EQUITY DISCOUNTED CASH FLOW MODELS

In the last three chapters, we considered the basic principles governing the estimation of discount rates and cash flows. In the process, we drew a distinction between valuing the equity in a business and valuing the entire business. In this chapter, we turn our attention to discounted cash flow models that value equity directly.

The first set of models examined take a strict view of equity cash flows and consider only dividends to be cashflows to equity. These dividend discount models represent the oldest variant of discounted cashflow models. While abandoned by many analysts as old-fashioned, we will argue that they are still useful in a wide range of circumstances. We then consider broader definitions of cash flows to equity, by first including stock buybacks in cashflows to equity and by then expanding out analysis to cover potential dividends or free cash flows to equity. We will close the chapter by examining why the different approaches may yield different values for equity per share.

I. Dividend Discount Models

The oldest discounted cash flow models in practice tend to be dividend discount models. While many analysts have turned away from dividend discount models on the premise that they yield estimates of value that are far too conservative, many of the fundamental principles that come through with dividend discount models apply when we look at other discounted cash flow models.

**Underlying Principle**

When investors buy stock in publicly traded companies, they generally expect to get two types of cashflows - dividends during the holding period and an expected price at the end of the holding period. Since this expected price is itself determined by future dividends, the value of a stock is the present value of dividends through infinity.

\[
\text{Value per share of stock} = \sum_{t=1}^{\infty} \frac{\text{E}(DPS_t)}{(1+k_e)^t}
\]
where,

\[ \text{DPS}_t = \text{Expected dividends per share in period } t \]

\[ k_e = \text{Cost of equity} \]

The rationale for the model lies in the present value rule - the value of any asset is the present value of expected future cash flows discounted at a rate appropriate to the riskiness of the cash flows.

There are two basic inputs to the model - expected dividends and the cost on equity. To obtain the expected dividends, we make assumptions about expected future growth rates in earnings and payout ratios. The required rate of return on a stock is determined by its riskiness, measured differently in different models - the market beta in the CAPM, and the factor betas in the arbitrage and multi-factor models. The model is flexible enough to allow for time-varying discount rates, where the time variation is caused by expected changes in interest rates or risk across time.

**Variations on the Dividend Discount Model**

Since projections of dollar dividends cannot be made through infinity, several versions of the dividend discount model have been developed based upon different assumptions about future growth. We will begin with the simplest – a model designed to value stock in a stable-growth firm that pays out what it can afford to in dividends and then look at how the model can be adapted to value companies in high growth that may be paying little or no dividends.

**I. The Gordon Growth Model**

The Gordon growth model relates the value of a stock to its expected dividends in the next time period, the cost of equity and the expected growth rate in dividends.

\[ \text{Value of Stock} = \frac{\text{DPS}_1}{k_e - g} \]

where,

\[ \text{DPS}_1 = \text{Expected Dividends next year} \]

\[ k_e = \text{Required rate of return for equity investors} \]

\[ g = \text{Growth rate in dividends forever} \]
While the Gordon growth model is a simple and powerful approach to valuing equity, its use is limited to firms that are growing at a stable rate. There are two insights worth keeping in mind when estimating a 'stable' growth rate. First, since the growth rate in the firm's dividends is expected to last forever, the firm's other measures of performance (including earnings) can also be expected to grow at the same rate. To see why, consider the consequences in the long term of a firm whose earnings grow 3% a year forever, while its dividends grow at 4%. Over time, the dividends will exceed earnings. On the other hand, if a firm's earnings grow at a faster rate than dividends in the long term, the payout ratio, in the long term, will converge towards zero, which is also not a steady state. Thus, though the model's requirement is for the expected growth rate in dividends, analysts should be able to substitute in the expected growth rate in earnings and get precisely the same result, if the firm is truly in steady state.

The second issue relates to what growth rate is reasonable as a 'stable' growth rate. As noted in Chapter 4, this growth rate has to be less than or equal to the growth rate of the economy in which the firm operates. This does not, however, imply that analysts will always agree about what this rate should be even if they agree that a firm is a stable growth firm for three reasons.

• Given the uncertainty associated with estimates of expected inflation and real growth in the economy, there can be differences in the benchmark growth rate used by different analysts, i.e., analysts with higher expectations of inflation in the long term may project a nominal growth rate in the economy that is higher.
• The growth rate of a company cannot be greater than that of the economy but it can be less. Firms can become smaller over time relative to the economy. Thus, even though the cap on the growth rate may be the nominal growth rate of the economy, analysts may use growth rates much lower than this value for individual companies.
• There is another instance in which an analyst may stray from a strict limit imposed on the 'stable growth rate'. If a firm is likely to maintain a few years of 'above-stable' growth rates, an approximate value for the firm can be obtained by adding a premium to the stable growth rate, to reflect the above-average growth in the initial years. Even in this case, the flexibility that the analyst has is limited. The sensitivity of the model to growth implies that the stable growth rate cannot be more than 0.25% to 0.5%
above the growth rate in the economy. If the deviation becomes larger, the analyst will be better served using a two-stage or a three-stage model to capture the 'super-normal' or 'above-average' growth and restricting the Gordon growth model to when the firm becomes truly stable.

The assumption that the growth rate in dividends has to be constant over time is a difficult assumption to meet, especially given the volatility of earnings. If a firm has an average growth rate that is close to a stable growth rate, the model can be used with little real effect on value. Thus, a cyclical firm that is expected to have year-to-year swings in growth rates, but has an average growth rate that is 3%, can be valued using the Gordon growth model, without a significant loss of generality. There are two reasons for this result. First, since dividends are smoothed even when earnings are volatile, they are less likely to be affected by year-to-year changes in earnings growth. Second, the mathematical effects of using an average growth rate rather than a constant growth rate are small.

In summary, the Gordon growth model is best suited for firms growing at a rate comparable to or lower than the growth rate in the economy and that have well established dividend payout policies that they intend to continue into the future. The dividend payout of the firm has to be consistent with the assumption of stability, since stable firms generally pay substantial dividends. In particular, this model will underestimate the value of the stock in firms that consistently pay out less than they can afford and accumulate cash in the process.

Illustration 5.1: Valuation with Stable Growth DDM: J.P. Morgan Chase

J.P. Morgan Chase has large stakes in both commercial and investment banking. In recent years, the firm has grown through acquisitions, some of which it has had problems digesting. In the most recent fiscal year, the firm paid $1.36 in dividends per share on earning per share of $2.08, resulting in a dividend payout ratio of 65.38%. If we assume that the firm will maintain its return on equity from the most recent year of 11.16% in perpetuity, we can estimate an expected growth rate in earnings per share:

\[
\text{Expected growth rate in EPS} = \text{Return on equity} \times \text{Retention Ratio}
\]

---

1 The average payout ratio for large stable firms in the United States is about 60%.
= 11.16% * (1-.6538) = 3.86%

Assuming a beta of 0.80 for the firm, based upon the betas of large commercial banks, with a riskfree rate of 4.5% and risk premium of 4% results in a cost of equity of 7.70%:

Cost of Equity = Riskfree Rate + Beta * Risk Premium = 4.5% + 0.8*4% = 7.7%

The value of equity per share can then be computed:

Value of equity per share at J.P. Morgan Chase = Expected Dividends next year/ (Cost of equity – Expected growth rate) = $1.36 (1.0386)/ (.077 - .0386) = $36.78

The stock was trading at $ 38 in early November of 2005, very close to our estimated value per share.

II. Two-stage Dividend Discount Model

The two-stage growth model allows for two stages of growth - an initial phase where the growth rate is not a stable growth rate and a subsequent steady state where the growth rate is stable and is expected to remain so for the long term. While, in most cases, the growth rate during the initial phase is higher than the stable growth rate, the model can be adapted to value companies that are expected to post low or even negative growth rates for a few years and then revert back to stable growth. In the dividend discount model, the value of equity can be written as:

Value of the Stock = PV of Dividends during extraordinary phase + PV of terminal price

\[
P_0 = \sum_{t=1}^{n} \frac{DPS_t}{(1+k_{e,fg})^t} + \frac{P_n}{(1+k_{e,fg})^n} \text{ where } P_n = \frac{DPS_{n+1}}{(k_{e,fg} - g_n)}
\]

where,

- \(DPS_t\) = Expected dividends per share in year \(t\)
- \(k_e\) = Cost of Equity (fg: High Growth period; st: Stable growth period)
- \(P_n\) = Price (terminal value) at the end of year \(n\)
- \(g\) = Extraordinary growth rate for the first \(n\) years
- \(g_n\) = Steady state growth rate forever after year \(n\)

In the case where the extraordinary growth rate (\(g\)) and payout ratio are unchanged for the first \(n\) years, this formula can be simplified.
where the inputs are as defined above.

The same constraint that applies to the growth rate for the Gordon Growth Rate model, i.e., that the growth rate in the firm is less than or equal to the nominal growth rate in the economy, applies for the terminal growth rate \( g_n \) in this model as well. In addition, the payout ratio has to be consistent with the estimated growth rate. If the growth rate is expected to drop significantly after the initial growth phase, the payout ratio should be higher in the stable phase than in the growth phase. A stable firm can pay out more of its earnings in dividends than a growing firm. One way of estimating this new payout ratio is to use the fundamental growth model described in Chapter 4.

\[
\text{Expected Growth} = (1 - \text{Payout Ratio}) \times \text{Return on equity}
\]

Algebraic manipulation yields the following stable period payout ratio:

\[
\text{Stable Payout ratio} = 1 - \frac{\text{Stable growth rate}}{\text{Stable period return on equity}}
\]

Thus, a firm with a 5% growth rate and a return on equity of 15% will have a stable period payout ratio of 66.67%. The other characteristics of the firm in the stable period should be consistent with the assumption of stability. For instance, it is reasonable to assume that a high growth firm has a beta of 2.0, but unreasonable to assume that this beta will remain unchanged when the firm becomes stable. In fact, the rule of thumb that we developed in the last chapter – that stable period betas should be between 0.8 and 1.2 – is worth repeating here. Similarly, the return on equity, which can be high during the initial growth phase, should come down to levels commensurate with a stable firm in the stable growth phase. What is a reasonable stable period return on equity? The industry average return on equity and the firm’s own stable period cost of equity provide useful information to make this judgment.

Since the two-stage dividend discount model is based upon two clearly delineated growth stages, high growth and stable growth, it is best suited for firms which are in high growth and expect to maintain that growth rate for a specific time period, after which the sources of the high growth are expected to disappear. One scenario, for instance, where
this may apply is when a company has patent rights to a very profitable product for the next few years and is expected to enjoy super-normal growth during this period. Once the patent expires, it is expected to settle back into stable growth. Another scenario where it may be reasonable to make this assumption about growth is when a firm is in an industry that is enjoying super-normal growth, because there are significant barriers to entry (either legal or as a consequence of infra-structure requirements), which can be expected to keep new entrants out for several years.

Illustration 5.2: Valuing a firm with the two-stage dividend discount model: Goldman Sachs

Goldman Sachs is one of the leading investment banks in the world. Assuming that it can maintain its brand name edge for a few years, we value Goldman using a two-stage dividend discount model, with five years of high growth and stable growth thereafter.

