Estimating Discount Rates

DCF Valuation
Estimating Inputs: Discount Rates

- **Critical ingredient** in discounted cashflow valuation. Errors in estimating the discount rate or mismatching cashflows and discount rates can lead to serious errors in valuation.

- At an intuitive level, the discount rate used should be consistent with both the **riskiness** and the **type of cashflow** being discounted.
  - **Equity versus Firm**: If the cash flows being discounted are cash flows to equity, the appropriate discount rate is a cost of equity. If the cash flows are cash flows to the firm, the appropriate discount rate is the cost of capital.
  - **Currency**: The currency in which the cash flows are estimated should also be the currency in which the discount rate is estimated.
  - **Nominal versus Real**: If the cash flows being discounted are nominal cash flows (i.e., reflect expected inflation), the discount rate should be nominal.
The cost of equity should be higher for riskier investments and lower for safer investments.

While risk is usually defined in terms of the variance of actual returns around an expected return, risk and return models in finance assume that the risk that should be rewarded (and thus built into the discount rate) in valuation should be the risk perceived by the marginal investor in the investment.

Most risk and return models in finance also assume that the marginal investor is well diversified, and that the only risk that he or she perceives in an investment is risk that cannot be diversified away (i.e., market or non-diversifiable risk).
## The Cost of Equity: Competing Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Expected Return</th>
<th>Inputs Needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPM</td>
<td>$E(R) = R_f + \beta (R_m - R_f)$</td>
<td>Riskfree Rate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Beta relative to market portfolio</td>
</tr>
<tr>
<td>Market Risk Premium</td>
<td></td>
<td></td>
</tr>
<tr>
<td>APM</td>
<td>$E(R) = R_f + \sum_{j=1}^{\infty} \beta_j (R_j - R_f)$</td>
<td>Riskfree Rate; # of Factors;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Betas relative to each factor</td>
</tr>
<tr>
<td>Factor</td>
<td></td>
<td>Factor risk premiums</td>
</tr>
<tr>
<td>Multi</td>
<td>$E(R) = R_f + \sum_{j=1}^{\infty} \beta_j (R_j - R_f)$</td>
<td>Riskfree Rate; Macro factors</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Betas relative to macro factors</td>
</tr>
<tr>
<td>Proxy</td>
<td>$E(R) = a + \sum_{j=1}^{\infty} b_j Y_j$</td>
<td>Proxies</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Regression coefficients</td>
</tr>
</tbody>
</table>
The CAPM: Cost of Equity

Consider the standard approach to estimating cost of equity:

\[ \text{Cost of Equity} = R_f + \text{Equity Beta} \times (E(R_m) - R_f) \]

where,

- \( R_f \) = Riskfree rate
- \( E(R_m) \) = Expected Return on the Market Index (Diversified Portfolio)

In practice,
- Short term government security rates are used as risk free rates
- Historical risk premiums are used for the risk premium
- Betas are estimated by regressing stock returns against market returns
Short term Governments are not riskfree in valuation…. 

- On a riskfree asset, the actual return is equal to the expected return. Therefore, there is no variance around the expected return.
- For an investment to be riskfree, then, it has to have:
  - No default risk
  - No reinvestment risk
- Thus, the riskfree rates in valuation will depend upon when the cash flow is expected to occur and will vary across time.
- In valuation, the time horizon is generally infinite, leading to the conclusion that a long-term riskfree rate will always be preferable to a short term rate, if you have to pick one.
Riskfree Rates in 2004

Riskfree Rates: An Exploration

0.00% 2.00% 4.00% 6.00% 8.00% 10.00% 12.00%
Germany 10-year (Euro) 4.30%
Greece 10-year (Euro) 4.15%
Italy 10-year (Euro) 4.12%
US 10-year Treasury ($) 4.25%
Brazil 10-year C Bond ($) 10.26%
Mexican 10-year (Peso) 11.50%
Japanese 10-Year (Yen) 1.50%
Estimating a Riskfree Rate when there are no default free entities....

- Estimate a range for the riskfree rate in local terms:
  - Approach 1: Subtract default spread from local government bond rate:
    Government bond rate in local currency terms - Default spread for Government in local currency
  - Approach 2: Use forward rates and the riskless rate in an index currency (say Euros or dollars) to estimate the riskless rate in the local currency.

