Game Theory and Business Strategy

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A Calculus of Games?

“Games combining chance and skill give the best representation of human life”

--Leibniz (1710)

“It would be desirable to have a complete study made of games, treated mathematically”

--Leibniz (1715)

Quoted in “Game Theory,” by Oskar Morgenstern, in *Dictionary of the History of Ideas*, at http://etext.lib.virginia.edu/DicHist/dict.html

Some Early History

Zermelo (1913), Konig (1927), and Kalmar (1928-29) showed that Chess is “strictly determined”

Borel (1921-27) studied random strategies

Von Neumann (1928) formulated the concept of “strategy,” proved the Minimax Theorem (two-player zero-sum games), and formulated cooperative game theory

Drawn in part from Morgenstern op. cit.

Cooperative Game Theory

The von Neumann-Morgenstern (1944) cooperative theory

Method of description of a game:

A set *N* of players

A function $v : \wp(N) \to \Re$, where, for each subset *S*, *v*(*S*) is interpreted as the value created by the players in *S*

"We call the functions … characteristic functions—even when they are viewed in themselves, without reference to any game" *

Methods of analysis:

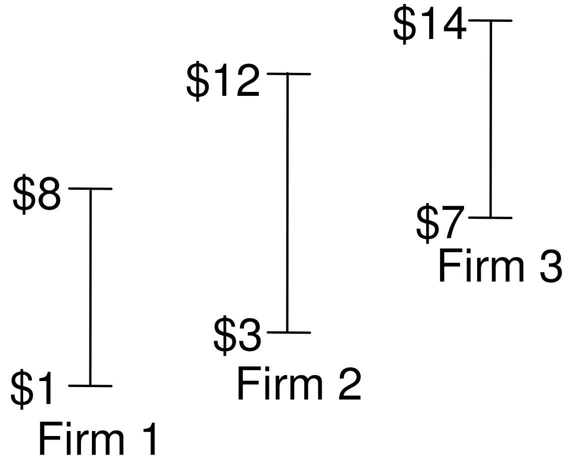
Core, Shapley Value, …

We will focus on the Core as the expression of competition

Note: Terminology here is misleading!

* *Theory of Games and Economic Behavior*, by John von Neumann and Oskar Morgenstern, Princeton University Press, 1994, p.245

Example of a Cooperative Game



3 firms, many suppliers, 2 buyers
Each firm can produce 1 unit
Each supplier can supply 1 firm
Each buyer can buy 1 unit

The pie $v(N)$ = \$16

Marginal contributions ("added values") $v(N) - v(N \setminus \{i\})$:

Firm 1 = Firm 3 = Any supplier = \$0

Firm 2 = \$2

Each buyer = \$7

This is a cooperative game-theoretic analysis of "positioning"

[How does it relate to the "generic strategies" (Porter 1980)?]

The Linguistic Connection

Some frameworks from business strategy:

Five Forces (Porter 1980)

Imitation-Substitution-Holdup-Slack (Ghemawat 1991)

...

Language from business strategy:

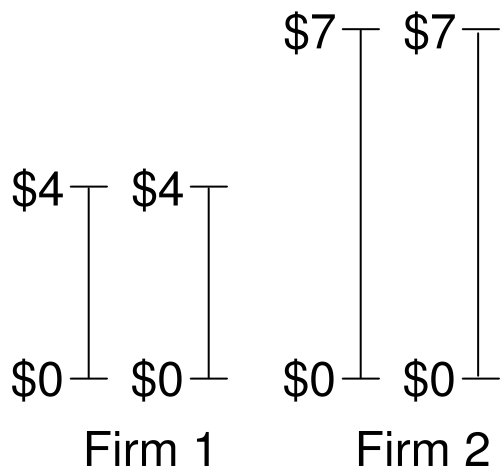
Value, power, bargaining, negotiation, …

Language of cooperative game theory:

Very similar!