- For the first five years, we will assume that Goldman Sachs will maintain its existing payout ratio of 9.07% and current return on equity of 18.49%. The resulting growth rate is computed below:

\[
\text{Expected growth rate in earnings per share} = \text{Return on equity} \times \text{Retention Ratio} \\
= 18.49\% \times (1 - 0.0907) = 16.82\%
\]

- Beyond year 5, we will assume that competitive pressures will bring the return on equity down to 12.00%. Assuming a growth rate of 4% yields a stable period payout ratio of 66/67%:

\[
\text{Stable period payout ratio} = 1 - g/\text{ROE} = 1 - 0.04/0.12 = 0.6667 \text{ or } 66.67\%
\]

- To compute the cost of equity, we will assume that Goldman Sachs will have a beta of 1.20 for the first 5 years of high growth and a beta of 1.00 beyond. With a riskfree rate of 4.50% and a risk premium of 4%, we can estimate the costs of equity in both time periods:

\[
\text{Cost of equity for first 5 years (high growth phase)} = 4.5\% + 1.2 \times (4\%) = 9.30\% \\
\text{Cost of equity in stable growth} = 4.5\% + 1.0 \times (4\%) = 8.5\%
\]
The first component of value is the present value of the expected dividends during the high growth period. Based upon the current earnings ($11.03), the expected growth rate (16.82%) and the expected dividend payout ratio (9.07%), the expected dividends can be computed for each year in the high growth period in Table 5.1.

**Table 5.1: Expected Dividends per share: Goldman Sachs**

<table>
<thead>
<tr>
<th>Year</th>
<th>EPS</th>
<th>DPS</th>
<th>Present Value @ 9.30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$12.88</td>
<td>$1.17</td>
<td>$1.07</td>
</tr>
<tr>
<td>2</td>
<td>$15.05</td>
<td>$1.36</td>
<td>$1.14</td>
</tr>
<tr>
<td>3</td>
<td>$17.58</td>
<td>$1.59</td>
<td>$1.22</td>
</tr>
<tr>
<td>4</td>
<td>$20.54</td>
<td>$1.86</td>
<td>$1.30</td>
</tr>
<tr>
<td>5</td>
<td>$23.99</td>
<td>$2.18</td>
<td>$1.39</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td>$6.12</td>
</tr>
</tbody>
</table>

The present value is computed using the cost of equity of 9.3% for the high growth period. The present value of the dividends can also be computed in short hand using the following computation (based upon current dividends per share of $1.00):

\[
PV \text{ of Dividends} = \frac{1.00(1.1682)\left(\frac{1 - (1.1682)^5}{0.093 - 1.1682}\right)}{(1.093)^5} = $6.12
\]

The price (terminal value) at the end of the high growth phase (end of year 5) can be estimated using the constant growth model.

\[
\text{Terminal price} = \frac{\text{Expected Dividends per share}_{n+1}}{k_{e,\text{st}} - g_n}
\]

Expected Earnings per share\(_6\) = $11.03 \times 1.1682^5 \times 1.04 = $24.96

Expected Dividends per share\(_6\) = \text{EPS}\(_6\) \times \text{Stable period payout ratio}

\[
= $24.96 \times 0.6667 = $16.64
\]

\[
\text{Terminal price} = \frac{\text{Dividends}_{6,n+1}}{k_{e,\text{st}} - g_n} = \frac{16.64}{0.085 - 0.04} = $369.78
\]

The terminal price has to be discounted back to today, using the high growth period cost of equity of 9.30% (and not at the stable growth period cost of equity of 8.5%). The reasoning is that investors have to live through the risk of the high growth period (and the concurrent cost of equity) to get to the terminal period. The present value of the terminal price, discounted back at the high growth period cost of equity, is:

\[
PV \text{ of Terminal Price} = \frac{369.78}{(1.093)^5} = $237.05
\]
The cumulated present value of dividends and the terminal price can then be calculated.

\[
P_0 = \frac{1.00(1.1682) \left( \frac{1 - (1.1682)^5}{(1.093)^5} \right)}{0.093 - 0.1682} = \frac{369.78}{(1.093)^5} = $6.12 + $237.05 = $243.17
\]

Goldman Sachs was trading at $128 at the time of this analysis in November 2005, making it significantly under valued.

Clearly, the market is less optimistic about Goldman’s future growth than we are. An interesting exercise in valuation is to estimate the growth rate that will yield the market price; this is called the implied growth rate. Figure 5.1 graphs the estimated value per share for Goldman Sachs as a function of the expected growth rate in earnings per share for the next 5 years:

*Figure 5.1: Value and Expected Growth: Goldman Sachs*

To arrive at the current market price of $128, we have to assume an expected growth rate of 2.6% for the next 5 years. We are holding all other inputs to the valuation including the growth rate after the fifth year and the costs of equity fixed in computing this number. The exercise can be repeated with any other input—return on equity, length of the growth period etc.
What does the difference between our assumptions about growth and the market’s implied growth rate tell us? One way to view the difference is as a margin for error: the actual growth rate in earnings per share can be substantially lower than our base case estimate of 16.82%, without hurting our assessment of the stock being under valued. The other is to consider it a potential clue that we may be missing key elements in the valuation. For instance, earnings at investment banks are notoriously volatile and 2004 happened to be a lucrative one for most of them. It is entirely possible that the market is considering the cyclicality in these earnings while valuing Goldman and we are being over optimistic in our assessment of good years to come.

III. The H Model for valuing Growth

The H model is a two-stage model for growth, but unlike the classical two-stage model, the growth rate in the initial growth phase is not constant but declines linearly over time to reach the stable growth rate in steady stage. This model was presented in Fuller and Hsia (1984) and is based upon the assumption that the earnings growth rate starts at a high initial rate \( g_a \) and declines linearly over the extraordinary growth period (which is assumed to last 2H periods) to a stable growth rate \( g_n \).\(^2\) It also assumes that the dividend payout and cost of equity are constant over time and are not affected by the shifting growth rates. Figure 5.2 graphs the expected growth over time in the H Model.

The value of expected dividends in the H Model can be written as:

$$P_0 = \frac{DPS_0 \times (1 + g_n)}{(k_e - g_n)} + \frac{DPS_0 \times H \times (g_a - g_n)}{(k_e - g_n)}$$

where,

- $P_0 =$ Value of the firm now per share,
- $DPS_t =$ DPS in year $t$
- $k_e =$ Cost of equity
- $g_a =$ Growth rate initially
- $g_n =$ Growth rate at end of $2H$ years, applies forever afterwards

This model avoids the problems associated with the growth rate dropping precipitously from the high growth to the stable growth phase, but it does so at a cost. First, the decline in the growth rate is expected to follow the strict structure laid out in the model -- it drops in linear increments each year based upon the initial growth rate, the stable growth rate and the length of the extraordinary growth period. While small deviations from this assumption do not affect the value significantly, large deviations can cause problems. Second, the assumption that the payout ratio is constant through both phases of growth exposes the analyst to an inconsistency -- as growth rates decline the payout ratio usually increases.
The allowance for a gradual decrease in growth rates over time may make this a useful model for firms which are growing rapidly right now, but where the growth is expected to decline gradually over time as the firms get larger and the differential advantage they have over their competitors declines. The assumption that the payout ratio is constant, however, makes this an inappropriate model to use for any firm that has low or no dividends currently. Thus, the model, by requiring a combination of high growth and high payout, may be quite limited in its applicability.

Illustration 5.3: Valuing with the H model: Barclays Bank

Barclays is an international bank with roots in the UK. It paid dividends per share of £0.240 on reported earnings per share of £0.512 in 2004. The firm’s earnings per share have grown at 8% over the prior 5 years but that growth rate is expected to decline linearly over the next 5 years to 3%, while the payout ratio remains unchanged. The beta for the stock is 0.9, the British pound riskfree rate is 4.2% and the market risk premium is 4%.

Cost of equity = 4.2% + 0.9*4% = 7.8%

The stock can be valued using the H model:

\[
\text{Value of stable growth} = \frac{(0.24)(1.03)}{0.078 - 0.03} = £5.15
\]

\[
\text{Value of extraordinary growth} = \frac{(0.24)(5/2)(0.08 - 0.03)}{0.078 - 0.03} = £0.63
\]

Value of stock = £5.15 + £0.63 = £5.78

The stock was trading at £5.84 in November 2005, making it again close to fairly valued.

IV. Three-stage Dividend Discount Model

The three-stage dividend discount model combines the features of the two-stage model and the H-model. It is the most general of the models because it does not impose any restrictions on the payout ratio and assumes an initial period of stable high growth, a second period of declining growth and a third period of stable low growth that lasts forever. Figure 5.3 graphs the expected growth over the three time periods.

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3 Proponents of the model would argue that using a steady state payout ratio for firms which pay little or no dividends is likely to cause only small errors in the valuation.
The value of the stock is then the present value of expected dividends during the high growth and the transitional periods and of the terminal price at the start of the final stable growth phase.

\[ P_0 = \sum_{i=1}^{n1} \frac{\text{EPS}_0 (1+g_a)^i \Pi_a}{(1+k_{e, hg})^i} + \sum_{i=n1+1}^{n2} \frac{\text{DPS}_i}{(1+k_e)^i} + \frac{\text{EPS}_{n2}(1+g_n)\Pi_n}{(k_{e, st} - g_n)(1+r)^n} \]

where,

- \( \text{EPS}_t \) = Earnings per share in year \( t \)
- \( \text{DPS}_t \) = Dividends per share in year \( t \)
- \( g_a \) = Growth rate in high growth phase (lasts \( n_1 \) periods)
- \( g_n \) = Growth rate in stable phase
- \( \Pi_a \) = Payout ratio in high growth phase
\[ \Pi_n = \text{Payout ratio in stable growth phase} \]

\[ k_e = \text{Cost of equity in high growth (hg), transition (t) and stable growth (st)} \]

This model's flexibility makes it a useful model for any firm, which in addition to changing growth over time is expected to change on other dimensions as well - in particular, payout policies and risk. It is best suited for firms which are growing at an extraordinary rate now and are expected to maintain this rate for an initial period, after which the differential advantage of the firm is expected to deplete leading to gradual declines in the growth rate to a stable growth rate. Practically speaking, this may be the more appropriate model to use for a firm whose earnings are growing at very high rates\(^4\), are expected to continue growing at those rates for an initial period, but are expected to start declining gradually towards a stable rate as the firm become larger and loses its competitive advantages.

**Illustration 5.4: Valuing with the Three-stage DDM model: Canara Bank**

Canara Bank is a mid-size bank in Southern India that is registering rapid growth as the overall banking market in India grows. Sheltered from competition from foreign banks, Canara Bank reported a return on equity of 23.22\% in 2004 and paid out dividends per share of Rs 5.50 that year (on reported earnings per share of Rs 33.27). We will assume that its protected position will allow the bank to maintain its current return on equity and retention ratio for the next 5 years, leading to an estimated expected growth rate in earnings per share of 19.38\%:

\[
\text{Payout Ratio} = \frac{\text{Dividend per share}}{\text{Earning per share}} = \frac{5.50}{33.27} = 16.53\%
\]

\[
\text{Expected Growth rate} = \text{Retention ratio} \times \text{ROE} = (1 - 0.1653) \times 23.22\% = 19.38\%
\]

The cost of equity for the high growth period is estimated using a beta of 1.10 for Canara Bank (based upon the betas of other Indian banks), the Indian rupee riskfree rate of 6\% and a market risk premium of 7\% (reflecting a mature market premium of 4\% and an additional country risk premium for India of 3\%).\(^5\)

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\(^4\) The definition of a 'very high' growth rate is largely subjective. As a rule of thumb, growth rates over 25\% would qualify as very high when the stable growth rate is 6-8\%.

