Valuation

Intuition Behind Present Value

- There are three reasons why a dollar tomorrow is worth less than a dollar today
  - Individuals prefer present consumption to future consumption. To induce people to give up present consumption you have to offer them more in the future.
  - When there is monetary inflation, the value of currency decreases over time. The greater the inflation, the greater the difference in value between a dollar today and a dollar tomorrow.
  - If there is any uncertainty (risk) associated with the cash flow in the future, the less that cash flow will be valued.
- Other things remaining equal, the value of cash flows in future time periods will decrease as
  - the preference for current consumption increases.
  - expected inflation increases.
  - the uncertainty in the cash flow increases.

Cash Flow Types and Discounting Mechanics

- There are five types of cash flows -
  - simple cash flows,
  - annuities,
  - growing annuities
  - perpetuities and
  - growing perpetuities

I.Simple Cash Flows

- A simple cash flow is a single cash flow in a specified future time period.

Cash Flow: CF_t

Time Period: t

- *The present value of this cash flow is-*

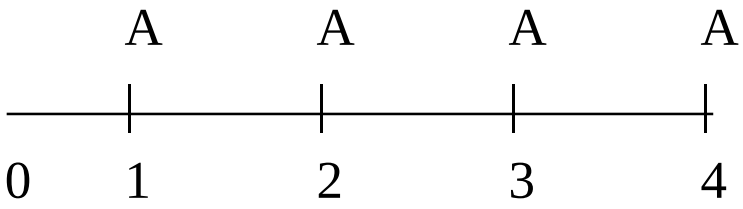
$$\text{PV of Simple Cash Flow} = CF_t / (1+r)^t$$

- *The future value of a cash flow is -*

$$\text{FV of Simple Cash Flow} = CF_0 (1+ r)^t$$

II. Annuities

- An annuity is a constant cash flow that occurs at regular intervals for a fixed period of time. Defining A to be the annuity,



Present Value of an Annuity

- The present value of an annuity can be calculated by taking each cash flow and discounting it back to the present, and adding up the present values. Alternatively, there is a short cut that can be used in the calculation [A = Annuity; r = Discount Rate; n = Number of years]

$$PV \text{ of an Annuity} = PV(A, r, n) = A\left[\frac{1 - \frac{1}{(1+r)^n}}{r}\right]$$

Example: PV of an Annuity

- The present value of an annuity of $1,000 at the end of each year for the next five years, assuming a discount rate of 10% is -

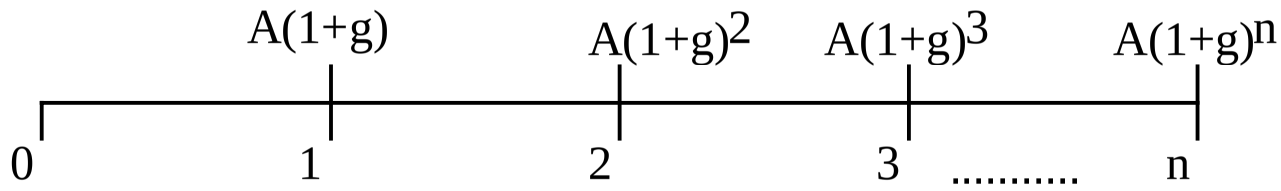
$$PV \text{ of \$1000 each year for next 5 years} = \$1000 \left[\frac{1 - \frac{1}{(1.10)^5}}{.10} \right] = \$3,791$$

- The notation that will be used in the rest of these lecture notes for the present value of an annuity will be PV(A,r,n).

III. Growing Annuity

- A growing annuity is a cash flow growing at a constant rate for a specified period of time. If A is the current cash flow, and g is the expected growth rate, the time line for a growing annuity looks as follows –

Figure 3.8: A Growing Annuity



Present Value of a Growing Annuity

- The present value of a growing annuity can be estimated in all cases, but one - where the growth rate is equal to the discount rate, using the following model:

$$PV \text{ of an Annuity} = PV(A,r,g,n) = A(1+g)\left[\frac{1 - \frac{(1+g)^n}{(1+r)^n}}{(r-g)}\right]$$

- In that specific case, the present value is equal to the nominal sums of the annuities over the period, without the growth effect.

