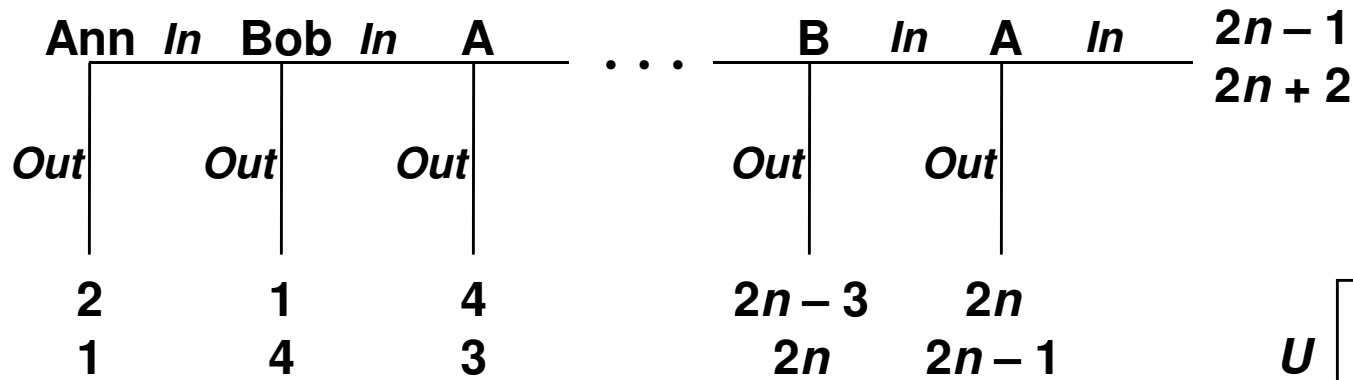


Origins of Epistemic Game Theory

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What is a Game?



“Centipede” (Rosenthal, 1981)

		Bob	
		L	R
Ann	U	1, 1	2, 3
	D	3, 2	0, 0

“The Battle of the Sexes”

“There are intuitions and representations, and the representations may not capture the intuitions.”

--“Weierstrass, Luzin, and Intuition,” by John Gray, *American Mathematical Monthly*, 108, 2001, 865-870

Can This Be Described as a Game?

“It is clear now, though, that the Truman administration’s assumptions—that Mao should logically fear the Russians more than he did the Americans, and would therefore welcome opportunities to improve relations with the United States—were incorrect. The Sino-Soviet Treaty of 1950 was what Mao wanted: it was in no way imposed on him, as the Soviet Union’s alliances with its East European neighbors had been. It was a means of discouraging what Mao believed to be an immediate dangerous adversary [viz., the U.S.] by seeking the protection of another great power that might someday become a danger but was now.

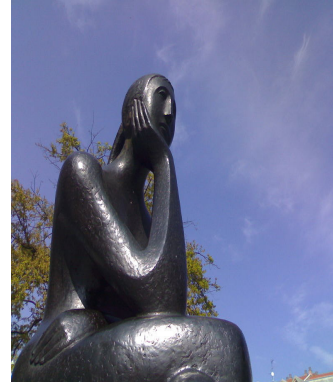
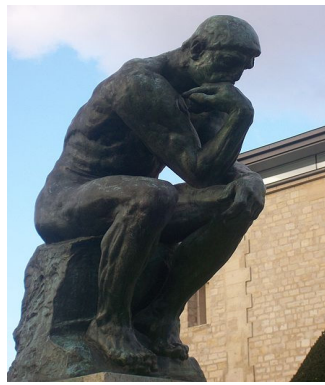
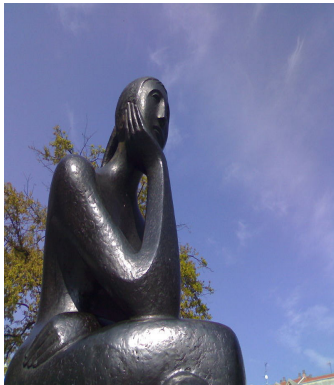
In this sense, it had an analog in the North Atlantic Treaty of April 1949, in which West Europeans had invited the Americans to guarantee their security against a feared Soviet invasion.... Both [treaties] were directed against the threat of an attack that was in fact remote, if it ever existed at all. But statesmen operate on the basis of what they believe at the time, not what historians may conclude decades later.”