\(^5\) The country risk premium for India is computed using the default spread for Indian bonds and relative equity market volatility; the approach was described in chapter 2. The default spread for India at the time of this valuation was 1.50\% and the standard deviation for Indian equity was approximately twice the standard deviation in the Indian government bond. The resulting country equity risk premium is 3\% (1.50\%*2).
Cost of equity in high growth = 6% + 1.10 (7%) = 13.70%

After year 5, we will assume that the beta will decline towards 1 in stable growth (which will occur after the 10th year) and that the risk premium for India will also drop to 5.50% (reflecting our assumptions that India will become a more stable economy).

Cost of equity in stable growth = 6% + 1.00 (5.50%) = 11.50%

We will assume that competition will pick up after year 5, pushing the return on equity down to the stable period cost of equity of 11.50% by the 10th year. The payout ratio in stable growth can then be estimated using the stable growth rate of 4%:

Stable period payout ratio = 1 - Expected Growth rate/ ROE = 1 - 4% / 11.50% = 65.22%

Table 5.2 summarizes the assumptions about payout ratios and expected growth rates and also shows the estimated earnings and dividends per share each year for the next 10 years:

<table>
<thead>
<tr>
<th>Year</th>
<th>EPS</th>
<th>Expected Growth Rate</th>
<th>Payout Ratio</th>
<th>DPS</th>
<th>Cost of Equity</th>
<th>Cumulated Cost of Equity</th>
<th>Present Value of DPS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Current</td>
<td>Rs 33.27</td>
<td>16.53%</td>
<td>Rs 5.50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Rs 39.72</td>
<td>19.38%</td>
<td>16.53%</td>
<td>Rs 6.57</td>
<td>13.70%</td>
<td>1.1370</td>
<td>Rs 5.77</td>
</tr>
<tr>
<td>2</td>
<td>Rs 47.41</td>
<td>19.38%</td>
<td>16.53%</td>
<td>Rs 7.84</td>
<td>13.70%</td>
<td>1.2928</td>
<td>Rs 6.06</td>
</tr>
<tr>
<td>3</td>
<td>Rs 56.60</td>
<td>19.38%</td>
<td>16.53%</td>
<td>Rs 9.36</td>
<td>13.70%</td>
<td>1.4699</td>
<td>Rs 6.37</td>
</tr>
<tr>
<td>4</td>
<td>Rs 67.57</td>
<td>19.38%</td>
<td>16.53%</td>
<td>Rs 11.17</td>
<td>13.70%</td>
<td>1.6713</td>
<td>Rs 6.68</td>
</tr>
<tr>
<td>5</td>
<td>Rs 80.66</td>
<td>19.38%</td>
<td>16.53%</td>
<td>Rs 13.34</td>
<td>13.70%</td>
<td>1.9002</td>
<td>Rs 7.02</td>
</tr>
<tr>
<td></td>
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<td>Present value of dividends in high growth phase = Rs 31.90</td>
</tr>
<tr>
<td>6</td>
<td>Rs 93.82</td>
<td>16.30%</td>
<td>26.27%</td>
<td>Rs 24.64</td>
<td>13.26%</td>
<td>2.1522</td>
<td>Rs 11.45</td>
</tr>
<tr>
<td>7</td>
<td>Rs 106.22</td>
<td>13.23%</td>
<td>36.01%</td>
<td>Rs 38.25</td>
<td>12.82%</td>
<td>2.4281</td>
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<tr>
<td>8</td>
<td>Rs 117.01</td>
<td>10.15%</td>
<td>45.74%</td>
<td>Rs 53.52</td>
<td>12.38%</td>
<td>2.7287</td>
<td>Rs 19.62</td>
</tr>
<tr>
<td>9</td>
<td>Rs 125.29</td>
<td>7.08%</td>
<td>55.48%</td>
<td>Rs 69.51</td>
<td>11.94%</td>
<td>3.0545</td>
<td>Rs 22.76</td>
</tr>
<tr>
<td>10</td>
<td>Rs 130.30</td>
<td>4.00%</td>
<td>65.22%</td>
<td>Rs 84.98</td>
<td>11.50%</td>
<td>3.4058</td>
<td>Rs 24.95</td>
</tr>
</tbody>
</table>

Present value of dividends in transition phase = Rs 94.53

During the transition phase, all of the inputs change in equal annual installments from the high growth period values to stable growth period values. Since the costs of equity change over time, the cumulated cost of equity is used to calculate the present value of dividends. To compute the cumulated cost of equity in year 8, for instance, we do the following:

Cumulated cost of equity in year 8 = \((1.137)^7(1.1326)(1.1282)(1.1238) = 2.7287\)
Dividing the dividend per share in year 8 by this value yields the present value for that year.

The terminal price at the end of year 10 can be calculated based upon the earnings per share in year 11, the stable growth rate of 4%, a cost of equity of 11.50% and the payout ratio of 65.22%

\[
\text{Terminal price} = \frac{\text{Rs 130.30} \times (1.04)^{(0.6522)}}{0.115 - 0.04} = \text{Rs 1178.41}
\]

To get the present value, we divide by the cumulated cost of equity in year 10 (from table 5.2):

Present value of terminal price = Rs 1178.41/ 3.4058 = Rs. 345.99

The components of value are as follows:

- Present Value of dividends in high growth phase: Rs 31.90
- Present Value of dividends in transition phase: Rs 94.53
- Present Value of terminal price at end of transition: Rs. 345.99

Value of Canara Bank stock: Rs. 472.42

Canara Bank trading at Rs 215 per share in November 2005, making it significantly under valued. Here, the biggest note of caution to an investor should center on the sustainability of the bank’s current high return on equity. If competition arrives sooner than expected the value of equity will drop drastically. For instance, the value of equity per share drops to Rs. 317 if the return on equity drops to 15% next year (instead of remaining at 23.22%).

**Applicability of the Dividend Discount Model**

While many analysts have abandoned the dividend discount model, arguing that its focus on dividends alone is too narrow, the model does have its proponents. In fact, many in the Ben Graham school of value investing swear by the dividend discount model and its soundness. In this section, we will begin by considering the advantages of the dividend discount model and then follow up by looking at its limitations. We will end the section by looking at scenarios where the dividend discount model is most applicable.
Strengths of the Model

The dividend discount model's primary attraction is its simplicity and its intuitive logic. After all, dividends represent the only cash flow from the firm that is tangible to investors. Estimates of free cash flows to equity and the firm remain estimates and conservative investors can reasonably argue that they cannot lay claim on these cash flows. Thus, Microsoft may have large free cash flows to equity but an investor in Microsoft cannot demand a share of Microsoft’s cash balance.

The second advantage of using the dividend discount model is that we need fewer assumptions to get to forecasted dividends than to forecasted free cashflows to either equity or debt. To get to the latter, we have to make assumptions about capital expenditures, depreciation and working capital. To get to the former, we can begin with dividends paid last year and estimate a growth rate in these dividends.

Finally, it can be argued that managers set their dividends at levels that they can sustain even with volatile earnings. Unlike cash flows that ebb and flow with a company’s earnings and reinvestments, dividends remain stable for most firms. Thus, valuations based upon dividends will be less volatile over time than cash flow based valuations.

Limitations of the Model

The dividend discount model’s strict adherence to dividends as cash flows does expose it to a serious problem. As we noted in the last chapter, many firms choose to hold back cash that they can pay out to stockholders. As a consequence, the free cash flows to equity at these firms exceed dividends and large cash balances build up. While stockholders may not have a direct claim on the cash balances, they do own a share of these cash balances and their equity values should reflect them. In the dividend discount model, we essentially abandon equity claims on cash balances and under value companies with large and increasing cash balances.

At the other end of the spectrum, there are also firms that pay far more in dividends than they have available in cash flows, often funding the difference with new debt or equity issues. With these firms, using the dividend discount model can generate
too optimistic an estimate of value because we are assuming that firms can continue to
draw on external funding to meet the dividend deficit in perpetuity.

Applicability

Notwithstanding its limitations, the dividend discount model can be useful in
three scenarios.

• It establishes a baseline or floor value for firms that have cash flows to equity that
  exceed dividends. For these firms, the dividend discount model will yield a
  conservative estimate of value, on the assumption that the cash not paid out by
  managers will be wasted in poor investments or acquisitions.

• It yields realistic estimates of value per share for firms that do pay out their free cash
  flow to equity as dividends, at least on average over time. There are firms, especially
  in mature businesses, with stable earnings, that try to calibrate their dividends to
  available cashflows. At least until very recently, regulated utility companies in the
  United States, such as phone and power, were good examples of such firms.

• In sectors where cash flow estimation is difficult or impossible, dividends are the only
  cash flows that can be estimated with any degree of precision. There are two reasons
  why all of the companies that we have valued using the dividend discount model in
  this chapter are financial service companies. The first is that estimating capital
  expenditures and working capital for a bank, an investment bank or an insurance
  company is difficult to do. The second is that retained earnings and book equity have
  real consequences for financial service companies since their regulatory capital ratios
  are computed on the basis of book value of equity.

In summary, then, the dividend discount model has far more applicability than its critics
concede. Even the conventional wisdom that the dividend discount model cannot be used
to value a stock that pays low or no dividends is wrong. If the dividend payout ratio is
adjusted to reflect changes in the expected growth rate, a reasonable value can be
obtained even for non-dividend paying firms. Thus, a high-growth firm, paying no

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6 This is true for any firm whose primary asset is human capital. Accounting conventions have generally
treated expenditure on human capital (training, recruiting etc.) as operating expenditures. Working capital
is meaningless for a bank, at least in its conventional form since current assets and liabilities comprise
much of what is on the balance sheet.
dividends currently, can still be valued based upon dividends that it is expected to pay out when the growth rate declines.

**Extensions of the Dividend Discount Model**

One reason for the fall of the dividend discount model from favor has been the increased used of stock buybacks as a way of returning cash to stockholders. A simple response to this trend is to expand the definition of dividends to include stock buybacks and to value stocks based on this composite number. In this section, we will consider the possibilities and limitations of this expanded dividend discount model and also examine whether the dividend discount model can be used to value entire markets or sectors.

*An Expanded Dividend Discount Model*

In recent years, firms in the United States have increasingly turned to stock buybacks as a way of returning cash to stockholders. Figure 5.4 presents the cumulative amounts paid out by firms in the form of dividends and stock buybacks from 1989 to 2002.

*Figure 5.4: Stock Buybacks and Dividends: Aggregate for US Firms - 1989-2002*
The trend towards stock buybacks is very strong, especially in the 1990s. By early 2000, more cash was being returned to stockholders in stock buybacks than in conventional dividends.

