turn imply that retailer $j$ prefers $\mathscr{C}(F_i, c_i)$ to $\mathscr{C}(F_j, c_j)$, which is a contradiction. ■

**Proof of Lemma 3.3.** We know from Lemmas 3.1 and **A**.1 that $r_a < r_{i_{\text{MIN}}} \leq \hat{r}_{i_{\text{MIN}}}$. Two scenarios can be constructed depending on the contract taken by $i_{\text{MIN}}$, namely

(1) $F_{i_{\text{MIN}}} = \hat{r}_{i_{\text{MIN}}}, c_{i_{\text{MIN}}} = p$: in this case $r_a \le F_{i_{\text{MIN}}}$. So retailer "$a$" will take the contract $\mathscr{C}(F_{i_{\text{MIN}}}, c_{i_{\text{MIN}}})$. This would imply $i_{\text{MIN}} \le a$, which is a contradiction.

(2) $F_{i_{\text{MIN}}} \le \hat{r}_{i_{\text{MIN}}}, c_{i_{\text{MIN}}} \le p$: in this case the distributor can offer retailer "a" the contract $F_a = r_a, c_a = p$. From Property 4 this contract will not be taken by $i_{\text{MIN}}$ or by any other retailer in $\mathscr{M}(\mathscr{Q})$. From the remarks following Equation (3) and Property 8, such a contract will also provide additional profit to the distributor. ■

**Proof of Lemma 3.4.** To prove this lemma, we shall construct a new contract $(F_a, c_a)$ which is: (i) profitable to the distributor; and (ii) one that will be taken by retailer "$a$" but none of the retailers in $\mathscr{M}(\mathscr{Q})$ will switch to the new contract.

From Properties 1 and 2, $c_i \ne c_j$ and $c_i > c_j$. Using Lemma 3.2, we can find an $\epsilon > 0$, such that,

$$\frac{r_a - \hat{r}_i + \epsilon}{\rho_i - \rho_a} = \frac{\hat{r}_j - r_a - \epsilon}{\rho_a - \rho_j} = \delta \ge 0. \tag{A8}$$

Using this value for $\delta$, we construct the new contract,

$$c_a = p - \left(\frac{2\delta}{\sigma^2}\right)^{0.5}, \tag{A9}$$

$$F_a + (p - c_a)\mu - \rho_a (p - c_a)^2 \sigma^2/2 = r_a. \tag{A10}$$

By Assumption 9 and Equation (A10), retailer "$a$" will take the new contract. By our choice of the value of $\delta$, retailers $i$ and $j$ obtain EU's of just $(\hat{r}_i - \epsilon)$ and $(\hat{r}_j - \epsilon)$ by taking the contract $(F_a, c_a)$. Thus their preferences for the contracts $(F_i, c_i)$ and $(F_j, c_j)$ over the new contract $(F_a, c_a)$ are strict. Using Property 2, strict preference implies, $c_i > c_a > c_j$. Thus, using Properties 4 and 5, neither these two retailers, nor any of the retailers in $\mathscr{M}(\mathscr{Q})$ will prefer the new contract over their current contracts. Finally, by Property 8, the distributor will stand to profit by offering this new contract. ■

**Proof of Lemma 3.5.** It suffices to consider the contract given by,

$$c_a = p - \left(\frac{2(r_a - \hat{r}_{i_{\text{MAX}}})}{(\rho_{i_{\text{MAX}}} - \rho_a)\sigma^2}\right)^{0.5}, \tag{A11}$$

$$F_a + (p - c_a)\mu - \rho_a (p - c_a)^2 \sigma^2/2 = r_a. \tag{A12}$$

In Equation (A11), we have used the fact that by arguments similar to the ones given in Lemmas 3.1 and **A**.1, $r_a \ge \hat{r}_{i_{\text{MAX}}}$. From construction the EU to retailer $i_{\text{MAX}}$ from the contract $(F_a, c_a)$ is equal to $\hat{r}_{i_{\text{MAX}}}$. We now show that the expected value of the contract $(F_a, c_a)$ is greater than the expected value of the contract, $(F_{i_{\text{MAX}}}, c_{i_{\text{MAX}}})$. (This is equivalent to showing that $c_a < c_{i_{\text{MAX}}}$, by Property 7.)