-- *We Now Know: Rethinking Cold War History*, by John Lewis Gaddis,
Oxford University Press, 1997, pp.69-70

Games with Epistemics

We would want a formalism in which to write down statements of the form

“Ann thinks that Bob thinks that Ann chooses the strategy U ”



Source:
Wikimedia Commons

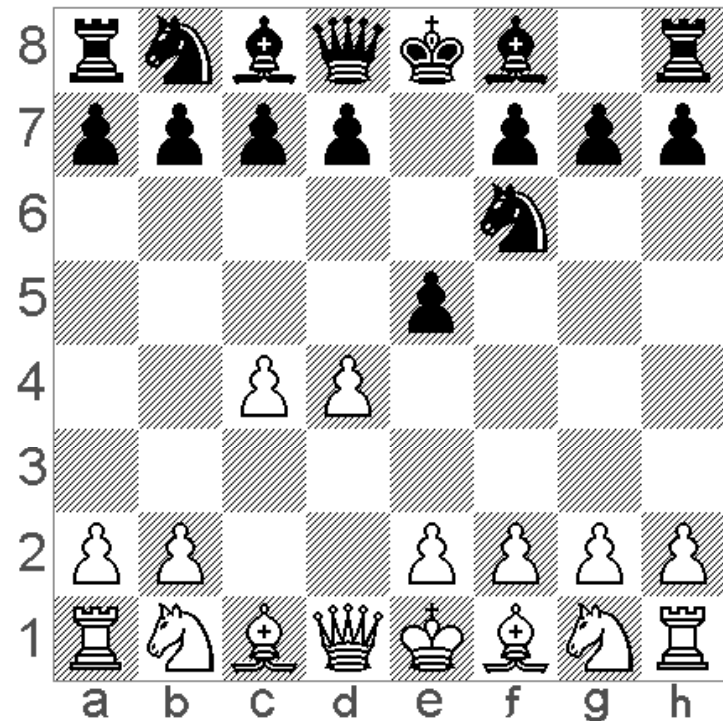
But game theory did not begin here!

Why not?

To see, let us go back to the beginnings ...

Origins: Chess in Budapest

“In the early 20th century, the Royal Game was important throughout much of Europe, particularly in the countries of the Austro-Hungarian Empire.... Against a background of high tournament drama, chessmasters wrote manuals on strategy; psychologists investigated the thought processes required in the game; and mathematicians wondered whether so human an activity could be made amenable to formal treatment. Others speculated about the relationship of chess to life in general, and the game was source of inspiration for several writers, including Vladimir Nabokov ... and ... Stefan Zweig.”



Source: Wikimedia Commons

From: “New Light on von Neumann: Politics, Psychology and the Creation of Game Theory,” by Robert Leonard, at www.cesmep.unito.it/WP/2007/7_WP_Cesmep.pdf

Lasker on Strategy

Lasker was World Chess Champion from 1897 to 1921

*“The magnitude of the work that a group of [players] can perform under all varying possible conditions that may present themselves ... is an index of the ... **value** of that group.”* [emphasis added]

--*Struggle*, by Emanuel Lasker, Lasker's Publishing Company, New York, 1907, p.31



Source: Wikimedia Commons

Von Neumann's 1928 Paper

1. The concept of **strategy**

*"[I]t is possible to bring all games ... into a much simpler normal form Each player S_m ($m = 1, 2, \dots, n$) chooses a number $1, 2, \dots, N_m$ without knowing the choices of the others."**

2. The **Minimax Theorem**

"[H]e is protected against his adversary 'finding him out'"

3. The concept of a **cooperative** game

"[T]he three-person game is essentially different from a game between two persons.... It is [now] a question of which of the three equally possible coalitions $S_1, S_2; S_1, S_3; S_2, S_3$ has been formed. A new element enters, which is entirely foreign to the stereotyped and well-balanced two-person game: struggle."

* But the map from trees to matrices is not injective. So, does this move lose information?