What are the implications for the dividend discount model? Focusing strictly on dividends paid as the only cash returned to stockholders exposes us to the risk that we might be missing significant cash returned to stockholders in the form of stock buybacks. The simplest way to incorporate stock buybacks into a dividend discount model is to add them on to the dividends and compute a modified payout ratio:

\[
\text{Modified dividend payout ratio} = \frac{\text{Dividends} + \text{Stock Buybacks}}{\text{Net Income}}
\]

While this adjustment is straightforward, the resulting ratio for any year can be skewed by the fact that stock buybacks, unlike dividends, are not smoothed out. In other words, a firm may buy back $3 billion in stock in one year and not buy back stock for the next 3 years. Consequently, a much better estimate of the modified payout ratio can be obtained by looking at the average value over a four or five year period. In addition, firms may sometimes buy back stock as a way of increasing financial leverage. If this is a concern, we could adjust for this by netting out new debt issued from the calculation above:

\[
\text{Modified dividend payout} = \frac{\text{Dividends} + \text{Stock Buybacks} - \text{Long Term Debt issues}}{\text{Net Income}}
\]

Adjusting the payout ratio to include stock buybacks will have ripple effects on the estimated growth and the terminal value. In particular, the modified growth rate in earnings per share can be written as:

\[
\text{Modified growth rate} = (1 - \text{Modified payout ratio}) \times \text{Return on equity}
\]

Even the return on equity can be affected by stock buybacks. Since the book value of equity is reduced by the market value of equity bought back, a firm that buys back stock can reduce its book equity (and increase its return on equity) dramatically. If we use this return on equity as a measure of the marginal return on equity (on new investments), we will overstate the value of a firm. Adding back stock buybacks in recent years to the book equity and re-estimating the return on equity can sometimes yield a more reasonable estimate of the return on equity on investments.
Illustration 5.5: Valuing with modified dividend discount model: Exxon Mobil

In November 2005, Exxon Mobil was the largest market cap company in the world. With the surge in cash flows generated by rising oil prices over the previous four years, Exxon had augmented dividends with stock buybacks each year. Table 5.3 summarizes the dividends and buybacks between 2001 and 2004.

Table 5.3: Dividends and Stock Buybacks: Exxon Mobil

<table>
<thead>
<tr>
<th></th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Income</td>
<td>15320</td>
<td>11460</td>
<td>21510</td>
<td>25330</td>
<td>73620</td>
</tr>
<tr>
<td>Dividends</td>
<td>6254</td>
<td>6217</td>
<td>6515</td>
<td>6896</td>
<td>25882</td>
</tr>
<tr>
<td>Buybacks</td>
<td>5721</td>
<td>4798</td>
<td>5881</td>
<td>9951</td>
<td>26351</td>
</tr>
<tr>
<td>Dividends+Buybacks</td>
<td>11975</td>
<td>11015</td>
<td>12396</td>
<td>16847</td>
<td>52233</td>
</tr>
<tr>
<td>Payout ratio</td>
<td>40.82%</td>
<td>54.25%</td>
<td>30.29%</td>
<td>27.22%</td>
<td>35.16%</td>
</tr>
<tr>
<td>Modified payout ratio</td>
<td>78.17%</td>
<td>96.12%</td>
<td>57.63%</td>
<td>66.51%</td>
<td>70.95%</td>
</tr>
</tbody>
</table>

Over the four-year period, the conventional payout ratio is only 35.16% but the modified payout ratio is 70.95%; the modified retention ratio is only 29.05%. We can estimate the expected growth in earnings for Exxon in the long term by taking the product of this modified retention ratio and the return on equity of 15% that Exxon reported in 2004:

Expected growth rate = (1- Modified payout ratio) ROE = (1-0.7095)(0.15) = 4.36%

To estimate the cost of equity, we will assume that Exxon has a beta of 0.80 and that the riskfree rate of 4.5% and a market risk premium of 4% apply:

Cost of equity = 4.50% + 0.80 (4%) = 7.70%

We can value Exxon Mobil, using a stable growth dividend discount model, but using the modified dividends per share:

Modified dividends per share = Earnings per share in 2004 * Modified payout ratio

\[ = \$ 5.00 \times 0.7095 = \$3.55 \]

Value of equity per share = Modified dividends per share (1+g)/(Cost of equity – g)

\[ = \$3.55 \times \frac{1.0436}{0.077 - 0.0436} = \$110.76 \]

At its prevailing market price of $ 60 a share (in November 2005), Exxon looks under valued.
Valuing entire markets or sectors

All our examples of the dividend discount model so far have involved individual companies, but there is no reason why we cannot apply the same model to value a sector or even the entire market. The market price of the stock would be replaced by the cumulative market value of all of the stocks in the sector or market. The expected dividends would be the cumulated dividends of all these stocks and could be expanded to include stock buybacks by all firms. The expected growth rate would be the growth rate in cumulated earnings and dividends of the index. There would be no need for a beta or betas, if you are looking at the entire market (which should have a beta of 1) and the sector beta can be used when valuing a sector to estimate a cost of equity. You could use a two-stage model, where the expected earnings growth rate is greater than the growth rate of the economy, but you should be cautious about setting the growth rate too high or the growth period too long when valuing the entire market because it will be difficult for cumulated earnings growth of all firms in an economy to run ahead of the growth rate in the economy for extended periods.

Consider a simple example. Assume that you have an index trading at 700 and that the average dividend yield of stocks in the index is 5%. Earnings and dividends can be expected to grow at 4% a year forever and the riskless rate is 5.4%. If you use a market risk premium of 4%, the value of the index can be estimated.

\[
\text{Cost of equity} = \text{Riskless rate} + \text{Risk premium} = 5.4\% + 4\% = 9.4\% \\
\text{Expected dividends next year} = (\text{Dividend yield } \times \text{Value of the index})(1 + \text{expected growth rate}) = (0.05 \times 700)(1.04) = 36.4 \\
\text{Value of the index} = \frac{\text{Expected dividends next year}}{\text{Cost of equity} - \text{Expected growth rate}} = \frac{36.4}{0.094 - 0.04} = 674
\]

At its existing level of 700, the market is slightly over priced.

Illustration 5.6: Valuing the S&P 500 using a dividend discount model: January 1, 2005

On January 1, 2005, the S&P 500 index was trading at 1211.92. The dividend yield on the index was only 1.81%, but including stock buybacks increases the modified dividend yield to 2.90%. Analysts were estimating that the earnings of the stocks in the index would increase 8.5% a year for the next 5 years. Beyond year 5, the expected growth rate in earnings and dividends is expected to be 4.22%, set equal to the treasury
bond rate today on the assumption that the treasury bond rate is a reasonably proxy for nominal long term growth in the economy. We will use a market risk premium of 4%, leading to a cost of equity of 8.22%:

Cost of equity = 4.22% + 4% = 8.22%

The expected dividends (and stock buybacks) on the index for the next 5 years can be estimated from the current dividends and expected growth of 8.50%.

Current modified dividends = 2.90% of 1211.92 = 35.148

<table>
<thead>
<tr>
<th>Year</th>
<th>Expected Dividends</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$38.13</td>
<td>$35.24</td>
</tr>
<tr>
<td>2</td>
<td>$41.37</td>
<td>$35.33</td>
</tr>
<tr>
<td>3</td>
<td>$44.89</td>
<td>$35.42</td>
</tr>
<tr>
<td>4</td>
<td>$48.71</td>
<td>$35.51</td>
</tr>
<tr>
<td>5</td>
<td>$52.85</td>
<td>$35.60</td>
</tr>
</tbody>
</table>

The present value is computed by discounting back the dividends at 8.22%. To estimate the terminal value, we estimate modified dividends in year 6 on the index:

Expected dividends in year 6 = $ 52.85 (1.0422) = $ 55.08

Terminal value of the index = \( \frac{\text{Expected Dividends}_6}{r - g} = \frac{\$55.08}{0.0822 - 0.0422} = \$1376.93 \)

Present value of Terminal value = \( \frac{\$1376.93}{1.0822^5} = \$927.63 \)

The value of the index can now be computed:

Value of index = Present value of dividends during high growth + Present value of terminal value = $35.24+35.33+35.42+$35.51+ $35.60+ $927.63 = $ 1104.73

Based upon this analysis, we would have concluded that the index was over valued by about 10% at 1211.92.

**II. FCFE (Potential Dividend) Discount Models**

The free cash flow to equity model does not represent a radical departure from the traditional dividend discount model. In fact, one way to describe a free cash flow to equity model is that it represents a model where we discount potential dividends rather than actual dividends. Consequently, the three versions of the FCFE valuation model presented in this section are simple variants on the dividend discount model, with one significant change - free cashflows to equity replace dividends in the models.
Underlying Principle

When we replace the dividends with FCFE to value equity, we are doing more than substituting one cash flow for another. We are implicitly assuming that the FCFE will be paid out to stockholders. There are two consequences.

1. There will be no future cash build-up in the firm, since the cash that is available after debt payments and reinvestment needs is paid out to stockholders each period.
2. The expected growth in FCFE will include growth in income from operating assets and not growth in income from increases in marketable securities. This follows directly from the last point.

How does discounting free cashflows to equity compare with the modified dividend discount model, where stock buybacks are added back to dividends and discounted? You can consider stock buybacks to be the return of excess cash accumulated largely as a consequence of not paying out their FCFE as dividends. Thus, FCFE represent a smoothed out measure of what companies can return to their stockholders over time in the form of dividends and stock buybacks.

The FCFE model treats the stockholder in a publicly traded firm as the equivalent of the owner in a private business. The latter can lay claim on all cash flows left over in the business after taxes, debt payments and reinvestment needs have been met. Since the free cash flow to equity measures the same for a publicly traded firm, we are assuming that stockholders are entitled to these cash flows, even if managers do not choose to pay them out. In essence, the FCFE model, when used in a publicly traded firm, implicitly assumes that there is a strong corporate governance system in place. Even if stockholders cannot force managers to return free cash flows to equity as dividends, they can put pressure on managers to ensure that the cash that does not get paid out is not wasted.

Inputs to the FCFE Model

Free cash flows to equity, like dividends, are cash flows to equity investors and we could use the same approach that we used to estimate the fundamental growth rate in dividends per share.

Expected Growth rate = Retention Ratio * Return on Equity
The use of the retention ratio in this equation implies that whatever is not paid out as dividends is reinvested back into the firm. There is a strong argument to be made, though, that this is not consistent with the assumption that free cash flows to equity are paid out to stockholders which underlies FCFE models. It is far more consistent to replace the retention ratio with the equity reinvestment rate, which measures the percent of net income that is invested back into the firm.

\[
\text{Equity Reinvestment Rate} = 1 - \frac{\text{Net Cap Ex} + \text{Change in Working Capital} - (\text{New Debt Issues} - \text{Repayments})}{\text{Net Income}}
\]

The return on equity may also have to be modified to reflect the fact that the conventional measure of the return includes interest income from cash and marketable securities in the numerator and the book value of equity also includes the value of the cash and marketable securities. In the FCFE model, there is no excess cash left in the firm and the return on equity should measure the return on non-cash investments. You could construct a modified version of the return on equity that measures the non-cash aspects.

\[
\text{Non-cash ROE} = \frac{\text{Net Income} - \text{After tax income from cash and marketable securities}}{\text{Book Value of Equity} - \text{Cash and Marketable Securities}}
\]

The product of the equity reinvestment rate and the modified ROE will yield the expected growth rate in FCFE.

