



I. Estimating Discount Rates

DCF Valuation

Estimating Inputs: Discount Rates

- **Critical ingredient** in discounted cashflow valuation. Errors in estimating the discount rate or mismatching cashflows and discount rates can lead to serious errors in valuation.
- At an intuitive level, the discount rate used should be consistent with both the **riskiness** and the **type of cashflow** being discounted.
 - Equity versus Firm: If the cash flows being discounted are cash flows to equity, the appropriate discount rate is a cost of equity. If the cash flows are cash flows to the firm, the appropriate discount rate is the cost of capital.
 - Currency: The currency in which the cash flows are estimated should also be the currency in which the discount rate is estimated.
 - Nominal versus Real: If the cash flows being discounted are nominal cash flows (i.e., reflect expected inflation), the discount rate should be nominal

Cost of Equity

- The cost of equity should be higher for riskier investments and lower for safer investments
- While risk is usually defined in terms of the variance of actual returns around an expected return, risk and return models in finance assume that the risk that should be rewarded (and thus built into the discount rate) in valuation should be the risk perceived by the marginal investor in the investment
- Most risk and return models in finance also assume that the marginal investor is well diversified, and that the only risk that he or she perceives in an investment is risk that cannot be diversified away (I.e, market or non-diversifiable risk)

The Cost of Equity: Competing Models

<i>Model</i>	<i>Expected Return</i>	<i>Inputs Needed</i>
CAPM	$E(R) = R_f + \beta (R_m - R_f)$	Riskfree Rate Beta relative to market portfolio Market Risk Premium
APM	$E(R) = R_f + \sum_{j=1}^n \beta_j (R_j - R_f)$	Riskfree Rate; # of Factors; Betas relative to each factor Factor risk premiums
Multi factor	$E(R) = R_f + \sum_{j=1, \dots, N} \beta_j (R_j - R_f)$	Riskfree Rate; Macro factors Betas relative to macro factors Macro economic risk premiums
Proxy	$E(R) = a + \sum_{j=1 \dots N} b_j Y_j$	Proxies Regression coefficients

The CAPM: Cost of Equity

- Consider the standard approach to estimating cost of equity:

$$\text{Cost of Equity} = R_f + \text{Equity Beta} * (E(R_m) - R_f)$$

where,

R_f = Riskfree rate

$E(R_m)$ = Expected Return on the Market Index (Diversified Portfolio)

- In practice,
 - Short term government security rates are used as risk free rates
 - Historical risk premiums are used for the risk premium
 - Betas are estimated by regressing stock returns against market returns

Short term Governments are not riskfree

- On a riskfree asset, the actual return is equal to the expected return. Therefore, there is no variance around the expected return.
- For an investment to be riskfree, then, it has to have
 - No default risk
 - No reinvestment risk
- Thus, the riskfree rates in valuation will depend upon when the cash flow is expected to occur and will vary across time
- A simpler approach is to match the duration of the analysis (generally long term) to the duration of the riskfree rate (also long term)
- In emerging markets, there are two problems:
 - The government might not be viewed as riskfree (Brazil, Indonesia)
 - There might be no market-based long term government rate (China)

Estimating a Riskfree Rate

- Estimate a range for the riskfree rate in local terms:
 - Approach 1: Subtract default spread from local government bond rate:
Government bond rate in local currency terms - Default spread for Government in local currency
 - Approach 2: Use forward rates and the riskless rate in an index currency (say Euros or dollars) to estimate the riskless rate in the local currency.
- Do the analysis in real terms (rather than nominal terms) using a real riskfree rate, which can be obtained in one of two ways –
 - from an inflation-indexed government bond, if one exists
 - set equal, approximately, to the long term real growth rate of the economy in which the valuation is being done.
- Do the analysis in another more stable currency, say US dollars.

A Simple Test

- You are valuing Ambev, a Brazilian company, in U.S. dollars and are attempting to estimate a riskfree rate to use in the analysis. The riskfree rate that you should use is
 - ❑ The interest rate on a Brazilian Real denominated long term Government bond
 - ❑ The interest rate on a US \$ denominated Brazilian long term bond (C-Bond)
 - ❑ The interest rate on a US \$ denominated Brazilian Brady bond (which is partially backed by the US Government)
 - ❑ The interest rate on a US treasury bond

Everyone uses historical premiums, but..

- The historical premium is the premium that stocks have historically earned over riskless securities.
- Practitioners never seem to agree on the premium; it is sensitive to
 - How far back you go in history...
 - Whether you use T.bill rates or T.Bond rates
 - Whether you use geometric or arithmetic averages.
- For instance, looking at the US:

Historical Period	<i>Arithmetic average</i>		<i>Geometric Average</i>	
	T.Bills	T.Bonds	T.Bills	T.Bonds
1928-2001	8.09%	6.84%	6.21%	5.17%
1962-2001	5.89%	4.68%	4.74%	3.90%
1991-2001	10.62%	6.90%	9.44%	6.17%

If you choose to use historical premiums....