Origins: Logic in Vienna



Source: Wikimedia Commons

“[T]here is exhibited an endless chain of reciprocally conjectural reactions and counter-reactions.... The remedy would lie in analogous employment of the so-called Russell theory of types in logistics. This would mean that on the basis of the assumed knowledge by the economic subjects of theoretical tenets of Type I, there can be formulated higher propositions of the theory; thus, at least, of Type II. On the basis of information about tenets of Type II, propositions of Type III, at least, may be set up, etc.”

--“Perfect Foresight and Economic Equilibrium,” by Oskar Morgenstern, Zeitschrift für Nationalökonomie, 1935

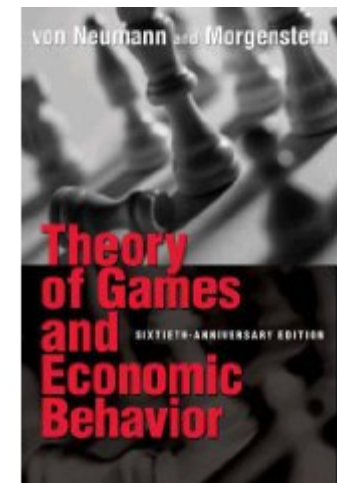
Theory of Games and Economic Behavior

Von Neumann and Morgenstern (1944) studied

two-player zero-sum games via the “protective” or “defensive” maximin strategies
 n -player zero-sum and general-sum games via the coalitional form

“Nor are our results for one player based upon any belief in the rational conduct of the other.” (TGEB, p.160)

“The theory of mechanics for 2, 3, 4, ... bodies is well known, and in its general theoretical (as distinguished from its special and computational) form is the foundation of the statistical theory for great numbers. For the social exchange economy—i.e., for the equivalent ‘games of strategy’—the theory of 2, 3, 4, ... participants was heretofore lacking.... A fundamental reopening of this subject is the more desirable because it is neither certain nor probable that a mere increase in the number of participants will always lead in fine to the conditions of free competition. (TGEB, pp.14-15)



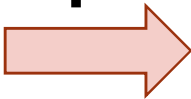
Indeterminacy in *TGEB*

Does the matrix (or tree) determine how a game is played?

Von Neumann and Morgenstern said no!

“[W]e shall in most cases observe a multiplicity of solutions. Considering what we have said about interpreting solutions as stable ‘standards of behavior’ this has a simple and not unreasonable meaning, namely that given the same physical background different ‘established orders of society’ or ‘accepted standards of behavior’ can be built.... (TGEB, p.42)

		Bob	
		L	R
Ann	U	1, 1	2, 3
	D	3, 2	0, 0

?  Play

Nash and Determinacy

Nash put the question of what is rational individual play without coalitions (back) on the table

He assumed the answer is unique—and made a verbal argument for the implication

*“We proceed by investigating the question: what would be a “rational” prediction of the behavior to be expected of rational[ly] playing the game in question? By using the principles that a rational prediction should be **unique**, that the players should be able to deduce and make use of it, and that such knowledge on the part of each player of what to expect the others to do should not lead him to act out of conformity with the prediction, one is led to the concept of a solution defined before.” [emphasis added]*

--doctoral dissertation (1950)

Von Neumann's View in 1955

“Von Neumann pointed out that the enormous variety of solutions which may obtain for n -person games was not surprising in view of the correspondingly enormous variety of observed stable social structures; many differing conventions can endure, existing today for no better reason than that they were here yesterday.”

--Von Neumann, round-table discussion of research in (cooperative) n -person games, Princeton, 2/1/55; reported by Philip Wolfe, as quoted in Kuhn and Tucker (*Bull. Amer. Math. Soc.*, 1958)

Ellsberg on Uncertainty

“These particular uncertainties—as to the other players’ beliefs about oneself—are almost universal, and it would constrict the application of a game theory fatally to rule them out.”

