FORECASTING CASH FLOWS

In the last chapter, we focused on the question of how best to measure cash flows. In this chapter, we turn to the more difficult question of how best to estimate expected future cash flows. We will begin by looking at the practice of using historical growth rates to forecast future cash flows and then look at the equally common approach of using estimates of growth either from management or other analysts tracking the company. As a final variation, we will describe a more consistent way of tying growth to a firm’s investment and financing policies.

In the second part of the chapter, we will examine different ways of bringing closure to valuation by estimating the terminal value and how to keep this number from becoming unbounded. In particular, we will look at the connection between terminal growth and reinvestment assumptions. In the final section of the chapter, we will consider three variations on cash flow forecasting - expected value estimates, scenario analysis and simulations.

The Structure of DCF Valuation

To value an asset, we have to forecast the expected cash flows over its life. This can become a problem when valuing a publicly traded firm, which at least in theory can have a perpetual life. In discounted cash flow models, we usually resolve this problem by estimating cash flows for a period (usually specified to be an extraordinary growth period) and a terminal value at the end of the period. While we will look at alternative approaches, the most consistent way of estimating terminal value in a discounted cash flow model is to assume that cash flows will grow at a stable growth rate that can be sustained forever after the terminal year. In general terms, the value of a firm that expects to sustain extraordinary growth for n years can be written as:

$$ \text{Value of a firm} = \sum_{t=1}^{n} \frac{\text{Expected Cash Flow}_t}{(1 + r)^t} + \frac{\text{Terminal Value}_n}{(1 + r)^n} $$

In keeping with the distinction between valuing equity and valuing the business that we made in the previous chapters, we can value equity in a firm by discounting expected
cash flows to equity and the terminal value of equity at the cost of equity or we can value the entire firm by discounting expected cash flows to the firm and the terminal value of the firm at the cost of capital.

There are three components to forecasting cash flows. The first is to determine the length of the extraordinary growth period: different firms, depending upon where they stand in their life cycles and the competition they face, will have different growth periods. The second is estimating the cash flows during the high growth period, using the measures of cash flows we derived in the last chapter. The third is the terminal value calculation, which should be based upon the expected path of cash flows after the terminal year.

I. Length of Extraordinary Growth Period

The question of how long a firm will be able to sustain high growth is perhaps one of the more difficult questions to answer in a valuation, but two points are worth making. One is that it is not a question of whether but when firms hit the stable growth wall. All firms ultimately become stable growth firms, in the best case, because high growth makes a firm larger and the firm’s size will eventually become a barrier to further high growth. In the worst-case scenario, firms may not survive and will be liquidated. The second is that high growth in valuation, or at least high growth that creates value\(^1\), comes from firms earning excess returns on their marginal investments. In other words, increased value comes from firms having a return on capital that is well in excess of the cost of capital (or a return on equity that exceeds the cost of equity). Thus, when you assume that a firm will experience high growth for the next 5 or 10 years, you are also implicitly assuming that it will earn excess returns (over and above the required return) during that period. In a competitive market, these excess returns will eventually draw in new competitors and the excess returns will disappear.

We should look at three factors when considering how long a firm will be able to maintain high growth.

1. Size of the firm: Smaller firms are much more likely to earn excess returns and maintain these excess returns than otherwise similar larger firms. This is because

---

\(^1\) Growth without excess returns will make a firm larger but not more valuable.
they have more room to grow and a larger potential market. Small firms in large markets should have the potential for high growth (at least in revenues) over long periods. When looking at the size of the firm, you should look not only at its current market share, but also at the potential growth in the total market for its products or services. A firm may have a large market share of its current market, but it may be able to grow in spite of this because the entire market is growing rapidly.

2. *Existing growth rate and excess returns:* Momentum does matter, when it comes to projecting growth. Firms that have been reporting rapidly growing revenues are more likely to see revenues grow rapidly at least in the near future. Firms that are earnings high returns on capital and high excess returns in the current period are likely to sustain these excess returns for the next few years.

3. *Magnitude and Sustainability of Competitive Advantages:* This is perhaps the most critical determinant of the length of the high growth period. If there are significant barriers to entry and sustainable competitive advantages, firms can maintain high growth for longer periods. If, on the other hand, there are no or minor barriers to entry or if the firm’s existing competitive advantages are fading, you should be far more conservative about allowing for long growth periods. The quality of existing management also influences growth. Some top managers² have the capacity to make the strategic choices that increase competitive advantages and create new ones.

*Illustration 4.1: Length of High Growth Period*

To illustrate the process of estimating the length of the high growth period, we will consider all of the companies that we will be valuing in the next two chapters and make subjective judgments about how long each one will be able to maintain high growth.

<table>
<thead>
<tr>
<th>Company</th>
<th>Competitive Advantage</th>
<th>Potential threats</th>
<th>Length of Growth period</th>
</tr>
</thead>
<tbody>
<tr>
<td>J.P. Morgan Chase</td>
<td>Size of firm and range of financial services.</td>
<td>Little pricing power; Out maneuvered by smaller and nimbler competitors.</td>
<td>No high growth period.</td>
</tr>
</tbody>
</table>

² Jack Welch at GE and Robert Goizueta at Coca Cola are good examples of CEOs who made a profound difference in the growth of their firms, which were perceived as mature firms when they took the reins.
<table>
<thead>
<tr>
<th>Company</th>
<th>Current ROE</th>
<th>Investment Banking Brand Name</th>
<th>Markets in the US and Europe are saturated and are volatile.</th>
<th>High Growth Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goldman Sachs</td>
<td>18.49%</td>
<td>Investment banking brand name. Market know-how and trading expertise.</td>
<td>Markets in the US and Europe are saturated and are volatile.</td>
<td>High growth period of 5 years.</td>
</tr>
<tr>
<td>Canara Bank</td>
<td>23.22%</td>
<td>Significant presence in a high growth market (India) with restrictions on new entrants.</td>
<td>Easing of bank entry allowing foreign banks to compete in market.</td>
<td>High growth period of 10 years.</td>
</tr>
<tr>
<td>Exxon Mobil</td>
<td>19.73%</td>
<td>Economies of scale and ownership of undeveloped oil reserves.</td>
<td>Oil is a non-renewable resource and alternative energy sources are becoming more feasible.</td>
<td>No high growth period.</td>
</tr>
<tr>
<td>Toyota Motors</td>
<td>10.18%</td>
<td>Healthiest and most efficient company in a troubled sector. Leader in energy efficient hybrids.</td>
<td>Overall growth in auto business slowing and competition increasing from Chinese and Indian automakers.</td>
<td>High growth period of 5 years.</td>
</tr>
<tr>
<td>Tsingtao Breweries</td>
<td>8.06%</td>
<td>Strong brand name in Asia, where beer consumption is growing rapidly.</td>
<td>Establish breweries in the US and Europe and other breweries in Asia competing for same market.</td>
<td>High growth period of 10 years.</td>
</tr>
<tr>
<td>Nintendo</td>
<td>8.54%</td>
<td>Early entrant with proprietary technology in gaming business.</td>
<td>Intense competition from larger competitors with own proprietary technologies (Sony and Microsoft)</td>
<td>No high growth period.</td>
</tr>
<tr>
<td>Target</td>
<td>9.63%</td>
<td>“Cool” retailer with good management.</td>
<td>In a business that is subject to fads; Market in the US can become saturated.</td>
<td>High growth period of 5 years.</td>
</tr>
<tr>
<td>Embraer</td>
<td>16.93%</td>
<td>Strong presence in small corporate and executive jet market. Cost advantages over developed market competitors.</td>
<td>Developed market competitors like Boeing and Airbus trying to move production to cheaper locales.</td>
<td>High growth period of 10 years.</td>
</tr>
<tr>
<td>Sirius Radio</td>
<td>Negative</td>
<td>Pioneer in high growth satellite radio</td>
<td>Competition is likely to be intense not</td>
<td>High growth period of 10 years.</td>
</tr>
</tbody>
</table>
business. only from other companies in sector but also from alternative technologies (internet radio etc.) years.

Note that these are subjective judgments and it is entirely possible that another analyst looking at these companies could have to come very different conclusions about these firms, with the same information.

II. Detailed Cash Flow Forecasts

Once the length of the extraordinary growth period has been established, we have to forecast cash flows over that period. It is in this stage of the process that we will be called upon to make our best judgments on how the company being valued will evolve over the coming years. We will begin this section by looking at the most logical source for these estimates, which is the company’s own past, but pinpoint some dangers associated with relying on history. We will also consider using estimates for the future provided by those we view as more in the know, which would include the company’s management and analysts tracking the company. We will close the section by presenting the link between growth and a company’s fundamentals.

I. Past as Prologue

When estimating the expected growth for a firm, we generally begin by looking at the firm’s history. How rapidly have the firm’s operations as measured by revenues or earnings grown in the recent past? While past growth is not always a good indicator of future growth, it does convey information that can be valuable while making estimates for the future. In this section, we begin by looking at measurement issues that arise when estimating past growth and then consider how past growth can be used in projections.

Estimating Historical Growth

Given a firm’s earnings history, estimating historical growth rates may seem like a simple exercise but there are several measurement problems that may arise. In particular, we have to consider the following:
a. Computational Choices: The average growth rate can vary depending upon whether it is an arithmetic average or a geometric average. The arithmetic average is the simple average of past growth rates, while the geometric mean takes into account the compounding that occurs from period to period.

\[
\text{Arithmetic Average} = \frac{\sum_{t=1}^{n} g_t}{n} \quad \text{where } g_t = \text{growth rate in year } t
\]

\[
\text{Geometric Average} = \left[ \frac{\text{Earnings}_0}{\text{Earnings}_{n}} \right]^{(1/n)} - 1 \quad \text{where } \text{Earnings}_n = \text{earnings in } n \text{ years ago}
\]

The two estimates can be very different, especially for firms with volatile earnings. The geometric average is a much more accurate measure of true growth in past earnings, especially when year-to-year growth has been erratic. In fact, the point about arithmetic and geometric growth rates also applies to revenues, though the difference between the two growth rates tend to be smaller for revenues than for earnings. For firms with volatile earnings and revenues, the caveats about using arithmetic growth carry even more weight.

b. Period of Estimation: The average growth rate for a firm can be very different, depending upon the starting and ending points for the estimation. If we begin the estimation calculation in a “bad earnings year” for the firm and end with a “good earnings year”, we will not surprisingly find that growth was healthy during the intermediate period.

c. Negative Earnings: Measures of historical growth are distorted by the presence of negative earnings numbers. The percentage change in earnings on a year-by-year basis is defined as:

\[
\% \text{ change in EPS in period } t = \frac{\text{EPS}_t - \text{EPS}_{t-1}}{\text{EPS}_{t-1}} = \frac{\text{EPS}_t}{\text{EPS}_{t-1}} - 1
\]

If \(\text{EPS}_{t-1}\) is negative or zero, this calculation yields a meaningless number. This extends into the calculation of the geometric mean. If the EPS in the initial time period is negative or zero, the geometric mean is not meaningful. While there are fall-back measures that will yield growth estimates even when earnings are negative, they do not provide any useful information about future growth. It is not incorrect and, in fact, it may be
appropriate, to conclude that the historical growth rate is 'not meaningful' when earnings are negative and to ignore it in predicting future growth.

