DEPARTMENT OF ECONOMICS

MICROECONOMICS, SPRING 2010

PROBLEM SET 10: GAME THEORY AND BERTRAND COMPETITION

DUE in Class on April 14 (Skreta Sections) or April 15 (Bowmaker and Collard-Wexler Sections)

1. SHORT ANSWER PROBLEMS

1. Let us assume the following game where player 1 and 2 think about playing UP or DOWN. Are there any dominant strategies in this game and what is a dominant strategy? What is the Nash equilibrium in this game?

\[
\begin{array}{c|cc}
\text{1} & \text{Up} & \text{Down} \\
\hline
\text{Up} & 10,5 & 7,6 \\
\text{Down} & 4,9 & 2,5 \\
\end{array}
\]

2. True or false. Entry deterrence can only occur in a game where one player moves before the other.

3. Take the game described in the following matrix. And let player 1 now move before player 2. What will be the credible Nash equilibrium? Use the extensive form of the game to solve this.

\[
\begin{array}{c|cc}
\text{1} & \text{Up} & \text{Down} \\
\hline
\text{Up} & 10,5 & 7,6 \\
\text{Down} & 4,9 & 2,5 \\
\end{array}
\]

2. LONG-ANSWER PROBLEMS

QUESTION 1

1. Rock, Paper, Scissors!

4. Rock, Paper, Scissors!


<table>
<thead>
<tr>
<th>Pikachu (aka, 'Princess')</th>
<th>Rock</th>
<th>Paper</th>
<th>Scissors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock</td>
<td>(0,0)</td>
<td>(1,-1)</td>
<td>(-1,1)</td>
</tr>
<tr>
<td>Paper</td>
<td>(-1,1)</td>
<td>(0,0)</td>
<td>(1,-1)</td>
</tr>
<tr>
<td>Scissors</td>
<td>(1,-1)</td>
<td>(-1,1)</td>
<td>(0,0)</td>
</tr>
</tbody>
</table>

(a) In the payoff schedule, it is stipulated that the winner receives $1 from the loser, under the hierarchy (akin to the 'intransitivity' of what is stronger, rock bends scissors, while scissors shears through paper, and paper covers rock), can you produce a flow diagram of the one dollar passed on for each scenario? Hint: can you see how this matches the payoffs in the matrix?

(b) What can you state about the zero profiles along the diagonal and what payments are made if they adopt the strategy? Can you illustrate the flow diagram for these kinds of ties?

(c) List all the strategic profiles that are not stable.

(d) What are the mixed strategies?

c) What is the Nash Equilibrium and why or why not?

7) Bon Jovi, Ozzy Osbourne, Shania Twain?: Battle of the Sexes!

2. Credible Strategies

<table>
<thead>
<tr>
<th>player A\player B</th>
<th>L</th>
<th>M</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>-10,-10</td>
<td>0,10</td>
<td>10,20</td>
</tr>
<tr>
<td>M</td>
<td>10,0</td>
<td>-20,-20</td>
<td>-5,15</td>
</tr>
<tr>
<td>R</td>
<td>20,10</td>
<td>15,-5</td>
<td>-30,-30</td>
</tr>
</tbody>
</table>

(a) Are there any dominant strategies in this game?

(b) Are there any dominated strategies?

(c) Find the pure strategy Nash equilibria.

(d) Find the credible equilibrium of the game if player A move first.

(e) Find the credible equilibrium if player B moves first.

QUESTION 2: BERTRAND - THE HOMOGENOUS PRODUCTS PRICE GAME OF OLIGOPOLY THEORY

Following the Bertrand model of oligopoly, assume that firms make simultaneous price decisions with constant marginal costs. Suppose that there are two producers of Cement for the U.S. market. The demand for cement is:

\[ Q^d = 100 - \frac{p}{4} \]

There are two producers, Holcim which produces in the United States and Cemex which produces in Mexico. Holcim has costs \( TC = 20Q \), while Cemex which has to import Cement from abroad has costs \( TC = 40Q \).

(a) What are the marginal costs of each firm?

(b) What is Cemex's best response given the price that Holcim is charging? Remember that there are 3 cases we need to consider...
(c) What is Holcim's best response given the price that Cemex is charging? Note how Cemex and Holcim's best responses are different.

(d) Plot Holcim and Cemex’s Best-Response Curves. What will be the Nash Equilibrium in this Game? At the Nash Equilibrium what is the quantity of cement imported from Mexico.

(e) What would happen if Cemex was unable to ship cement to the U.S. so Holcim was a monopolist. Compare consumer surplus when Holcim is a Monopolist versus when it competes against Cemex.

(f) What is the Nash Equilibrium if Cemex's Cost Function is $C(Q) = 240Q$?