- Go back as far as you can. A risk premium comes with a standard error. Given the annual standard deviation in stock prices is about 25%, the standard error in a historical premium estimated over 25 years is roughly:

$$\text{Standard Error in Premium} = 25\% / \sqrt{25} = 25\% / 5 = 5\%$$

- Be consistent in your use of the riskfree rate. Since we argued for long term bond rates, the premium should be the one over T.Bonds
- Use the geometric risk premium. It is closer to how investors think about risk premiums over long periods.

Country Risk Premiums

- Historical risk premiums are almost impossible to estimate with any precision in markets with limited history - this is true not just of emerging markets but also of many Western European markets.
- For such markets, we can estimate a modified historical premium beginning with the U.S. premium as the base:
 - *Relative Equity Market approach*: The country risk premium is based upon the volatility of the market in question relative to U.S market.
Country risk premium = Risk Premium_{US}* Country Equity / US Equity
 - *Country Bond approach*: In this approach, the country risk premium is based upon the default spread of the bond issued by the country.
Country risk premium = Risk Premium_{US}+ Country bond default spread
 - *Combined approach*: In this approach, the country risk premium incorporates both the country bond spread and equity market volatility.

Step 1: Assessing Country Risk Using Country Ratings: Latin America - March 2001

<i>Country</i>	<i>Rating</i>	<i>Typical Spread</i>	<i>Market Spread</i>
Argentina	B1	450	563
Bolivia	B1	450	551
Brazil	B1	450	537
Colombia	Ba2	300	331
Ecuador	Caa2	750	787
Guatemala	Ba2	300	361
Honduras	B2	550	581
Mexico	Baa3	145	235
Paraguay	B2	550	601
Peru	Ba3	400	455
Uruguay	Baa3	145	193
Venezuela	B2	550	631

Step 2: From Bond Default Spreads to Equity Spreads

- Country ratings measure default risk. While default risk premiums and equity risk premiums are highly correlated, one would expect equity spreads to be higher than debt spreads.
 - One way to adjust the country spread upwards is to use information from the US market. In the US, the equity risk premium has been roughly twice the default spread on junk bonds.
 - Another is to multiply the bond spread by the relative volatility of stock and bond prices in that market. For example,
 - Standard Deviation in Bovespa (Equity) = 32.6%
 - Standard Deviation in Brazil C-Bond = 17.1%
 - Adjusted Equity Spread = 5.37% $(32.6/17.1\%) = 10.24\%$
- Ratings agencies make mistakes. They are often late in recognizing and building in risk.

Another Example: Assessing Country Risk Using Currency Ratings: Western Europe

<i>Country</i>	<i>Rating</i>	<i>Typical Spread</i>	<i>Actual Spread</i>
• Austria	Aaa	0	
• Belgium	Aaa	0	
• Denmark	Aaa	0	
• Finland	Aaa	0	
• France	Aaa	0	
• Germany	Aaa	0	
• Greece	A3	95	50
• Ireland	AA2	65	35
• Italy	Aa3	70	30
• Netherlands	Aaa	0	
• Norway	Aaa	0	
• Portugal	A3	95	55
• Spain	Aa1	60	30
• Sweden	Aa1	60	25
• Switzerland	Aaa	0	

Greek Country Risk Premium

- Country ratings measure default risk. While default risk premiums and equity risk premiums are highly correlated, one would expect equity spreads to be higher than debt spreads.
 - One way to adjust the country spread upwards is to use information from the US market. In the US, the equity risk premium has been roughly twice the default spread on junk bonds.
 - Another is to multiply the bond spread by the relative volatility of stock and bond prices in that market. For example,
 - Standard Deviation in Greek ASE(Equity) = 40.5%
 - Standard Deviation in Greek GDr Bond = 26.1%
 - Adjusted Equity Spread = 0.95% $(40.5\%/26.1\%) = 1.59\%$
- Ratings agencies make mistakes. They are often late in recognizing and building in risk.