- Do the analysis in real terms (rather than nominal terms) using a real riskfree rate, which can be obtained in one of two ways –
  - from an inflation-indexed government bond, if one exists
  - set equal, approximately, to the long term real growth rate of the economy in which the valuation is being done.

- Do the analysis in a currency where you can get a riskfree rate, say US dollars.
A Simple Test

You are valuing Embraer, a Brazilian company, in U.S. dollars and are attempting to estimate a riskfree rate to use in the analysis. The riskfree rate that you should use is

A. The interest rate on a Brazilian Real denominated long term bond issued by the Brazilian Government (15%)
B. The interest rate on a US $ denominated long term bond issued by the Brazilian Government (C-Bond) (10.30%)
C. The interest rate on a US $ denominated Brazilian Brady bond (which is partially backed by the US Government) (10.15%)
D. The interest rate on a dollar denominated bond issued by Embraer (9.25%)
E. The interest rate on a US treasury bond (4.29%)
Everyone uses historical premiums, but..

- The historical premium is the premium that stocks have historically earned over riskless securities.
- Practitioners never seem to agree on the premium; it is sensitive to
  - How far back you go in history…
  - Whether you use T.bill rates or T.Bond rates
  - Whether you use geometric or arithmetic averages.
- For instance, looking at the US:

<table>
<thead>
<tr>
<th>Historical Period</th>
<th>Arithmetic average</th>
<th>Geometric Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stocks - T.Bills</td>
<td>Stocks - T.Bills</td>
</tr>
<tr>
<td>1928-2004</td>
<td>7.92%</td>
<td>6.02%</td>
</tr>
<tr>
<td>1964-2004</td>
<td>5.82%</td>
<td>4.59%</td>
</tr>
<tr>
<td>1994-2004</td>
<td>8.60%</td>
<td>6.85%</td>
</tr>
</tbody>
</table>
If you choose to use historical premiums....

- Go back as far as you can. A risk premium comes with a standard error. Given the annual standard deviation in stock prices is about 25%, the standard error in a historical premium estimated over 25 years is roughly:
  
  \[
  \text{Standard Error in Premium} = \frac{25\%}{\sqrt{25}} = \frac{25\%}{5} = 5\% 
  \]

- Be consistent in your use of the riskfree rate. Since we argued for long term bond rates, the premium should be the one over T.Bonds

- Use the geometric risk premium. It is closer to how investors think about risk premiums over long periods.
Risk Premium for a Mature Market? Broadening the sample

**Equity Risk Premiums - By Country**

Average Risk Premium across all mature equity markets = 4%

![Bar Chart]

- Australia
- Belgium
- Canada
- Denmark
- France
- Germany
- Ireland
- Italy
- Japan
- Netherlands
- South Africa
- Spain
- Sweden
- Switzerland
- UK
- USA
- World
Two Ways of Estimating Country Equity Risk Premiums for other markets..

- **Default spread on Country Bond**: In this approach, the country equity risk premium is set equal to the default spread of the bond issued by the country (but only if it is denominated in a currency where a default free entity exists).
  - Brazil was rated B2 by Moody’s and the default spread on the Brazilian dollar denominated C.Bond at the end of August 2004 was 6.01%. (10.30%-4.29%)

- **Relative Equity Market approach**: The country equity risk premium is based upon the volatility of the market in question relative to U.S market.
  
  Total equity risk premium = Risk Premium_{US} \times \frac{\sigma_{Country Equity}}{\sigma_{US Equity}}

  Using a 4.82% premium for the US, this approach would yield:
  
  Total risk premium for Brazil = 4.82\% \times \frac{34.56\%}{19.01\%} = 8.76\%

  Country equity risk premium for Brazil = 8.76\% - 4.82\% = 3.94\%

  (The standard deviation in weekly returns from 2002 to 2004 for the Bovespa was 34.56\% whereas the standard deviation in the S&P 500 was 19.01\%)
And a third approach

- Country ratings measure default risk. While default risk premiums and equity risk premiums are highly correlated, one would expect equity spreads to be higher than debt spreads.

