Liquidity and Congestion

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Motivation
Is Liquidity an Asset Characteristic?

Table: Hasbrouck (JF 2009) Table 3 Panel A: Pearson correlation of $c^{TAQ}$, PropZero and $\lambda$.

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<tr>
<th></th>
<th>$c$</th>
<th>%0Vol</th>
<th>$\lambda$</th>
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<tr>
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<td>$\lambda$</td>
<td>0.51</td>
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I survived the crash of 2:45 pm.

I survived the crash tshirts

Size:
Select a size

Quantity:
1 shirt. Only $11.02 in bulk!

As low as $12.95 on a Value T-Shirt

Add to cart $16.95 per shirt

Choose your style and color

Basic T-Shirt: White

Over 60 more!

Men
Women
Kid
Baby
Rough Idea

Start with a matching framework, e.g. Duffie, Garleanu and Pedersen (2005).

Introduce an outside option.

Increasing search efficiency:

1. makes market participation more valuable,
2. but also attracts more traders which increases search costs.

Agents don’t account for the negative externality.
Solution Strategy

Define state transition equations taking number of entrants as given for the steady state.

Define utility flows from being each state and asset price taking number of entrants as given for the steady state.

Compute value of entering the market.

Make an assumption about the distribution of outside options and compute fixed point for number of entrants.
State Transition Equations
1 asset in supply $S$.

3 kinds of traders:

1. Buyers: $\eta_b$
2. Sellers: $\eta_s$
3. Happy: $\eta_h$

Happy traders become discontent with probability $\gamma$.

Search efficiency characterized by $\lambda$. $\lambda\eta_b$ sellers find buyers. $\lambda\eta_s$ buyers find sellers. $\lambda\eta_b\eta_s$ matches take place.
Market clearing implies that $\eta_s + \eta_h = S$.

$$\lambda \eta_b \eta_s = \gamma \eta_h \quad \Rightarrow \quad \eta_b = \frac{\gamma}{\lambda} \left( \frac{\eta_h}{S - \eta_h} \right)$$

$g$ traders enter the market each period.

$$g = \gamma \eta_b + \lambda \eta_b \eta_s = \gamma \eta_h \left[ 1 + \frac{\gamma}{\lambda} \left( \frac{1}{S - \eta_h} \right) \right]$$
\[\eta_h = A\]
\[\eta_s = S - A\]
\[\eta_b = \frac{\gamma}{\lambda} \left( \frac{A}{S - A} \right)\]

\[A = \frac{\left[ g + \gamma S + \frac{\gamma^2}{\lambda} \right] - \sqrt{\left[ g + \gamma S + \frac{\gamma^2}{\lambda} \right]^2 - 4\gamma gS}}{2\gamma}\]
Corollary 1: More investors entering the market increases number of buyers and happy owners but decreases number of sellers.
Corollary 2: More efficient search decreases the number of buyers and sellers and increases the number of happy owners.
Utility and Pricing
Asset has price $p$, pays dividend $d$, and is worth $d - x$ after liquidity shock.

Discount rate $r > 0$.

Expected utilities: $v_b$, $v_h$ and $v_s$. Expected utility of outside investors is 0.

\[rv_b = \gamma (0 - v_b) + \lambda \eta_s ([v_h - v_b] - p)\]
\[rv_h = d + \gamma (v_s - v_h)\]
\[rv_s = (d - x) + \lambda \eta_b ([0 - v_s] + p)\]
Buyer has bargaining power $z \in (0, \infty)$.

$$p = \frac{z}{1 + z} (v_s - 0) + \frac{1}{1 + z} (v_h - v_b)$$
\[ v_b = k \left( \frac{x}{(r + \gamma + \lambda \eta_s)z + \gamma} \right) \frac{\lambda \eta_s z}{r + \gamma} \]

\[ v_h = \frac{d}{r} - k \left( \frac{x}{r} + \frac{x}{(r + \gamma + \lambda \eta_s)z + \gamma} \right) \frac{\gamma}{r + \gamma} \]

\[ v_s = \frac{d}{r} - k \left( \frac{x}{r} + \frac{x}{(r + \gamma + \lambda \eta_s)z + \gamma} \right) \]

\[ p = \frac{d}{r} - k \frac{x}{r} \]

\[ k = \frac{(r + \gamma + \lambda \eta_s)z + \gamma}{(r + \gamma + \lambda \eta_s)z + (r + \gamma + \lambda \eta_b)} \]
Corollary 3 (i): The price of the asset increases in the dividend flow.
**Corollary 3 (i):** The price of the asset increases in the traders entering the market.
Corollary 3 (ii): The price of the asset decreases with buyer bargaining power.
Equilibrium Entry
New agents have outside option $k \in [\underline{k}, \bar{k}]$ drawn from $f(k)$.

Fraction $\theta$ of new agents enter the market. Let $k^*$ denotes the maximum outside option for which new agents will enter the market.

$$g^* = \int_{\underline{k}}^{\bar{k}} f(k) \, dk$$
An **equilibrium** is a set of allocations $\theta$ and $(\eta_b, \eta_s, \eta_h)$ and a price $p$ such that:

1. $(\eta_b, \eta_s, \eta_h)$ solve the market clearing conditions,
2. $p$ solves the flow value equations, and
3. $\theta$ solves the market entry fixed point problem.
\[ W = W_{\text{Current}} + W_{\text{Future}} \]

\[ W_{\text{Current Inside}} = \eta_b v_b + \eta_h v_h + \eta_s v_s \]

\[ W_{\text{Current Outside}} = \int_{k^*}^{k} k f(k) \, dk \]

\[ W_{\text{Future Inside}} = \frac{1}{r} \int_{k}^{k^*} v_b f(k) \, dk \]

\[ W_{\text{Future Outside}} = \frac{1}{r} \int_{k^*}^{k} k f(k) \, dk \]
Search Efficiency Parameter ($\lambda$)

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Price

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Solution

- Equilibrium
- Exogenous