From Country Spreads to Corporate Risk premiums

- Approach 1: Assume that every company in the country is equally exposed to country risk. In this case,
$$E(\text{Return}) = \text{Riskfree Rate} + \text{Country Spread} + \text{Beta (US premium)}$$

Implicitly, this is what you are assuming when you use the local Government's dollar borrowing rate as your riskfree rate.
- Approach 2: Assume that a company's exposure to country risk is similar to its exposure to other market risk.
$$E(\text{Return}) = \text{Riskfree Rate} + \text{Beta (US premium + Country Spread)}$$
- Approach 3: Treat country risk as a separate risk factor and allow firms to have different exposures to country risk (perhaps based upon the proportion of their revenues come from non-domestic sales)
$$E(\text{Return}) = \text{Riskfree Rate} + \text{Beta (US premium)} + \text{Beta (Country Spread)}$$

Estimating Company Exposure to Country Risk

- Different companies should be exposed to different degrees to country risk. For instance, a Brazilian firm that generates the bulk of its revenues in the United States should be less exposed to country risk in Brazil than one that generates all its business within Brazil.
- The factor “ ” measures the relative exposure of a firm to country risk. One simplistic solution would be to do the following:
$$= \% \text{ of revenues domestically}_{\text{firm}} / \% \text{ of revenues domestically}_{\text{avg firm}}$$

For instance, if a firm gets 35% of its revenues domestically while the average firm in that market gets 70% of its revenues domestically

$$= 35\% / 70\% = 0.5$$
- There are two implications
 - A company’s risk exposure is determined by where it does business and not by where it is located
 - Firms might be able to actively manage their country risk exposures

Estimating E(Return) for Embraer

- Assume that the beta for Embraer is 0.88, and that the riskfree rate used is 4.5%. (Real Riskfree Rate)
- Approach 1: Assume that every company in the country is equally exposed to country risk. In this case,
$$E(\text{Return}) = 4.5\% + 10.24\% + 0.88 (5.51\%) = 19.59\%$$
- Approach 2: Assume that a company's exposure to country risk is similar to its exposure to other market risk.
$$E(\text{Return}) = 4.5\% + 0.88 (5.51\% + 10.24\%) = 18.36\%$$
- Approach 3: Treat country risk as a separate risk factor and allow firms to have different exposures to country risk (perhaps based upon the proportion of their revenues come from non-domestic sales)
$$E(\text{Return}) = 4.5\% + 0.8(5.51\%) + 0.50 (10.24\%) = 14.47\%$$

Embraer is less exposed to country risk than the typical Brazilian firm since much of its business is overseas.

Implied Equity Premiums

- If we use a basic discounted cash flow model, we can estimate the implied risk premium from the current level of stock prices.
- For instance, if stock prices are determined by a variation of the simple Gordon Growth Model:
 - $\text{Value} = \text{Expected Dividends next year} / (\text{Required Returns on Stocks} - \text{Expected Growth Rate})$
 - Dividends can be extended to include expected stock buybacks and a high growth period.
 - Plugging in the current level of the index, the dividends on the index and expected growth rate will yield a “implied” expected return on stocks. Subtracting out the riskfree rate will yield the implied premium.
- This model can be extended to allow for two stages of growth - an initial period where the entire market will have earnings growth greater than that of the economy, and then a stable growth period.

Estimating Implied Premium for U.S. Market: Jan 1, 2002

- Level of the index = 1148
- Treasury bond rate = 5.05%
- Expected Growth rate in earnings (next 5 years) = 10.3% (Consensus estimate for S&P 500)
- Expected growth rate after year 5 = 5.05%
- Dividends + stock buybacks = 2.74% of index (Current year)

	Year 1	Year 2	Year 3	Year 4	Year 5
Expected Dividends =	\$34.72	\$38.30	\$42.24	\$46.59	\$51.39
+ Stock Buybacks					