- Another is to multiply the bond default spread by the relative volatility of stock and bond prices in that market. In this approach:
  - Country Equity risk premium = Default spread on country bond* $\sigma_{\text{Country Equity}} / \sigma_{\text{Country Bond}}$
  - Standard Deviation in Bovespa (Equity) = 34.56%
  - Standard Deviation in Brazil C-Bond = 26.34%
  - Default spread on C-Bond = 6.01%
  - Country Equity Risk Premium = 6.01% (34.56%/26.34%) = 7.89%
Can country risk premiums change? Updating Brazil in January 2004

Brazil’s financial standing and country rating improved dramatically towards the end of 2004. Its rating improved to B1. In January 2005, the interest rate on the Brazilian C-Bond dropped to 7.73%. The US treasury bond rate that day was 4.22%, yielding a default spread of 3.51% for Brazil.

- Standard Deviation in Bovespa (Equity) = 25.09%
- Standard Deviation in Brazil C-Bond = 15.12%
- Default spread on C-Bond = 3.51%
- Country Risk Premium for Brazil = 3.51% (25.09%/15.12%) = 5.82%
From Country Equity Risk Premiums to Corporate Equity Risk premiums

- **Approach 1:** Assume that every company in the country is equally exposed to country risk. In this case,
  \[
  E(\text{Return}) = \text{Riskfree Rate} + \text{Country ERP} + \beta \times (\text{US premium})
  \]
  Implicitly, this is what you are assuming when you use the local Government’s dollar borrowing rate as your riskfree rate.

- **Approach 2:** Assume that a company’s exposure to country risk is similar to its exposure to other market risk.
  \[
  E(\text{Return}) = \text{Riskfree Rate} + \beta \times (\text{US premium} + \text{Country ERP})
  \]

- **Approach 3:** Treat country risk as a separate risk factor and allow firms to have different exposures to country risk (perhaps based upon the proportion of their revenues come from non-domestic sales)
  \[
  E(\text{Return}) = \text{Riskfree Rate} + \beta \times (\text{US premium}) + \lambda \times (\text{Country ERP})
  \]

*ERP: Equity Risk Premium*
Estimating Company Exposure to Country Risk: Determinants

- **Source of revenues:** Other things remaining equal, a company should be more exposed to risk in a country if it generates more of its revenues from that country. A Brazilian firm that generates the bulk of its revenues in Brazil should be more exposed to country risk than one that generates a smaller percent of its business within Brazil.

- **Manufacturing facilities:** Other things remaining equal, a firm that has all of its production facilities in Brazil should be more exposed to country risk than one which has production facilities spread over multiple countries. The problem will be accentuated for companies that cannot move their production facilities (mining and petroleum companies, for instance).

- **Use of risk management products:** Companies can use both options/futures markets and insurance to hedge some or a significant portion of country risk.
Estimating Lambdas: The Revenue Approach

- The easiest and most accessible data is on revenues. Most companies break their revenues down by region. One simplistic solution would be to do the following:
  \[ \lambda = \frac{\text{% of revenues domestically}_{\text{firm}}}{\text{% of revenues domestically}_{\text{avg firm}}} \]

- Consider, for instance, Embraer and Embratel, both of which are incorporated and traded in Brazil. Embraer gets 3% of its revenues from Brazil whereas Embratel gets almost all of its revenues in Brazil. The average Brazilian company gets about 77% of its revenues in Brazil:
  - \( \lambda_{\text{Embraer}} = \frac{3\%}{77\%} = 0.04 \)
  - \( \lambda_{\text{Embratel}} = \frac{100\%}{77\%} = 1.30 \)

- There are two implications
  - A company’s risk exposure is determined by where it does business and not by where it is located
  - Firms might be able to actively manage their country risk exposures
Estimating Lambdas: Earnings Approach

Figure 2: EPS changes versus Country Risk: Embraer and Embratel
Estimating Lambdas: Stock Returns versus C-Bond Returns

\[ \text{Return}_{\text{Embraer}} = 0.0195 + 0.2681 \text{Return}_{\text{C Bond}} \]
\[ \text{Return}_{\text{Embratel}} = -0.0308 + 2.0030 \text{Return}_{\text{C Bond}} \]
Assume that the beta for Embraer is 1.07, and that the riskfree rate used is 4.29%. Also assume that the risk premium for the US is 4.82% and the country risk premium for Brazil is 7.89%.

