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Lexicographic Probabilities and Iterated Admissibility

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The implications of common knowledge of rationality in normal-form games have been explored in recent years by Aumann (1987), Bernheim (1984, 1986), Brandenburger and Dekel (1987, 1989), Pearce (1984), Tan and Werlang (1988), and others. This line of research is concerned with providing what Aumann has termed "analytic" foundations for game-theoretic solution concepts.

The most conservative approach is due to Bernheim and Pearce. Bernheim and Pearce's notion of rationalizability assumes that it is common knowledge that each player is a subjective expected utility maximizer (Savage 1954; Anscombe and Aumann 1963), but that the precise subjective probability distribution held by each player is not necessarily common knowledge. Bernheim and Pearce show that this assumption implies that players choose iteratively undominated actions.[1]

The hypothesis of subjective expected utility maximization does not preclude players from choosing actions that are weakly dominated (inadmissible). Within decision theory, admissibility has long been advanced as an important criterion for choice (see, for example, Luce and Raiffa 1957, chapter 13). On the premise that the criterion is equally reasonable in a game-theoretic context, Kohlberg and Mertens (1986, p. 1014) view "admissibility of the players' strategies as a basic requirement" of a satisfactory solution concept. Moreover, they argue in favor of an iterated admissibility requirement.[2]

This leads naturally to the following question: What are the implications of assuming, in addition to common knowledge of rationality, that it is common knowledge that players choose only admissible actions? Samuelson (1989) and Börgers (1990) have addressed this and other questions concerning the interplay between the concepts of common knowledge and admissibility. The present paper was inspired by Börgers' work in particular, and uses lexicographic probabilities to derive a result (proposition 2 below) which is analogous to a result of Börgers. Both results address the question posed at the beginning of this paragraph.

By analogy with the result linking common knowledge of subjective expected utility maximization and iterated deletion of dominated actions, one might conjecture that assuming it to be common knowledge that players choose *admissible* actions implies that they choose *iteratively admissible* actions. However, as Samuelson has demonstrated, establishing such a result seems problematic. Börgers shows that a weaker result is achievable. He shows that *approximate* common knowledge of admissibility implies that players choose actions that survive one round of deletion of weakly dominated actions followed by iterated deletion of strongly dominated actions. This latter procedure was proposed by Dekel and Fudenberg (1990), who showed it to be equivalent to iterated admissibility when players may be uncertain about other players' payoffs.

Börgers is led to assume approximate common knowledge since, in order to incorporate admissibility, he supposes that a player's subjective probability distribution assigns strictly positive weight to *all* choices of the other players. If there are any dominance relationships in the game, this condition will violate common knowledge of rationality since it requires players to place positive probability on dominated choices by other players. In order to resolve this conflict, Börgers replaces common knowledge with a notion of approximate common knowledge that builds on the definitions of Monderer and Samet (1989) and Stinchcombe (1988).

The objective of the present paper is to show how lexicographic probabilities can be used to prove a result analogous to that of Börgers.[3] Thus, instead of approximate common knowledge, I use a lexicographic analogue, which I call common first-order knowledge. To define common first-order knowledge of admissibility, I employ the non-Archimedean version of subjective expected utility theory developed by Blume, Brandenburger, and Dekel (1991a). According to this theory, a decision maker possesses, in addition to a utility function on outcomes, a *hierarchy* of probability distributions on uncertain events that is used lexicographically in ranking actions. (In conventional subjective expected utility theory, a decision maker possesses a *single* probability distribution.) Admissibility of the decision maker's choices is ensured by requiring that every event receive positive measure under some probability distribution in the hierarchy. As for common first-order knowledge, one begins by saying that the decision maker "first-order knows an event" if that event is assigned probability 1 by the first-order probability distribution in the decision maker's hierarchy. An event is then common first-order knowledge if everyone first-order knows it, everyone first-order knows that everyone first-order knows it, and so on.