Disasterization: A Simple Way to Fix the Asset Pricing Properties of Macroeconomic Models

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Introduction

- It's difficult to get good asset pricing properties (high and volatile excess returns for stocks and bonds) in macro models (RBC, New Keynesian).
- Variants a la habit formation (Campbell Cochrane '95) or Epstein-Zin-Weil utilities (Bansal Yaron '04) work in endowment economies, but don't work well in production economies (Fernandez-Villaverde, Koijen, Rubio-Ramirez, van Binsbergen 08).
- I use the "Rare disaster" hypothesis of Rietz ('88) and Barro ('06)
- Existing work largely rests on endowment economies: it's unclear how to mix them with traditional production economies.
- Here, I show a fairly generic way to fix properties of RBC models

Related literatures

- Rare events and fat tails: Rietz ('88), Brown Goetzman and Ross ('95), Longstaff and Piazzesi ('04), Veronesi '04, Barro ('06), Weitzman ('07), Santa-Clara and Yan ('06), Gabaix et al. ('03, '06), Barro and Ursua ('08), Barro, Nakamura and Steinsson and Ursua ('08), Gourio '08, Wachter ('08), Julliard and Ghosh ('08).
- Variable rare disasters:
  - *Methodology*: "Linearity-Generating processes: A modelling tool yielding closed forms for asset prices" (Gabaix '09a)
  - *Closed economy*: "Variable Rare Disasters: An Exactly Solved Model for Ten Puzzles in Macro-Finance" (Gabaix '0bb)
  - *Open economy*: "Rare Disasters and Exchange Rates" (Farhi and Gabaix '08), "Crash risk in currency Markets" (w/ Farhi, Ranciere and Verdehlan)
- Papers on many assets: Lettau and Wachter ('07), Bansal and Shaliastovich ('08).
- Macro-style papers: Boldrin, Christiano Fisher '01 (but highly volatile interest rate), Tallarini (2000).

A way to fix the asset pricing properties of RBC models

- Start from a real business cycle model that can generate realistic macro dynamics, fix it, and get a new model with the *same* business cycle properties, but *different* asset pricing properties.

$$\max \sum_t \rho^t \frac{C_t^{1-\gamma}}{1-\gamma} \phi\left(L_t\right)$$

$$\begin{aligned}
K_{t+1} &= \left[\left(1-\delta\right) K_t + I_t\right] \cdot \Delta_{t+1} \\
Y_t &= \left(a_t e^{X_t} L_t\right)^{\beta} K_t^{1-\beta} = C_t + I_t \\
a_{t+1} &= e^{g} a_t \cdot \Delta_{t+1} = \text{permanent part of productivity} \\
X_{t+1} &= \rho X_t + \eta_{t+1} = \text{ transitory part of productivity}
\end{aligned}$$

- In original economy, $\Delta_{t+1} = 1$ for all t (no disaster)
- In the new economy: If there is no disaster, $\Delta_{t+1} = 1$, otherwise Δ_{t+1} =disaster impact. E.g. $\Delta_{t+1} = 0.8$
- Key ingredient: Disaster affects all the extensive variables (K, a), only. So if econ. was on equilibrium path before disaster, it is also on it

Doing regular macro with this

A mapping theorem

Suppose $E_{t-1}\left[\Delta_t^{1-\gamma}\right]$ constant.

- **Theorem**. The above economy, with disasters, can be solved in the following two-step procedure

1. Solve for the "0" economy with no disasters, i.e. with $\forall t, \Delta_t = 1$, and $\rho_0 = \rho / E\left[\Delta_t^{1-\gamma}\right]$. Call $C_t^0, K_t^0, a_t^0, L_t^0, X_t^0$ the solution for each t.
2. Then, the solution of the economy 1 with disasters is

$$\begin{aligned} (C_t, K_t, a_t) &= \left(\mathcal{D}_t C_t^0, \mathcal{D}_t K_t^0, \mathcal{D}_t a_t^0\right) \\ (L_t, X_t) &= \left(L_t^0, X_t^0\right) \end{aligned}$$

where $\mathcal{D}_t = \Delta_1 ... \Delta_t$ is the cumulative disaster.

- In other terms, the *extensive* variables (C_t, K_t, a_t) are scaled by disaster $\mathcal{D}_t$, while the *intensive* variables (L_t, X_t) are left unchanged.
- Commodities (and labor) prices are the same, asset prices are unchanged.

A mapping theorem, II

- Conditionally on no disasters: "macro" variables (investment, labor, capital,consumptions) don't change at all, but asset prices change.

E.g., one derives yield curve, stock prices etc., while keeping macro side constant.