Expected Growth in FCFE = Equity Reinvestment Rate * Non-cash ROE

This growth rate can then be applied to the non-cash net income to value the equity in the operating assets. Adding cash and marketable securities to this number will yield the total value of equity in the company.

**Variations on FCFE Models**

As with the dividend discount model, there are variations on the free cashflow to equity model, revolving around assumptions about future growth and reinvestment needs. In this section, we will examine versions of the FCFE model that parallel our earlier discussion of the dividend discount model.

1. **The constant growth FCFE model**

   The constant growth FCFE model is designed to value firms that are growing at a stable rate and are hence in steady state. The value of equity, under the constant growth
model, is a function of the expected FCFE in the next period, the stable growth rate and the required rate of return.

\[ P_0 = \frac{FCFE_1}{k_e - g_n} \]

where,

- \( P_0 \) = Value of equity today
- \( FCFE_1 \) = Expected FCFE next year
- \( k_e \) = Cost of equity of the firm
- \( g_n \) = Growth rate in FCFE for the firm forever

The model is very similar to the Gordon growth model in its underlying assumptions and works under some of the same constraints. The growth rate used in the model has to be less than or equal to the expected nominal growth rate in the economy in which the firm operates. The assumption that a firm is in steady state also implies that it possesses other characteristics shared by stable firms. This would mean, for instance, that capital expenditures, relative to depreciation, are not disproportionately large and the firm is of 'average' risk. (If the capital asset pricing model is used, the beta of the equity should not significantly different from one.) To estimate the reinvestment for a stable growth firm, you can use one of two approaches.

- You can use the typical reinvestment rates for firms in the industry to which the firm belongs. A simple way to do this is to use the average capital expenditure to depreciation ratio for the industry (or better still, just stable firms in the industry) to estimate a normalized capital expenditure for the firm.
- Alternatively, you can use the relationship between growth and fundamentals developed in Chapter 4 to estimate the required reinvestment. The expected growth in net income can be written as:

\[ \text{Expected growth rate in net income} = \text{Equity Reinvestment Rate} \times \text{Return on equity} \]

This allows us to estimate the equity reinvestment rate:

\[ \text{Equity reinvestment rate} = \frac{\text{Expected growth rate}}{\text{Return on Equity}} \]

To illustrate, a firm with a stable growth rate of 4% and a return on equity of 12% would need to reinvest about a third of its net income back into net capital expenditures and
working capital needs. Put another way, the free cash flows to equity should be two thirds of net income.

This model, like the Gordon growth model, is best suited for firms growing at a rate comparable to or lower than the nominal growth in the economy. It is, however, the better model to use for stable firms that pay out dividends that are unsustainably high (because they exceed FCFE by a significant amount) or are significantly lower than the FCFE. Note, though, that if the firm is stable and pays outs its FCFE as dividend, the value obtained from this model will be the same as the one obtained from the Gordon growth model.

**Illustration 5.7: FCFE Stable Growth Model: Exxon Mobil**

Earlier in this chapter, we valued Exxon Mobil using a modified dividend discount model and found it to be significantly under valued at its current price of $60 a share. In this illustration, we will value Exxon Mobil using a stable growth FCFE model instead, with the following assumptions:

- To estimate Exxon’s cost of equity, we will continue to use the same parameters we used in the dividend discount model: a beta of 0.80, a riskfree rate of 4.5% and a market risk premium of 4%, resulting in a cost of equity of 7.70%.
  \[
  \text{Cost of equity} = 4.5\% + 0.80 (4\%) = 7.70\%
  \]

- High and rising oil prices have clearly pushed up Exxon’s income in 2004 but it is unlikely that oil prices will continue to rise forever at this pace. Rather than use the net income from 2004 of $25.322 billion as our measure of earnings, we will use the average net income of $18.405 billion over the last 5 years as a measure of normalized net income. Netting out the interest income from cash from these earnings yields the non-cash net income value for the base year.
  \[
  \text{Non-cash Net Income} = \text{Net Income} – \text{Interest Income from Cash}
  \]
  \[
  = 18,405 – 321 = $18,086 \text{ million}
  \]

- Based upon the normalized net income of $18.086 billion and the non-cash book value of equity at the end of 2003, we estimated a return on equity of 21.88%.
  \[
  \text{Non-cash ROE} = \frac{\text{Non-cash Net Income}_{2004}}{\text{(Book value of equity – Cash)}_{2003}}
  \]
  \[
  = 18086/ (93297 – 10626) = 21.88\%
  \]
To estimate the reinvestment rate, we looked at net capital expenditures and working capital investments over the last 5 years and estimated a normalized equity reinvestment rate of 16.98%. The expected growth rate in perpetuity can then be computed to be 3.71%:

\[ \text{Expected growth rate in net income} = \text{Return on equity} \times \text{Equity Reinvestment Rate} = 21.88\% \times 0.1698 = 0.0371 \]

The value of Exxon Mobil equity can then be estimated as follows:

Value of equity in operating assets = Non-cash Net Income \( (1 - \text{Reinvestment Rate}) \) \( (1 + g)/ (\text{Cost of equity} - g) = 18086 (1 - 0.1698) (1.0371)/ (0.077 - 0.0371) = 390.69 \text{ billion} \)

Adding the value of cash and marketable securities ($18.5 billion) to this number and dividing by the number of shares yields the value of equity per share:

Value of equity per share = \( (390.69 + 18.5)/ 6.2224 = 65.77 \)

Based upon this model, Exxon is only slightly under valued at $60 a share. There are two reasons this valuation is more realistic than the modified dividend discount model valuation. First, the net income is normalized and allows for the cycles that are usually seen in commodity prices. Second, the reinvestment is measured directly in this valuation by looking at capital expenditures and working capital investments rather than indirectly through a retention ratio.

II. The Two-stage FCFE Model

The two-stage FCFE model is designed to value a firm that is expected to grow much faster than a mature firm in the initial period and at a stable rate after that. In this model, the value of any stock is the present value of the FCFE per year for the extraordinary growth period plus the present value of the terminal price at the end of the period.

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7 We computed the average of the net capital expenditures each year for the last 5 years and divided this number by the average operating income over the last 5 years. The resulting ratio of 11.83% was then multiplied by the current year’s operating income of $35.872 billion to arrive at the normalized net capital expenditure for the current year of $4,243 million. To estimate the normalized non-cash working capital change, we first computed non-cash working capital as a percent of revenues for the last 5 years (0.66%) and multiplied this value by the change in revenues over the last year ($50.79 billion) to arrive at the non-cash working capital change of $336 million. Finally, the normalized change in debt of $333 million was estimated using the current book value debt to capital ratio (7.27%) of the total normalized reinvestment (4,243+336). The resulting normalized equity reinvestment is $4,246 million (4,243+336-333). Dividing by the non-cash net income in 2004 of $25,011 million yields the equity reinvestment rate of 16.98%.
Value of equity = \sum \frac{FCFE_t}{(1 + k_e)^t} + \frac{P_n}{(1 + k_e)}

where,

FCFE_t = Free Cashflow to Equity in year t
P_n = Value of equity at the end of the extraordinary growth period
k_e = Cost of equity in high growth (hg) and stable growth (st) periods

The terminal value for equity is generally calculated using the stable growth rate model,

P_n = \frac{FCFE_{n+1}}{r - g_n}

where g_n = Growth rate after the terminal year forever.

The same caveats that apply to the growth rate for the stable growth rate model, described in the previous section, apply here as well. In addition, the assumptions made to derive the free cashflow to equity, after the terminal year, have to be consistent with the assumption of stability. For instance, while capital spending may be much greater than depreciation in the initial high growth phase, the difference should narrow as the firm enters its stable growth phase. We can use the two approaches described for the stable growth model – industry average capital expenditure requirements or the fundamental growth equation (equity reinvestment rate = g/ROE) to make this estimate. The beta and debt ratio may also need to be adjusted in stable growth to reflect the fact that stable growth firms tend to have average risk (betas closer to one) and use more debt than high growth firms.

This model makes the same assumptions about growth as the two-stage dividend discount model, i.e., that growth will be high and constant in the initial period and drop abruptly to stable growth after that. It is different because of its emphasis on FCFE rather than dividends. Consequently, it provides much better results than the dividend discount model when valuing firms which either have dividends which are unsustainable (because they are higher than FCFE) or which pay less in dividends than they can afford to (i.e., dividends are less than FCFE).

Illustration 5.9: Two-Stage FCFE Model: Toyota

Toyota Motors is one of the largest automobile companies in the world. In 2005, it was also the most profitable with its new hybrids capturing market share from the
SUVs and minivans made by U.S. auto manufacturers. To value the company, we made the following assumptions:

- Toyota reported net income of 1,171 billion yen in 2004, of which 29.68 billion yen reflected interest income from cash holdings. Based upon the book value of equity and cash holdings at the beginning of 2004, we computed a non-cash return on equity of 16.55%.

\[
\text{Non-cash ROE} = \frac{\text{Non-cash Net Income}_{2004}}{\text{Book value of equity} - \text{Cash}}_{2003} = \frac{1171.00 - 29.68}{8625 - 1730} = 16.55\%
\]

- In 2004, Toyota reported capital expenditures of 1,923 billion yen, depreciation of 998 billion yen and a decrease in non-cash working capital of 50 billion yen. The firm increased its total debt by 140 billion yen during the year. The resulting equity reinvestment rate is 64.40%.

\[
\text{Equity Reinvestment Rate} = \frac{\text{Cap Ex} - \text{Depreciation} + \text{Chg in WC} - \text{Net Debt CF}}{\text{Non-cash Net Income}} = \frac{1923 - 998 - 50 - 140}{1171 - 29.68} = 64.40\%
\]

- We will assume that Toyota will be able to maintain its current non-cash return on equity and equity reinvestment rate for the next 5 years, resulting in an expected growth rate in net income of 10.66%:

\[
\text{Expected growth rate in Net Income} = \text{Non-cash ROE} * \text{Equity Reinvestment Rate} = .1655 *.644 = .1066 \text{ or 10.66}\%
\]

- To estimate the cost of equity, we will assume that Toyota’s beta will be 1.10 in perpetuity. To estimate the market risk premium, we break down Toyota’s sales by region of the world (using 2005 data) and estimate a composite risk premium of 4.69%.