Since retailer $i_{\text{MAX}}$ obtains the same expected utility from these two contracts,

$$\begin{aligned}&(F_a + (p - c_a)\mu - \rho_{i_{\text{MAX}}}(p - c_a)^2\sigma^2/2)\\ &\quad = (F_{i_{\text{MAX}}} + (p - c_{i_{\text{MAX}}})\mu - \rho_{i_{\text{MAX}}}(p - c_{i_{\text{MAX}}})^2\sigma^2/2).\end{aligned} \tag{A13}$$

Assume instead that the EV of $(F_a, c_a)$ is less than the EV of $(F_{i_{\text{MAX}}}, c_{i_{\text{MAX}}})$. Combining this assumption with Equation (A13) we get

$$\begin{aligned}0 &\ge \text{Difference in EV's}\\ &= (F_a + (p - c_a)\mu) - (F_{i_{\text{MAX}}} + (p - c_{i_{\text{MAX}}})\mu),\\ &= \rho_{i_{\text{MAX}}}(p - c_a)^2\sigma^2/2 - \rho_{i_{\text{MAX}}}(p - c_{i_{\text{MAX}}})^2\sigma^2/2,\\ &= \rho_{i_{\text{MAX}}}((p - c_a)^2\sigma^2/2 - (p - c_{i_{\text{MAX}}})^2\sigma^2/2),\\ &\Rightarrow ((p - c_a)^2\sigma^2/2 - (p - c_{i_{\text{MAX}}})^2\sigma^2/2) \le 0.\end{aligned} \tag{A14}$$

However, because $\rho_a < \rho_{i_{\text{MAX}}}$, inequality (A14) will imply,

$$\begin{aligned}&(F_a + (p - c_a)\mu - \rho_a(p - c_a)^2\sigma^2/2)\\ &\quad - (F_{i_{\text{MAX}}} + (p - c_{i_{\text{MAX}}})\mu - \rho_a(p - c_{i_{\text{MAX}}})^2\sigma^2/2)\\ &\quad = (F_a + (p - c_a)\mu - \rho_{i_{\text{MAX}}}(p - c_a)^2\sigma^2/2)\\ &\qquad - (F_{i_{\text{MAX}}} + (p - c_{i_{\text{MAX}}})\mu - \rho_{i_{\text{MAX}}}(p - c_{i_{\text{MAX}}})^2\sigma^2/2)\\ &\qquad + (\rho_{i_{\text{MAX}}} - \rho_a)\Big((p - c_a)^2\sigma^2/2\\ &\qquad - (p - c_{i_{\text{MAX}}})^2\sigma^2/2\Big) \le 0.\end{aligned} \tag{A15}$$

Equation (A15) implies that retailer "$a$" prefers (or is indifferent to) $(F_{i_{\text{MAX}}}, c_{i_{\text{MAX}}})$ over $(F_a, c_a)$, which is a contradiction. Therefore, (A14) must be false. This implies, $c_a < c_{i_{\text{MAX}}}$.

However $c_a < c_{i_{\text{MAX}}}$ also implies that $F_a < F_{i_{\text{MAX}}}$, else $(F_a, c_a)$ will dominate $(F_{i_{\text{MAX}}}, c_{i_{\text{MAX}}})$ (due to the higher fixed payment and lower unit cost). Combining this with Assumption 9 we get that retailer $i_{\text{MAX}}$ prefers the contract $(F_{i_{\text{MAX}}}, c_{i_{\text{MAX}}})$ to $(F_a, c_a)$ (even though both give $i_{\text{MAX}}$ the same expected utility). To complete the proof we see from Property 4 that no other retailer in $\mathscr{M}(\mathscr{Q})$ will choose the new contract, $(F_a, c_a)$. (The proof is unaffected if the number of retailers with reservation utility $r_a$ is greater than one.) ■

## Biographies

Vipul Agrawal is an Assistant Professor of Operations Management, Leonard N. Stern School of Business, New York University. He

received his Ph.D. in Operations Management from The Wharton School of the University of Pennsylvania. He holds a M.S. in Industrial Engineering from Stanford University and a B.S. in Mechanical Engineering from the University of Delhi. He currently teaches courses on Operations Management and Supply Chain Management at the Stern School. He has published in *Operations Research* and *IIE Transactions* and is on the Editorial Review Board of *Production and Operations Management Journal*. His current research interests include design and control of supply chains, pricing and contracts in supply chains, performance evaluation and benchmarking, and impact of electronic commerce on supply chains.

Sridhar Seshadri is an Associate Professor of Operations Management, Leonard N. Stern School of Business, New York University. He received his Bachelor of Technology degree in 1978 from the Indian Institute of Technology, Madras, India, his Post Graduate Diploma in Management in 1980 from the Indian Institute of Management, Ahmedabad, India, and his Ph.D. degree in Management Science in 1993 from the University of California at Berkeley. His primary area of expertise is Stochastic Modeling and Optimization with applications to manufacturing, distribution, telecommunications system design, database design and finance. He joined the Stern School of Business in 1993. He taught at the Administrative Staff College of India, Hyderabad during 1985–1989. Prior to that he worked in industry as a Production Manager and Project Engineer. He serves as an Associate Editor for *Naval Research Logistics*, *Operations Research Letters*, *Telecommunication Systems Journal*, and the *Production and Operations Management Journal*.