Origins of Epistemics

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Version 07-01-09

Beginnings

"[T]here is exhibited an endless chain of reciprocally conjectural reactions and counter-reactions…. The remedy would lie in analogous employment of the so-called Russell theory of types in logistics. This would mean that on the basis of the assumed knowledge by the economic subjects of theoretical tenets of Type I, there can be formulated higher propositions of the theory; thus, at least, of Type II. On the basis of information about tenets of Type II, propositions of Type III, at least, may be set up, etc."

--Morgenstern, "Perfect Foresight and Economic Equilibrium" (*ZfN*, 1935)

A Change in Focus: Competition and Cooperation

Von Neumann's interest was in

the "protective" or "defensive" maximin strategies in two-player zero-sum games

the implications of coalition formation in *n*-player (general-sum) games—i.e., the implications of the interplay between competition and cooperation

"Nor are our results for one player based upon any belief in the rational conduct of the other"

--*TGEB*, p.160

A Change in Direction: Determinism

Nash put the question of what is rational individual play (without coalitions) on the table

But, very different from the von Neumann and Morgenstern philosophy of games, he assumed the answer is unique—and made a verbal argument for the implication

"We proceed by investigating the question: what would be a "rational" prediction of the behavior to be expected of rational[ly] playing the game in question? By using the principles that a rational prediction should be unique, that the players should be able to deduce and make use of it, and that such knowledge on the part of each player of what to expect the others to do should not lead him to act out of conformity with the prediction, one is led to the concept of a solution defined before."

--Nash (doctoral dissertation, 1950)

Von Neumann and Indeterminism

"Von Neumann pointed out that the enormous variety of solutions which may obtain for n-person games was not surprising in view of the correspondingly enormous variety of observed stable social structures; many differing conventions can endure, existing today for no better reason than that they were here yesterday."

--Von Neumann, round-table discussion of research in (cooperative) *n*-person games, Princeton, 2/1/55; reported by Philip Wolfe, as quoted in Kuhn and Tucker (*Bull. Amer. Math. Soc.*, 1958)

Ellsberg and Uncertainty

(no, not that paper!)

"These particular uncertainties—as to the other players' beliefs about oneself—are almost universal, and it would constrict the application of a game theory fatally to rule them out."

--Ellsberg (*The Review of Economics and Statistics*, 1959)

[Recall (Aumann and Brandenburger, 1995):

Every two-player Nash equilibrium can arise in an epistemic structure where, at the true state, there is mutual (even common) belief of the actual conjectures

Every *n*-player Nash equilibrium can arise in an epistemic structure where, at the true state, there is common belief of the actual conjectures]

Harsanyi:
(Almost) the Creator of Epistemic Game Theory

A Harsanyi structure is a collection

<$S_1, \ldots, S_n; T_1, \ldots, T_n; \lambda_1, \ldots, \lambda_n; f_1, \ldots, f_n; \pi_1, \ldots, \pi_n$>

where

S_i and T_i are player i's strategy and type spaces

$\lambda_i : T_i \to \mathcal{M}(T_{-i})$

$f_i : T_i \to \mathcal{M}(S_i)$ (or: purify via expansion of the type spaces)

$\pi_i : S \times T \to \mathbb{R}$

At this level of generality, a Harsanyi structure can describe uncertainty about both

the structure of the game (the payoff functions)

and

the play of the game

… although Harsanyi's interest was only in the former

Example

Ann \ Bob	L	R
U	2, 2	0, 0
D	0, 0	1, 1

	$L \leftarrow t_b$	$R \leftarrow u_b$	$R \leftarrow v_b$
$U \leftarrow t_a$	½, ½	½, ½	0, 0
$D \leftarrow u_a$	½, ½	0, 0	½, ½
$D \leftarrow v_a$	0, 0	½, ½	½, ½

[At the state (v_a, v_b):

Ann is rational

Ann assigns probability 1 to Bob's actual strategy choice

Ann assigns probability ½ to Bob's being irrational

Ann assigns probability ½ to Bob's assigning probability ½ to Ann's choosing *U* not *D*