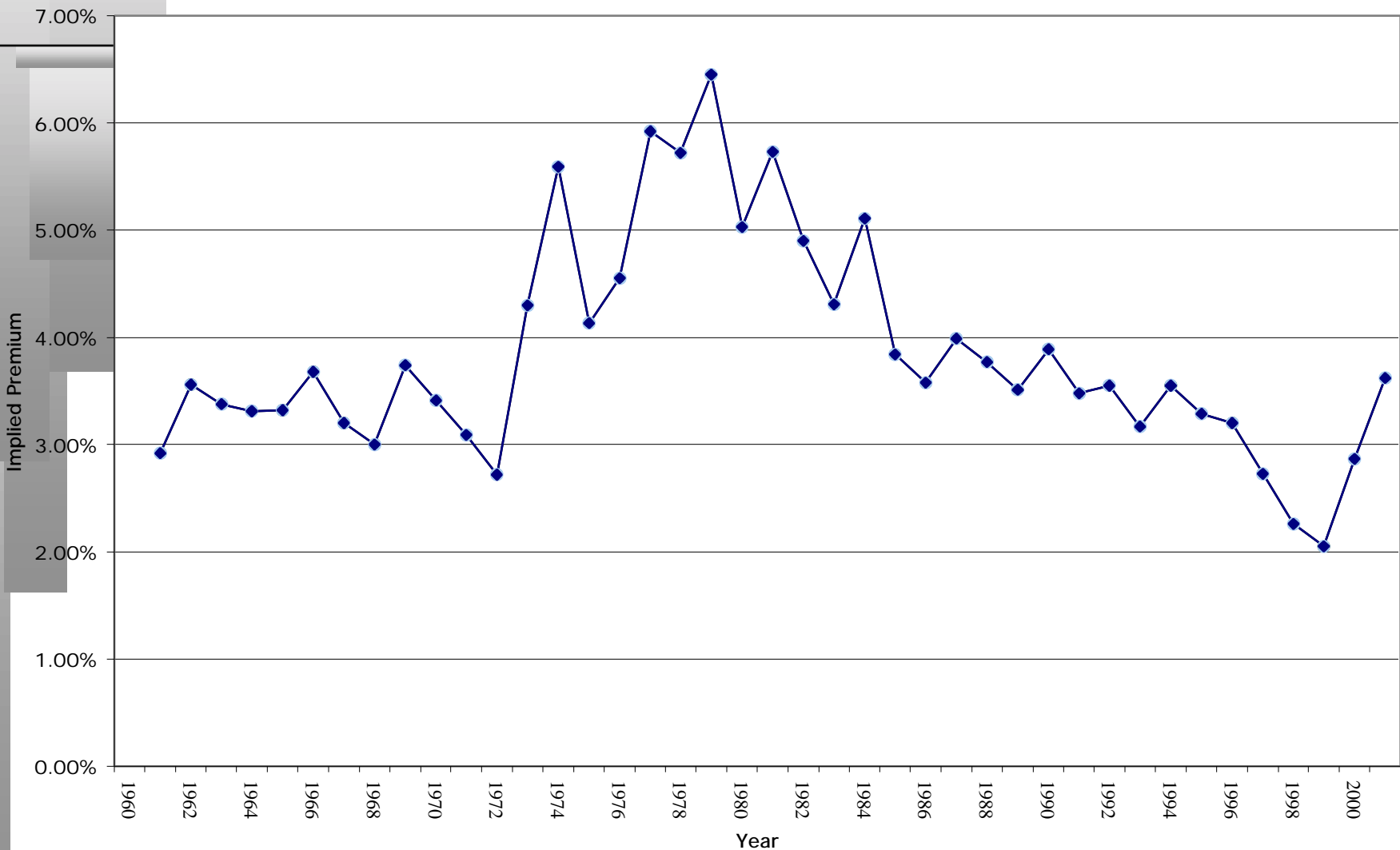
Expected dividends + buybacks in year 6 = $51.39 (1.0505) = \$ 54.73$

$$1148 = 34.72/(1+r) + 38.30/(1+r)^2 + 42.24/(1+r)^3 + 46.59/(1+r)^4 + (51.39 + (54.73/(r - .0505)))/(1+r)^5$$

Solving for r, $r = 8.67\%$. (Only way to do this is trial and error)

Implied risk premium = $8.67\% - 5.05\% = \mathbf{3.62\%}$

Implied Premium for US Equity Market



Implied Premium for Brazilian Market: March 1, 2001

- Level of the Index = 16417
- Dividends on the Index = 4.40% of (Used weighted yield)
- Other parameters
 - Riskfree Rate = 4.5% (real riskfree rate)
 - Expected Growth
 - Next 5 years = 13.5% (Used expected real growth rate in Earnings)
 - After year 5 = 4.5% (real growth rate in long term)
- Solving for the expected return:
 - Expected return on Equity = 11.16%
 - Implied Equity premium = $11.16\% - 4.5\% = 6.66\%$

The Effect of Using Implied Equity Premiums on Value

- Embraer's value per share (using historical premium + country risk adjustment) = 11.22 BR
- Embraer's value per share (using implied equity premium of 6.66%) = 20.02 BR
- Embraer's stock price (at the time of the valuation) = 15.25 BR