**Illustration 4.2: Differences between Arithmetic and Geometric Averages: Ryanair**

Table 4.1 reports the revenues, EBITDA, EBIT and net income for Ryanair, the Ireland-based discount European airline, for each year from 1999 to 2004. The arithmetic and geometric average growth rates in each series are reported at the bottom of the table:

**Table 4.1: Arithmetic and Geometric Average Growth Rates: Ryanair**

<table>
<thead>
<tr>
<th>Year</th>
<th>Revenues</th>
<th>Growth Rate</th>
<th>EBITDA</th>
<th>EBIT</th>
<th>Net Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>€203,803.17</td>
<td></td>
<td>€81,420.71</td>
<td>€56,281.16</td>
<td>€45,525.20</td>
</tr>
<tr>
<td>1999</td>
<td>€258,973.00</td>
<td>27.07%</td>
<td>€104,070.00</td>
<td>€67,861.00</td>
<td>€57,471.00</td>
</tr>
<tr>
<td>2000</td>
<td>€330,571.00</td>
<td>27.65%</td>
<td>€128,107.00</td>
<td>€84,055.00</td>
<td>€72,518.00</td>
</tr>
<tr>
<td>2001</td>
<td>€432,940.00</td>
<td>30.97%</td>
<td>€173,186.00</td>
<td>€114,011.00</td>
<td>€104,483.00</td>
</tr>
<tr>
<td>2002</td>
<td>€550,991.00</td>
<td>27.27%</td>
<td>€221,943.00</td>
<td>€162,933.00</td>
<td>€150,375.00</td>
</tr>
<tr>
<td>2003</td>
<td>€731,591.00</td>
<td>32.78%</td>
<td>€340,339.00</td>
<td>€263,474.00</td>
<td>€238,398.00</td>
</tr>
<tr>
<td>2004</td>
<td>€1,074,224.00</td>
<td>46.83%</td>
<td>€368,981.00</td>
<td>€270,851.00</td>
<td>€206,611.00</td>
</tr>
<tr>
<td>2005</td>
<td>€1,336,586.00</td>
<td>24.42%</td>
<td>€428,192.00</td>
<td>€329,489.00</td>
<td>€266,741.00</td>
</tr>
<tr>
<td>Arithmetic Average</td>
<td>31.00%</td>
<td>27.44%</td>
<td>29.88%</td>
<td>30.68%</td>
<td></td>
</tr>
<tr>
<td>Geometric Average</td>
<td>30.82%</td>
<td>26.76%</td>
<td>28.72%</td>
<td>28.73%</td>
<td></td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>7.50%</td>
<td>14.59%</td>
<td>18.88%</td>
<td>22.77%</td>
<td></td>
</tr>
</tbody>
</table>

Geometric Average = \((\text{Earnings}_{2005}/\text{Earnings}_{1998})^{1/7}\) - 1

The arithmetic average growth rate is higher than the geometric average growth rate for all four items, but the difference is larger with net income and operating income (EBIT) than it is with revenues and EBITDA. This is because the net and operating income are the more volatile of the numbers. Looking at the net and operating income in 1999 and 2004, it is also quite clear that the geometric averages are much better indicators of true growth.

**The Usefulness of Historical Growth**

Is the growth rate in the past a good indicator of growth in the future? Not necessarily. In a study of the relationship between past growth rates and future growth rates, Little (1960) coined the term "Higgledy Piggledy Growth" because he found little
evidence that firms that grew fast in one period continued to grow fast in the next period. In the process of running a series of correlations between growth rates in earnings in consecutive periods of different length, he frequently found negative correlations between growth rates in the two periods and the average correlation across the two periods was close to zero (0.02).

If past growth in earnings is not a reliable indicator of future growth at many firms, it becomes even less so at smaller firms. The growth rates at smaller firms tend to be even more volatile than growth rates at other firms in the market. The correlation between growth rates in earnings in consecutive time periods (five-year, three-year and one-year) for firms in the United States, categorized by market value, is reported in Figure 4.1.

While the correlations tend to be higher across the board for one-year growth rates than for 3-year or 5-year growth rates in earnings, they are also consistently lower for smaller firms than they are for the rest of the market. This would suggest that you should be more cautious about using past growth, especially in earnings, for forecasting future growth at these firms.
In general, revenue growth tends to be more persistent and predictable than earnings growth. This is because accounting choices have a far smaller effect on revenues than they do on earnings. In fact, there are some analysts who use historical growth rates for individual items in the cash flow forecast: revenues, operating expenses, capital expenditures, depreciation and so on. The danger of doing this is that allowing each item to grow at different rates may result in significant internal inconsistencies. For instance, allowing revenues to grow at 10% a year while operating expenses grow 6% a year will increase operating margins to unsustainable levels, if continued long enough.

**The Effects of Firm Size**

Since the growth rate is stated in percentage terms, the role of size has to be weighed in the analysis. It is easier for a firm with $10 million in earnings to generate a 50% growth rate than it is for a firm with $500 million in earnings to generate the same growth. Since it becomes harder for firms to sustain high growth rates as they become larger, past growth rates for firms that have grown dramatically in size may be difficult to sustain in the future. While this is a problem for all firms, it is a particular problem when analyzing small and growing firms. While the fundamentals at these firms, in terms of management, products and underlying markets, may not have changed, it will still be difficult to maintain historical growth rates as the firms double or triple in size.

The true test for a small firm lies in how well it handles growth. Some firms have been able to continue to deliver their products and services efficiently as they have grown. In other words, they have been able to scale up successfully. Other firms have had much more difficulty replicating their success as they become larger. In analyzing small firms, therefore, it is important that you look at plans to increase growth but it is even more critical that you examine the systems in place to handle this growth.

**II. Outside Estimates of Growth**

Some analysts evade their responsibility for estimating growth by using growth estimates that are provided to them either by the management of the company that they are valuing or by other analysts tracking the firm. In this section, we consider this practice and whether the resulting valuations are more precise.
Management Estimates

A surprising number of valuations use forecasts for revenues and earnings provided by the company management. This practice does have two advantages: it makes estimation simple because the numbers are provided by managers, and it allows valuation analysts to blame others when the forecasts are not delivered. The dangers are manifold:

- In chapter 1, we talked about the dangers of bias in valuation. The management of a company cannot be expected to be unbiased about the company’s future prospects and by extension, their own management skills. All too often, management forecasts represent wish lists rather than expectations for the future.
- There is a different problem that is created when management compensation is tied to meeting or beating the forecasts provided. In this case, there will be a tendency to play down expectations, with the intent of beating forecasts and generating rewards.
- Finally, management forecasts can represent combinations of assumptions that are inconsistent. For instance, management may forecast revenue growth of 10% a year for the next 10 years with little or no new capital expenditures over the period. While utilizing existing assets more efficiently may generate some short-term growth, it is difficult to see how it can be the basis for long term growth.

We are not arguing that management forecasts should be ignored. There is clearly useful information in these estimates and the key is to make sure that management forecasts are feasible and internally consistent.

Analyst Estimates

When valuing publicly traded firms, we do have access to forecasts of growth that other analysts tracking these firms have made. Services like I/B/E/S and Zack’s aggregate and summarize analyst forecasts and make them widely accessible. Thus, we can easily find out what analysts following Google are expecting its earnings growth to be over the next 5 years.

The Information Advantages

There are a number of reasons to believe that analyst forecasts of growth should be better than using historical growth rates.
Analysts, in addition to using historical data, can use information that has come out about both the firm and the overall economy since the last earnings report, to make predictions about future growth. This information can sometimes lead to significant re-evaluation of the firm's expected cash flows.

Analysts can also condition their growth estimates for a firm on information revealed by competitors on pricing policy and future growth. For instance, a negative earnings report by one telecommunications firm can lead to a reassessment of earnings for other telecommunication firms.

Analysts sometimes have access to private information about the firms they follow which may be relevant in forecasting future growth. This avoids answering the delicate question of when private information becomes illegal inside information. There is no doubt, however, that good private information can lead to significantly better estimates of future growth. In an attempt to restrict this type of information leakage, the SEC issued new regulations in 2000 preventing firms from selectively revealing information to a few analysts or investors. Outside the United States, however, firms routinely convey private information to analysts following them.

Models for forecasting earnings that depend entirely upon past earnings data may ignore other publicly available information that is useful in forecasting future earnings. It has been shown, for instance, that other financial variables such as earnings retention, profit margins and asset turnover are useful in predicting future growth. Analysts can incorporate information from these variables into their forecasts.

**The Quality of Earnings Forecasts**

If firms are followed by a large number of analysts and these analysts are indeed better informed than the rest of the market, the forecasts of growth that emerge from analysts should be better than estimates based upon either historical growth or other publicly available information. But is this presumption justified? Are analyst forecasts of growth superior to other forecasts?

The general consensus from studies that have looked at short-term forecasts (one quarter ahead to four quarters ahead) of earnings is that analysts provide better forecasts of earnings than models that depend purely upon historical data. The mean relative absolute error, which measures the absolute difference between the actual earnings and
the forecast for the next quarter, in percentage terms, is smaller for analyst forecasts than it is for forecasts based upon historical data. Two other studies shed further light on the value of analysts’ forecasts. Crichfield, Dyckman and Lakonishok (1978) examine the relative accuracy of forecasts in the Earnings Forecaster, a publication from Standard and Poors that summarizes forecasts of earnings from more than 50 investment firms. They measure the squared forecast errors by month of the year and compute the ratio of analyst forecast error to the forecast error from time-series models of earnings. They find that the time series models actually outperform analyst forecasts from April until August, but under perform them from September through January. They hypothesize that this is because there is more firm-specific information available to analysts during the latter part of the year. The other study by O'Brien (1988) compares consensus analyst forecasts from the Institutions Brokers Estimate System (I/B/E/S) with time series forecasts from one quarter ahead to four quarters ahead. The analyst forecasts outperform the time series model for one-quarter ahead and two-quarter ahead forecasts, do as well as the time series model for three-quarter ahead forecasts and do worse than the time series model for four-quarter ahead forecasts. Thus, the advantage gained by analysts from firm-specific information seems to deteriorate as the time horizon for forecasting is extended.

In valuation, the focus is more on long-term growth rates in earnings than on next quarter's earnings. There is little evidence to suggest that analysts provide superior forecasts of earnings when the forecasts are over three or five years. An early study by Cragg and Malkiel compared long term forecasts by five investment management firms in 1962 and 1963 with actual growth over the following three years to conclude that analysts were poor long term forecasters. This view is contested by Vander Weide and Carleton (1988) who find that the consensus prediction of five-year growth in the I/B/E/S is superior to historically oriented growth measures in predicting future growth. There is an intuitive basis for arguing that analyst predictions of growth rates must be better than time-series or other historical-data based models simply because they use more information. The evidence indicates, however, that this superiority in forecasting is surprisingly small for long-term forecasts and that past growth rates play a significant role in determining analyst forecasts.

There is one final consideration. Analysts generally forecast earnings per share and most services report these estimates. When valuing a firm, you need forecasts of
operating income and the growth in earnings per share will not be equal to the growth in operating income. In general, the growth rate in operating income should be lower than the growth rate in earnings per share. Thus, even if you decide to use analyst forecasts, you will have to adjust them down to reflect the need to forecast operating income growth.

Analyst forecasts may be useful in coming up with a predicted growth rate for a firm but there is a danger to blindly following consensus forecasts. Analysts often make significant errors in forecasting earnings, partly because they depend upon the same data sources (which might have been erroneous or misleading) and partly because they sometimes overlook significant shifts in the fundamental characteristics of the firm. The secret to successful valuation often lies in discovering inconsistencies between analysts' forecasts of growth and a firm's fundamentals. The next section examines this relationship in more detail.

**III. Fundamental Growth**

With both historical and analyst estimates, growth is an exogenous variable that affects value but is divorced from the operating details of the firm. The soundest way of incorporating growth into value is to make it endogenous, i.e., to make it a function of how much a firm reinvests for future growth and the quality of its reinvestment. We will begin by considering the relationship between fundamentals and growth in equity income, and then move on to look at the determinants of growth in operating income.

*Growth In Equity Earnings*

When estimating cash flows to equity, we usually begin with estimates of net income, if we are valuing equity in the aggregate, or earnings per share, if we are valuing equity per share. In this section, we will begin by presenting the fundamentals that determine expected growth in earnings per share and then move on to consider a more expanded version of the model that looks at growth in net income.

*Growth in Earnings Per Share*

The simplest relationship determining growth is one based upon the retention ratio (percentage of earnings retained in the firm) and the return on equity on its projects. Firms that have higher retention ratios and earn higher returns on equity should have
much higher growth rates in earnings per share than firms that do not share these characteristics. To establish this, note that

\[
g_t = \frac{NI_t - NI_{t-1}}{NI_{t-1}}
\]

where,

\[
g_t = \text{Growth Rate in Net Income}
\]
\[
NI_t = \text{Net Income in year } t
\]

Given the definition of return on equity, the net income in year \( t-1 \) can be written as:

\[
NI_{t-1} = \text{Book Value of Equity}_{t-2} \times ROE_{t-1}
\]

where,

\[
ROE_{t-1} = \text{Return on equity in year } t-1
\]

The net income in year \( t \) can be written as:

\[
NI_t = (\text{Book Value of Equity}_{t-2} + \text{Retained Earnings}_{t-1}) \times ROE_t
\]

Assuming that the return on equity is unchanged, i.e., \( ROE_t = ROE_{t-1} = ROE \),

\[
g_t = \left( \frac{\text{Retained Earnings}_{t-1}}{NI_{t-1}} \right)(ROE) = (\text{Retained Ratio})(ROE) = (b)(ROE)
\]

where \( b \) is the retention ratio. Note that the firm is not being allowed to raise equity by issuing new shares. Consequently, the growth rate in net income and the growth rate in earnings per share are the same in this formulation.