The Value of a Gold Mine

- Consider the example of a gold mine, where you have the rights to the mine for the next 20 years, over which period you plan to extract 5,000 ounces of gold every year. The price per ounce is $300 currently, but it is expected to increase 3% a year. The appropriate discount rate is 10%. The present value of the gold that will be extracted from this mine can be estimated as follows –

$$PV \text{ of extracted gold} = \$300 * 5000 * (1.03) \left[\frac{1 - \frac{(1.03)^{20}}{(1.10)^{20}}}{.10 - .03} \right] = \$16,145,980$$

IV. Perpetuity

- A perpetuity is a constant cash flow at regular intervals forever. The present value of a perpetuity is-

$$PV \text{ of Perpetuity} = \frac{A}{r}$$

Valuing a Console Bond

- A console bond is a bond that has no maturity and pays a fixed coupon. Assume that you have a 6% coupon console bond. The value of this bond, if the interest rate is 9%, is as follows -

  Value of Console Bond = $60 / .09 = $667

V. Growing Perpetuities

- A growing perpetuity is a cash flow that is expected to grow at a constant rate forever. The present value of a growing perpetuity is -

$$PV \text{ of Growing Perpetuity} = \frac{CF_1}{(r - g)}$$

where

- CF_1 is the expected cash flow next year,
- g is the constant growth rate and
- r is the discount rate.

Discounted Cashflow Valuation: Basis for Approach

$$\text{Value} = \sum_{t=1}^{t=n} \frac{CF_t}{(1+r)^t}$$

- where,
- n = Life of the asset
- CF_t = Cashflow in period t
- r = Discount rate reflecting the riskiness of the estimated cashflows

I. Valuing Riskless Cashflows

- When cashflows are riskless, you can value them by discounting the cashflows at the riskless rate.

- For a cashflow to be riskless, you have to be guaranteed the cashflow by an entity with no default risk.

Valuing a Zero Coupon Government Bond

- To see an example of this valuation at work, assume that the ten-year interest rate on riskless investments is 4.55%, and that you are pricing a zero-coupon treasury bond, with a maturity of ten years and a face value of $ 1000. The price of the bond can be estimated as follows:

  Price of the Bond =

- Note that the face value is the only cash flow, and that this bond will be priced well below the face value of $ 1,000. Such a bond is said to be trading below par.

Valuing a default-free coupon bond

- Consider now a five-year treasury bond with a coupon rate of 5.50%, with coupons paid every 6 months. To value this bond initially we will use the default-free interest rate for each cash flow.

Time	*Coupon*	*Default-free Rate*	*Present Value*
0.5	$ 27.50	4.15%	$ 26.95
1	$ 27.50	4.30%	$ 26.37
1.5	$ 27.50	4.43%	$ 25.77
2	$ 27.50	4.55%	$ 25.16
2.5	$ 27.50	4.65%	$ 24.55
3	$ 27.50	4.74%	$ 23.93
3.5	$ 27.50	4.82%	$ 23.32
4	$ 27.50	4.90%	$ 22.71
4.5	$ 27.50	4.97%	$ 22.11
5	$ 1,027.50	5.03%	$ 803.92
			$ 1,024.78

Valuing a bond with default risk

- To value a bond with default risk, you have to discount the promised cashflows (coupons and principal) at an interest rate that reflects the default risk. (Riskless rate + Default Spread)
- Alternatively, you could adjust the coupons and principal for the likelihood of default (use expected cashflows) and discount back at the riskless rate.

Example: A Corporate Bond

- Consider, for instance a bond issued by Boeing with a coupon rate of 8.75%, maturing in 35 years. Based upon its default risk (measured by a bond rating assigned to Boeing by Standard and Poor's at the time of this analysis), the market interest rate on Boeing's debt is 0.5% higher than the treasury bond rate of 5.5% for default-free bonds of similar maturity.

$$\text{Price of Boeing bond} = \sum_{t=0.5}^{t=35} \frac{43.875}{(1.06)^t} + \frac{1,000}{(1.06)^{35}} = \$1,404.25$$

Valuing Equity

- Equity represents a residual cashflow rather than a promised cashflow.
- You can value equity in one of two ways:
  - By discounting cashflows to equity at the cost of equity to arrive at the value of equity directly.
  - By discounting cashflows to the firm at the cost of capital to arrive at the value of the business. Subtracting out the firm's outstanding debt should yield the value of equity.