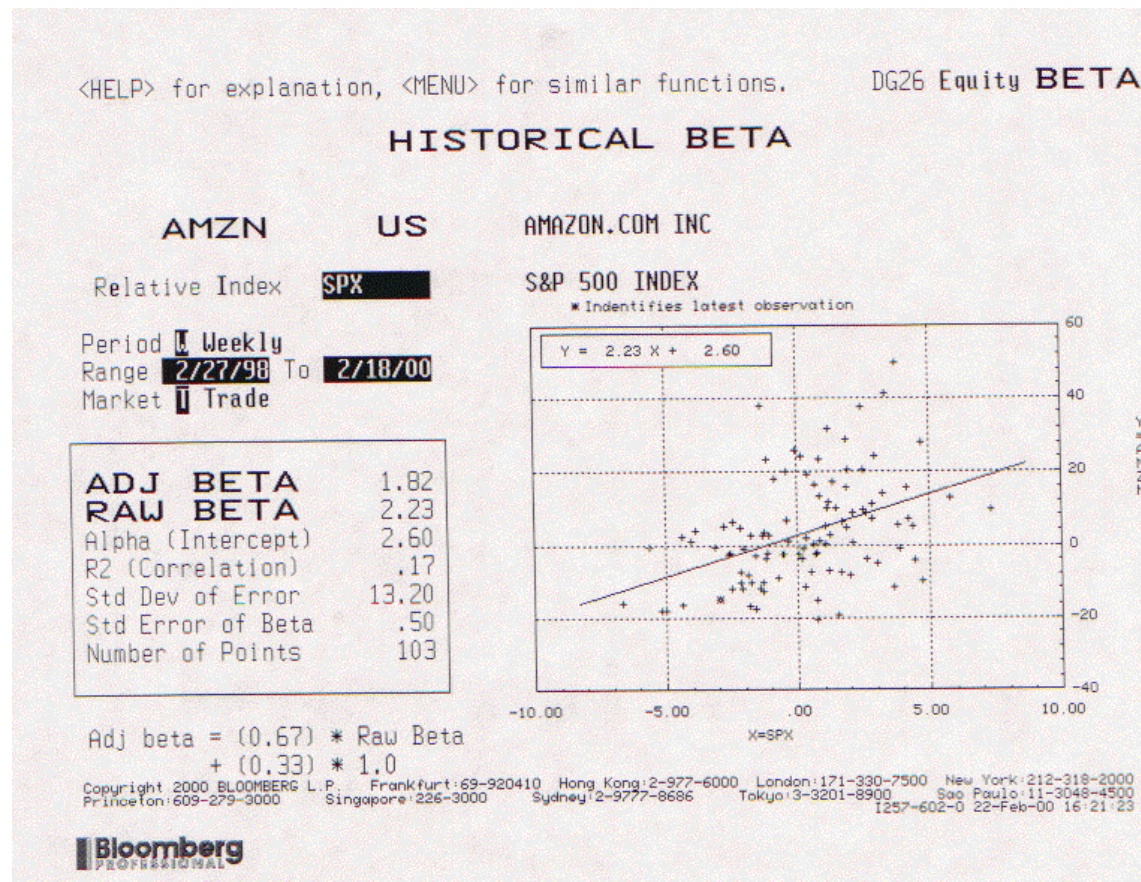
Estimating Beta

- The standard procedure for estimating betas is to regress stock returns (R_j) against market returns (R_m) -