*Illustration 4.3: Growth in Earnings Per Share: Examples*

In this illustration, we will consider the expected growth rate in earnings based upon the retention ratio and return on equity for two financial service firms – Goldman Sachs and J.P. Morgan Chase, a real estate investment trust (Vornado) and a telecommunication firm (Verizon). In Table 4.2, we summarize the returns on equity, retention ratios and expected growth rates in earnings for the four firms (assuming that they can maintain their existing fundamentals).

*Table 4.2: Fundamental Growth Rates in Earnings per Share*

<table>
<thead>
<tr>
<th></th>
<th>Return on Equity</th>
<th>Retention Ratio</th>
<th>Expected Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>J.P. Morgan Chase</td>
<td>11.16%</td>
<td>34.62%</td>
<td>3.86%</td>
</tr>
</tbody>
</table>
Goldman Sachs has the highest expected growth rate in earnings per share, because of its high return on equity and retention ratio. Verizon has the highest return on equity, but retains less of its earnings, leading to a lower expected growth rate. Chase’s low return on equity and retention ratio act as a drag on expected growth, whereas Vornado’s expected growth rate is depressed by the requirement that it pay out most of its earnings as dividends.

Growth in Net Income

If we relax the assumption that the only source of equity is retained earnings, the growth in net income can be different from the growth in earnings per share. Intuitively, note that a firm can grow net income significantly by issuing new equity to fund new projects while earnings per share stagnates. To derive the relationship between net income growth and fundamentals, we need a measure of how investment that goes beyond retained earnings. One way to obtain such a measure is to estimate directly how much equity the firm reinvests back into its businesses in the form of net capital expenditures and investments in working capital.

Equity reinvested in business = (Capital Expenditures – Depreciation) + Change in Working Capital - (New Debt Issued – Debt Repaid)

Dividing this number by the net income gives us a much broader measure of the equity reinvestment rate:

Equity Reinvestment Rate = \frac{\text{Equity reinvested}}{\text{Net Income}}

Unlike the retention ratio, this number can be well in excess of 100% because firms can raise new equity. The expected growth in net income can then be written as:

Expected Growth in Net Income = (\text{Equity Reinvestment Rate})(\text{Return on Equity})

Illustration 4.4: Growth in Net Income: Toyota and Exxon Mobil

To estimate growth in net income based upon fundamentals, we look at Toyota, the Japanese automaker, and at Exxon Mobil, the world’s largest oil company. In Table 4.3, we first estimate the components of equity reinvestment and use it to estimate the
reinvestment rate for each of the firms. We also present the return on equity and the expected growth rate in net income at each of these firms.

Table 4.3: Expected Growth in Net Income

<table>
<thead>
<tr>
<th></th>
<th>Non-cash Net Income</th>
<th>Change in Net Cap Ex</th>
<th>Equity Reinvestment Rate</th>
<th>ROE</th>
<th>Expected Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exxon Mobil (in millions)</td>
<td>$25,011</td>
<td>$4,243</td>
<td>$336</td>
<td>16.98%</td>
<td>21.88%</td>
</tr>
<tr>
<td>Toyota (in billions of yen)</td>
<td>1,141</td>
<td>925</td>
<td>-50</td>
<td>64.40%</td>
<td>16.55%</td>
</tr>
</tbody>
</table>

The pluses and minuses of this approach are visible in the table above. The approach much more accurately captures the true reinvestment in the firm by focusing not on what was retained but on what was reinvested. The limitation of the approach is that the ingredients that go into the reinvestment – capital expenditures, working capital change and net debt issued – are all volatile numbers. It is usually much more realistic to look at the average reinvestment rate over three or five years, rather than just the current year. We will return to examine this question in more depth when we look at growth in operating income.

Determinants of Return on Equity

Both earnings per share and net income growth are affected by the return on equity of a firm. The return on equity is affected by the leverage decisions of the firm. In the broadest terms, increasing leverage will lead to a higher return on equity if the pre-interest, after-tax return on capital exceeds the after-tax interest rate paid on debt. This is captured in the following formulation of return on equity:

\[
ROE = ROC + \frac{D}{E} (ROC - i(1 - t))
\]

where,

\[
ROC = \frac{EBIT(1 - t)}{BV \text{ of Debt} + +BV \text{ of Equity}}
\]
\[
\frac{D}{E} = \frac{\text{BV of Debt}}{\text{BV of Equity}}
\]
\[
i = \frac{\text{Interest Expense on Debt}}{\text{BV of Debt}}
\]
\[t = \text{Tax rate on ordinary income}\]

The derivation is simple. Using this expanded version of ROE, the growth rate can be written as:

\[
g = b \left( \frac{D}{E} \left( \frac{\text{ROC} - i (1 - t)}{1 + i} \right) \right)
\]

The advantage of this formulation is that it allows explicitly for changes in leverage and the consequent effects on growth.

*Illustration 4.5: Breaking down Return on Equity: Exxon Mobil and Toyota*

To consider the components of return on equity, we look, in Table 4.4, at Exxon Mobil and Toyota, two firms whose returns on equity we looked at in Illustration 4.4.

<table>
<thead>
<tr>
<th></th>
<th>ROC</th>
<th>Book D/E</th>
<th>Book Interest Rate</th>
<th>Tax Rate</th>
<th>ROE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exxon Mobil</td>
<td>15.10%</td>
<td>10.23%</td>
<td>6.68%</td>
<td>35.00%</td>
<td>16.20%</td>
</tr>
<tr>
<td>Toyota</td>
<td>8.28%</td>
<td>87.66%</td>
<td>2.51%</td>
<td>33.00%</td>
<td>14.06%</td>
</tr>
</tbody>
</table>

Comparing these numbers to those reported in Illustration 4.4, note that the return on equity is lower for both firms, using this extended calculation. One reason for the difference is the use of marginal tax rates to compute returns on capital and equity in this illustration, whereas we used the reported net income in illustration 4.4. Note also that a significant portion of Toyota’s high return on equity comes from its use of debt (and the resulting high debt to equity ratio).
Average and Marginal Returns

The return on equity is conventionally measured by dividing the net income in the most recent year by the book value of equity at the end of the previous year. Consequently, the return on equity measures both the quality of both older projects that have been on the books for a substantial period and new projects from more recent periods. Since older investments represent a significant portion of the earnings, the average returns may not shift substantially for larger firms that are facing a decline in returns on new investments, either because of market saturation or competition. In other words, poor returns on new projects will have a lagged effect on the measured returns. In valuation, it is the returns that firms are making on their newer investments that convey the most information about a quality of a firm’s projects. To measure these returns, we could compute a marginal return on equity by dividing the change in net income in the most recent year by the change in book value of equity in the prior year:

\[
\text{Marginal Return on Equity} = \frac{\Delta \text{Net Income}_i}{\Delta \text{Book Value of Equity}_{i-1}}
\]

For example, Goldman Sachs reported a return on equity of 18.49% in 2005, based upon net income of $4,972 million in 2005 and book value of equity of $26,888 million at the end of 2004:

Return on Equity in 2005 = $4,972/26,888 = 18.49%

The marginal return on equity for Goldman in 2005 is computed using the change in net income and book value of equity:

Change in net income from 2004 to 2005 = $4,972 - $4,553 = $419 million
Change in Book value of equity from 2003 to 2004 = 26888 – 22913 = $ 3,975 million
Marginal Return on Equity = $419 / $3,975 = 10.55%

To the extent that the marginal return on equity represents the returns on new investments, this offers a cautionary note that the return on equity on new investments may be lower than the historical returns.

The Effects of Changing Return on Equity

So far in this section, we have operated on the assumption that the return on equity remains unchanged over time. If we relax this assumption, we introduce a new component to growth – the effect of changing return on equity on existing investment
over time. Consider, for instance, a firm that has a book value of equity of $100 million and a return on equity of 10%. If this firm improves its return on equity to 11%, it will post an earnings growth rate of 10% even if it does not reinvest any money. This additional growth can be written as a function of the change in the return on equity.

Addition to Expected Growth Rate = \( \frac{\text{ROE}_t - \text{ROE}_{t-1}}{\text{ROE}_{t-1}} \)

where ROE\(_t\) is the return on equity in period \( t \). This will be in addition to the fundamental growth rate computed as the product of the return on equity in period \( t \) and the retention ratio.

Total Expected Growth Rate = \((b)(\text{ROE}_t) + \frac{\text{ROE}_t - \text{ROE}_{t-1}}{\text{ROE}_{t-1}}\)

While increasing return on equity will generate a spurt in the growth rate in the period of the improvement, a decline in the return on equity will create a more than proportional drop in the growth rate in the period of the decline.

It is worth differentiating at this point between returns on equity on new investments and returns on equity on existing investments. The additional growth that we are estimating above comes not from improving returns on new investments but by changing the return on existing investments. For lack of a better term, you could consider it “efficiency generated growth”.

**Illustration 4.6: Effects of Changing Return on Equity: J.P. Morgan Chase**

In Illustration 4.3, we looked at Chase’s expected growth rate based upon its return on equity of 11.16% and its retention ratio of 34.62%. Assume that the firm will be able to improve its overall return on equity (on both new and existing investments) to 12% next year and that the retention ratio remains at 34.62%. The expected growth rate in earnings per share next year can then be written as:

\[
\text{Expected Growth rate in EPS} = \left( \text{ROE}_t \right)\text{(Retention Ratio)} + \frac{\text{ROE}_t - \text{ROE}_{t-1}}{\text{ROE}_{t-1}}
\]

\[
= (0.12)(0.3462) + \frac{0.12 - 0.1116}{0.1116}
\]

\[
= 0.12 \times 0.3462 + \frac{0.12 - 0.1116}{0.1116}
\]

\[
= 0.1168 = 11.68\%
\]

After next year, the growth rate will subside to a more sustainable 4.15% (0.12*0.3462).

How would the answer be different if the improvement in return on equity were only on new investments but not on existing assets? The expected growth rate in earnings per share can then be written as:
Expected Growth rate in EPS = ROE\textsubscript{t} \times Retention Ratio = 0.12 \times 0.3462 = 0.0415

Thus, there is no additional growth created in this case. What if the improvement had been only on existing assets and not on new investments? Then, the expected growth rate in earnings per share can be written as:

\[
(\text{ROE}\textsubscript{t})(\text{Retention Ratio}) + \frac{\text{ROE}\textsubscript{t} - \text{ROE}\textsubscript{t+1}}{\text{ROE}\textsubscript{t+1}}
\]

Expected Growth rate in EPS = \((0.1116)(0.3462) + \frac{0.12 - 0.1116}{0.1116}\)

= 0.1139 = 11.39%

**Growth in Operating Income**

Just as equity income growth is determined by the equity reinvested back into the business and the return made on that equity investment, you can relate growth in operating income to total reinvestment made into the firm and the return earned on capital invested. We will consider three separate scenarios, and examine how to estimate growth in each, in this section. The first is when a firm is earning a high return on capital that it expects to sustain over time. The second is when a firm is earning a positive return on capital that is expected to increase over time. The third is the most general scenario, where a firm expects operating margins to change over time, sometimes from negative values to positive levels.

**A. Stable Return on Capital Scenario**

When a firm has a stable return on capital, its expected growth in operating income is a product of the reinvestment rate, i.e., the proportion of the after-tax operating income that is invested in net capital expenditures and non-cash working capital, and the quality of these reinvestments, measured as the return on the capital invested.

Expected Growth\textsubscript{EBIT} = \text{Reinvestment Rate} \times \text{Return on Capital}

where,

\[
\text{Reinvestment Rate} = \frac{\text{Capital Expenditure} - \text{Depreciation} + \Delta \text{Non - cash WC}}{\text{EBIT} (1 - \text{tax rate})}
\]

\[
\text{Return on Capital} = \frac{\text{EBIT}(1 - \tau)}{\text{Capital Invested}}
\]

In making these estimates, you use the adjusted operating income and reinvestment values that you computed in Chapter 4. Both measures should be forward looking and the return on capital should represent the expected return on capital on future investments. In
the rest of this section, you consider how best to estimate the reinvestment rate and the return on capital.