Two Measures of Cash Flows

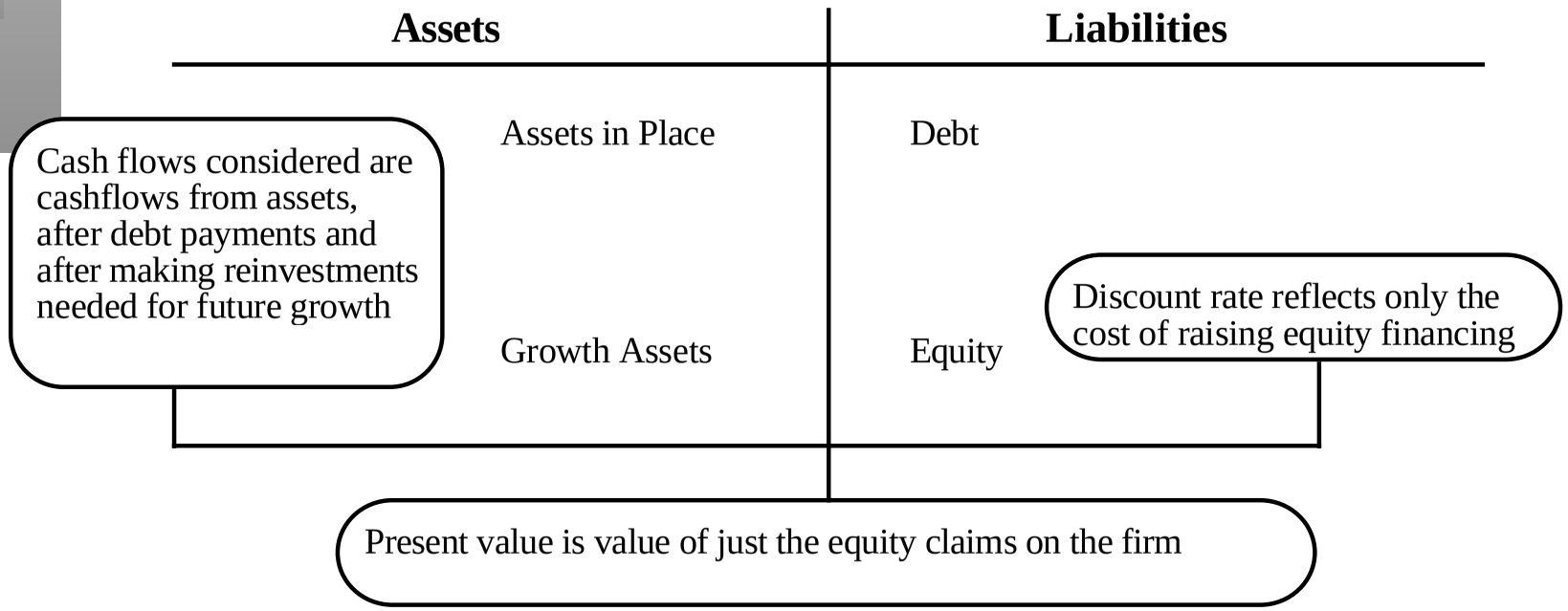
- **Cash flows to Equity**: Thesea are the cash flows generated by the asset after all expenses and taxes, and also after payments due on the debt. This cash flow, which is after debt payments, operating expenses and taxes, is called the **cash flow to equity investors**.
- **Cash flow to Firm**: There is also a broader definition of cash flow that we can use, where we look at not just the equity investor in the asset, but at the total cash flows generated by the asset for both the equity investor and the lender. This cash flow, which is before debt payments but after operating expenses and taxes, is called the **cash flow to the firm**

Two Measures of Discount Rates

- **Cost of Equity**: This is the rate of return required by equity investors on an investment. It will incorporate a premium for equity risk -the greater the risk, the greater the premium.
- **Cost of capital**: This is a composite cost of all of the capital invested in an asset or business. It will be a weighted average of the cost of equity and the after-tax cost of borrowing.