- So, you can simulate the model (with many other variables) by approximation around the steady state.
- Disasterization result in Gabaix ('07), subsequently used (with many other things) in Gourio ('09): time-varying proba. of disaster, Epstein-Zin-Weil...
- Extends to other things: with habit $u\left(C_t, \overline{C}_t, L_t\right) \to \overline{C}_t$ multiplied by Δ_t too.
- In this economy, there's no high slope of the yield curve (Barro '06), nor a time-varying slope of the yield curve: So let's see how to get that.

Bonds – Setup from "Ten Puzzles" Paper

- Normalize baseline inflation to 0. Real value of money: $\frac{Q_{t+1}}{Q_t} = 1 - i_t$

$$i_{t+1} = \frac{e^{-\phi_i} i_t + 1_{\{\text{Disaster at } t+1\}} j_t}{1 - i_t} + \varepsilon^i_{t+1}$$

- Inflation mean-reverts with a linearity–generating twist, and if there is a disaster, inflation jumps by j_t.
- j_t can be constant or mean-reverting:

$$j_{t+1} = j_* + \frac{e^{-\phi_j} (j_t - j_*)}{1 - i_t} + \varepsilon^j_{t+1}$$

- Notations: $j_* = \frac{\kappa(\phi_i - \kappa)}{pB^{-\gamma} F}$; $\pi_t \equiv pB^{-\gamma} F_t (j_t - j_*)$; $i_{**} \equiv i_* + \kappa$

Nominal Bonds

$$Z_t^1(T) = E_t\left[\frac{M_{t+T}^0}{M_t^0}\frac{M_{t+T}^*}{M_t^*}\frac{Q_{t+T}}{Q_t}\right] = E_t\left[\frac{M_{t+T}^0}{M_t^0}\right]E_t\left[\frac{M_{t+T}^*}{M_t^*}\frac{Q_{t+T}}{Q_t}\right] = Z_t^0($$

$Z_t^0(T) = E\left[\rho_0^T C_{0t+T}^{-\gamma}/C_{0t}^{-\gamma}\right]$ = Price of a bond in regular economy with no

$$Z_t^*(T) = e^{-\alpha T}\left(1 - \frac{1-e^{-\psi_i T}}{\psi_i}i_t - \frac{\frac{1-e^{-\psi_i T}}{\psi_i} - \frac{1-e^{-\psi_\pi T}}{\psi_\pi}}{\psi_\pi - \psi_i}\pi_t\right)$$

$Z_t^*(T)$ =Price of a bond in a disaster endowment economy. It exhibits bond risk premia, upward sloping yield curve etc., as in Gabaix ('08b).

- So we get (i) Upward sloping yield curve,and (ii) varying slope →Fama-Bliss, Campell-Shiller, Cochrane-Piazzesi puzzles.
- Note that if the central bank is less credible (e.g.,high debt/GDP ratio), the yield curve is more steep →Impact of the central bank on long term rates, even in a Ricardian world.

Derivations from that framework

- Bottomline:
  - Nominal Bond prices change
  - In general, asset prices change
- With the model, one could study other interesting feedback from asset prices to the real economy:
  - Increase in long term rates (due to increased nominal bond premium, perhaps because of higher Debt/GDP) $\longrightarrow$ lower investment
  - Increase in equity valuations $\longrightarrow$ more R&D and higher growth, and also a crash ahead.
- However, how do we model a stock? Here the price of capital (Q) remains constant and equal to 1. Let's see how to enrich the model to fix that.

On Potential Paradigms

- 3 rational, representative-agent paradigms for high risk premia
  - External habit (Abel, Campbell-Cochrane)
  - Epstein-Zin-Weil utilites and long run risk (Bansal Yaron)
  - Disasters
- Other frameworks: models with heterogeneous agents, non-rational beliefs, robustness...
- I conclude that: Disasters are the most tractable, and the most amenable to fitting with macro

Neoclassical model with variable Tobin's Q

$$\max \sum_t \rho^t \frac{C_t^{1-\gamma}}{1-\gamma}$$

$$\begin{aligned} K_{t+1} &= \left(\left(1-\delta_K\right) K_t + I_t \right) \cdot \Delta_{t+1} \\ Y_t &= \left(\int_0^1 Q_{it}^{1/(1+\mu)} di \right)^{1+\mu} = C_t + I_t, \quad Q_{it} = A_t K_{it}^{\alpha} L_{it}^{1-\alpha} \end{aligned}$$

$$A_{t+1} = A_t \cdot \Delta_{t+1}, \quad \Delta_{t+1} = \begin{cases} 1 & \text{in normal times} \\ B_{t+1},\ B_{t+1} < 1 & \text{if disaster} \end{cases}$$

- Each firm is a Dixit-Stiglitz monopolist $\rightarrow$Monopoly Profits are: $D_t = \mu Y_t$.
- Corrective Taxes lead to first best allocation / production.