<table>
<thead>
<tr>
<th>Region</th>
<th>Units sold</th>
<th>% of Sales</th>
<th>Risk premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan</td>
<td>2381</td>
<td>32.14%</td>
<td>4%</td>
</tr>
<tr>
<td>North America</td>
<td>2271</td>
<td>30.66%</td>
<td>4%</td>
</tr>
<tr>
<td>Europe</td>
<td>979</td>
<td>13.22%</td>
<td>4%</td>
</tr>
<tr>
<td>Asia</td>
<td>834</td>
<td>11.26%</td>
<td>7%</td>
</tr>
<tr>
<td>Central and South America</td>
<td>185</td>
<td>2.50%</td>
<td>10%</td>
</tr>
<tr>
<td>Oceania</td>
<td>239</td>
<td>3.23%</td>
<td>6%</td>
</tr>
<tr>
<td>Others</td>
<td>519</td>
<td>7.01%</td>
<td>6%</td>
</tr>
<tr>
<td>Total</td>
<td>7408</td>
<td></td>
<td>4.69%</td>
</tr>
</tbody>
</table>

With a riskfree rate of 2% (in yen) the cost of equity for Toyota is 7.16%:

\[
\text{Cost of equity} = \text{Riskfree Rate} + \text{Beta (Risk Premium)} = 2\% + 1.1 (4.69\%) = 7.16\%
\]
- Beyond the fifth year, we will assume that the expected growth rate in net income will drop to 2% (set equal to the riskfree rate in yen) and that the return on equity will drop to the stable period cost of equity of 7.16%. The resulting equity reinvestment rate is 27.93%.

Stable period equity reinvestment rate = Expected growth/ Return on Equity = 2%/7.16% = 27.93%

In table 5.4, we compute the free cash flows to equity each year for the next 5 years assuming earnings growth of 10.66% and an equity reinvestment rate of 64.40%. We also calculate the present value of the cash flows using the cost of equity of 7.16% as the discount rate:

Table 5.4: Estimated Free Cash Flows to Equity: Toyota (in billions of yen)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Growth Rate</td>
<td>10.66%</td>
<td>10.66%</td>
<td>10.66%</td>
<td>10.66%</td>
<td>10.66%</td>
</tr>
<tr>
<td>Net Income</td>
<td>1,262.98</td>
<td>1,397.62</td>
<td>1,546.60</td>
<td>1,711.47</td>
<td>1,893.91</td>
</tr>
<tr>
<td>Equity Reinvestment Rate</td>
<td>64.40%</td>
<td>64.40%</td>
<td>64.40%</td>
<td>64.40%</td>
<td>64.40%</td>
</tr>
<tr>
<td>FCFE</td>
<td>449.63</td>
<td>497.56</td>
<td>550.60</td>
<td>609.30</td>
<td>674.25</td>
</tr>
<tr>
<td>Cost of Equity</td>
<td>7.16%</td>
<td>7.16%</td>
<td>7.16%</td>
<td>7.16%</td>
<td>7.16%</td>
</tr>
<tr>
<td>Cumulative Cost of Equity</td>
<td>107.16%</td>
<td>114.84%</td>
<td>123.06%</td>
<td>131.87%</td>
<td>141.32%</td>
</tr>
<tr>
<td>Present Value</td>
<td>419.58</td>
<td>433.28</td>
<td>447.43</td>
<td>462.04</td>
<td>477.12</td>
</tr>
</tbody>
</table>

The sum of the present value of free cashflows to equity over the high growth period is 2239.49 billion yen. To estimate the terminal value, we first estimate the free cash flows to equity in year 6.

Expected Net Income in year 6 = Net Income₅*(1 + g) = 1893.91*(1.02) = 1931.79

Equity Reinvestment in year 6 = Net Income₆*Stable Equity reinvestment rate
= 1931.79 * 0.2793 = 539.50

Expected FCFE in year 6= EPS₆-Equity Reinvestment₆
= 1931.79 – 539.50 = 1392.29

Terminal value of equity = FCFE₁₁/(Cost of equity₁₁-g)
= \frac{1392.29}{0.0716-0.02} = 26,974

Present value of terminal value of equity = 26,974/1.0716⁵ = 19088.21
The value of the equity in the operating assets can be obtained by adding the present value of the free cash flows to equity in the high growth period to the present value of the terminal value of equity. Adding cash and marketable securities to this value and dividing by the number of shares yields the value of equity per share:

\[
\text{Value of equity in operating assets} = 2239 + 19088 = 21,327 \text{ billion Yen} \\
+ \text{Cash and Marketable Securities} = 1,484 \text{ billion Yen} \\
= \text{Value of Equity} = 22,811 \text{ billion Yen} \\
/ \text{Number of Shares} = 3.61 \text{ billion} \\
= \text{Value of equity per share} = 6,319 \text{ Yen}
\]

The stock was trading 5600 Yen in November 2005, at the time of this valuation, making it slightly under valued.

**III. The E-Model - A Three Stage FCFE Model**

The E model is designed to value firms that are expected to go through three stages of growth - an initial phase of high growth rates, a transitional period where the growth rate declines and a steady state period where growth is stable. In this model, the value of a stock is the present value of expected free cash flow to equity over all three stages of growth:

\[
P_0 = \sum_{t=1}^{n_1} \frac{FCFE_t}{(1 + k_{e,kg})^t} + \sum_{t=n_1+1}^{n_2} \frac{FCFE_t}{(1 + k_{e,t})^t} + \frac{P_{n_2}}{(1 + k_{e,st})^n}
\]

where,

- \(P_0\) = Value of equity today
- \(FCFE_t\) = FCFE in year \(t\)
- \(k_e\) = Cost of equity
- \(P_{n_2}\) = Value of equity at the end of transitional period = \(\frac{FCFE_{n_2+1}}{r - g_n}\)
- \(n_1\) = End of initial high growth period
- \(n_2\) = End of transition period

Since the model assumes that the growth rate goes through three distinct phases - high growth, transitional growth and stable growth - it is important that assumptions about other variables are consistent with these assumptions about growth.
• It is reasonable to assume that as the firm goes from high growth to stable growth, the relationship between capital spending and depreciation will change. In the high growth phase, capital spending is likely to be much larger than depreciation. In the transitional phase, the difference is likely to narrow. Finally, the difference between capital spending and depreciation will be lower still in stable growth, reflecting the lower expected growth rate.

• As the growth characteristics of a firm change, so do its risk characteristics. In the context of the CAPM, as the growth rate declines, the beta of the firm can be expected to change. The tendency of betas to converge towards one in the long term has been confirmed by empirical observation of portfolios of firms with high betas. Over time, as these firms get larger and more diversified, the average betas of these portfolios move towards one.

Since the model allows for three stages of growth, and for a gradual decline from high to stable growth, it is the appropriate model to use to value firms with very high growth rates currently. The assumptions about growth are similar to the ones made by the three-stage dividend discount model, but the focus is on FCFE instead of dividends, making it more suited to value firms whose dividends are significantly higher or lower than the FCFE. In particular, it gives more realistic estimates of value for equity for high growth firms that are expected to have negative cash flows to equity in the near future. The discounted value of these negative cash flows, in effect, captures the effect of the new shares that will be issued to fund the growth during the period, and thus indirectly captures the dilution effect of value of equity per share today.

Illustration 5.10: Three Stage FCFE Model: Tsingtao Breweries (China)

Tsingtao Breweries produces and distributes beer and other alcoholic beverages in China and around the world under the Tsingtao brand name. As beer consumption in Asia grows, Tsingtao has high growth potential and we will value it using a three stage FCFE model, using the following assumptions:

- In 2004, Tsingtao reported net income 285.20 million CY, of which 25.50 million CY was income from cash and marketable securities. The resulting non-cash return on equity, based upon the book value of equity and cash at the start of 2004, is 8.06%:
  \[
  \text{Non-cash ROE} = \frac{\text{Non-cash Net Income}_{2004}}{\text{Book value of equity} - \text{Cash}}_{2003} = \frac{(285.20-25.50)}{(4071-850)} = 8.06\%
  \]
To compute the equity reinvestment rate, we looked at the average capital expenditure and working capital investments over the last five years, as well as new debt issues over the period:

Normalized net capital expenditures = CY 170.38 million

Normalized non-cash working capital change = CY 39.93 million

Normalized net debt cash flows = $ 92.17 million (Debt issues – Repayments)

Normalized equity reinvestment rate = (Cap Ex – Depreciation + Chg in WC – Net Debt CF)/ Non-cash Net Income = (170.38 + 39.93 – 92.17)/ (285.20-25.50) = 45.49%

We will assume that the return on equity will increase to 12% (from 8.06%) over the next 5 years, resulting in an expected growth rate of 13.74%

Expected growth rate = ROE * Equity Reinvestment Rate + \[1 + (ROE_{\text{target}} - \text{Current ROE})/\text{ROE}\]^{1/n-1} = .12 * . 4549 + (1 + (.12-.0806)/.0806)^{1/5-1}) = 13.74%

Note that the second term in the equation measures growth related to using existing assets more efficiently over the next 5 years. We are also assuming that new investments will generate returns on equity of 12% starting next year.

To estimate the cost of equity, we will use a beta of 0.80 for Tsingtao in perpetuity. In conjunction with a riskfree rate of 5.50% in Chinese Yuan and a risk premium of 5.60% (composed of a mature market premium of 4% and a country risk premium of 1.60% for China\(^8\)), the resulting cost of equity is 9.98%:

Cost of equity = 5.50% + 0.8 (5.60%) = 9.98%

Starting in year 6, Tsingtao will transition to a stable growth rate of 5.50% in year 10.

To compute the equity reinvestment rate in perpetuity we will assume that the return on equity will drop in stable growth to the cost of equity of 9.98%.

Stable Equity Reinvestment rate = g/ROE = .055/.098 = .5511 or 55.11%

To value Tsingtao, we will begin by projecting the free cash flows to equity during the high growth and transition phases, using an expected growth rate of 13.74% in net income and an equity reinvestment rate of 45.49% for the first 5 years. The following

---

\(^8\) The country risk premium for China was estimated using the default spread for China (1%) and the relative equity market volatility (std deviation of Chinese equities/ std deviation of Chinese bonds) for China of 1.60.

\(^9\) This may seem like a high growth rate for the stable phase but it is being estimated in Chinese Yuan. The higher inflation rate in that currency will make nominal growth higher.
5 years represent a transition period, where the growth drops in linear increments from 13.74% to 5.50% and the equity reinvestment rate moves from 45.49% to 55.11%. The resulting free cash flows to equity are shown in Table 5.5.