*Reinvestment Rate*

The reinvestment rate measures how much a firm is plowing back to generate future growth. The reinvestment rate is often measured using the most recent financial statements for the firm. Although this is a good place to start, it is not necessarily the best estimate of the future reinvestment rate. A firm’s reinvestment rate can ebb and flow, especially in firms that invest in relatively few, large projects or acquisitions. For these firms, looking at an average reinvestment rate over time may be a better measure of the future. In addition, as firms grow and mature, their reinvestment needs (and rates) tend to decrease. For firms that have expanded significantly over the last few years, the historical reinvestment rate is likely to be higher than the expected future reinvestment rate. For these firms, industry averages for reinvestment rates may provide a better indication of the future than using numbers from the past. Finally, it is important that you continue treating R&D expenses and operating lease expenses consistently. The R&D expenses, in particular, need to be categorized as part of capital expenditures for purposes of measuring the reinvestment rate.

The reinvestment rate for a firm can be negative if its depreciation exceeds its capital expenditures or if the working capital declines substantially during the course of the year. For most firms, this negative reinvestment rate will be a temporary phenomenon reflecting lumpy capital expenditures or volatile working capital. For these firms, the current year’s reinvestment rate (which is negative) can be replaced with an average reinvestment rate over the last few years. For some firms, though, the negative reinvestment rate may be a reflection of the policies of the firms and how we deal with it will depend upon why the firm is embarking on this path:

- Firms that have over invested in capital equipment or working capital in the past may be able to live off past investment for a number of years, reinvesting little and generating higher cash flows for that period. If this is the case, we should not use the negative reinvestment rate in forecasts and estimate growth based upon improvements in return on capital. Once the firm has reached the point where it is efficiently using its resources, though, we should change the reinvestment rate to reflect industry averages.
• The more extreme scenario is a firm that has decided to liquidate itself over time, by not replacing assets as they become run down and by drawing down working capital. In this case, the expected growth should be estimated using the negative reinvestment rate. Not surprisingly, this will lead to a negative expected growth rate and declining earnings over time.

*Return on Capital*

The return on capital is often based upon the firm's return on existing investments, where the book value of capital is assumed to measure the capital invested in these investments. Implicitly, you assume that the current accounting return on capital is a good measure of the true returns earned on existing investments and that this return is a good proxy for returns that will be made on future investments. This assumption, of course, is open to question for the following reasons.

- The book value of capital might not be a good measure of the capital invested in existing investments, since it reflects the historical cost of these assets and accounting decisions on depreciation. When the book value understates the capital invested, the return on capital will be overstated; when book value overstates the capital invested, the return on capital will be understated. This problem is exacerbated if the book value of capital is not adjusted to reflect the value of the research asset or the capital value of operating leases.

- The operating income, like the book value of capital, is an accounting measure of the earnings made by a firm during a period. All the problems in using unadjusted operating income described in Chapter 4 continue to apply.

- Even if the operating income and book value of capital are measured correctly, the return on capital on existing investments may not be equal to the marginal return on capital that the firm expects to make on new investments, especially as you go further into the future.

Given these concerns, you should consider not only a firm’s current return on capital, but any trends in this return as well as the industry average return on capital. If the current return on capital for a firm is significantly higher than the industry average, the forecasted return on capital should be set lower than the current return to reflect the erosion that is likely to occur as competition responds.
Finally, any firm that earns a return on capital greater than its cost of capital is earning an excess return. The excess returns are the result of a firm’s competitive advantages or barriers to entry into the industry. High excess returns locked in for very long periods imply that this firm has a permanent competitive advantage.

*Illustration 4.7: Measuring the Reinvestment Rate, Return on Capital and Expected Growth Rate – Titan Cement and SAP*

In this Illustration, we will estimate the reinvestment rate, return on capital and expected growth rate for Titan Cement, a Greek cement company, and SAP, the enterprise software company. We begin by presenting the inputs for the return on capital computation in Table 4.5.

*Table 4.5: Return on Capital*

<table>
<thead>
<tr>
<th></th>
<th>EBIT</th>
<th>EBIT (1-t)</th>
<th>BV of Debt</th>
<th>BV of Equity (net of cash)</th>
<th>Return on Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>Titan Cement</td>
<td>232</td>
<td>173</td>
<td>399</td>
<td>445</td>
<td>20.49%</td>
</tr>
<tr>
<td>SAP</td>
<td>2161</td>
<td>1414</td>
<td>530</td>
<td>6565</td>
<td>19.93%</td>
</tr>
</tbody>
</table>

Return on capital = EBIT (1-t)/ (BV of Debt + BV of Equity – Cash)

We use the effective tax rate for computing after-tax operating income and the book value of debt and equity from the end of the prior year. For SAP, we use the operating income and book value of equity, adjusted for the capitalization of the research asset, as described in the last chapter. The after-tax returns on capital are computed in the last column.

We follow up by estimating capital expenditures, depreciation and the change in non-cash working capital from the most recent year in Table 4.6.

*Table 4.6: Reinvestment Rate*

<table>
<thead>
<tr>
<th></th>
<th>EBIT(1-t)</th>
<th>Capital expenditures</th>
<th>Depreciation</th>
<th>Change in Working Capital</th>
<th>Reinvestment</th>
<th>Reinvestment Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Titan Cement</td>
<td>173</td>
<td>110</td>
<td>60</td>
<td>52</td>
<td>=102</td>
<td>102/173 = 58.5%</td>
</tr>
<tr>
<td>SAP</td>
<td>1414</td>
<td>2027</td>
<td>1196</td>
<td>-19</td>
<td>812</td>
<td>812/1414= 57.4%</td>
</tr>
</tbody>
</table>

Finally, we compute the expected growth rate by multiplying the after-tax return on capital by the reinvestment rate in Table 4.7.
Table 4.7: Expected Growth Rate in Operating Income

<table>
<thead>
<tr>
<th></th>
<th>Reinvestment Rate</th>
<th>Return on Capital</th>
<th>Expected Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Titan Cement</td>
<td>58.5%</td>
<td>20.49%</td>
<td>11.99%</td>
</tr>
<tr>
<td>SAP</td>
<td>57.4%</td>
<td>19.93%</td>
<td>11.44%</td>
</tr>
</tbody>
</table>

If Titan Cement can maintain the return on capital and reinvestment rate that they had last year, it would be able to grow at 11.99% a year. With similar assumptions, the earnings at SAP can grow 11.44% a year.

Illustration 4.8: Current, Historical Average and Industry Averages

The reinvestment rate is a volatile number and often shifts significantly from year to year. Consider Titan Cement’s reinvestment rate in Table 4.8 over the last five years.

Table 4.8: Reinvestment and Reinvestment Rate: Titan Cement

<table>
<thead>
<tr>
<th></th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBIT</td>
<td>€ 162.78</td>
<td>€ 186.39</td>
<td>€ 200.60</td>
<td>€ 222.00</td>
<td>€ 231.80</td>
<td>€ 1,003.57</td>
</tr>
<tr>
<td>Tax rate</td>
<td>25.47%</td>
<td>25.47%</td>
<td>25.47%</td>
<td>25.47%</td>
<td>25.47%</td>
<td></td>
</tr>
<tr>
<td>EBIT (1-t)</td>
<td>€ 121.32</td>
<td>€ 138.92</td>
<td>€ 149.51</td>
<td>€ 154.42</td>
<td>€ 172.76</td>
<td>€ 736.92</td>
</tr>
<tr>
<td>Capital Expenditures</td>
<td>€ 50.54</td>
<td>€ 81.00</td>
<td>€ 113.30</td>
<td>€ 102.30</td>
<td>€ 109.50</td>
<td>€ 456.64</td>
</tr>
<tr>
<td>Depreciation</td>
<td>€ 39.26</td>
<td>€ 40.87</td>
<td>€ 80.94</td>
<td>€ 73.70</td>
<td>€ 60.30</td>
<td>€ 295.07</td>
</tr>
<tr>
<td>Change in Non-cash working capital</td>
<td>€ 9.93</td>
<td>€ 59.90</td>
<td>€ 8.85</td>
<td>-€ 0.07</td>
<td>€ 11.42</td>
<td>-€ 183.66</td>
</tr>
<tr>
<td>Reinvestment</td>
<td>€ 21.21</td>
<td>€ 100.03</td>
<td>€ 41.21</td>
<td>€ 28.53</td>
<td>€ 60.62</td>
<td>€ 251.60</td>
</tr>
<tr>
<td>Reinvestment Rate</td>
<td>17.48%</td>
<td>72.01%</td>
<td>27.56%</td>
<td>18.48%</td>
<td>35.09%</td>
<td>34.14%</td>
</tr>
</tbody>
</table>

The reinvestment rate over the last 5 years has ranged from 17.48 in 2000 to 72.01% in 2001. We computed the average reinvestment rate over the five years, by dividing the total reinvestment over the 5 years by the total after-tax operating income over the last 5 years.4

We also computed Titan Cement’s return on capital each year for the last 5 years in Table 4.9:

Table 4.9: Return on Capital: Titan Cement

<table>
<thead>
<tr>
<th></th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBIT (1-t)</td>
<td>€ 121.32</td>
<td>€ 138.92</td>
<td>€ 149.51</td>
<td>€ 154.42</td>
<td>€ 172.76</td>
</tr>
<tr>
<td>BV of Capital</td>
<td>€ 353.00</td>
<td>€ 787.00</td>
<td>€ 743.00</td>
<td>€ 786.00</td>
<td>€ 843.00</td>
</tr>
<tr>
<td>Return on capital</td>
<td>34.37%</td>
<td>17.65%</td>
<td>20.12%</td>
<td>19.65%</td>
<td>20.49%</td>
</tr>
</tbody>
</table>

4 This tends to work better than averaging the reinvestment rate over 5 years. The reinvestment rate tends to be much more volatile than the dollar values.
With the return in 2000 as the outlier, the return on capital at Titan Cement has approximated about 20% in the last three years.

Clearly, the estimates of expected growth are a function of what you assume about future investments. For Titan Cement, if you assume that the average reinvestment rate over the last 5 years and the current return on capital are better measures for the future, your expected growth rate would be:

\[
\text{Expected Growth rate} = \text{Reinvestment Rate} \times \text{Return on Capital}
\]

\[
= 0.3414 \times 0.2049 = 0.07 \text{ or } 7\%
\]

In the case of Titan Cement, we believe that this estimate is a much more reasonable one given what we know about the firm and its growth potential.

**B. Positive and Changing Return on Capital Scenario**

The analysis in the previous section is based upon the assumption that the return on capital remains stable over time. If the return on capital changes over time, the expected growth rate for the firm will have a second component, which will increase the growth rate if the return on capital increases and decrease the growth rate if the return on capital decreases.

\[
\text{Expected Growth Rate} = \left( \text{ROC}_t \right) \times \text{Reinvestment rate} + \frac{\text{ROC}_t - \text{ROC}_{t-1}}{\text{ROC}_t}
\]

For example, a firm that sees its return on capital improves from 10% to 11% while maintaining a reinvestment rate of 40% will have an expected growth rate of:

\[
\text{Expected Growth Rate} = \left( 0.11 \right) \times 0.40 + \frac{0.11 - 0.10}{0.10} = 14.40\%
\]

In effect, the improvement in the return on capital increases the earnings on existing assets and this improvement translates into an additional growth of 10% for the firm.

**Marginal and Average Returns on Capital**

So far, you have looked at the return on capital as the measure that determines return. In reality, however, there are two measures of returns on capital. One is the return earned by firm collectively on all of its investments, which you define as the average return on capital. The other is the return earned by a firm on just the new investments it makes in a year, which is the marginal return on capital.
Changes in the marginal return on capital do not create a second-order effect and
the value of the firm is a product of the marginal return on capital and the reinvestment
rate. Changes in the average return on capital, however, will result in the additional
impact on growth chronicled above.

*Candidates for Changing Average Return on Capital*

What types of firms are likely to see their return on capital change over time? One
category would include firms with poor returns on capital that improve their operating
efficiency and margins, and consequently their return on capital. In these firms, the
expected growth rate will be much higher than the product of the reinvestment rate and
the return on capital. In fact, since the return on capital on these firms is usually low
before the turn-around, small changes in the return on capital translate into big changes in
the growth rate. Thus, an increase in the return on capital on existing assets of 1% to 2%
doubles the earnings (resulting in a growth rate of 100%).