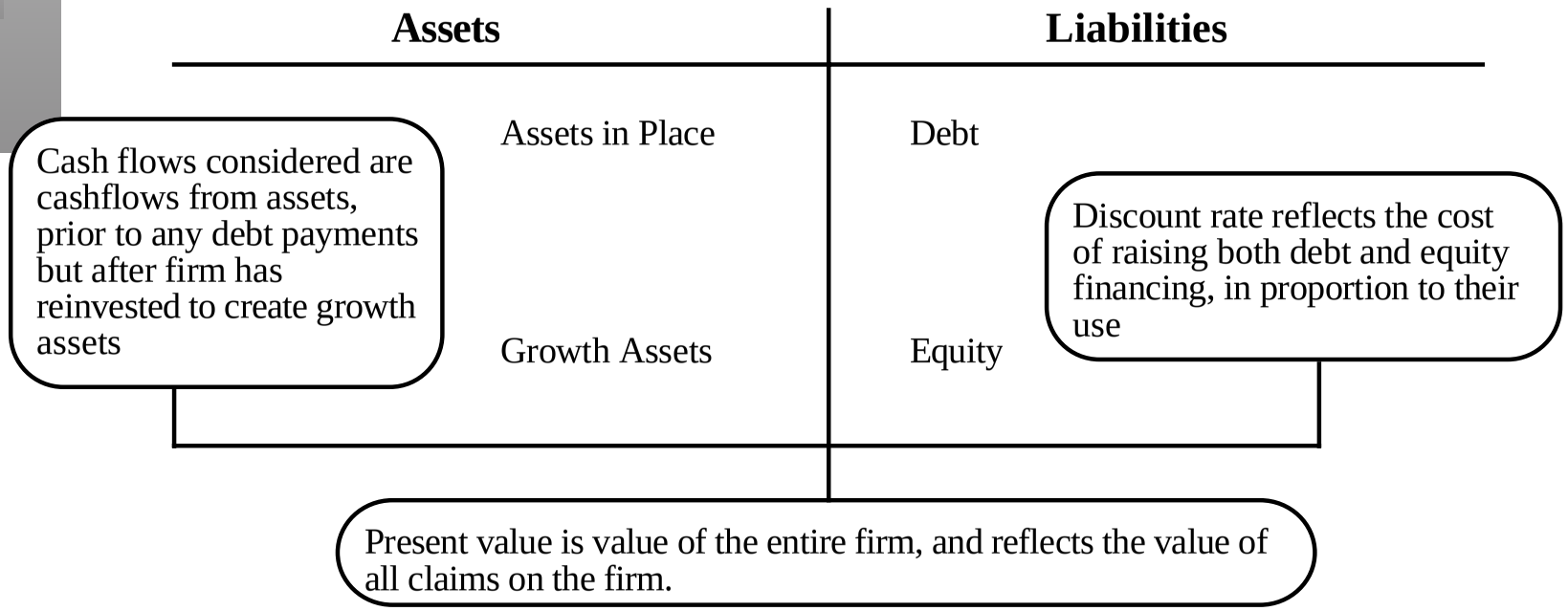
Equity Valuation

Figure 5.5: Equity Valuation



Firm Valuation

Figure 5.6: Firm Valuation



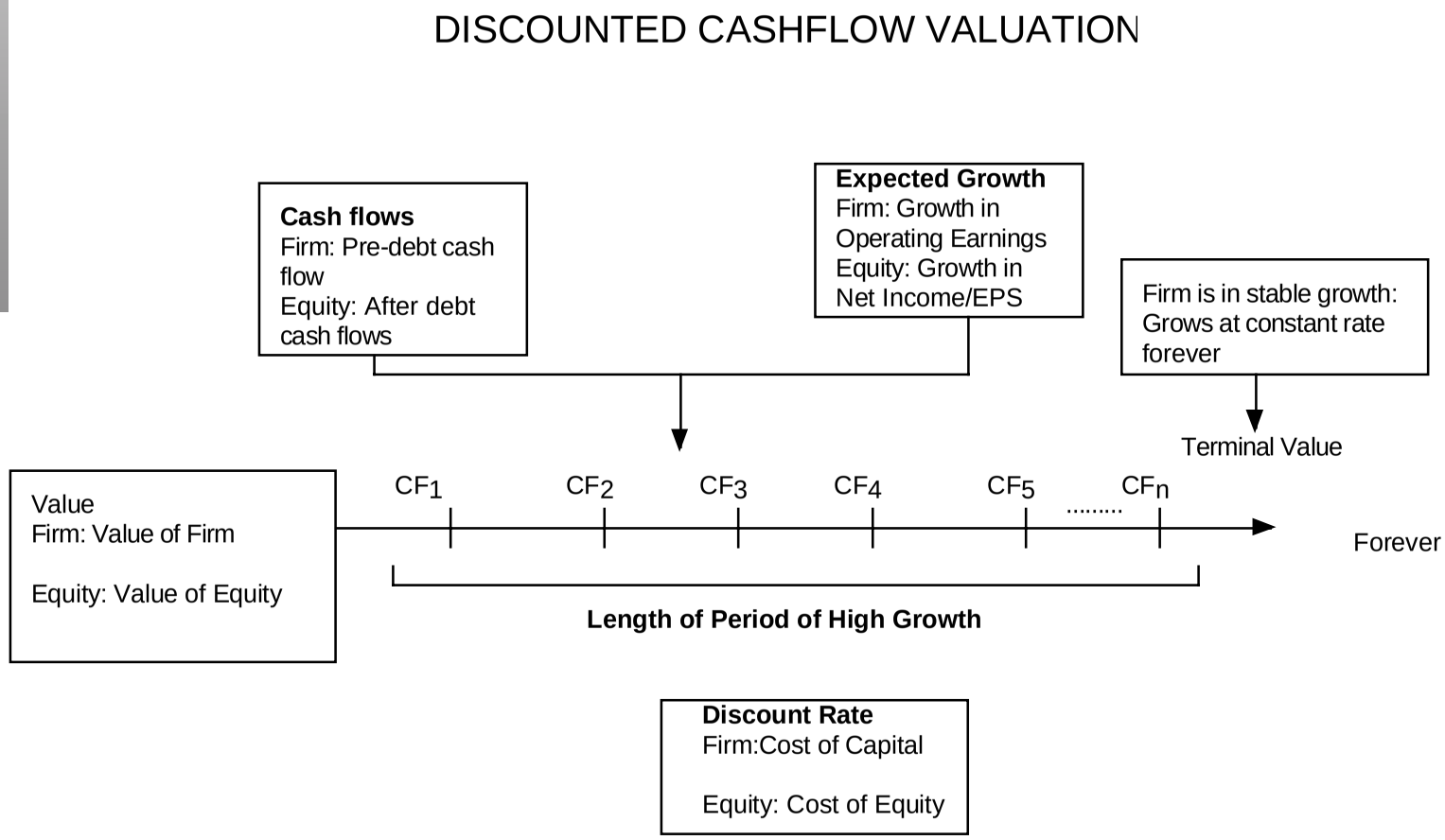
Valuing a Finite-Life Asset

- Consider a rental building that you are considering for acquisition. The building is assumed to have a finite life of 12 years and is expected to have cash flows *before debt payments and after reinvestment needs* of $ 1 million, growing at 5% a year for the next 12 years.
- The building is also expected to have a value of $ 2.5 million at the end of the 12^{th} year (called the salvage value).
- The cost of capital is 9.51%.

Expected Cash Flows and present value

Year	Expected Cash Flows	Value at End	PV at 9.51%
1	$ 1,050,000		$ 958,817
2	$ 1,102,500		$ 919,329
3	$ 1,157,625		$ 881,468
4	$ 1,215,506		$ 845,166
5	$ 1,276,282		$ 810,359
6	$ 1,340,096		$ 776,986
7	$ 1,407,100		$ 744,987
8	$ 1,477,455		$ 714,306
9	$ 1,551,328		$ 684,888
10	$ 1,628,895		$ 656,682
11	$ 1,710,339		$ 629,638
12	$ 1,795,856	$ 2,500,000	$ 1,444,124
		Value of Store =	$ 10,066,749

Valuation with Infinite Life



I. Dividend Discount Model

- The simplest measure of cashflow to equity is the expected dividend. In a dividend discount model, the value of equity is the present value of expected dividends, discounted back at the cost of equity.