References: "Value-Based Business Strategy," and "Biform Games," by Adam Brandenburger and Harborne Stuart, *Journal of Economics & Management Strategy*, 1996, and *Management Science*, 2007

A Second Example



2 firms, 3 buyers
Each firm can produce 2 units
Each buyer can buy 1 unit

The pie $v(N)$ = \$18

Firm 1 gets \$0

Firm 2 gets \$6

Each buyer gets \$4

Reference: "Disadvantageous Syndicates," by Andrew Postlewaite and Robert Rosenthal, *Journal of Economic Theory*, 1974

Choosing the Game

What about “strategic moves” such as the decision

whether to enter a market

where to position a product

what brand to build

how much capacity to install

how much money to devote to R&D

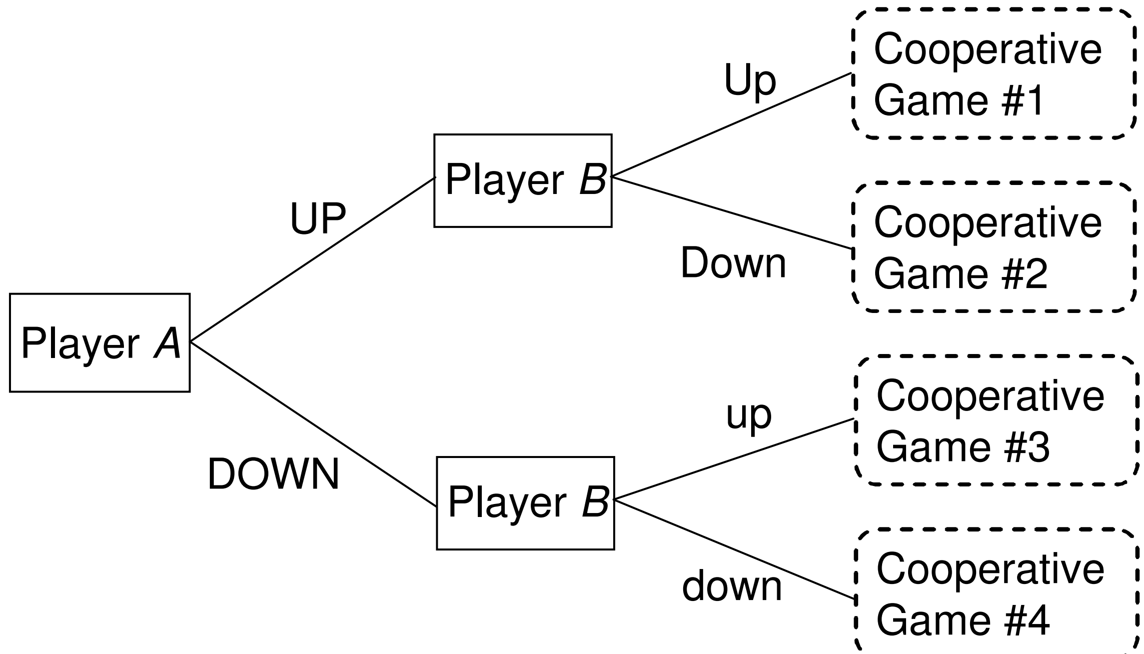
...

Such moves and countermoves are formalized via non-cooperative theory

But the consequences are to be analyzed via cooperative theory

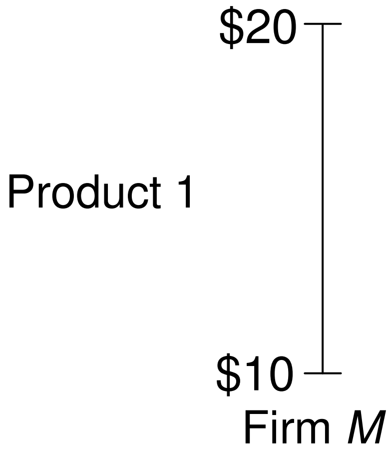
So, we need a hybrid formalism

Biform Games*

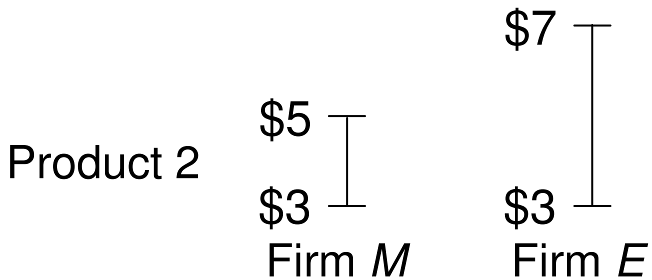


"Biform Games," by Adam Brandenburger and Harborne Stuart, *Management Science*, 2007