The other category would include firms that have very high returns on capital on
their existing investments but are likely to see these returns slip as competition enters the
business, not only on new investments but also on existing investments.

*Illustration 4.9: Estimating Expected Growth with Changing Return on Capital -
Blockbuster*

In 2004, Blockbuster, the video rental company, reported an after-tax return on
capital of 4.06% and a reinvestment rate of 26.46%. If it maintains these numbers in
perpetuity, its expected growth rate can be estimated as follows:
Expected Growth Rate = Return on capital * Reinvestment Rate = .0406*.2646 = 1.07%
Assume that the firm will see its return on capital increase on both its existing assets and
its new investments to 6.20% next year and that its reinvestment rate will stay at 26.46%.
The expected growth rate next year can be estimated.

\[
\text{Expected growth rate} = (0.062)(0.2646) + \frac{0.062 - 0.0406}{0.0406} = 54.35\%
\]

If the improvement in return on capital on existing assets occurs more gradually over the
next 5 years, the expected annual growth rate for the next 5 years can be estimated as
follows:

\[
\text{Expected growth rate} = (0.062)(0.2646) + \left[ 1 + \frac{0.062 - 0.0406}{0.0406} \right]^{1/5} - 1 = 10.48\%
\]
The first term in the equation represents expected growth in earnings from new investments and the second

*C. Negative Return on Capital Scenario*

The third and most difficult scenario for estimating growth is when a firm is losing money and has a negative return on capital. Since the firm is losing money, the reinvestment rate is also likely to be negative. To estimate growth in these firms, you have to move up the income statement and first project growth in revenues. Next, you use the firm’s expected operating margin in future years to estimate the operating income in those years. If the expected margin in future years is positive, the expected operating income will also turn positive, allowing us to apply traditional valuation approaches in valuing these firms. You also estimate how much the firm has to reinvest to generate revenue growth, by linking revenues to the capital invested in the firm.

*Growth in Revenues*

Many high growth firms, while reporting losses, also show large increases in revenues from period to period. The first step in forecasting cash flows is forecasting revenues in future years, usually by forecasting a growth rate in revenues each period. In making these estimates, there are five points to keep in mind.

- The rate of growth in revenues will decrease as the firm’s revenues increase. Thus, a ten-fold increase in revenues is entirely feasible for a firm with revenues of $2 million but unlikely for a firm with revenues of $2 billion.
- Compounded growth rates in revenues over time can seem low, but appearances are deceptive. A compounded growth rate in revenues of 40% over ten years will result in a 40-fold increase in revenues over the period.
- While growth rates in revenues may be the mechanism that you use to forecast future revenues, you do have to keep track of the dollar revenues to ensure that they are reasonable, given the size of the overall market that the firm operates in. If the projected revenues for a firm ten years out would give it a 90% or 100% share (or greater) of the overall market in a competitive market place, you clearly should reassess the revenue growth rate.
- Assumptions about revenue growth and operating margins have to be internally consistent. Firms can post higher growth rates in revenues by adopting more
aggressive pricing strategies but the higher revenue growth will then be accompanied by lower margins.

• In coming up with an estimate of revenue growth, you have to make a number of subjective judgments about the nature of competition, the capacity of the firm that you are valuing to handle the revenue growth and the marketing capabilities of the firm.

Estimating revenue growth rates for a young firm in a new business may seem like an exercise in futility. While it is difficult to do, there are ways in which you can make the process easier.

• One is to work backwards by first considering the share of the overall market that you expect your firm to have once it matures and then determining the growth rate you would need to arrive at this market share. For instance, assume that you are analyzing an online toy retailer with $100 million in revenues currently. Assume also that the entire toy retail market had revenues of $70 billion last year. Assuming a 3% growth rate in this market over the next 10 years and a market share of 5% for your firm, you would arrive at expected revenues of $4.703 billion for the firm in ten years and a compounded revenue growth rate of 46.98%.

\[
\text{Expected Revenues in 10 years} = 70 \text{ billion} \times 1.03^{10} \times 0.05 = 4.703 \text{ billion}
\]

\[
\text{Expected compounded growth rate} = \left(\frac{4,703}{100}\right)^{1/10} - 1 = 0.4698
\]

• The other approach is to forecast the expected growth rate in revenues over the next 3 to 5 years based upon past growth rates. Once you estimate revenues in year 3 or 5, you can then forecast a growth rate based upon companies with similar revenues growth currently. For instance, assume that the online toy retailer analyzed above had revenue growth of 200% last year (revenues went from $33 million to $100 million). You could forecast growth rates of 120%, 100%, 80% and 60% for the next 4 years, leading to revenues of $1.267 billion in four years. You could then look at the average growth rate posted by retail firms with revenues between $1 and $1.5 billion last year and use that as the growth rate commencing in year 5.

Illustration 4.10: Estimating Revenues at Sirius

In earlier illustrations, we had considered Sirius Radio, the satellite radio pioneer. In Table 4.10, we forecast revenues for the firm for the next 10 years.
Table 4.10: Revenue Growth Rates and Revenues: Sirius

<table>
<thead>
<tr>
<th>Year</th>
<th>Revenue growth rate</th>
<th>Revenues</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td>$187</td>
<td>$187</td>
</tr>
<tr>
<td>1</td>
<td>200.00%</td>
<td>$562</td>
</tr>
<tr>
<td>2</td>
<td>100.00%</td>
<td>$1,125</td>
</tr>
<tr>
<td>3</td>
<td>80.00%</td>
<td>$2,025</td>
</tr>
<tr>
<td>4</td>
<td>60.00%</td>
<td>$3,239</td>
</tr>
<tr>
<td>5</td>
<td>40.00%</td>
<td>$4,535</td>
</tr>
<tr>
<td>6</td>
<td>25.00%</td>
<td>$5,669</td>
</tr>
<tr>
<td>7</td>
<td>20.00%</td>
<td>$6,803</td>
</tr>
<tr>
<td>8</td>
<td>15.00%</td>
<td>$7,823</td>
</tr>
<tr>
<td>9</td>
<td>10.00%</td>
<td>$8,605</td>
</tr>
<tr>
<td>10</td>
<td>5.00%</td>
<td>$9,035</td>
</tr>
</tbody>
</table>

We based our estimates of growth for the firms in the initial years on the growth in revenues over the last year – Sirius reported revenue growth of 250% in 2004-05. As the revenues increased, we tempered our estimates of revenue growth (in percent) to reflect the size of the company. As a check, we also examined how much the revenues at each of these firms would be in ten years relative to more mature companies in the sector now. Clear Channel, which is the largest competitor in the radio business, is a mature company with revenues of $9.34 billion in 2004. Based upon our projections, Sirius will rival Clear Channel in terms of size and revenues ten years from now.

Operating Margin Forecasts

Before considering how best to estimate the operating margins, let us begin with an assessment of where many high growth firms, early in the life cycle, stand when the valuation begins. They usually have low revenues and negative operating margins. If revenue growth translates low revenues into high revenues and operating margins stay negative, these firms will not only be worth nothing but are unlikely to survive. For firms to be valuable, the higher revenues eventually have to deliver positive earnings. In a valuation model, this translates into positive operating margins in the future. A key input in valuing a high growth firm then is the operating margin you would expect it to have as it matures.

In estimating this margin, you should begin by looking at the business that the firm is in. While many new firms claim to be pioneers in their businesses and some
believe that they have no competitors, it is more likely that they are the first to find a new way of delivering a product or service that was delivered through other channels before. Thus, Amazon might have been one of the first firms to sell books online, but Barnes and Noble and Borders preceded them as book retailers. In fact, one can consider online retailers as logical successors to catalog retailers such as L.L. Bean or Lillian Vernon. Similarly, Yahoo! might have been one of the first (and most successful) internet portals but they are following the lead of newspapers that have used content and features to attract readers and used their readership to attract advertising. Using the average operating margin of competitors in the business may strike some as conservative. After all, they would point out, Amazon can hold less inventory than Borders and does not have the burden of carrying the operating leases that Barnes and Noble does (on its stores) and should, therefore, be more efficient about generating its revenues and subsequently earnings. This may be true but it is unlikely that the operating margins for internet retailers can be persistently higher than their brick-and-mortar counterparts. If they were, you would expect to see a migration of traditional retailers to online retailing and increased competition among online retailers on price and products driving the margin down.

While the margin for the business in which a firm operates provides a target value, there are still two other estimation issues that you need to confront. Given that the operating margins in the early stages of the life cycle are negative, you first have to consider how the margin will improve from current levels to the target values. Generally, the improvements in margins will be greatest in the earlier years (at least in percentage terms) and then taper off as the firm approaches maturity. The second issue is one that arises when talking about revenue growth. Firms may be able to post higher revenue growth with lower margins but the trade off has to be considered. While firms generally want both higher revenue growth and higher margin, the margin and revenue growth assumptions have to be consistent.

Illustration 4.11: Estimating Operating Margins - Sirius

To estimate the operating margins for Sirius Radio, we begin by estimating the operating margins of other firms in the radio business. In 2004, the average pre-tax
operating margin for firms in this business was approximately 20%. We will assume that Sirius will move toward its target margins, with greater marginal improvements in the earlier years and smaller ones in the later years. Table 4.11 summarizes the expected operating margins and resulting operating income over time for Sirius Radio.

Table 4.11: Expected Operating Margins

<table>
<thead>
<tr>
<th>Year</th>
<th>Revenues</th>
<th>Operating Margin</th>
<th>Operating Income (Loss)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td>$187</td>
<td>-419.92%</td>
<td>-$787</td>
</tr>
<tr>
<td>1</td>
<td>$562</td>
<td>-199.96%</td>
<td>-$1,125</td>
</tr>
<tr>
<td>2</td>
<td>$1,125</td>
<td>-89.98%</td>
<td>-$1,012</td>
</tr>
<tr>
<td>3</td>
<td>$2,025</td>
<td>-34.99%</td>
<td>-$708</td>
</tr>
<tr>
<td>4</td>
<td>$3,239</td>
<td>-7.50%</td>
<td>-$243</td>
</tr>
<tr>
<td>5</td>
<td>$4,535</td>
<td>6.25%</td>
<td>$284</td>
</tr>
<tr>
<td>6</td>
<td>$5,669</td>
<td>13.13%</td>
<td>$744</td>
</tr>
<tr>
<td>7</td>
<td>$6,803</td>
<td>16.56%</td>
<td>$1,127</td>
</tr>
<tr>
<td>8</td>
<td>$7,823</td>
<td>18.28%</td>
<td>$1,430</td>
</tr>
<tr>
<td>9</td>
<td>$8,605</td>
<td>19.14%</td>
<td>$1,647</td>
</tr>
<tr>
<td>10</td>
<td>$9,035</td>
<td>19.57%</td>
<td>$1,768</td>
</tr>
</tbody>
</table>

Based upon our projections, Sirius Radio can expect to continue reporting operating losses for the next four years but the margins will improve over time.

**Sales to Capital Ratio**

High revenue growth is clearly a desirable objective, especially when linked with positive operating margins in future years. Firms do, however, have to invest to generate both revenue growth and positive operating margins in future years. This investment can take traditional forms (plant and equipment) but it should also include acquisitions of other firms, partnerships, investments in distribution and marketing capabilities and research and development.

To link revenue growth with reinvestment needs, you look at the revenues that every dollar of capital that you invest generates. This ratio, called the sales to capital ratio, allows us to estimate how much additional investment the firm has to make to generate the projected revenue growth. This investment can be in internal projects, acquisitions or working capital. To estimate the reinvestment needs in any year, you

---

5 The average pre-tax operating margin for the sector was 24.49% but Clear Channel, the largest player, had a pre-tax operating margin of 16.50%. The weighted average for the sector was roughly 20%.
6 The margin each year is computed as follows:
(Margin this year + Target margin)/2
divide the revenue growth that you have projected (in dollar terms) by the sales to capital ratio. Thus, if you expect revenues to grow by $1 billion and you use a sales to capital ratio of 2.5, you would estimate a reinvestment need for this firm of $400 million ($1 billion/2.5). Lower sales to capital ratios increase reinvestment needs (and reduce cash flows) while higher sales to capital ratios decrease reinvestment needs (and increase cash flows).