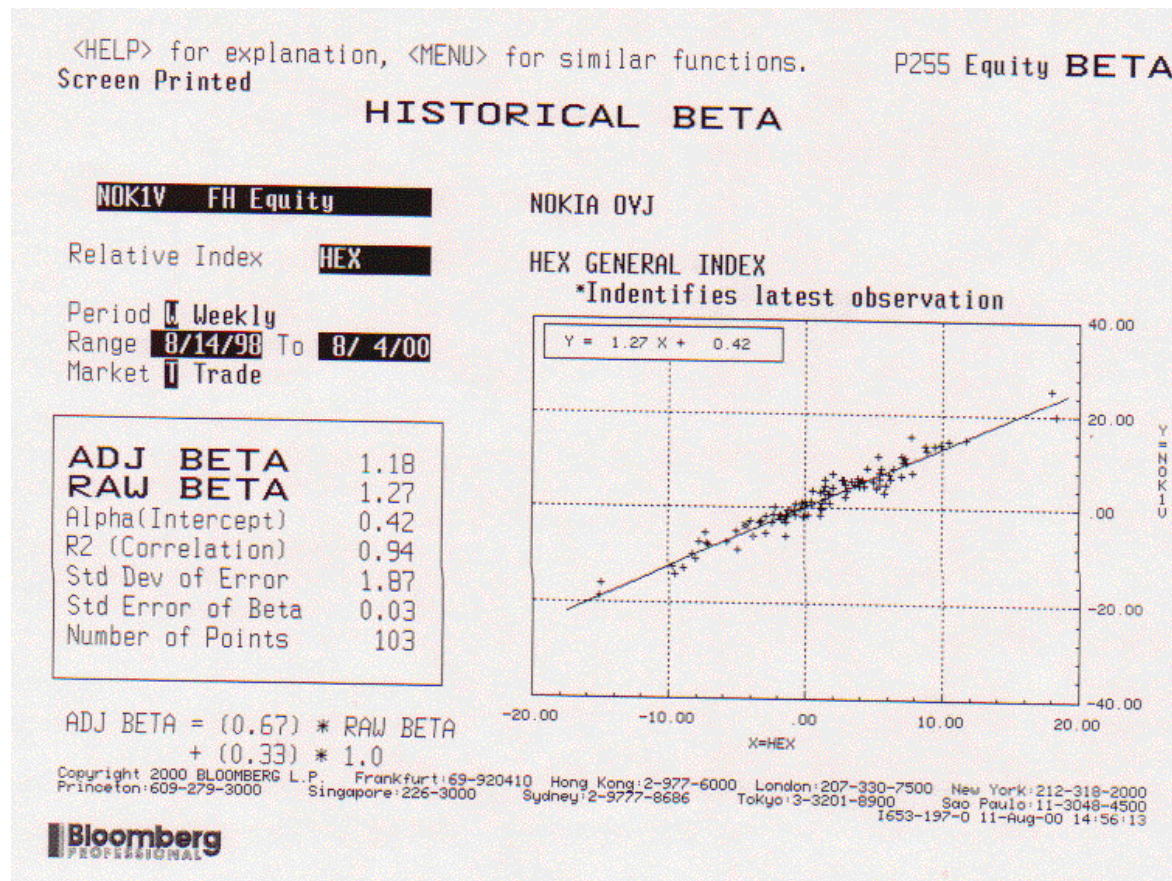
$$R_j = a + b R_m$$

- where a is the intercept and b is the slope of the regression.
- The slope of the regression corresponds to the beta of the stock, and measures the riskiness of the stock.
- This beta has three problems:
 - It has high standard error
 - It reflects the firm's business mix over the period of the regression, not the current mix
 - It reflects the firm's average financial leverage over the period rather than the current leverage.

Beta Estimation: The Noise Problem



Beta Estimation: The Index Effect



Determinants of Betas

- **Product or Service:** The beta value for a firm depends upon the sensitivity of the demand for its products and services and of its costs to macroeconomic factors that affect the overall market.
 - Cyclical companies have higher betas than non-cyclical firms
 - Firms which sell more discretionary products will have higher betas than firms that sell less discretionary products
- **Operating Leverage:** The greater the proportion of fixed costs in the cost structure of a business, the higher the beta will be of that business. This is because higher fixed costs increase your exposure to all risk, including market risk.
- **Financial Leverage:** The more debt a firm takes on, the higher the beta will be of the equity in that business. Debt creates a fixed cost, interest expenses, that increases exposure to market risk.

Equity Betas and Leverage

- The beta of equity alone can be written as a function of the unlevered beta and the debt-equity ratio

$$\beta_L = \beta_u (1 + ((1-t)D/E))$$

where

β_L = Levered or Equity Beta

β_u = Unlevered Beta (Asset Beta)

t = Corporate marginal tax rate

D = Market Value of Debt

E = Market Value of Equity

- While this beta is estimated on the assumption that debt carries no market risk (and has a beta of zero), you can have a modified version:

$$\beta_L = \beta_u (1 + ((1-t)D/E)) - \beta_{\text{debt}} (1-t) (D/E)$$

Solutions to the Regression Beta Problem

- Modify the regression beta by
 - changing the index used to estimate the beta
 - adjusting the regression beta estimate, by bringing in information about the fundamentals of the company
- Estimate the beta for the firm using
 - the standard deviation in stock prices instead of a regression against an index.
 - accounting earnings or revenues, which are less noisy than market prices.
- Estimate the beta for the firm from the bottom up without employing the regression technique. This will require
 - understanding the business mix of the firm
 - estimating the financial leverage of the firm
- Use an alternative measure of market risk that does not need a regression.

Bottom-up Betas

- The bottom up beta can be estimated by :
 - Taking a weighted (by sales or operating income) average of the unlevered betas of the different businesses a firm is in.

$$\beta_j \frac{\text{Operating Income}_j}{\text{Operating Income}_{\text{Firm}}}$$

(The unlevered beta of a business can be estimated by looking at other firms in the same business)

- Lever up using the firm's debt/equity ratio

$$\beta_{\text{levered}} = \beta_{\text{unlevered}} [1 + (1 - \text{tax rate}) (\text{Current Debt/Equity Ratio})]$$

- The bottom up beta will give you a better estimate of the true beta when

- It has lower standard error ($SE_{\text{average}} = SE_{\text{firm}} / \sqrt{n}$ (n = number of firms))
- It reflects the firm's current business mix and financial leverage
- It can be estimated for divisions and private firms.

Bottom-up Beta: Firm in Multiple Businesses

Boeing in 1998

<i>Segment</i>	<i>Estimated Value</i>	<i>Unlevered Beta</i>	<i>Segment Weight</i>
Commercial Aircraft	30,160.48	0.91	70.39%
Defense	12,687.50	0.80	29.61%

Estimated Value = Revenues of division * Enterprise Value/Sales_{Business}

Unlevered Beta of firm = 0.91 (.7039) + 0.80 (.2961) = 0.88

Levered Beta Calculation

Market Value of Equity = \$ 33,401

Market Value of Debt = \$8,143

Market Debt/Equity Ratio = 24.38%

Tax Rate = 35%

Levered Beta for Boeing = 0.88 (1 + (1 - .35) (.2438)) = 1.02

Siderar's Bottom-up Beta

Siderar is an Argentine steel company.