$$\text{Value of Equity} = \sum_{t=1}^{t=\infty} \frac{\text{Expected Dividends}_t}{(1+\text{ Cost of Equity})^t}$$

Example: A stable growth dividend paying stock

- Consolidated Edison, the utility that produces power for much of New York city, paid dividends per share of $ 2.12 in 1998. The dividends are expected to grow 5% a year in the long term, and the company has a cost of equity of 9.40%. The value per share can be estimated as follows:

  Value of Equity per share = $2.12 (1.05) / (.094 - .05) = $ 50.59

- The stock was trading at $ 54 per share at the time of this valuation. We could argue that based upon this valuation, the stock was mildly overvalued.

Example: A high growth dividend paying stock

- A assume that you were trying to value Coca Cola. The company paid $0.69 as dividends per share during 1998, and these dividends are expected to grow 25% a year for the next 10 years.
- Beyond that, the expected growth rate is expected to be 6% a year forever.
- The cost of equity is 11% for Coca Cola.

Expected Dividends on Coca Cola

Year	*Dividends per Share*	*Present Value*
1	$ 0.86	$ 0.78
2	$ 1.08	$ 0.88
3	$ 1.35	$ 0.99
4	$ 1.68	$ 1.11
5	$ 2.11	$ 1.25
6	$ 2.63	$ 1.41
7	$ 3.29	$ 1.58
8	$ 4.11	$ 1.78
9	$ 5.14	$ 2.01
10	$ 6.43	$ 2.26
	PV of Dividends	$ 14.05

Expected Terminal Price and value per share today

- Terminal Price (at the end of year 10)
  - Expected Dividends per share in year 11 = $ 6.43 *1.06 = $ 6.81
  - Expected Terminal Price = $ 6.81 / (.11 - .06) = $ 136.24
- Value of Stock today

  = PV of Dividends in high growth + PV of Terminal Price

  = $ 14.05 + $ 136.24/$(1.11)^{10}$ = $62.03

A Measure of Potential Dividends: Free Cashflows to Equity

- Dividends are discretionary and are set by managers of firms. Not all firms pay out what they can afford to in dividends.
- We consider a broader definition of cash flow to which we call **free cash flow to equity**, defined as the cash left over after operating expenses, interest expenses, net debt payments and reinvestment needs. By **net debt payments**, we are referring to the difference between new debt issued and repayments of old debt. If the new debt issued exceeds debt repayments, the free cash flow to equity will be higher.

  Free Cash Flow to Equity (FCFE) = Net Income – Reinvestment Needs – (Debt Repaid – New Debt Issued)

Valuing the Home Depot's Equity

- Assume that we expect the free cash flows to equity at the Home Depot to grow for the next 10 years at rates much higher than the growth rate for the economy. To estimate the free cash flows to equity for the next 10 years, we make the following assumptions:
  - The net income of $1,614 million will grow 15% a year each year for the next 10 years.
  - The firm will reinvest 75% of the net income back into new investments each year, and its net debt issued each year will be 10% of the reinvestment.
  - To estimate the terminal price, we assume that net income will grow 6% a year forever after year 10. Since lower growth will require less reinvestment, we will assume that the reinvestment rate after year 10 will be 40% of net income; net debt issued will remain 10% of reinvestment.

Estimating cash flows to equity: The Home Depot

Year	Net Income	Reinvestment Needs	Net Debt Issued	FCFE	PV of FCFE
1	$ 1,856	$ 1,392	$ (139)	$ 603	$ 549
2	$ 2,135	$ 1,601	$ (160)	$ 694	$ 576
3	$ 2,455	$ 1,841	$ (184)	$ 798	$ 603
4	$ 2,823	$ 2,117	$ (212)	$ 917	$ 632
5	$ 3,246	$ 2,435	$ (243)	$ 1,055	$ 662
6	$ 3,733	$ 2,800	$ (280)	$ 1,213	$ 693
7	$ 4,293	$ 3,220	$ (322)	$ 1,395	$ 726
8	$ 4,937	$ 3,703	$ (370)	$ 1,605	$ 761
9	$ 5,678	$ 4,258	$ (426)	$ 1,845	$ 797
10	$ 6,530	$ 4,897	$ (490)	$ 2,122	$ 835
Sum of PV of FCFE =					$6,833