Approach 1: Assume that every company in the country is equally exposed to country risk. In this case,

\[ E(\text{Return}) = 4.29\% + 1.07 \times (4.82\%) + 7.89\% = 17.34\% \]

Approach 2: Assume that a company’s exposure to country risk is similar to its exposure to other market risk.

\[ E(\text{Return}) = 4.29\% + 1.07 \times (4.82\% + 7.89\%) = 17.89\% \]

Approach 3: Treat country risk as a separate risk factor and allow firms to have different exposures to country risk (perhaps based upon the proportion of their revenues come from non-domestic sales)

\[ E(\text{Return})= 4.29\% + 1.07(4.82\%) + 0.27 \times (7.89\%) = 11.58\% \]
Valuing Emerging Market Companies with significant exposure in developed markets

The conventional practice in investment banking is to add the country equity risk premium on to the cost of equity for every emerging market company, notwithstanding its exposure to emerging market risk. Thus, Embraer would have been valued with a cost of equity of 17.34% even though it gets only 3% of its revenues in Brazil. As an investor, which of the following consequences do you see from this approach?

A. Emerging market companies with substantial exposure in developed markets will be significantly over valued by equity research analysts.

B. Emerging market companies with substantial exposure in developed markets will be significantly under valued by equity research analysts.

Can you construct an investment strategy to take advantage of the misvaluation?
Implied Equity Premiums

- If we assume that stocks are correctly priced in the aggregate and we can estimate the expected cashflows from buying stocks, we can estimate the expected rate of return on stocks by computing an internal rate of return. Subtracting out the riskfree rate should yield an implied equity risk premium.
- This implied equity premium is a forward looking number and can be updated as often as you want (every minute of every day, if you are so inclined).
### Implied Equity Premiums

We can use the information in stock prices to back out how risk averse the market is and how much of a risk premium it is demanding.

In 2004, dividends & stock buybacks were 2.90% of the index, generating 35.15 in cashflows

<table>
<thead>
<tr>
<th>Year</th>
<th>Cashflow</th>
<th>Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>38.13</td>
<td>8.5%</td>
</tr>
<tr>
<td>2</td>
<td>41.37</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>44.89</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>48.71</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>52.85</td>
<td></td>
</tr>
</tbody>
</table>

After year 5, we will assume that earnings on the index will grow at 4.22%, the same rate as the entire economy.

January 1, 2005
S&P 500 is at 1211.92

If you pay the current level of the index, you can expect to make a return of 7.87% on stocks (which

\[
1211.92 = \frac{38.13}{(1 + r)} + \frac{41.37}{(1 + r)^2} + \frac{44.89}{(1 + r)^3} + \frac{48.71}{(1 + r)^4} + \frac{52.85}{(1 + r)^5} + \frac{52.85(1.0422)}{(r - 0.0422)(1 + r)^5}
\]

Implied Equity risk premium = Expected return on stocks - Treasury bond rate = 7.87% - 4.22% = 3.65%
Implied Risk Premium Dynamics

- Assume that the index jumps 10% on January 2 and that nothing else changes. What will happen to the implied equity risk premium?
  - Implied equity risk premium will increase
  - Implied equity risk premium will decrease

- Assume that the earnings jump 10% on January 2 and that nothing else changes. What will happen to the implied equity risk premium?
  - Implied equity risk premium will increase
  - Implied equity risk premium will decrease

- Assume that the riskfree rate increases to 5% on January 2 and that nothing else changes. What will happen to the implied equity risk premium?
  - Implied equity risk premium will increase
  - Implied equity risk premium will decrease
Implied Premiums in the US

Implied Premium for US Equity Market

Year

Implied Premium

0.00% 1.00% 2.00% 3.00% 4.00% 5.00% 6.00% 7.00%
Implied Premium versus RiskFree Rate

Expected Return on Stocks = T.Bond Rate + Equity Risk Premium

T. Bond Rate

ERP

T.Bond Rate  Implied Premium (FCFE)
Implied Premiums: From Bubble to Bear Market… January 2000 to January 2003