… and likewise for Bob]

Harsanyi's Restricted Formulation

The Bayesian equilibrium concept in his papers:

- Every (positive-probability) type optimizes
- In epistemic terms, there is (almost-)everywhere rationality

The examples in his papers:

- All types are 'physical'/payoff types (the formulation that led to information economics)
- In particular, if there is no uncertainty about payoff functions, then Bayesian equilibrium reduces to mixed-strategy equilibrium

Bob

Ann	"Weak"	"Strong"
"Weak"	2/5	1/10
"Strong"	1/5	3/10

(My thanks to Willemien Kets for discussions on this topic)

(Re-)Emergence of the Nash Non-Conformists

Nash non-conformists have been present all along (the term is from Luce and Raiffa, 1954)

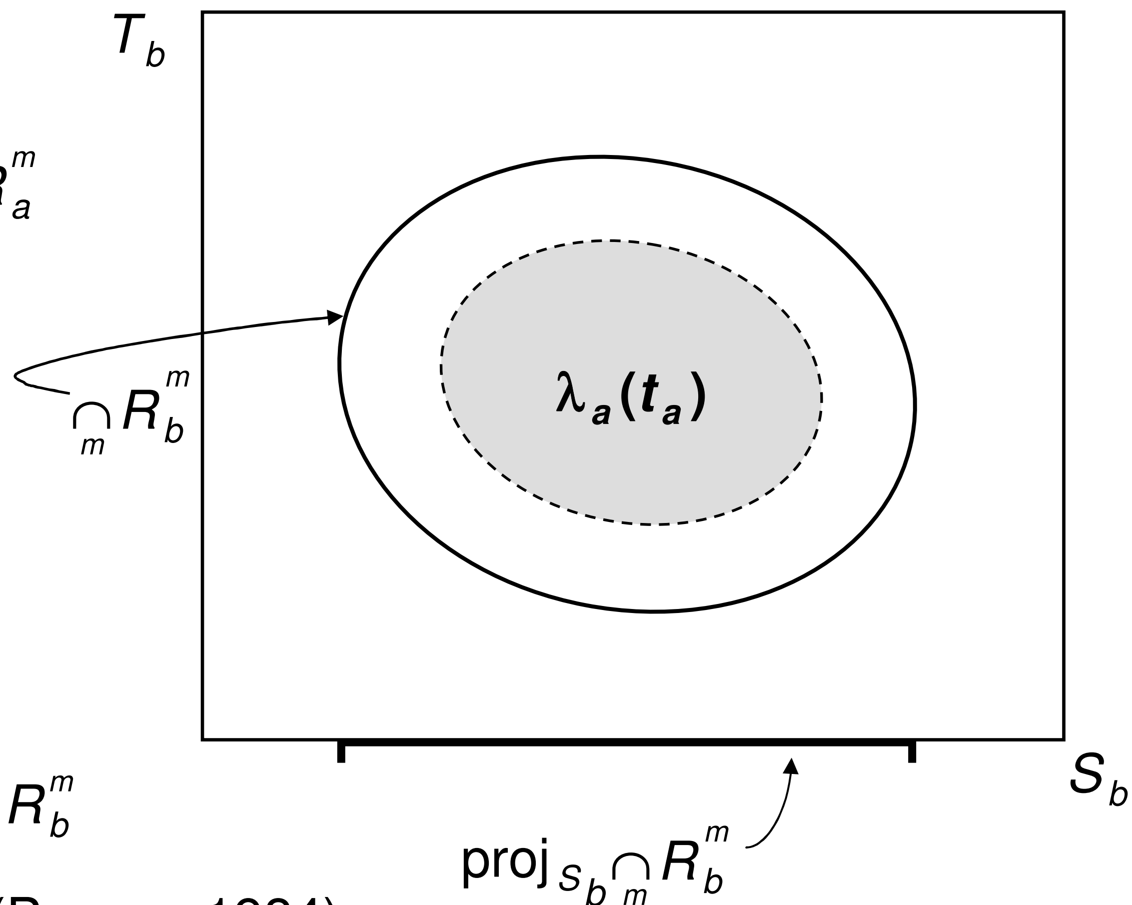
Aumann (1976) provided a crucial missing piece of language