Introducing firm-level disasters

- If there's a disaster, expropriation of *rents* (not capital), with probability $1 - F_t$. The firm loses its patents, not its physical capital. Rents are redistributed. So earnings are:

$$D_t = \mu Y_t \Delta_1^\pi ... \Delta_t^\pi$$

$$\Delta_t^\pi = \begin{cases} 1 & \text{in normal times} \\ 0 \text{ with probability } 1 - F_t\text{, otherwise } 1 & \text{if disaster} \end{cases}$$

- If rents are redistributed, that affects the value of the firms, but not the productive capacity of the economy.
- NPV of future profits: $V_t^\pi = E_t \left[\sum_{s=0}^\infty \frac{M_{t+s}}{M_t} D_{t+s} \right]$
- Resilience of a stock like "Ten Puzzles":

$$H_t \equiv p_t E_t \left[B_{t+1}^{-\gamma} F_t - 1 \right] \equiv H_* + \widehat{H}_t$$

Postulate a linearity-generating process (Gabaix '08) for variations in resilience:

$$\widehat{H}_{t+1} = \frac{1 + H_*}{1 + H_t} e^{-\phi_H} \widehat{H}_t + \varepsilon_{t+1}^H$$

Stock Market Value

With $r_e \equiv R - \ln(1 + H_*)$,

$$
\begin{aligned}
V_t &= V_t^{\pi} + K_t = \frac{\mu Y_t}{r_e}\left(1 + \frac{\widehat{H}_t}{r_e + \phi}\right) + K_t \\
&= \text{PV of rents } + \text{ physical capital stock}
\end{aligned}
$$

- Intuition:
  - The stock market fluctuates, but in ways unrelated to the real economy.
  - Why? It's just about values of future rents, but it's not related to the real economy
- This is why this framework sticks to Exp. Ut., and doesn't use EZW. (Barro '08).

Return on the stock

- Expected return, for a sample with no disaster:

$$E_t \left[R_{t+1} \mid \text{Normal times} \right] = R - H_t$$

- Riskless rate $r = R - pE\left[B^{-\gamma}\right] \simeq 1\%$.
- So, we get time-varying stock returns
- Conclusion so far: we have a model with
    - The usual good RBC properties
    - Time-varying price-earnings ratios
    - High equity premium

Tobin's Q

$$V_t = V_t^{\pi} + K_t = \frac{\mu Y_t}{r_e}\left(1 + \frac{\widehat{H}_t}{r_e + \phi}\right) + K_t$$

- Tobin's Q:

$$Q_t = \frac{V_t}{K_t} = 1 + \frac{\mu}{r_e}\left(1 + \frac{\widehat{H}_t}{r_e + \phi}\right)\frac{Y_t}{K_t}$$

- Profits give $Q > 1$
- Physical capital has always $p_K = 1$ (zero adjustment cost for physical capital)
- Physical investment is unrelated to Tobin's Q. It's always the RBC investment.
- That is in sync with the empirical evidence (e.g., Philippon 08).
  - It's only the Q of intangibles / property rights that varies.
  - When Q is high, no desire to invest (infinite cost of adjustment for patents)

Calibration

- Macro parameters (annual values): $\delta = 4\%$, $\gamma = 4$, $g_c = 2.5\%$
- Disaster (numbers from Barro): $p = 1.7\%$ $\rightarrow$The importance of disaster events is multiplied by $E\left[B^{-\gamma}\right] = 10$.
- Speed of mean-reversion: $\phi = 0.2$
- Markup of 15%: $\mu = 0.15$
- On average:

$$\frac{V_t}{Y_t} = \frac{\mu}{r_e} + \frac{K_t}{Y_t}$$

- $r_e = 5\%$, so typical values are:

$$\frac{\mu}{r_e} = 3 \simeq \frac{K_t}{Y_t}$$

  so half the stock market valuation is for physical capital, half for rents.
- Debt / Equity mix is 50%, $\sigma_F = 0.09 \rightarrow$ Volatility of stock market is about 15% a year.

Conclusion

- A way to fix asset pricing properties of RBC models (and New Keynesian models)
- The simplicity of this approach contrasts with the difficulty of doing the same with habit formation or Epstein-Zin-Weil utility.
- Economic intuition: investors experience time-varying perceptions of risk of rents
- However, this is uncorrelated with economic activity
- To a good first order, this is what reality looks like.
- Still, to do list:
  - What if higher stock market values spur more entrepreneurship?
  - Impact of collateral values.
- It's useful to have a benchmark model with no effects (i) to be able to study the (perhaps small) effect of stock market on economic activity (ii) that can be used (via linearization around steady state) with a wide variety of factors.