Table 5.5: Estimated FCFE for Tsingtao Breweries

<table>
<thead>
<tr>
<th>Year</th>
<th>Net Income</th>
<th>Expected Growth</th>
<th>Equity Reinvestment Rate</th>
<th>FCFE</th>
<th>Cost of equity</th>
<th>Cumulated cost of equity</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td>CY259.70</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>CY295.37</td>
<td>13.74%</td>
<td>45.49%</td>
<td>$161.00</td>
<td>9.98%</td>
<td>1.0998</td>
<td>CY146.39</td>
</tr>
<tr>
<td>2</td>
<td>CY335.95</td>
<td>13.74%</td>
<td>45.49%</td>
<td>$183.12</td>
<td>9.98%</td>
<td>1.2096</td>
<td>CY151.40</td>
</tr>
<tr>
<td>3</td>
<td>CY382.10</td>
<td>13.74%</td>
<td>45.49%</td>
<td>$208.28</td>
<td>9.98%</td>
<td>1.3303</td>
<td>CY156.57</td>
</tr>
<tr>
<td>4</td>
<td>CY434.59</td>
<td>13.74%</td>
<td>45.49%</td>
<td>$236.89</td>
<td>9.98%</td>
<td>1.4630</td>
<td>CY161.92</td>
</tr>
<tr>
<td>5</td>
<td>CY494.29</td>
<td>13.74%</td>
<td>45.49%</td>
<td>$269.43</td>
<td>9.98%</td>
<td>1.6090</td>
<td>CY167.45</td>
</tr>
<tr>
<td>6</td>
<td>CY554.04</td>
<td>12.09%</td>
<td>47.42%</td>
<td>$291.34</td>
<td>9.98%</td>
<td>1.7696</td>
<td>CY164.64</td>
</tr>
<tr>
<td>7</td>
<td>CY611.90</td>
<td>10.44%</td>
<td>49.34%</td>
<td>$309.99</td>
<td>9.98%</td>
<td>1.9462</td>
<td>CY159.28</td>
</tr>
<tr>
<td>8</td>
<td>CY665.71</td>
<td>8.79%</td>
<td>51.26%</td>
<td>$324.45</td>
<td>9.98%</td>
<td>2.1405</td>
<td>CY151.58</td>
</tr>
<tr>
<td>9</td>
<td>CY713.29</td>
<td>7.15%</td>
<td>53.19%</td>
<td>$333.92</td>
<td>9.98%</td>
<td>2.3541</td>
<td>CY141.85</td>
</tr>
<tr>
<td>10</td>
<td>CY752.53</td>
<td>5.50%</td>
<td>55.11%</td>
<td>$337.81</td>
<td>9.98%</td>
<td>2.5890</td>
<td>CY130.48</td>
</tr>
</tbody>
</table>

Present value of FCFE during high growth phase = CY1,531.53

To estimate the terminal value of equity, we used the net income in the year 11, reduce it by the equity reinvestment needs in that year and then assume a perpetual growth rate to get to a value.

Expected stable growth rate = 5.50%

Equity reinvestment rate in stable growth = 55.11%

Cost of equity in stable growth = 9.98%

Expected FCFE in year 11

\[ \text{FCFE}_{11} = (\text{Net Income}_{11})(1 - \text{Stable period equity reinvestment rate}) \]

\[ (752.53)(1.055)(1 - 0.551) = 356.39 \text{ million CY} \]

Terminal Value of equity in Tsingtao Breweries:

\[ \text{Terminal Value} = \frac{\text{FCFE}_{11}}{\text{Stable period cost of equity} \times \text{Stable growth rate}} \]

\[ = \frac{356.39}{0.0998 - 0.055} = 7,955 \text{ million CY} \]

To estimate the value of equity today, we sum up the present values of the FCFE over the high growth period and transition period and add to it the present value of the terminal value of equity.
Value of Equity in operating assets = $1531.53 + \frac{7955}{(1.0998)^{10}}$

= 4,604 million CY

Adding the current cash balance and dividing by the number of shares yields the value of equity per share:

Value of equity per share = \frac{\text{Value of equity in operating assets} + \text{Cash}}{\# \text{ Shares}}

= \frac{4604 + 1330}{1346.79} = 4.41 \text{ CY/share}

The stock was trading at 7.78 Yuan per share in November 2005, which would make it overvalued, based upon this valuation.

**Evaluating FCFE Models**

The FCFE model is a more general version of the dividend discount model and allows analysts more freedom in estimating cash flows. In a sense, it substitutes potential dividends for actual dividends paid and should yield more realistic estimates of value for firms where the two numbers deviate. In this section, we consider the strengths and weaknesses of FCFE models.

**Strengths of the Model**

The most significant advantage from using FCFE models is that we are no longer bound by the judgments of managers on dividend policy. We can substitute the free cash flows to equity – what could have been returned to stockholders – for what actually gets returned. Thus, we get more realistic estimates of value for firms that consistently pay out less or more than they could have paid out. With the former, the free cash flow to equity model will yield a value for equity that is higher than the dividend discount model value, whereas with the latter, it will generate a value that is lower.

The second advantage with FCFE models is that, unlike dividends, they are not constrained to be non-negative values. In fact, the free cash flows to equity can be negative, and usually are for growth companies with significant reinvestment needs. Firms that have negative free cash flows to equity can be expected to make new stock issues in the future. The expected dilution that will occur is already built into the value of equity through the negative free cash flows to equity.
One final aspect of the model bears repeating. In FCFE models, we are implicitly assuming that cash flows to equity will be withdrawn from the firm each year. Thus, there will be no cash buildup in the firm and we do not need to keep track of future cash balances. A common mistake in FCFE models is double counting, where analysts estimate the value of the equity by discounting FCFE to the firm and then also keep track of the cash buildup in the firm because the firm is paying out less than its FCFE as dividends.10

**Limitations of the Model**

While free cash flows to equity models relax the constraints on measuring cashflows to equity placed by dividend discount models, there is a cost. Analysts have to estimated net capital expenditures and non-cash working capital needs each year to get to cash flows. While this may be straight forward, analysts also have to estimate how much cash the firm will raise from new debt issues and how much they will use to repay old debt. This exercise is fairly straight forward when firms maintain stable debt ratios but becomes increasingly complicated as debt ratios are expected to change over time. In the former case, we can use the short cut for free cash flows to equity:

\[
\text{Free Cash Flow to Equity} = \text{Net Income} - (\text{Cap Ex} - \text{Depreciation}) (1 - \delta) - \text{Chg in non-cash WC} (1-\delta)
\]

In the latte case, we have to use the expanded version of the model:

\[
\text{Free Cash Flow to Equity} = \text{Net Income} - (\text{Cap Ex} - \text{Depreciation}) - \text{Chg in non-cash WC} + (\text{Debt repaid} - \text{New Debt issues})
\]

This calculation can become complicated for firms that are expected to change their debt ratios over time, since we have to compute new debt issues that the firm has to make to get their desired debt ratio.

---

10 Note that we would still add the current cash balance to the value of equity in the operating assets. What cannot be counted is the additional cash build up that will occur because the firm is paying out less in dividends than it has available in FCFE.
Applicability of FCFE Models

Clearly, free cash flows to equity models cannot be used when the inputs needed to compute free cash flows to equity – capital expenditures, depreciation, working capital and net debt cash flows – are difficult or impossible to estimate. As noted earlier in the discussion of dividend discount models, this is often the case with financial service companies and can sometimes be an issue when there is incomplete or unreliable financial information available on the company. If this occurs, falling back on the dividend discount model will yield more reliable estimates of value.

If free cashflows to equity can be estimated, there is no reason why we cannot use free cash flow to equity models to value all companies. However, the practical problems associated with estimating cash flows to equity when debt ratios are expected to change over time can make a difference in whether we use equity or firm valuation models. With firm valuation models, changes in the debt ratios are easier to incorporate into the valuation because they affect the discount rate (through the weights in the cost of capital calculation). As we will see in the next section, we should arrive at the same equity value using either approach, though there are implicit assumptions we make in each one that can cause deviations.

FCFE versus Dividend Discount Model Valuation

The FCFE model can be viewed as an alternative to the dividend discount model. Since the two approaches sometimes provide different estimates of value for equity, it is worth examining when they provide similar estimates of value, when they provide different estimates of value and what the difference tells us about the firm.

a. When they are similar

There are two conditions under which the value from using the FCFE in discounted cashflow valuation will be the same as the value obtained from using the dividend discount model. The first is the obvious one, where the dividends are equal to the FCFE. There are firms that maintain a policy of paying out excess cash as dividends either because they have pre-committed to doing so or because they have investors who expect this policy of them.
The second condition is more subtle, where the FCFE is greater than dividends, but the excess cash (FCFE - Dividends) is invested in fairly priced assets (i.e. assets that earn a fair rate of return and thus have zero net present value). For instance, investing in financial assets that are fairly priced should yield a net present value of zero. To get equivalent values from the two approaches, though, we have to keep track of accumulating cash in the dividend discount model and add it to the value of equity (as shown in illustration 5.11 at the end of this section).

b. When they are different

There are several cases where the two models will provide different estimates of value. First, when the FCFE is greater than the dividend and the excess cash either earns below-market interest rates or is invested in negative net present value assets, the value from the FCFE model will be greater than the value from the dividend discount model. There is reason to believe that this is not as unusual as it would seem at the outset. There are numerous case studies of firms, which having accumulated large cash balances by paying out low dividends relative to FCFE, have chosen to use this cash to finance unwise takeovers (where the price paid is greater than the value received from the takeover). Second, the payment of dividends less than FCFE lowers debt-equity ratios and may lead the firm to become under levered, causing a loss in value.

In the cases where dividends are greater than FCFE, the firm will have to issue either new stock or debt to pay these dividends or cut back on its investments, leading to at least one of three negative consequences for value. If the firm issues new equity to fund dividends, it will face substantial issuance costs that decrease value. If the firm borrows the money to pay the dividends, the firm may become over levered (relative to the optimal) leading to a loss in value. Finally, if paying too much in dividends leads to capital rationing constraints where good projects are rejected, there will be a loss of value (captured by the net present value of the rejected projects).

There is a third possibility and it reflects different assumptions about reinvestment and growth in the two models. If the same growth rate used in the dividend discount and FCFE models, the FCFE model will give a higher value than the dividend discount model whenever FCFE are higher than dividends and a lower value when dividends exceed FCFE. In reality, the growth rate in FCFE should be different from the growth rate in
dividends, because the free cash flow to equity is assumed to be paid out to stockholders. This will affect the equity reinvestment rate of the firm. In addition, the return on equity used in the FCFE model should reflect the return on equity on non-cash investments, whereas the return on equity used in the dividend discount model should be the overall return on equity. Table 5.6 summarizes the differences in assumptions between the two models.

Table 5.6: Differences between DDM and FCFE Model

<table>
<thead>
<tr>
<th></th>
<th>Dividend Discount Model</th>
<th>FCFE Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Implicit Assumption</td>
<td>Only dividends are paid. Remaining portion of earnings is invested back into the firm, some in operating assets and some in cash &amp; marketable securities.</td>
<td>The FCFE is paid out to stockholders. The remaining earnings are invested only in operating assets.</td>
</tr>
<tr>
<td>Expected Growth</td>
<td>Measures growth in income from both operating and cash assets. In terms of fundamentals, it is the product of the retention ratio and the return on equity</td>
<td>Measures growth only in income from operating assets. In terms of fundamentals, it is the product of the equity reinvestment rate and the non-cash return on equity.</td>
</tr>
<tr>
<td>Dealing with cash and marketable securities</td>
<td>The income from cash and marketable securities is built into earnings and ultimately into dividends. Therefore, cash and marketable securities do not need to be added in</td>
<td>You have two choices: 1. Build in income from cash and marketable securities into projections of income and estimate the value of equity. 2. Ignore income from cash and marketable securities, and add their value to equity value in model</td>
</tr>
</tbody>
</table>
In general, when firms pay out much less in dividends than they have available in FCFE, the expected growth rate and terminal value will be higher in the dividend discount model, but the year-to-year cash flows will be higher in the FCFE model.

3. What does it mean when they are different?

When the value using the FCFE model is different from the value using the dividend discount model, with consistent growth assumptions, there are two questions that need to be addressed - What does the difference between the two models tell us? Which of the two models is the appropriate one to use in evaluating the market price?