--Daniel Ellsberg (*The Review of Economics and Statistics*, 1959)

A player’s **conjecture** is a probability measure on the strategy profiles chosen by the other players

Theorem: Every (mixed-strategy) Nash equilibrium can arise in an epistemic structure where, at the true state, each player assigns probability 1 to the actual conjectures, each player assigns probability 1 to this event, and so on

--Robert Aumann and Adam Brandenburger (*Econometrica*, 1995)

So, Nash equilibrium does not (intrinsically) allow for uncertainty of the type Ellsberg wanted



Epistemic Game Theory (Nearly)

A **Harsanyi structure** (1967-8) is a collection

$$\langle S_1, \dots, S_n; T_1, \dots, T_n; \lambda_1, \dots, \lambda_n; f_1, \dots, f_n; \pi_1, \dots, \pi_n \rangle$$

where

S_i and T_i are player i 's strategy and type spaces

$$\lambda_i : T_i \rightarrow \mathcal{M}(T_{-i})$$

$f_i : T_i \rightarrow \mathcal{M}(S_i)$ (or: purify via expansion of the type spaces)

$$\pi_i : S \times T \rightarrow \mathbb{R}$$

This structure can be used to describe uncertainty about

the structure of the game (the payoff functions)

the strategies chosen in the game

both

Example of “Strategic” Uncertainty

		Bob		
		<i>L</i>	<i>C</i>	<i>R</i>
Ann	<i>U</i>	7, 0	0, 5	0, 7
	<i>M</i>	5, 0	2, 2	5, 0
	<i>D</i>	0, 7	0, 5	7, 0

		<i>L</i>	<i>R</i>
		↑ t_b	↑ u_b
$U \leftarrow t_a$		1, 0	0, 1
$D \leftarrow u_a$		0, 1	1, 0

At the state (t_a, u_b) :

Ann is rational

Ann assigns probability 0 to Bob's actual strategy choice

Ann assigns probability 1 to Bob's being rational

Ann assigns probability 1 to Bob's assigning probability 0 to her actual choice

... and similarly for Bob

Payoff Uncertainty

Harsanyi and his followers treated payoff uncertainty only, not strategic uncertainty

In particular, in the absence of payoff uncertainty, Harsanyi's analysis reduces to the use of (mixed-strategy) Nash equilibrium

The equilibrium mindset persisted

Emergence of EGT

1980s

Characterization of basic epistemic condition of “rationality and common belief of rationality” (RCBR) under strategic uncertainty

Leads to (non-equilibrium) solution concepts on the matrix defined by fixed points and iterations of a monotone map

2000s

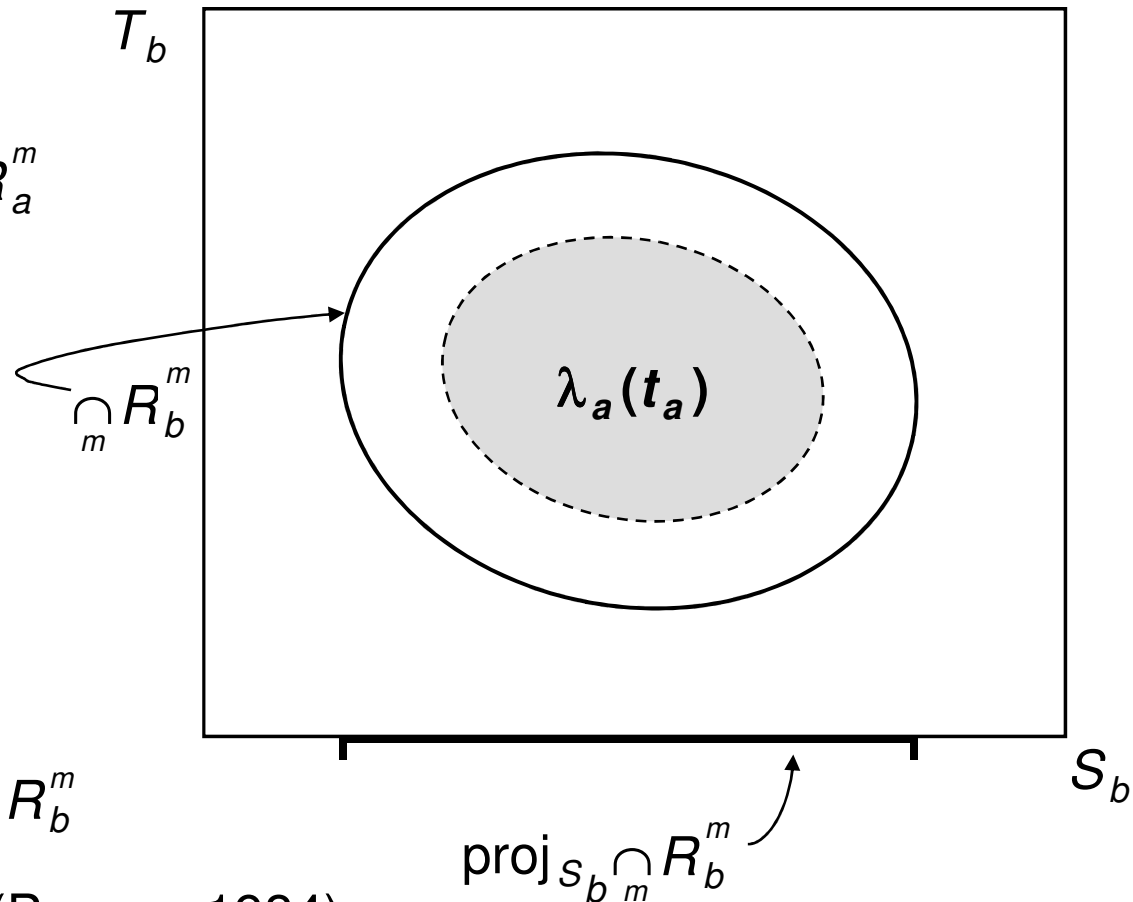
Modifications to RCBR (involving much deeper epistemics) to treat the tree and admissibility on the matrix