With this, Bernheim (1984) and Pearce (1984) formulated a quasi-epistemic analysis of "common knowledge of rationality"

The idea was clear, but further progress required a formalization

The Fundamental Theorem of EGT

Consider $(s_a, t_a) \in \bigcap_m R_a^m$



$\text{proj}_{S_a} \bigcap_m R_a^m \times \text{proj}_{S_b} \bigcap_m R_b^m$

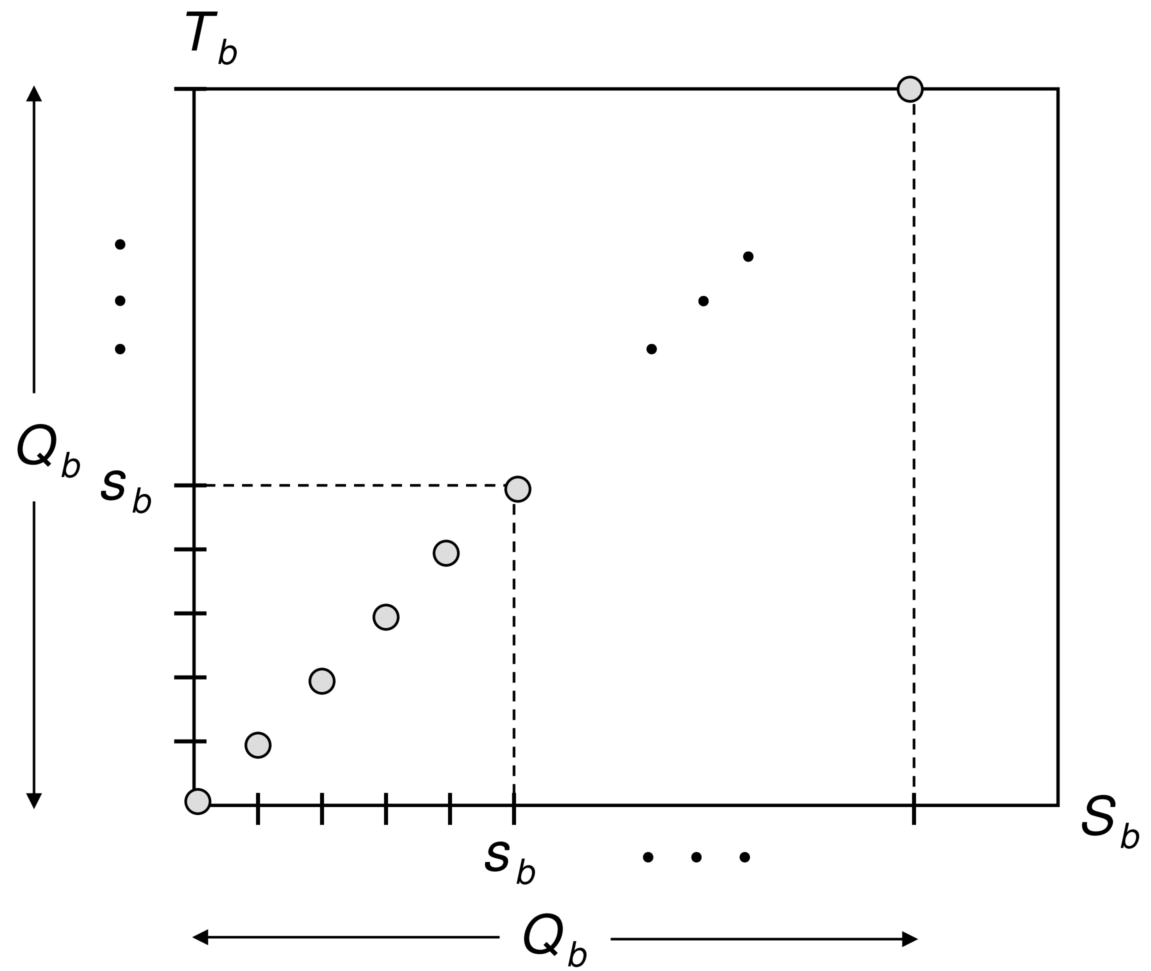
is a best-response set (Pearce, 1984)

(Note the use of conjunction!)