To estimate the sales to capital ratio, you look at both a firm’s past and the business it operates in. To measure this ratio historically, you look at changes in revenue each year and divide it by the reinvestment made that year. You also look at the average ratio of sales to book capital invested in the business in which the firm operates.

Linking operating margins to reinvestment needs is much more difficult to do, since a firm’s capacity to earn operating income and sustain high returns comes from the competitive advantages that it acquires, partly through internal investment and partly through acquisitions. Firms that adopt a two-track strategy in investing, where one track focuses on generating higher revenues and the other on building up competitive strengths should have higher operating margins and values than firms that concentrate only on revenue growth.

**Link to Return on Capital**

One of the dangers that you face when using a sales-to-capital ratio to generate reinvestment needs is that you might under-estimate or over-estimate your reinvestment needs. You can keep tabs on whether this is happening and correct it when it does by also estimating the after-tax return on capital on the firm each year through the analysis. To estimate the return on capital in a future year, you use the estimated after-tax operating income in that year and divide it by the total capital invested in that firm in that year. The former number comes from your estimates of revenue growth and operating margins, while the latter can be estimated by aggregating the reinvestments made by the firm all the way through the future year. For instance, a firm that has $500 million in capital invested today and is required to reinvest $300 million next year and $400 million the year after will have capital invested of $1.2 billion at the end of the second year.

For firms losing money today, the return on capital will be a negative number when the estimation begins but improve as margins improve. If the sales-to-capital ratio is set too high, the return-on-capital in the later years will be too high, while if it is set too
low, it will be too low. Too low or high relative to what, you ask? There are two comparisons that are worth making. The first is to the average return-on-capital for mature firms in the business in which your firm operates – mature specialty and brand name retailers, in the case of Ashford.com. The second is to the firm’s own cost of capital. A projected return on capital of 40% for a firm with a cost of capital of 10% in a sector where returns on capital hover around 15% is an indicator that the firm is investing too little for the projected revenue growth and operating margins. Decreasing the sales to capital ratio until the return on capital converges on 15% would be prudent.

*Illustration 4.12: Estimated Sales to Capital Ratio - Sirius*

To estimate how much Sirius Radio will have to invest to generate the expected revenue growth, we estimate the current sales to capital ratio and the average sales to capital ratio for the firm.

Current sales to capital ratio for Sirius = Revenues/ Book value of capital = 187/ 1657 = 0.11

Average sales to capital ratio for peer group = 1.50

We used a sales to capital ratio of 1.50 for Sirius, reflecting the industry average. Based upon this estimate, we can now calculate how much Sirius will have to reinvest each year for the next 10 years in Table 4.12.

*Table 4.12: Estimated Reinvestment Needs – Sirius*

<table>
<thead>
<tr>
<th>Year</th>
<th>Change in revenue</th>
<th>Sales/Capital Ratio</th>
<th>Reinvestment</th>
<th>Capital Invested</th>
<th>Imputed ROC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$1,657</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>$375</td>
<td>1.50</td>
<td>$250</td>
<td>$1,907</td>
<td>-67.87%</td>
</tr>
<tr>
<td>2</td>
<td>$562</td>
<td>1.50</td>
<td>$375</td>
<td>$2,282</td>
<td>-53.08%</td>
</tr>
<tr>
<td>3</td>
<td>$900</td>
<td>1.50</td>
<td>$600</td>
<td>$2,882</td>
<td>-31.05%</td>
</tr>
<tr>
<td>4</td>
<td>$1,215</td>
<td>1.50</td>
<td>$810</td>
<td>$3,691</td>
<td>-8.43%</td>
</tr>
<tr>
<td>5</td>
<td>$1,296</td>
<td>1.50</td>
<td>$864</td>
<td>$4,555</td>
<td>7.68%</td>
</tr>
<tr>
<td>6</td>
<td>$1,134</td>
<td>1.50</td>
<td>$756</td>
<td>$5,311</td>
<td>16.33%</td>
</tr>
<tr>
<td>7</td>
<td>$1,134</td>
<td>1.50</td>
<td>$756</td>
<td>$6,067</td>
<td>21.21%</td>
</tr>
<tr>
<td>8</td>
<td>$1,020</td>
<td>1.50</td>
<td>$680</td>
<td>$6,747</td>
<td>23.57%</td>
</tr>
<tr>
<td>9</td>
<td>$782</td>
<td>1.50</td>
<td>$522</td>
<td>$7,269</td>
<td>17.56%</td>
</tr>
<tr>
<td>10</td>
<td>$430</td>
<td>1.50</td>
<td>$287</td>
<td>$7,556</td>
<td>15.81%</td>
</tr>
</tbody>
</table>

To examine whether the assumptions about reinvestment are reasonable, we keep track of the capital invested in the firm each year by adding the reinvestment in that year to the capital invested in the prior year. Dividing the estimated after-tax operating income from table 4.11 by the capital invested (at the end of the prior year) yields an imputed return on
capital for the firm each year. The return on capital at Sirius converges on the industry average of 12% by the terminal year. This suggests that our estimates of sales to capital ratios are reasonable.

III. Terminal Value

Since you cannot estimate cash flows forever, you generally impose closure in discounted cash flow valuation by stopping your estimation of cash flows sometime in the future and then computing a terminal value that reflects the value of the firm at that point.

\[
\text{Value of a Firm} = \sum_{t=1}^{n} \frac{CF_t}{(1 + k_c)^t} + \frac{\text{Terminal Value}}{(1 + k_c)^n}
\]

You can find the terminal value in one of three ways. One is to assume a liquidation of the firm’s assets in the terminal year and estimate what others would pay for the assets that the firm has accumulated at that point. The other two approaches value the firm as a going concern at the time of the terminal value estimation. One applies a multiple to earnings, revenues or book value to estimate the value in the terminal year. The other assumes that the cash flows of the firm will grow at a constant rate forever – a stable growth rate. With stable growth, the terminal value can be estimated using a perpetual growth model.

Liquidation Value

In some valuations, we can assume that the firm will cease operations at a point in time in the future and sell the assets it has accumulated to the highest bidders. The estimate that emerges is called a liquidation value. There are two ways in which the liquidation value can be estimated. One is to base it on the book value of the assets, adjusted for any inflation during the period. Thus, if the book value of assets ten years from now is expected to be $2 billion, the average age of the assets at that point is 5 years and the expected inflation rate is 3%, the expected liquidation value can be estimated.

Expected Liquidation value = Book Value of Assets_{Term yr} \times (1 + \text{inflation rate})^{\text{Average life of assets}}

= $2 billion \times (1.03)^5 = $2.319 billion
The limitation of this approach is that it is based upon accounting book value and does not reflect the earning power of the assets.

The alternative approach is to estimate the value based upon the earning power of the assets. To make this estimate, we would first have to estimate the expected cash flows from the assets and then discount these cash flows back to the present, using an appropriate discount rate. In the example above, for instance, if we assumed that the assets in question could be expected to generate $400 million in after-tax cash flows for 15 years (after the terminal year) and the cost of capital was 10%, your estimate of the expected liquidation value would be:

\[
\text{Expected Liquidation value} = \left( \frac{1}{\text{(1.10)}^{15}} \right) \times \frac{1}{0.10} \times 400 = 3.042 \text{ billion}
\]

When valuing equity, there is one additional step that needs to be taken. The estimated value of debt outstanding in the terminal year has to be subtracted from the liquidation value to arrive at the liquidation proceeds for equity investors.

**Multiple Approach**

In this approach, the value of a firm in a future year is estimated by applying a multiple to the firm’s earnings or revenues in that year. For instance, a firm with expected revenues of $6 billion ten years from now will have an estimated terminal value in that year of $12 billion if a value to sales multiple of 2 is used. If valuing equity, we use equity multiples such as price earnings ratios to arrive at the terminal value.

While this approach has the virtue of simplicity, the multiple has a huge effect on the final value and where it is obtained can be critical. If, as is common, the multiple is estimated by looking at how comparable firms in the business today are priced by the market. The valuation becomes a relative valuation rather than a discounted cash flow valuation. If the multiple is estimated using fundamentals, it converges on the stable growth model that will be described in the next section.

All in all, using multiples to estimate terminal value, when those multiples are estimated from comparable firms, results in a dangerous mix of relative and discounted cash flow valuation. While there are advantages to relative valuation, and we will consider these in a later chapter, a discounted cash flow valuation should provide you with an estimate of intrinsic value, not relative value. Consequently, the only consistent
way of estimating terminal value in a discounted cash flow model is to use either a liquidation value or a stable growth model.

**Stable Growth Model**

In the liquidation value approach, we are assuming that your firm has a finite life and that it will be liquidated at the end of that life. Firms, however, can reinvest some of their cash flows back into new assets and extend their lives. If we assume that cash flows, beyond the terminal year, will grow at a constant rate forever, the terminal value can be estimated as.

\[
\text{Terminal Value}_t = \frac{\text{Cash Flow}_{t+1}}{r - g_{\text{stable}}}
\]

where the cash flow and the discount rate used will depend upon whether you are valuing the firm or valuing the equity. If we are valuing the equity, the terminal value of equity can be written as:

\[
\text{Terminal value of Equity}_n = \frac{\text{Cashflow to Equity}_{n+1}}{\text{Cost of Equity}_{n+1} - g_n}
\]

The cashflow to equity can be defined strictly as dividends (in the dividend discount model) or as free cashflow to equity. If valuing a firm, the terminal value can be written as:

\[
\text{Terminal value}_n = \frac{\text{Cashflow to Firm}_{n+1}}{\text{Cost of Capital}_{n+1} - g_n}
\]

where the cost of capital and the growth rate in the model are sustainable forever.

In this section, we will begin by considering how high a stable growth rate can be, how to best estimate when your firm will be a stable growth firm and what inputs need to be adjusted as a firm approaches stable growth.

**Constraints on Stable Growth**

Of all the inputs into a discounted cash flow valuation model, none can affect the value more than the stable growth rate. Part of the reason for it is that small changes in the stable growth rate can change the terminal value significantly and the effect gets larger as the growth rate approaches the discount rate used in the estimation. Not surprisingly, analysts often use it to alter the valuation to reflect their biases.
The fact that a stable growth rate is constant forever, however, puts strong constraints on how high it can be. Since no firm can grow forever at a rate higher than the growth rate of the economy in which it operates, the constant growth rate cannot be greater than the overall growth rate of the economy. In making a judgment on what the limits on stable growth rate are, we have to consider the following questions.

1. **Is the company constrained to operate as a domestic company or does it operate (or have the capacity) to operate multi-nationally?** If a firm is a purely domestic company, either because of internal constraints (such as those imposed by management) or external (such as those imposed by a government), the growth rate in the domestic economy will be the limiting value. If the company is a multinational or has aspirations to be one, the growth rate in the global economy (or at least those parts of the globe that the firm operates in) will be the limiting value. Note that the difference will be small for a U.S. firm, since the U.S economy still represents a large portion of the world economy. It may, however, mean that you could use a stable growth rate that is slightly higher (say 1/2 to 1%) for a Coca Cola than a Consolidated Edison.

2. **Is the valuation being done in nominal or real terms?** If the valuation is a nominal valuation, the stable growth rate should also be a nominal growth rate, i.e. include an expected inflation component. If the valuation is a real valuation, the stable growth rate will be constrained to be lower. Again, using Coca Cola as an example, the stable growth rate can be as high as 5.5% if the valuation is done in nominal U.S. dollars but only 3% if the valuation is done in real dollars.

3. **What currency is being used to estimate cash flows and discount rates in the valuation?** The limits on stable growth will vary depending upon what currency is used in the valuation. If a high-inflation currency is used to estimate cash flows and discount rates, the limits on stable growth will be much higher, since the expected inflation rate is added on to real growth. If a low-inflation currency is used to estimate cash flows, the limits on stable growth will be much lower. For instance, the stable growth rate that would be used to value Titan Cements, the Greek cement company, will be much higher if the valuation is done in drachmas than in euros.
While the stable growth rate cannot exceed the growth rate of the economy in which a firm operates, it can be lower. There is nothing that prevents us from assuming that mature firms will become a smaller part of the economy and it may, in fact, be the more reasonable assumption to make. Note that the growth rate of an economy reflects the contributions of both young, higher-growth firms and mature, stable growth firms. If the former grow at a rate much higher than the growth rate of the economy, the latter have to grow at a rate that is lower.