Example of a Biform Game



2 firms, 1 buyer
Independent products



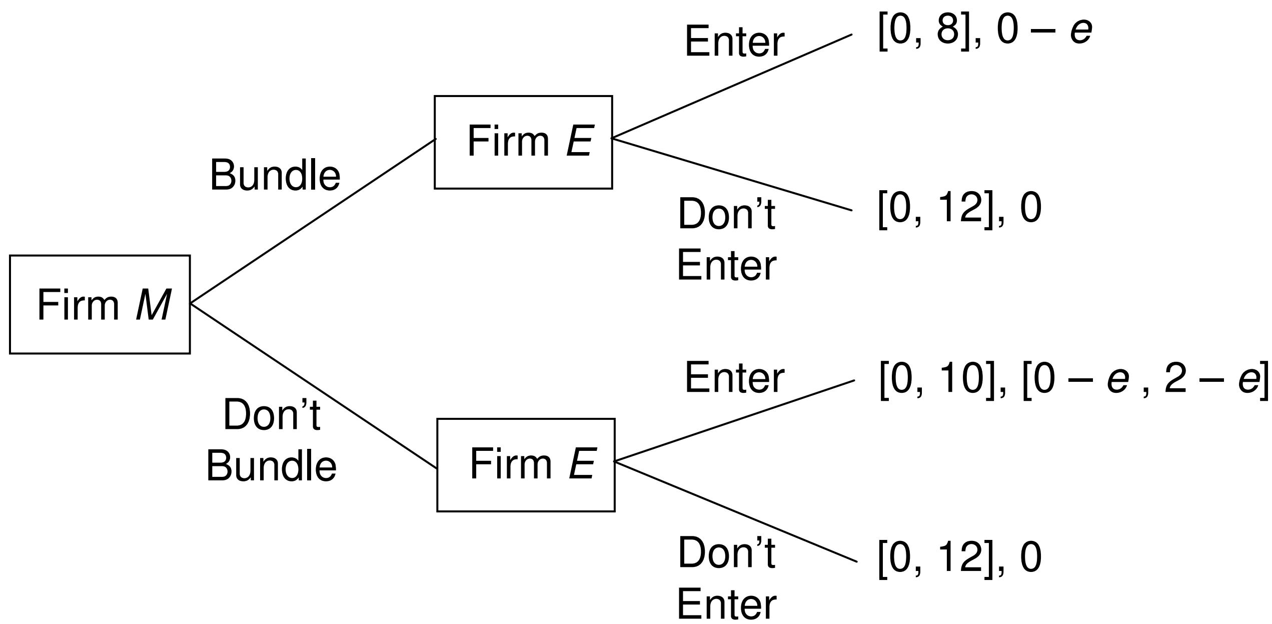
Firm *M* does not bundle:

Pie = \$14, $MC_{\text{Firm } M}$ = \$10, $MC_{\text{Firm } E}$ = \$2, MC_{Buyer} = \$14

Firm *M* does bundle:

Pie = \$12, $MC_{\text{Firm } M}$ = \$8, $MC_{\text{Firm } E}$ = \$0, MC_{Buyer} = \$12

Example continued



Aside on Non-Cooperative Theory

Use of the biform model is not restricted to

Nash Equilibrium

Backward Induction

The model is neutral to what non-cooperative solution concept is used

The “epistemic program” in game theory establishes that

Nash Equilibrium

Backward Induction

do not follow from the assumption that the players are rational (but from much more stringent conditions)

The central feature of this program is to allow uncertainty over not just what the game is (Harsanyi 1967-8) but also what strategies are chosen