Business	Unlevered Beta	D/E Ratio	Levered Beta
Steel	0.68	5.97%	0.71

Proportion of operating income from steel = 100%

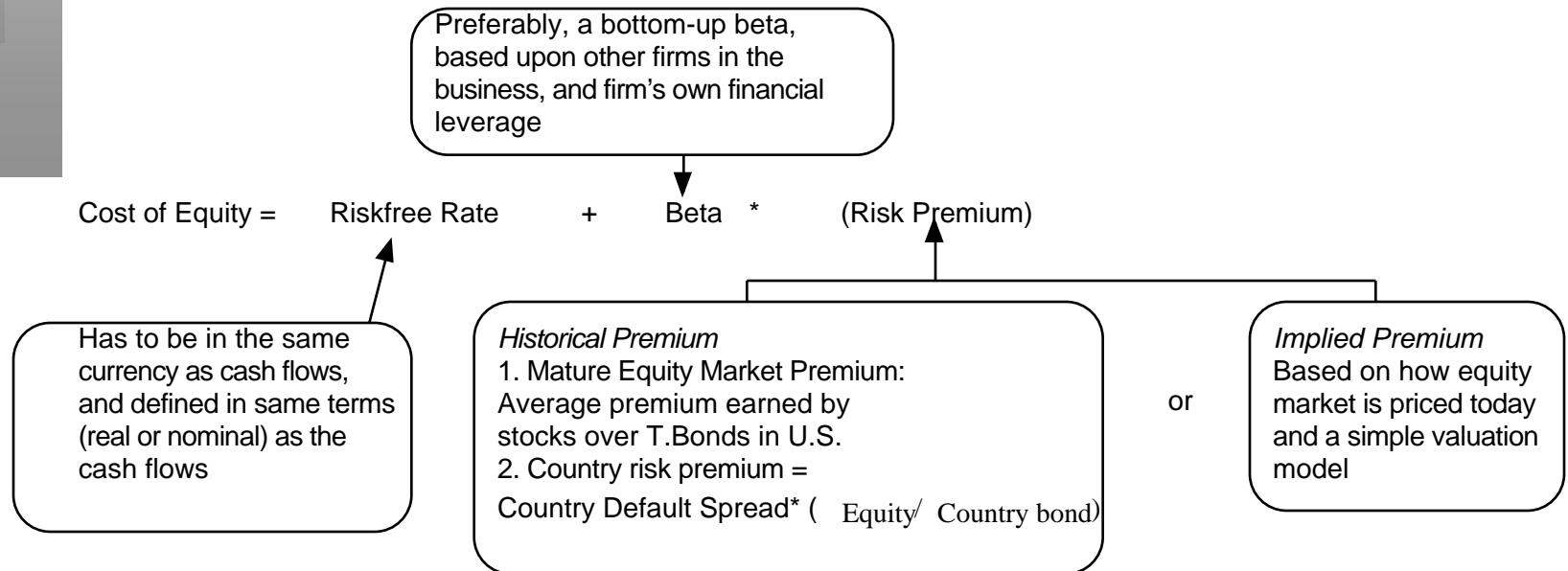
Levered Beta for Siderar = 0.71

Comparable Firms?

Can an unlevered beta estimated using U.S. steel companies be used to estimate the beta for an Argentine steel company?

- Yes
- No

The Cost of Equity: A Recap



Estimating the Cost of Debt

- The cost of debt is the rate at which you can borrow at currently, It will reflect not only your default risk but also the level of interest rates in the market.
- The two most widely used approaches to estimating cost of debt are:
 - Looking up the yield to maturity on a straight bond outstanding from the firm. The limitation of this approach is that very few firms have long term straight bonds that are liquid and widely traded
 - Looking up the rating for the firm and estimating a default spread based upon the rating. While this approach is more robust, different bonds from the same firm can have different ratings. You have to use a median rating for the firm
- When in trouble (either because you have no ratings or multiple ratings for a firm), estimate a synthetic rating for your firm and the cost of debt based upon that rating.