Setting the stable growth rate to be less than or equal to the growth rate of the economy is not only the consistent thing to do but it also ensures that the growth rate will be less than the discount rate. This is because of the relationship between the riskless rate that goes into the discount rate and the growth rate of the economy. Note that the riskless rate can be written as:

\[
\text{Nominal riskless rate} = \text{Real riskless rate} + \text{Expected inflation rate}
\]

In the long term, the real riskless rate will converge on the real growth rate of the economy and the nominal riskless rate will approach the nominal growth rate of the economy. In fact, a simple rule of thumb on the stable growth rate is that it should not exceed the riskless rate used in the valuation.

*Key Assumptions about Stable Growth*

In every discounted cash flow valuation, there are two critical assumptions you need to make on stable growth. The first relates to what the characteristics of the firm will be in stable growth, in terms of return on investments and costs of equity and capital. The second assumption relates to how the firm that you are valuing will make the transition from high growth to stable growth.

*I. Characteristics of Stable Growth Firm*

As firms move from high growth to stable growth, you need to give them the characteristics of stable growth firms. A firm in stable growth is different from that same firm in high growth on a number of dimensions. In general, you would expect stable growth firms to be less risky, use more debt, have lower (or even no) excess returns and reinvest less than high growth firms. In this section, we will consider how best to adjust each of these variables.
a. Equity Risk

When looking at the cost of equity, high growth firms tend to be more exposed to market risk (and have higher betas) than stable growth firms. Part of the reason for this is that they tend to be niche players, providers of discretionary products and services and a high leverage operation. Thus, firms like Commerce One or NTT Docomo may have betas that exceed 1.5 or even 2. As these firms and their corresponding markets mature, you would expect them to have less exposure to market risk and betas that are closer to one – the average for the market. One option is to set the beta in stable growth to one for all firms, arguing that firms in stable growth should all be average risk. Another is to allow for small differences to persist even in stable growth with firms in more volatile businesses having higher betas than firms in more stable businesses. We would recommend that, as a rule of thumb, stable period betas should not exceed 1.2.7

But what about firms that have betas well below 1, such as commodity companies? If you are assuming that these firms will stay in their existing businesses, there is no harm in assuming that the beta remains at existing levels. However, if your estimates of growth in perpetuity8 will require them to branch out into other business, you should adjust the beta upwards towards one.

b. Project Returns

High growth firms tend to have high returns on capital (and equity) and earn excess returns. In stable growth, it becomes much more difficult to sustain excess returns. There are some who believe that the only assumption consistent with stable growth is to assume no excess returns; the return on capital is set equal to the cost of capital. While, in principle, excess returns in perpetuity are not feasible, it is difficult in practice to assume that firms will suddenly lose the capacity to earn excess returns. Since entire industries often earn excess returns over long periods, assuming a firm’s returns on equity and capital will move towards industry averages will yield more reasonable estimates of value.

7 Two thirds of U.S. firms have betas that fall between 0.8 and 1.2. That becomes the range for stable period betas.
8 If you are valuing a commodity company and assuming any growth rate that exceeds inflation, you are assuming that your firm will branch into other businesses and you have to adjust the beta accordingly.
c. Debt Ratios and Costs of Debt

High growth firms tend to use less debt than stable growth firms. As firms mature, their debt capacity increases. When valuing firms, this will change the debt ratio that we use to compute the cost of capital. When valuing equity, changing the debt ratio will change both the cost of equity and the expected cash flows. The question whether the debt ratio for a firm should be moved towards a more sustainable level in stable growth cannot be answered without looking at the incumbent managers’ views on debt and how much power stockholders have in these firms. If managers are willing to change their debt ratios and stockholders retain some power, it is reasonable to assume that the debt ratio will move to a higher level in stable growth; if not, it is safer to leave the debt ratio at existing levels.

As earnings and cash flows increase, the perceived default risk in the firm will also change. A firm that is currently losing $10 million on revenues of $100 million may be rated B, but its rating should be much better if your forecasts of $10 billion in revenues and $1 billion in operating income come to fruition. In fact, internal consistency requires that you re-estimate the rating and the cost of debt for a firm as you change its revenues and operating income.

On the practical question of what debt ratio and cost of debt to use in stable growth, you should look at the financial leverage of larger and more mature firms in the industry. One solution is to use the industry average debt ratio and cost of debt as the debt ratio and cost of debt for the firm in stable growth.

d. Reinvestment and Retention Ratios

Stable growth firms tend to reinvest less than high growth firms and it is critical that we both capture the effects of lower growth on reinvestment and that we ensure that the firm reinvests enough to sustain its stable growth rate in the terminal phase. The actual adjustment will vary depending upon whether we are discounting dividends, free cash flows to equity or free cash flows to the firm. In the dividend discount model, note that the expected growth rate in earnings per share can be written as a function of the retention ratio and the return on equity.

\[
\text{Expected Growth Rate} = \text{Retention ratio} \times \text{Return on Equity}
\]
Algebraic manipulation can allow us to state the retention ratio as a function of the expected growth rate and return on equity:

Retention ratio = \frac{\text{Expected Growth rate}}{\text{Return on Equity}}

If we assume, for instance, a stable growth rate of 4% (based upon the growth rate of the economy) for Goldman Sachs and a return on equity of 12% (based upon industry averages), we would be able to compute the retention ratio in stable growth:

Retention ratio = \frac{4\%}{12\%} = 33.33\%

Goldman Sachs will have to reinvest 33.33% of its earnings into the firm to generate its expected growth of 4%; it can pay out the remaining 66.67%.

In a free cash flow to equity model, where we are focusing on net income growth, the expected growth rate is a function of the equity reinvestment rate and the return on equity.

Expected Growth Rate = \text{Equity Reinvestment rate} \times \text{Return on Equity}

The equity reinvestment rate can then be computed as follows:

\text{Equity Reinvestment rate} = \frac{\text{Expected Growth rate}}{\text{Return on Equity}}

If, for instance, we assume that Toyota will have a stable growth rate of 2% and that its return on equity in stable growth is 8%, we can estimate an equity reinvestment rate:

\text{Equity Reinvestment rate} = \frac{2\%}{8\%} = 12\%

Finally, looking at free cash flows to the firm, we estimated the expected growth in operating income as a function of the return on capital and the reinvestment rate:

Expected Growth rate = \text{Reinvestment rate} \times \text{Return on Capital}

Again, algebraic manipulation yields the following measure of the reinvestment rate in stable growth.

\text{Reinvestment Rate in stable growth} = \frac{\text{Stable growth rate}}{\text{ROC}_n}

where the $\text{ROC}_n$ is the return on capital that the firm can sustain in stable growth. This reinvestment rate can then be used to generate the free cash flow to the firm in the first year of stable growth.
Linking the reinvestment rate and retention ratio to the stable growth rate also makes the valuation less sensitive to assumptions about stable growth. While increasing the stable growth rate, holding all else constant, can dramatically increase value, changing the reinvestment rate as the growth rate changes will create an offsetting effect. The gains from increasing the growth rate will be partially or completely offset by the loss in cash flows because of the higher reinvestment rate. Whether value increases or decreases as the stable growth increases will entirely depend upon what you assume about excess returns. If the return on capital is higher than the cost of capital in the stable growth period, increasing the stable growth rate will increase value. If the return on capital is equal to the stable growth rate, increasing the stable growth rate will have no effect on value. This can be proved quite easily.

\[
\text{Terminal Value} = \frac{\text{EBIT}_{n+1}(1-t)(1-\text{Reinvestment Rate})}{\text{Cost of Capital}_n - \text{Stable Growth Rate}}
\]

Substituting in the stable growth rate as a function of the reinvestment rate, from above, you get:

\[
\text{Terminal Value} = \frac{\text{EBIT}_{n+1}(1-t)(1-\text{Reinvestment Rate})}{\text{Cost of Capital}_n - (\text{Reinvestment Rate} \times \text{Return on Capital})}
\]

Setting the return on capital equal to the cost of capital, you arrive at:

\[
\text{Terminal Value} = \frac{\text{EBIT}_{n+1}(1-t)(1-\text{Reinvestment Rate})}{\text{Cost of Capital}_n - (\text{Reinvestment Rate} \times \text{Cost on Capital})}
\]

Simplifying, the terminal value can be stated as:

\[
\text{Terminal Value}_{\text{ROC=WACC}} = \frac{\text{EBIT}_{n+1}(1-t)}{\text{Cost of Capital}_n}
\]

You could establish the same proposition with equity income and cash flows and show that a return on equity equal to the cost of equity in stable growth nullifies the positive effect of growth.

Illustration 4.13: Stable Growth rates and Excess Returns

Alloy Mills is a textile firm that is currently reporting after-tax operating income of $100 million. The firm has a return on capital currently of 20% and reinvests 50% of its earnings back into the firm, giving it an expected growth rate of 10% for the next 5 years:
Expected Growth rate = 20% * 50% = 10%
After year 5, the growth rate is expected to drop to 5% and the return on capital is expected to stay at 20%. The terminal value can be estimated as follows:
Expected operating income in year 6 = 100 \times (1.10)^5 (1.05) = $169.10 million

Expected reinvestment rate from year 5 = \frac{g}{ROC} = \frac{5\%}{20\%} = 25\%

Terminal value in year 5 = \frac{$169.10(1-0.25)}{0.10-0.05} = $2,537 million

The value of the firm today would then be:
Value of firm today =
\frac{\$55}{1.10} + \frac{\$60.5}{1.10^2} + \frac{\$66.55}{1.10^3} + \frac{\$73.21}{1.10^4} + \frac{\$80.53}{1.10^5} + \frac{\$2,537}{1.10^5} = $1,825 million

If we did change the return on capital in stable growth to 10% while keeping the growth rate at 5%, the effect on value would be dramatic:
Expected operating income in year 6 = 100 \times (1.10)^5 (1.05) = $169.10 million

Expected reinvestment rate from year 5 = \frac{g}{ROC} = \frac{5\%}{10\%} = 50\%

Terminal value in year 5 = \frac{$169.10(1-0.5)}{0.10-0.05} = $1,691 million

Value of firm today =
\frac{\$55}{1.10} + \frac{\$60.5}{1.10^2} + \frac{\$66.55}{1.10^3} + \frac{\$73.21}{1.10^4} + \frac{\$80.53}{1.10^5} + \frac{\$1,691}{1.10^5} = $1,300 million

Now consider the effect of lowering the growth rate to 4% while keeping the return on capital at 10% in stable growth:
Expected operating income in year 6 = 100 \times (1.10)^5 (1.04) = $167.49 million

Expected reinvestment rate in year 6 = \frac{g}{ROC} = \frac{4\%}{10\%} = 40\%

Terminal value in year 5 = \frac{$167.49(1-0.4)}{0.10-0.04} = $1,675 million

Value of firm today =
\frac{\$55}{1.10} + \frac{\$60.5}{1.10^2} + \frac{\$66.55}{1.10^3} + \frac{\$73.21}{1.10^4} + \frac{\$96.63}{1.10^5} + \frac{\$1,675}{1.10^5} = $1,300 million
Note that the terminal value decreases by $16 million but the cash flow in year 5 also increases by $16 million because the reinvestment rate at the end of year 5 drops to 40%. The value of the firm remains unchanged at $1,300 million. In fact, changing the stable growth rate to 0% has no effect on value:

Expected operating income in year 6 = 100 \( (1.10)^5 \) = $161.05 million

Expected reinvestment rate in year 6 = \( \frac{g}{\text{ROC}} = \frac{0\%}{10\%} = 0\% \)

Terminal value in year 5 = \( \frac{161.05(1-0.00)}{0.10 - 0.00} \) = $1,610.5 million

Value of firm today = $1,300

Illustration 4.14: Stable Growth Inputs

To illustrate how the inputs to valuation change as we go from high growth to stable growth, we will consider three firms – Goldman Sachs, with the dividend discount model, Toyota with a free cashflow to equity model and Titan Cement, with a free cashflow to the firm model.

Consider Goldman Sachs first in the context of the dividend discount model. While we will do the valuation in the next chapter, note that there are only three real inputs to the dividend discount model – the payout ratio (which determines dividends), the expected return on equity (which determines the expected growth rate) and the beta (which affects the cost of equity). In Illustration 4.1, we argued that Goldman Sachs would have a five-year high growth period. Table 4.13 summarizes the inputs into the dividend discount model for the valuation of Goldman Sachs.