The more common occurrence is for the value from the FCFE model to exceed the value from the dividend discount model. The difference between the value from the FCFE model and the value using the dividend discount model can be considered one component of the value of controlling a firm - it measures the value of controlling dividend policy. In a hostile takeover, the bidder can expect to control the firm and change the dividend policy (to reflect FCFE), thus capturing the higher FCFE value.

As for which of the two values is the more appropriate one for use in evaluating the market price, the answer lies in the openness of the market for corporate control. If there is a sizable probability that a firm can be taken over or its management changed, the market price will reflect that likelihood and the appropriate benchmark to use is the value from the FCFE model. As changes in corporate control become more difficult, either because of a firm's size and/or legal or market restrictions on takeovers, the value from the dividend discount model will provide the appropriate benchmark for comparison.

Illustration 5.11: Equivalence (or not) of FCFE and DDM models

To illustrate the implicit assumptions that we need to make for the dividend discount and FCFE models to converge, let us consider a hypothetical company. Tivoli Enterprises paid out dividends of $30 million on net income of $100 million in the most recent financial year; revenues were $1,000 million for the year. During the same year, capital expenditures amounted to $75 million, depreciation was $50 million and non-cash working capital was 5% of revenues. In addition, new debt issues exceeded debt repayments by $10 million. Finally, let us assume that the firm had no cash on hand at the time of the valuation.
We will assume that this firm is of average risk and has beta of 1. With a riskfree rate of 5% and a risk premium of 4%, the cost of equity that we compute for Tivoli Enterprises is 9%:

Cost of equity = Riskfree Rate + Beta * Risk Premium = 5% + 4% = 9%

We will also assume that this cost of equity will hold forever.

To value this firm, we will assume that revenues, net income, dividends capital expenditures, depreciation and net debt cash flows will grow at 10% a year for the next 5 years. In addition, we will assume that non-cash working capital will remain at its existing proportion of revenues (5%). In table 5.7, we estimate the free cash flows to equity and dividends each year for the next 5 years:

Table 5.7: Expected FCFE and Dividends: High Growth Period

<table>
<thead>
<tr>
<th>Current</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues</td>
<td>$1000.00</td>
<td>$1100.00</td>
<td>$1210.00</td>
<td>$1331.00</td>
<td>$1464.10</td>
</tr>
<tr>
<td>Net Income</td>
<td>$100.00</td>
<td>$110.00</td>
<td>$121.00</td>
<td>$133.10</td>
<td>$146.41</td>
</tr>
<tr>
<td>- (CapEx-Depreciation)</td>
<td>$25.00</td>
<td>$27.50</td>
<td>$30.25</td>
<td>$33.28</td>
<td>$36.60</td>
</tr>
<tr>
<td>- Change in Working Capital</td>
<td>$5.00</td>
<td>$5.50</td>
<td>$6.05</td>
<td>$6.66</td>
<td>$7.32</td>
</tr>
<tr>
<td>+ Net Debt Cash flow</td>
<td>$10.00</td>
<td>$11.00</td>
<td>$12.10</td>
<td>$13.31</td>
<td>$14.64</td>
</tr>
<tr>
<td>Free Cashflow to Equity</td>
<td>$88.50</td>
<td>$97.35</td>
<td>$107.09</td>
<td>$117.79</td>
<td>$129.57</td>
</tr>
<tr>
<td>Dividends</td>
<td>$30.00</td>
<td>$33.00</td>
<td>$36.30</td>
<td>$39.93</td>
<td>$43.92</td>
</tr>
</tbody>
</table>

At the end of year 5, let us assume that the firm will be in stable growth, growing 4% a year in perpetuity and that the return on equity will be 12% in perpetuity as well. To estimate the terminal value of equity in the FCFE model, we first compute a stable period equity reinvestment rate:

Stable period equity reinvestment rate = \( g / \text{ROE} = 4\% / 12\% = 33.33\% \)

Value of equity at end of fifth year = \( \frac{\text{Net Income}_6 (1 - \text{Equity Reinvestment Rate})}{(\text{Cost of equity} - \text{Expected Growth Rate})} \)

= \( \frac{161.05 (1.04) (1 - .3333)}{(.09 -.04)} = \) \$ 2233.24 million

The computation of terminal value for equity in the dividend discount model mirrors this calculation, if the stable period payout ratio is estimated from the growth rate and return on equity:

Stable period payout ratio = \( 1 - g / \text{ROE} = 1 -.04/.12 = .6667 \) or 66.67%
Value of equity at end of fifth year = \frac{\text{Net Income} \times \text{(Payout Ratio)}}{\text{(Cost of equity} - \text{Expected Growth Rate)}}

= \frac{161.05 \times 1.04 \times 0.6667}{0.09 - 0.04} = $2233.24 million

While the terminal values of equity in the two models are the same, the value of equity that we derive today will be different if we focus just on dividends paid rather than the FCFE.

Value of equity\text{FCFE} = \frac{88.50}{(1.09)} + \frac{97.35}{(1.09)^2} + \frac{107.09}{(1.09)^3} + \frac{117.79}{(1.09)^4} + \frac{129.57}{(1.09)^5} + \frac{2233.24}{(1.09)^5} = $1864.93 million

Value of equity\text{DDM} = \frac{33.00}{(1.09)} + \frac{36.30}{(1.09)^2} + \frac{39.93}{(1.09)^3} + \frac{43.92}{(1.09)^4} + \frac{48.32}{(1.09)^5} + \frac{2233.24}{(1.09)^5} = $1,605.63 million

Since the firm pays out less in dividends than it has available in FCFE, the dividend discount model yields a lower value of equity. The flaw in this analysis, though, is that there will be cash building up in the firm in the dividend discount model. To measure that cash build-up, we will initially assume that whatever does not get paid out as dividends each year will be reinvested at the cost of equity of 9%. The resulting cash balance by the end of year 5 is shown in table 5.8:

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free Cashflow to Equity</td>
<td>$88.50</td>
<td>$97.35</td>
<td>$107.09</td>
<td>$117.79</td>
<td>$129.57</td>
</tr>
<tr>
<td>Dividends</td>
<td>$33.00</td>
<td>$36.30</td>
<td>$39.93</td>
<td>$43.92</td>
<td>$48.32</td>
</tr>
<tr>
<td>Cash held back (FCFE – Dividends)</td>
<td>$55.50</td>
<td>$61.05</td>
<td>$67.16</td>
<td>$73.87</td>
<td>$81.26</td>
</tr>
<tr>
<td>Cumulative Cash Build-up</td>
<td>$55.50</td>
<td>$121.55</td>
<td>$199.64</td>
<td>$291.48</td>
<td>$398.97</td>
</tr>
</tbody>
</table>

Note that the cumulative cash build up each year is obtained by adding the previous year’s cash balance, invested at 9%, to the cash held back in that year.

Cumulative cash build-up in year 2 = 55.50 \times (1.09) + 61.05 = $121.55 million

Cumulative cash build-up in year 3 = 121.55 \times (1.09) + 67.16 = $199.64 million

The value built up by the end of year 5 is $398.97 million and the present value can be computed by discounting back at 9% to today.

Present value of cumulated cash build up in year 5 = \frac{398.97 \text{ million}}{(1.09)^5} = $259.30 million

Adding this on to the value obtained in the dividend discount model gives us the composite value of equity for the firm:
Composite value of equity = DDM Value + PV of Cash Build up = 1605.63 + 259.30 = $1864.93 million

This is identical to the FCFE value. Note, though, the implicit assumptions that allowed the two values to converge:

1. The terminal values of equity in both models were computed using fundamentals – equity reinvestment rates in the FCFE model and payout ratios in the DDM. If analysts attach payout ratios or equity reinvestment rates that are not consistent with their growth and ROE assumptions in computing terminal values, the two models can yield very different values. (Using industry average payout ratios and equity reinvestment rates to compute terminal values, which is a common practice, will also have the same effect).

2. The cash not paid out as dividends is assumed to earn the cost of equity and thus is value neutral. In other words, the excess cash is invested in zero net present value investments.

The second assumption is a critical one. One concern that investors have with firms that build up cash balances is that the cash can be used to fund poor acquisitions. In other words, the cash can be invested in negative net present value investments. If, for instance, we assume in the example above that the cash build-up was invested to earn 7% (in risky investments with a cost of equity of 9%), table 5.9 summarizes the cash build up over time:

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free Cashflow to Equity</td>
<td>$88.50</td>
<td>$97.35</td>
<td>$107.09</td>
<td>$117.79</td>
<td>$129.57</td>
</tr>
<tr>
<td>Dividends</td>
<td>$33.00</td>
<td>$36.30</td>
<td>$39.93</td>
<td>$43.92</td>
<td>$48.32</td>
</tr>
<tr>
<td>Cash Build up (invested at 7%)</td>
<td>$55.50</td>
<td>$120.44</td>
<td>$196.02</td>
<td>$283.61</td>
<td>$384.72</td>
</tr>
</tbody>
</table>

Adding the present value of the cumulated cash build up at the end of the fifth year to the DDM value now yields a value for equity that is lower than the FCFE model:

Present value of cumulated cash build up in year 5 = \(\frac{$384.72 \text{ million}}{(1.09)^5}\) = $250.04 million

Value of equity = DDM Value + PV of Cash Build up = 1605.63 + 250.04 = $1855.68 million
The loss in value of $9.26 million relative to the FCFE model can be attributed to the firm’s negative net present value investments.

One way to think of the classic DDM model is to assume that cash is completely wasted. In this extreme scenario, the value of the cash build-up is effectively zero. That is why the dividend discount model can be viewed as a floor on the value.

**Per Share versus Aggregate Valuation**

In this chapter, some of the valuations that we did used per share values for earnings and cash flows and arrived at a per share estimate of value for equity. Other valuations used aggregate net income and cash flows and arrived at the aggregate value for equity. Why use one approach over the other and what are the pros and cons?

The per share approach tends to be a little simpler and information is usually more accessible. Most data services report earnings per share and analyst estimates of growth in earnings per share. There are two reasons, though, for sticking with aggregate valuation. The first is that it is easier to keep operating assets separate from cash, if we begin with net income rather than earnings per share, and break it down into net income from operating assets and cash income. The second is that the number of shares to use to compute per share values can be subject to debate when there are options, warrants and convertible bonds outstanding. These equity options issued by the firm can be converted into shares, thus altering the number of shares outstanding. Analysts do try to factor in these options by computing the partially diluted (where options in the money are counted as shares outstanding) or fully diluted (where all options are counted) per share values. However, options do not lend themselves easily to this characterization. A much more robust way of dealing with options is to value them as options and to subtract this value from the aggregate value of equity estimated for a firm to arrive at an equity value for common stock. Dividing this value by the actual number of shares outstanding should yield the correct value for equity per share. We will deal with this question much more extensively later in this book, when we look at employee stock options and their effects on value.
Conclusion

The primary difference between the dividend discount models and the free cashflow to equity models lies in the definition of cash flows - the dividend discount model uses a strict definition of cashflow to equity, i.e., the expected dividends on the stock, while the FCFE model uses an expansive definition of cashflow to equity as the residual cashflow after meeting all financial obligations and investment needs. When firms have dividends that are different from the FCFE, the values from the two models will be different. In valuing firms for takeovers or in valuing firms where there is a reasonable chance of changing corporate control, the value from the FCFE provides the better estimate of value.