Terminal Value and Value of Equity today

- $FCFE_{11}$ = Net Income$_{11}$ – Reinvestment$_{11}$ – Net Debt Paid (Issued)$_{11}$
  = \$6,530 (1.06) – \$6,530 (1.06) (0.40) – (-277) = \$ 4,430 million
- Terminal Price$_{10}$ = $FCFE_{11}/(k_e - g)$
  = \$ 4,430 / (.0978 - .06) = \$117,186 million
- The value per share today can be computed as the sum of the present values of the free cash flows to equity during the next 10 years and the present value of the terminal value at the end of the 10th year.

Value of the Stock today = \$ 6,833 million + \$ 117,186/$(1.0978)^{10}$

= \$52,927 million

Valuing Boeing as a firm

- Assume that you are valuing Boeing as a firm, and that Boeing has cash flows before debt payments but after reinvestment needs and taxes of $ 850 million in the current year.
- Assume that these cash flows will grow at 15% a year for the next 5 years and at 5% thereafter.
- Boeing has a cost of capital of 9.17%.

Expected Cash Flows and Firm Value

- Terminal Value = $ 1710 (1.05)/(.0917-.05) = $ 43,049 million

Year	Cash Flow	Terminal Value	Present Value
1	$978		$895
2	$1,124		$943
3	$1,293		$994
4	$1,487		$1,047
5	$1,710	$43,049	$28,864
Value of Boeing as a firm =			$32,743

Relative Valuation

- **What is it?**: The value of any asset can be estimated by looking at how the market prices "similar" or 'comparable" assets.
- **Philosophical Basis**: The intrinsic value of an asset is impossible (or close to impossible) to estimate. The value of an asset is whatever the market is willing to pay for it (based upon its characteristics)
- **Information Needed**: To do a relative valuation, you need
  - an identical asset, or a group of comparable or similar assets
  - a standardized measure of value (in equity, this is obtained by dividing the price by a common variable, such as earnings or book value)
  - and if the assets are not perfectly comparable, variables to control for the differences
- **Market Inefficiency**: Pricing errors made across similar or comparable assets are easier to spot, easier to exploit and are much more quickly corrected.

Categorizing Multiples

- Multiples of Earnings
  - Equity earnings multiples: Price earnings ratios and variants
  - Operating earnings multiples: Enterprise value to EBITDA or EBIT
  - Cash earnings multiples
- Multiples of Book Value
  - Equity book multiples: Price to book equity
  - Capital book multiples: Enterprise value to book capital
- Multiples of revenues
  - Price to Sales
  - Enterprise value to Sales

The Fundamentals behind multiple

- Every multiple has embedded in it all of the assumptions that underlie discounted cashflow valuation. In particular, your assumptions about growth, risk and cashflow determine your multiple.
- If you have an equity multiple, you can begin with an equity discounted cash flow model and work out the determinants.
- If you have a firm value multiple, you can begin with a firm valuation model and work out the determinants.

Equity Multiples and Fundamentals

- Gordon Growth Model: $P_0 = \frac{DPS_1}{r - g_n}$
- Dividing both sides by the earnings,

$$\frac{P_0}{EPS_0} = PE = \frac{\text{Payout Ratio} * (1 + g_n)}{r - g_n}$$

- Dividing both sides by the book value of equity,

$$\frac{P_0}{BV_0} = PBV = \frac{ROE * \text{Payout Ratio} * (1 + g_n)}{r - g_n}$$

- If the return on equity is written in terms of the retention ratio and the expected growth rate $\frac{P_0}{BV_0} = PBV = \frac{ROE - g_n}{r - g_n}$
- Dividing by the Sales per share,

$$\frac{P_0}{Sales_0} = PS = \frac{\text{Profit Margin} * \text{Payout Ratio} * (1 + g_n)}{r - g_n}$$

Firm Value Multiples and Determinants

- Begin with a firm valuation model

$$V_0 = \frac{FCFF_1}{k_c - g_n}$$

- You can derive the determinants of value to EBIT or EBITDA

What to control for...