<table>
<thead>
<tr>
<th>Table 4.13: Inputs to Dividend Discount Model – Goldman Sachs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>High Growth Period</strong></td>
</tr>
<tr>
<td>Payout ratio</td>
</tr>
<tr>
<td>Return on Equity</td>
</tr>
<tr>
<td>Expected Growth rate</td>
</tr>
<tr>
<td>Beta</td>
</tr>
<tr>
<td>Cost of equity (Riskfree rate=4.5%; Risk premium = 4%)</td>
</tr>
</tbody>
</table>
Note that the payout ratio and the beta for the high growth period are based upon the current year’s values. The return on equity for the next 5 years is set at 18.49 which is the current return on equity. The expected growth rate of 16.82% for the next 5 years is the product of the return on equity and retention ratio. In stable growth, we adjust the beta to one, lowering the cost of equity to 8.50%. We assume that the stable growth rate will be 4%, just slightly below the nominal growth rate in the economy (and the riskfree rate of 4.50%). We also assume that the return on equity will drop to 12%, still above the cost of equity in stable growth but reflecting Goldman’s substantial competitive advantages. The retention ratio decreases to 33.33%, as both growth and ROE drop.

To analyze Toyota in a free cash flow to equity model, we summarize our inputs for high growth and stable growth in Table 4.14.

*Table 4.14: Inputs to Free Cash flow to Equity Model – Toyota*

<table>
<thead>
<tr>
<th></th>
<th>High Growth</th>
<th>Stable Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return on Equity</td>
<td>16.55%</td>
<td>6.40%</td>
</tr>
<tr>
<td>Equity Reinvestment rate</td>
<td>64.40%</td>
<td>31.25%</td>
</tr>
<tr>
<td>Expected Growth</td>
<td>10.66%</td>
<td>2.00%</td>
</tr>
<tr>
<td>Beta</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td>Cost of equity (Riskfree rate= 2%; Risk premium=4%)</td>
<td>6.40%</td>
<td>6.00%</td>
</tr>
</tbody>
</table>

In high growth, the high equity reinvestment rate and high return on equity combine to generate an expected growth rate of 10.66% a year. In stable growth, we reduce the return on equity for Toyota to the cost of equity, assuming that it will be difficult to sustain excess returns for perpetuity in this business. Note also that the stable growth rate is low, reflecting the fact that the valuation is in Japanese yen (with the riskfree rate of 2% acting as the cap on growth). The beta for the firm is left unchanged at its existing level, since Toyota’s management has been fairly disciplined in staying focused on their core businesses.

Finally, let us consider Titan Cement. In Table 4.15, we report on the return on capital, reinvestment rate and expected growth for the firm in high growth (next five years) and stable growth period (beyond year 5).
Table 4.15: Inputs to Free Cash Flow to Firm Valuation: Titan Cement

<table>
<thead>
<tr>
<th></th>
<th>High Growth</th>
<th>Stable Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return on Capital</td>
<td>20.49%</td>
<td>6.57%</td>
</tr>
<tr>
<td>Reinvestment rate</td>
<td>34.14%</td>
<td>51.93%</td>
</tr>
<tr>
<td>Expected Growth</td>
<td>7.00%</td>
<td>3.41%</td>
</tr>
<tr>
<td>Beta</td>
<td>0.93</td>
<td>1.00</td>
</tr>
<tr>
<td>Cost of capital</td>
<td>6.78%</td>
<td>6.57%</td>
</tr>
</tbody>
</table>

The firm has a high return on capital currently but we will assume that the excess returns will disappear when the firm reaches its stable growth phase; the return on capital will drop to the cost of capital of 6.57%. Since the stable growth rate is 3.41%, the resulting reinvestment rate at Titan Cement will increase to 51.93% (3.41%/6.57%). We will also assume that the beta for Titan Cement will converge on the market average.

Assuming that excess returns continue in perpetuity, as we have for Goldman Sachs, is potentially troublesome. However, the competitive advantages that some firms have built up historically or will build up over the high growth phase will not disappear in an instant. The excess returns will fade over time, but moving them to or towards industry averages in stable growth seems like a reasonable compromise.

II. The Transition to Stable Growth

Once you have decided that a firm will be in stable growth at a point in time in the future, you have to consider how the firm will change as it approaches stable growth. There are three distinct scenarios. In the first, the firm will maintain its high growth rate for a period of time and then become a stable growth firm abruptly; this is a two-stage model. In the second, the firm will maintain its high growth rate for a period and then have a transition period where its characteristics change gradually towards stable growth levels; this is a three stage model. In the third, the firm’s characteristics change each year from the initial period to the stable growth period; this can be considered an n-stage model.

Which of these three scenarios gets chosen depends upon the firm being valued. Since the firm goes in one year from high growth to stable growth in the two-stage model, this model is more appropriate for firms with moderate growth rates, where the shift will not be too dramatic. For firms with very high growth rates in operating income,
a transition phase (in a 2 stage model) allows for a gradual adjustment not just of growth rates but also of risk characteristics, returns on capital and reinvestment rates towards stable growth levels. For very young firms or for firms with negative operating margins, allowing for changes in each year (in an n-stage model) is prudent.

Can you have high growth periods for firms that have expected growth rates that are less than or equal to the growth rate of the economy? The answer is yes, for some firms. This is because stable growth requires not just that the growth rate be less than the growth rate of the economy, but that the other inputs into the valuation are also appropriate for a stable growth firm. Consider, for instance, a firm whose operating income is growing at 4% a year but whose current return on capital is 20% and whose beta is 1.5. You would still need a transition period where the return on capital declined to more sustainable levels (say 12%) and the beta moved towards one. By the same token, you can have an extraordinary growth period, where the growth rate is less than the stable growth rate and then moves up to the stable growth rate. For instance, you could have a firm that is expected to see its earnings grow at 2% a year for the next 5 years (which would be the extraordinary growth period) and 4% thereafter.

**Estimation Approaches**

There are three approaches that are used to estimate cash flows in valuation. The simplest and most widely used is the expected value approach, where analysts estimate an expected cash flow for each time period, allowing implicitly or explicitly for good and bad scenarios. The second is a variant, where cash flows are estimated under different scenarios, ranging from best case to worst case, with values estimated under each scenario. The last and most information intensive is to estimate distributions for each input and to run simulations, where outcomes are drawn from each distribution and values estimated with each simulation.

**a. Expected Value**

In most valuations, analysts estimate expected cash flows in each time period from investing in a business or asset. The expected cash flow represents the single best estimate of the cash flow in a period, and computed correctly, should encapsulate the
likelihood both good and bad outcomes. This should therefore require a consideration of the probabilities of each scenario occurring and the cash flow under each scenario. In practice, however, such detailed analysis is almost never done, with analysts settling for an expected value for each variable (revenue growth, operating margin, tax rate etc.) that determines cash flows. In the process, we do expose ourselves to the following errors:

- Some analysts use “best case” or “conservative” estimates instead of true expected values for the cash flows. With the former, they will over estimate the value and with the latter, they will under estimate value.

- Even analysts who claim to use expected cash flows often fail to consider the full range of outcomes. For instance, many valuations of publicly traded firms seem to be based only upon cash flows if the firm continues as a going concern and do not factor in the very real possibility that the firm may cease operations. The resulting expected cash flows will be overstated, as will the values of firms with a significant likelihood of distress.

- Managers can alter the way they run businesses, after observing what occurs in the real world; an oil company will adjust exploration and production to reflect the price of oil in each period. Since analysts have to estimate the expected cash flows in all future periods, it is difficult to build in this learning into the model. This is why real options practitioners believe that discounted cash flow valuations, even done right, understate the values of businesses where this learning has significant value.

In summary, the expected cash flow approach is simple and surprisingly powerful (when used right), but it is also easily manipulated and misused.

b. Scenario Analysis

In scenario analysis, we estimate cash flows under different scenarios, ranging from optimistic to pessimistic, and report the resulting conclusions as a range of values rather than as a single estimate. In general, scenario analysis requires the following steps:

a. Identifying the Scenarios: The first and perhaps most critical step in scenario analysis is determining the scenarios. In its most naïve form, this can take the form of best case and worst-case scenarios, but in more sophisticated analysis, the scenarios can be built around either macro-economic or competitive factors. We can value an automotive
company under strong and weak economy scenarios and a bank under high and low interest rate scenarios.

b. **Estimating the cashflows and value under each scenario**: While the temptation at the first stage of the process is to create as many scenarios as we can, the second stage of the process acts as a natural check on the first stage. We have to estimate the expected cash flows under each scenario, and need to possess enough information to make these estimates. Presumably, the values will be very different under different scenarios; if they were not, the process would be pointless.

c. **Estimating the likelihood of each scenario**: Coupled with having different scenarios must be probabilities of each scenario occurring. Without this information, a decision maker has no way of weighing the different estimates of value. T

d. **Reporting the output**: The value of a business or asset will vary across scenarios and there are two choices when it comes to presenting the output from scenario analysis. The first is to compute an expected value across scenarios, estimated using the probabilities of scenarios occurring. The other is to report a range of values for an asset or business, with the lowest value (the highest value) across all scenarios representing the bottom (the top) of the range.

Scenario analysis allows us to see how the value of a business is affected by changes in the underlying fundamentals, but there is a danger in presenting valuations in a range rather than as an estimate. If the scenarios cover the spectrum, as is the case when we do best case and worst case scenarios, the resulting range of values will be so wide that it will be useless. After all, knowing that a stock is worth anywhere from $15 to $70 is not of much use in determining whether to buy it or sell it at a market price of $30. Taking an expected value across scenarios may be more useful but that expected value should be close (if not identical) to the single best estimate of value obtained using expected cash flows.

c. **Simulations**

Unlike scenario analysis, where we look at the values under discrete scenarios, simulations allow for more flexibility in how we deal with uncertainty. In its classic form, distributions of values are estimated for each parameter in the valuation (growth, market
share, operating margin, beta etc.), In each simulation, we draw one outcome from each
distribution to generate a unique set of cashflows and value. Across a large number of
simulations, we can derive a distribution for the value of a business or asset that will
reflect the underlying uncertainty we face in estimating the inputs to the valuation.

There have generally been two impediments to good simulations. The first is
informational: estimating distributions of values for each input into a valuation is difficult
to do. In other words, it is far easier to estimate an expected growth rate of 8% in
revenues for the next 5 years than it is to specify the distribution of expected growth rates
– the type of distribution, parameters of that distribution – for revenues. Simulations tend
to work best in cases where there is either historical data (different growth rates over
time) or cross sectional data (a range of growth rates across comparable companies) that
make it feasible to estimate distributions. The second is computational; until the advent of
personal computers, simulations tended to be too time and resource intensive for the
typical analyst. Both these constraints have eased in recent years and simulations have
become more feasible.

As simulations become more common, analysts have to confront three potential
problems. The first is that the distributions for inputs are often incorrectly specified both
in terms of type and parameters; it is garbage in, garbage out. The second is the
misconception that the cash flows from simulations are somehow risk adjusted because
they factor in the likelihood of poor outcomes. They are not, since expected cash flows
should factor in the likelihood of poor outcomes. We still need to use risk adjusted
discount rates to get to the value today. The third problem that both scenario analysis and
simulation share is that analysts often double count risk by first computing an expected
value using risk-adjusted discount rates and then considering the likelihood that the value
will be lower. For instance, a stock with an expected value of $ 40 is a good buy if the
stock price is $30, even if there is a 40% chance that the value is less than $ 30.

Conclusion

Forecasting future cash flows is key to valuing businesses. In making these
estimates, we can rely on the past history of the firm or on estimates supplied to us by
analysts or managers, but we do so at our own risk. Past growth rates are not reliable
forecasters of future growth and management/analyst estimates of growth are often biased. Tying expected growth to the investment policy of the firm – how much it reinvests and how well it chooses its investments – is not only prudent but preserves internal consistency in valuations.

When valuing equity, especially in high growth businesses, the bulk of the value will come from the terminal value. To keep terminal values bounded and reasonable, the growth rate used in perpetuity should be less than or equal to the growth rate of the economy and the reinvestment rate assumed has to be consistent with the growth rate.