Fundamental Theorem contd.

Let $Q_a \times Q_b$ be a best-response set

For a given $t_a = s_a$, put the weights along the diagonal

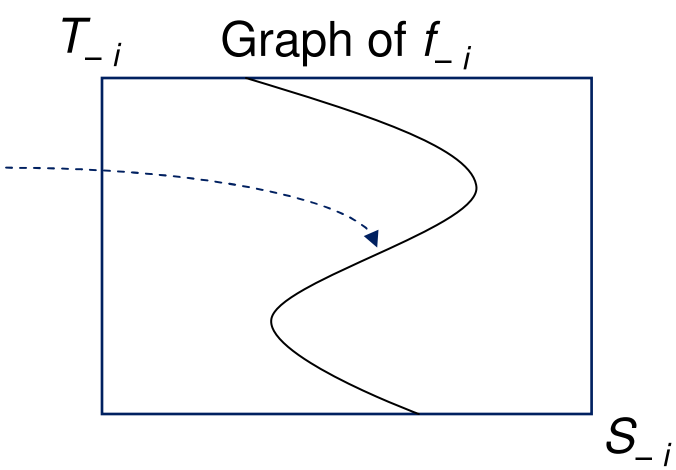


Map vs. Product Structures

Hierarchical constructions of type structures yield product structures

There is a natural transformation from map structures to product structures

$\bar{\lambda}_i(t_i)$ is concentrated on



From Product to Map Structures

Define new type spaces by

$$U_i = S_i \times T_i$$

Define new $\overline{\lambda}_i$ maps by

$$\overline{\lambda}_i(s_i, t_i) = \lambda_i(t_i)$$

Define the f_i maps by

$$f_i(s_i, t_i) = s_i$$

Question:

What is—and what is not—preserved under these transformations?

Other Structures

Of course, there are many other epistemic structures:

partition structures

logic-based structures

...

In the early days of a field, it is to be expected that various—prima facie, different—formalisms should arise

But, now, we want to know which structures are/are not equivalent

(See, e.g., “Belief Hierarchies in Standard State Space Models and Epistemic Equivalence of Belief Spaces,” by Elias Tsakas)

A Comment on Knowledge Structures

"It is inherent in the concept of "strategy" that all the information about the actions of the participants … a player is able to obtain or infer is already incorporated in the "strategy." Consequently, each player must choose his strategy in complete ignorance of the choices of the rest of the players…."
--Von Neumann (*Math. Annalen*, 1928)

EGT in Applications

Each different type structure reflects a different "context" for the game

To the extent that EGT is less "predictive" than equilibrium analysis (esp. refinements), this is in tune with von Neumann's philosophy

In fact, Nash equilibrium plays a smaller role in many applications than has often been thought

Iterated dominance concepts—e.g., extensive-form rationalizability, iterated admissibility—can be very powerful

Example: Iterated admissibility yields the desired outcome in Burn-a-Dollar (Ben Porath and Dekel, 1992) and Beer-Quiche* (Cho and Kreps, 1987)

*When the two payoff types for sender are treated as one player

Applications contd.

But we also want applications where EGT (proudly!) differs from conventional analysis

Example: “Rationalizable Bidding in First-Price Auctions” by Battigalli and Siniscalchi (2003)

The refinements program:

“[W]hat seems most urgently needed is more proofs, more properties, more theorems ***…****”*

--Mertens (1989)

The EGT program:

What seems most urgently needed is more applications, more applications, more applications ***…***