Leads to solution concepts defined by non-monotone maps

Survey: “The Power of Paradox: Some Recent Developments in Interactive Epistemology,” by Adam Brandenburger (*International Journal of Game Theory*, 2007)

First (Zeroth?) Theorem of EGT

Consider $(s_a, t_a) \in \bigcap_m R_a^m$



$$\text{proj}_{S_a} \bigcap_m R_a^m \times \text{proj}_{S_b} \bigcap_m R_b^m$$

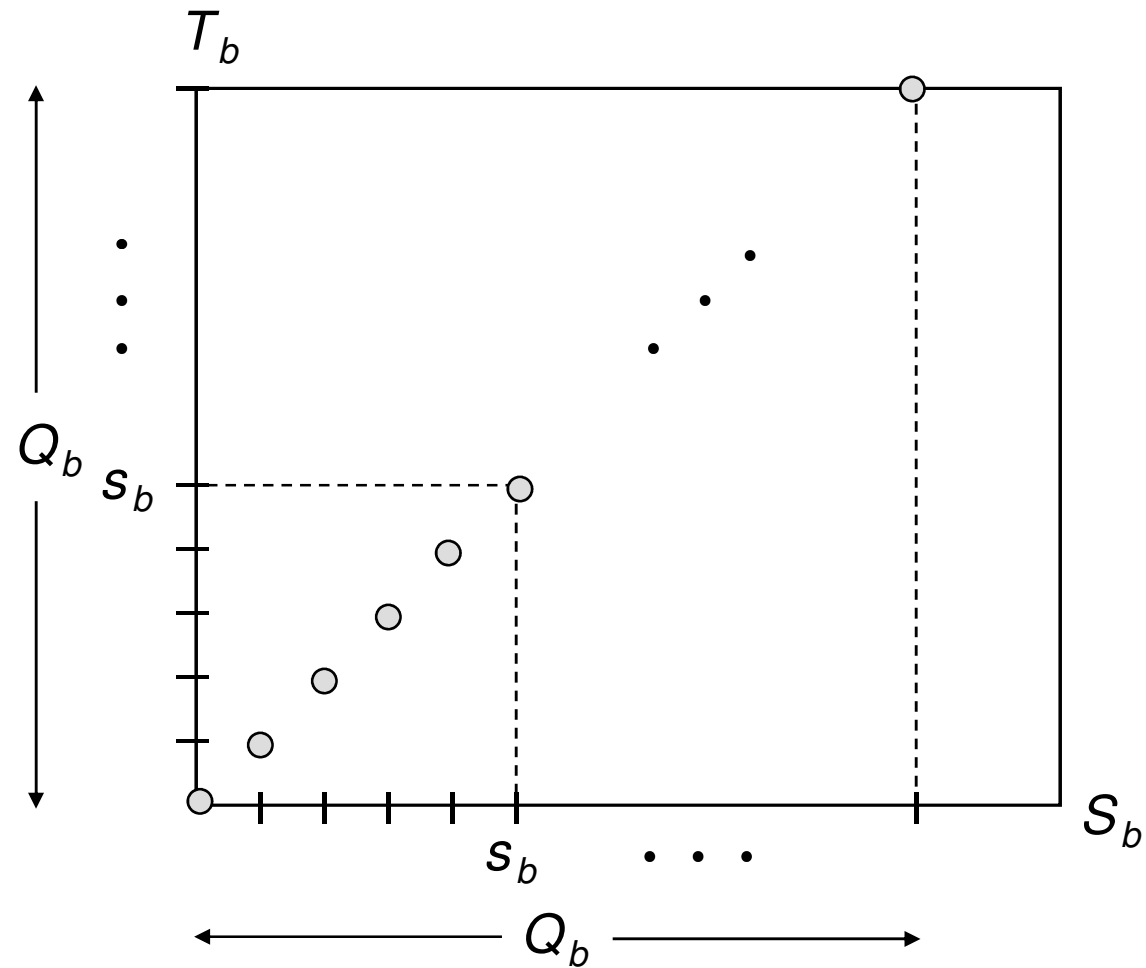
is a best-response set (Pearce, 1984)