Estimating Synthetic Ratings

- The rating for a firm can be estimated using the financial characteristics of the firm. In its simplest form, the rating can be estimated from the interest coverage ratio

$$\text{Interest Coverage Ratio} = \text{EBIT} / \text{Interest Expenses}$$

- For Siderar, in 1999, for instance

$$\text{Interest Coverage Ratio} = 161/48 = 3.33$$

- Based upon the relationship between interest coverage ratios and ratings, we would estimate a rating of A- for Siderar. With a default spread of 1.25% (given the rating of A-)
- For Titan's interest coverage ratio, we used the interest expenses and EBIT from 2000.

$$\text{Interest Coverage Ratio} = 55,467 / 4028 = 13.77$$

Interest Coverage Ratios, Ratings and Default Spreads

If Coverage Ratio is	Estimated Bond Rating	Default Spread(1/99)	Default Spread(1/01)
> 8.50	AAA	0.20%	0.75%
6.50 - 8.50	AA	0.50%	1.00%
5.50 - 6.50	A+	0.80%	1.50%
4.25 - 5.50	A	1.00%	1.80%
3.00 - 4.25	A-	1.25%	2.00%
2.50 - 3.00	BBB	1.50%	2.25%
2.00 - 2.50	BB	2.00%	3.50%
1.75 - 2.00	B+	2.50%	4.75%
1.50 - 1.75	B	3.25%	6.50%
1.25 - 1.50	B -	4.25%	8.00%
0.80 - 1.25	CCC	5.00%	10.00%
0.65 - 0.80	CC	6.00%	11.50%
0.20 - 0.65	C	7.50%	12.70%
< 0.20	D	10.00%	15.00%

Cost of Debt computations

- Companies in countries with low bond ratings and high default risk might bear the burden of country default risk
 - For Siderar, the rating estimated of A- yields a cost of debt as follows:
Pre-tax Cost of Debt in 1999
= US T.Bond rate + Country default spread + Company Default Spread
= 6% + 5.25% + 1.25% = 12.50%
- The synthetic rating for Titan is AAA. The default spread in 2001 is 0.75%.
Pre-tax Cost of Debt
= Riskfree Rate + Company Default Spread+ Country Spread
= 5.10% + 0.75% + 0.95% = 6.80%

Synthetic Ratings: Some Caveats

- The relationship between interest coverage ratios and ratings, developed using US companies, tends to travel well, as long as we are analyzing large manufacturing firms in markets with interest rates close to the US interest rate
- They are more problematic when looking at smaller companies in markets with higher interest rates than the US.

Weights for the Cost of Capital Computation

- The weights used to compute the cost of capital should be the market value weights for debt and equity.
- There is an element of circularity that is introduced into every valuation by doing this, since the values that we attach to the firm and equity at the end of the analysis are different from the values we gave them at the beginning.
- As a general rule, the debt that you should subtract from firm value to arrive at the value of equity should be the same debt that you used to compute the cost of capital.

Estimating Cost of Capital: Titan Cements

■ Equity

- Cost of Equity = 5.10% + 0.96 (4%+1.59%) = 10.47%
- Market Value of Equity = 739,217 million GDr (78.7%)

■ Debt

- Cost of debt = 5.10% + 0.75% + 0.95% = 6.80%
- Market Value of Debt = 199,766 million GDr (21.3 %)

■ Cost of Capital

$$\text{Cost of Capital} = 10.47 \% (.787) + 6.80\% (1 - .2449) (0.213) = 9.33 \%$$

Mature
market
premium

Greek country premium

Company default spread

Country default spread

Titan Cement: Book Value Weights

- Titan Cement has a book value of equity of 135,857 million GDR and a book value of debt of 200,000 million GDR. Estimate the cost of capital using book value weights instead of market value weights.

Converting a Dollar Cost of Capital into a Peso cost of capital

- Approach 1: Use a peso riskfree rate in all of the calculations above. For instance, if the peso riskfree rate was 10%, the cost of capital would be computed as follows:
 - Cost of Equity = 10.00% + 0.71 (4% + 10.53%) = 20.32%
 - Cost of Debt = 10.00% + 5.25% (Country default) + 1.25% (Company default) = 16.5%(This assumes the peso riskfree rate has no country risk premium embedded in it.)
- Approach 2: Use the differential inflation rate to estimate the cost of capital. For instance, if the inflation rate in pesos is 7% and the inflation rate in the U.S. is 3%

$$\begin{aligned}\text{Cost of capital} &= (1 + \text{Cost of Capital}_{\$}) \frac{1 + \text{Inflation}_{\text{Peso}}}{1 + \text{Inflation}_{\$}} \\ &= 1.1587 (1.07/1.03) = 1.2037 \rightarrow 20.37\%\end{aligned}$$

Dealing with Hybrids and Preferred Stock

- When dealing with hybrids (convertible bonds, for instance), break the security down into debt and equity and allocate the amounts accordingly. Thus, if a firm has \$ 125 million in convertible debt outstanding, break the \$125 million into straight debt and conversion option components. The conversion option is equity.
- When dealing with preferred stock, it is better to keep it as a separate component. The cost of preferred stock is the preferred dividend yield. (As a rule of thumb, if the preferred stock is less than 5% of the outstanding market value of the firm, lumping it in with debt will make no significant impact on your valuation).

Recapping the Cost of Capital