Multiple	*Determining Variables*
Price/Earnings Ratio	*Growth*, Payout, Risk
Price/Book Value Ratio	Growth, Payout, Risk, ***ROE***
Price/Sales Ratio	Growth, Payout, Risk, *Net Margin*
Value/EBIT Value/EBIT (1-t) Value/EBITDA	Growth, ***Reinvestment Needs***, Leverage, Risk
Value/Sales	Growth, Net Capital Expenditure needs, Leverage, Risk, ***Operating Margin***
Value/Book Capital	Growth, Leverage, Risk and ***ROC***

Choosing Comparable firms

- If life were simple, the value of a firm would be analyzed by looking at how an exactly identical firm - in terms of risk, growth and cash flows - is priced. In most analyses, however, a comparable firm is defined to be one in the same business as the firm being analyzed.
- If there are enough firms in the sector to allow for it, this list will be pruned further using other criteria; for instance, only firms of similar size may be considered. Implicitly, the assumption being made here is that firms in the same sector have similar risk, growth and cash flow profiles and therefore can be compared with much more legitimacy.

How to control for differences..

- Modify the basic multiple to adjust for the effects of the most critical variable determining that multiple. For instance, you could divide the PE ratio by the expected growth rate to arrive at the PEG ratio.

  PEG = PE / Expected Growth rate

- If you want to control for more than one variable, you can draw on more sophisticated techniques such as multiple regressions.

Example: PEG Ratios

Company	*PE*	*Expected Growth Rate*	*PE/Expected Growth (PEG)*
Acclaim Entertainment	13.70	23.60%	0.58
Activision	75.20	40.00%	1.88
Broderbund	32.30	26.00%	1.24
Davidson Associates	44.30	33.80%	1.31
Edmark	88.70	37.50%	2.37
Electronic Arts	33.50	22.00%	1.52
The Learning Co.	33.50	28.80%	1.16
Maxis	73.20	30.00%	2.44
Minnesota Educational	69.20	28.30%	2.45
Sierra On-Line	43.80	32.00%	1.37

Example: PBV ratios, ROE and Growth

Company Name	*P/BV*	*ROE*	*Expected Growth*
Total ADR B	0.90	4.10	9.50%
Giant Industries	1.10	7.20	7.81%
Royal Dutch Petroleum ADR	1.10	12.30	5.50%
Tesoro Petroleum	1.10	5.20	8.00%
Petrobras	1.15	3.37	15%
YPF ADR	1.60	13.40	12.50%
Ashland	1.70	10.60	7%
Quaker State	1.70	4.40	17%
Coastal	1.80	9.40	12%
Elf Aquitaine ADR	1.90	6.20	12%
Holly	2.00	20.00	4%
Ultramar Diamond Shamrock	2.00	9.90	8%
Witco	2.00	10.40	14%
World Fuel Services	2.00	17.20	10%
Elcor	2.10	10.10	15%
Imperial Oil	2.20	8.60	16%
Repsol ADR	2.20	17.40	14%
Shell Transport & Trading ADR	2.40	10.50	10%
Amoco	2.60	17.30	6%
Phillips Petroleum	2.60	14.70	7.50%
ENI SpA ADR	2.80	18.30	10%
Mapco	2.80	16.20	12%
Texaco	2.90	15.70	12.50%
British Petroleum ADR	3.20	19.60	8%
Tosco	3.50	13.70	14%

Results from Multiple Regression

- We ran a regression of PBV ratios on both variables:

PBV = -0.11 + 11.22 (ROE) + 7.87 (Expected Growth) R^2 = 60.88%

(5.79) (2.83)

- The numbers in brackets are t-statistics and suggest that the relationship between PBV ratios and both variables in the regression are statistically significant. The R-squared indicates the percentage of the differences in PBV ratios that is explained by the independent variables.
- Finally, the regression itself can be used to get predicted PBV ratios for the companies in the list. Thus, the predicted PBV ratio for Repsol would be:

$$\text{Predicted PBV}_{\text{Repsol}} = -0.11 + 11.22\ (.1740) + 7.87\ (.14) = 2.94$$

Since the actual PBV ratio for Repsol was 2.20, this would suggest that the stock was undervalued by roughly 25%.