(Note the use of infinite conjunction!)

First Theorem contd.

Let $Q_a \times Q_b$ be a best-response set

For a given $t_a = s_a$,
put the weights
along the diagonal



Type Structures: The Context of a Game

Each different type structure reflects a different **context** for the game

To the extent that EGT is less “predictive” than equilibrium analysis (esp. refinements), this is in tune with von Neumann’s philosophy

(In fact, Nash equilibrium plays a smaller role in many applications than has often been thought—iterated dominance concepts can be very powerful)

We also want applications where EGT (proudly!) differs from conventional analysis

Example: “Rationalizable Bidding in First-Price Auctions” by Pierpaolo Battigalli and Marciano Siniscalchi (*Games and Economic Behavior*, 2003)

Type Structures: Foundations

For a given matrix (or tree):

Is every hierarchy of beliefs induced by a type in some type structure?

Is there a type structure containing “all” possible beliefs?

An impossibility result (puzzle-book version):

Ann believes that Bob believes that Ann believes that what Bob believes (about Ann’s type) is incorrect

1. Does Ann believe that what Bob believes is incorrect?
2. Does Ann not believe that what Bob believes is incorrect?

Interpretation: If the analyst’s tools are available to the players, there are statements which the players can think about but cannot believe.

(“An Impossibility Theorem on Beliefs in Games,” by Adam Brandenburger and H. Jerome Keisler, *Studia Logica*, 2006)

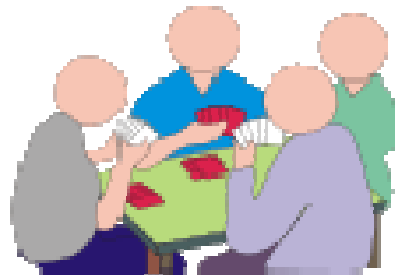
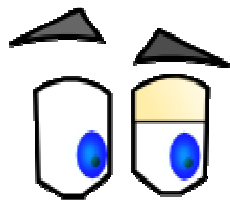
The Analyst vs. the Players

Remember:

“By using the principles that a rational prediction should be unique, that the players should be able to deduce and make use of it,” (Nash,1950)

But, why should an analyst’s prediction (even if there is a unique “rational”(?) one) be accessible to the players?

Shouldn’t we distinguish the analysis of a game from the game itself?



Source: Wikimedia Commons

A Critique of EGT

“A fundamental difficulty may make the decision-theoretic approach impossible to implement, however. To assess his subjective probability distribution over other players’ strategies, player i may feel that he should try to imagine himself in their situations. When he does so, he may realize that the other players cannot determine their optimal strategies until they have assessed their subjective probability distributions over i ’s possible strategies. Thus, player i may realize that he cannot predict his opponents’ behavior until he understands what an intelligent person would expect him rationally to do, which is, of course, the problem that he started with.”

--Game Theory: Analysis of Conflict, by Roger Myerson, 1991, p.114

But, why should a player think that what he does is necessarily what other players must think he does?

(My thanks to Willemien Kets for pointing me to this passage)