See: “The Power of Paradox: Some Recent Results in Interactive Epistemology,” by Adam Brandenburger, *International Journal of Game Theory*, 2007

Evaluation of Cooperative Games

The bargaining process in a cooperative game can be decomposed into two elements:

competition—captured via the Core

residual negotiation—if the Core is not a single point

A theory of residual negotiation:

For each player, calculate the Core projection (a closed bounded interval of $\mathfrak{R}$)

Take a (subjective) weighted average of the upper and lower endpoints

This measure can be axiomatized via a modification of the Milnor (1954) derivation of the Hurwicz (1951) optimism-pessimism index

Efficiency/Inefficiency of Strategies

AU (**Adding Up**): For all strategy profiles s

$$\sum_{i \in N} [\, V(s)(N) - V(s)(N \backslash \{i\}) \,] = V(s)(N)$$

NE (**No Externalities**): For each player i, pair of strategies r^i, s^i for player i, and strategy profile s^{-i} for the players other than i

$$V(r^i, s^{-i})(N \backslash \{i\}) = V(s^i, s^{-i})(N \backslash \{i\})$$

NC (**No Coordination**): For each player i, pair of strategies r^i, s^i for player i, and pair of profiles r^{-i}, s^{-i} for the players other than i

$$V(r^i, r^{-i})(N) > V(s^i, r^{-i})(N) \text{ if and only if } V(r^i, s^{-i})(N) > V(s^i, s^{-i})(N)$$

Efficiency/Inefficiency continued

Theorem: Fix a biform game satisfying AU, NE, and NC. Suppose that, for each strategy profile *r*, the cooperative game *V*(*r*) has a nonempty Core. Then, if a profile *s* is a (pure) Nash equilibrium, it is efficient.

Theorem: Fix a biform game satisfying AU and NE. Suppose that, for each strategy profile *r*, the cooperative game *V*(*r*) has a nonempty Core. Then, if a profile *s* is efficient, it is a (pure) Nash equilibrium.

Examples:

The positioning game (with a "status-quo" move) satisfies AU, NE, and NC

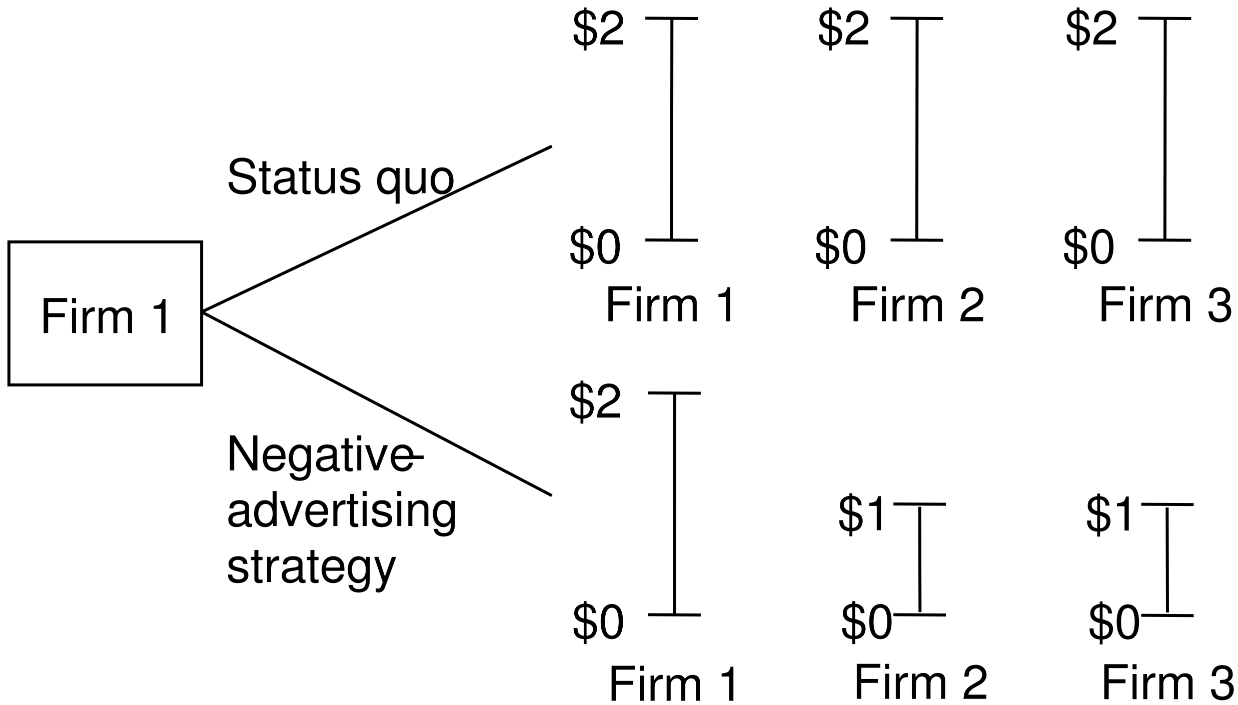
The bundling game fails AU and NC

Think of these theorems as game-theoretic analogs to the First and Second Welfare Theorems of General Equilibrium Theory

The theorems are closely related to Makowski-Ostroy (1994), (1995)

Additional Examples of Inefficiency:

A "Negative Advertising" Game



3 firms, many suppliers, 2 buyers

AU and NC are satisfied, but NE fails

Additional Examples of Inefficiency:

A "Technology Coordination" Game

No

	No	Yes
No	v(N) = 6 v(1, 2) = 4 v(2, 3) = 4 v(3, 1) = 4	v(N) = 5 v(1, 2) = 3 v(2, 3) = 3 v(3, 1) = 4
Yes	v(N) = 5 v(1, 2) = 3 v(2, 3) = 4 v(3, 1) = 3	v(N) = 6 v(1, 2) = 6 v(2, 3) = 3 v(3, 1) = 3

Yes

	No	Yes
No	v(N) = 5 v(1, 2) = 4 v(2, 3) = 3 v(3, 1) = 3	v(N) = 6 v(1, 2) = 3 v(2, 3) = 6 v(3, 1) = 3
Yes	v(N) = 6 v(1, 2) = 3 v(2, 3) = 3 v(3, 1) = 6	v(N) = 9 v(1, 2) = 6 v(2, 3) = 6 v(3, 1) = 6

Interpretation: The new technology costs $1 more per player, and is worth $2 more per player, provided at least two players adopt it

AU and NE are satisfied, but NC fails

Example continued

	No	Yes
No	2, 2, 2	2, 1, 2
Yes	1, 2, 2	3, 3, 0

No

	No	Yes
No	2, 2, 1	0, 3, 3
Yes	3, 0, 3	3, 3, 3

Yes

There are 2 (pure) Nash equilibria:

(*No*, *N*o, *No*) is inefficient

(*Yes*, *Yes*, *Yes*) is efficient

A Connection to Corporate ("Multibusiness") Strategy

Think of the multibusiness firm as a "mini-economy" that may exhibit inefficiencies

Our theorem gives us a classification:

Example of a failure of AU:

The Holdup Problem leading to a failure to invest

Example of a failure of NE:

Divisional vs. corporate management of brand, knowledge, … leading to a failure to account for spillovers

Example of a failure of NC:

Divisional vs. "coordinated" choice of supplier leading to a failure to defray a supplier's fixed costs, speed up its learning, …

Conclusion

The theory of mechanics for 2, 3, 4, … bodies is well known, and in its general theoretical (as distinguished from its special and computational) form is the foundation of the statistical theory for great numbers. For the social exchange economy—i.e. for the equivalent "games of strategy"—the theory of 2, 3, 4, … participants was heretofore lacking….

*The problem must be formulated, solved and understood for small numbers of participants before anything can be proved about the changes of its character in any limiting case of large numbers such as free [perfect] competition.**

**Theory of Games and Economic Behavior*, by John von Neumann and Oskar Morgenstern, Princeton University Press, 1994, p.14