- S&P 500 Index
- Implied Equity Risk Premium

Aswath Damodaran
Effect of Changing Tax Status of Dividends on Stock Prices - January 2003

- Expected Return on Stocks (Implied) in Jan 2003 = 7.91%
- Dividend Yield in January 2003 = 2.00%
- Assuming that dividends were taxed at 30% (on average) on 1/1/03 and that capital gains were taxed at 15%.
- After-tax expected return on stocks = 2%(1-.3)+5.91%(1-.15) = 6.42%
- If the tax rate on dividends drops to 15% and the after-tax expected return remains the same:
  2% (1-.15) + X% (1-.15) = 6.42%
  New Pre-tax required rate of return = 7.56%
  New equity risk premium = 3.75%
  Value of the S&P 500 at new equity risk premium = 965.11
  Expected Increase in index due to dividend tax change = 9.69%
Which equity risk premium should you use for the US?

- **Historical Risk Premium**: When you use the historical risk premium, you are assuming that premiums will revert back to a historical norm and that the time period that you are using is the right norm. You are also more likely to find stocks to be overvalued than undervalued (Why?)

- **Current Implied Equity Risk premium**: You are assuming that the market is correct in the aggregate but makes mistakes on individual stocks. If you are required to be market neutral, this is the premium you should use. (What types of valuations require market neutrality?)

- **Average Implied Equity Risk premium**: The average implied equity risk premium between 1960-2003 in the United States is about 4%. You are assuming that the market is correct on average but not necessarily at a point in time.
Level of the Index (S&P CNX Index) = 1219
- This is a market cap weighted index of the 500 largest companies in India and represents 90% of the market value of Indian companies

Dividends on the Index = 3.51% of 1219 (Simple average is 2.75%)

Other parameters
- Riskfree Rate = 5.50%
- Expected Growth (in Rs)
  - Next 5 years = 18% (Used expected growth rate in Earnings)
  - After year 5 = 5.5%

Solving for the expected return:
- Expected return on Equity = 11.76%
- Implied Equity premium = 11.76 - 5.5% = 6.16%
We can use the information in stock prices to back out how risk averse the market is and how much of a risk premium it is demanding.

Analysts are estimating an expected growth rate of 11.36% in earnings over the next 5 years for stocks in the DAX (Source: IBES)

Expected dividends and stock buybacks over next 5 years

<table>
<thead>
<tr>
<th>Year</th>
<th>Dividends</th>
<th>Buybacks</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>116.13</td>
<td></td>
<td>116.13</td>
</tr>
<tr>
<td>2</td>
<td>129.32</td>
<td></td>
<td>129.32</td>
</tr>
<tr>
<td>3</td>
<td>144.01</td>
<td></td>
<td>144.01</td>
</tr>
<tr>
<td>4</td>
<td>160.37</td>
<td></td>
<td>160.37</td>
</tr>
<tr>
<td>5</td>
<td>178.59</td>
<td></td>
<td>178.59</td>
</tr>
</tbody>
</table>

Assumed to grow at 3.95% a year forever after year 5.

Buy the index for 3905.65

Implied Equity Risk Premium: 7.78% - 3.95% = 3.83%

\[
\text{Implied Equity Risk Premium} = \frac{116.13}{(1+r)} + \frac{129.32}{(1+r)^2} + \frac{144.01}{(1+r)^3} + \frac{160.37}{(1+r)^4} + \frac{178.59}{(1+r)^5} + \frac{178.59(1.0395)}{(r-0.0395)(1+r)^5}
\]
Estimating Beta

- The standard procedure for estimating betas is to regress stock returns ($R_j$) against market returns ($R_m$) -
  \[ R_j = a + b R_m \]
  - where $a$ is the intercept and $b$ is the slope of the regression.
- The slope of the regression corresponds to the beta of the stock, and measures the riskiness of the stock.
- This beta has three problems:
  - It has high standard error
  - It reflects the firm’s business mix over the period of the regression, not the current mix
  - It reflects the firm’s average financial leverage over the period rather than the current leverage.
Beta Estimation: The Noise Problem
Beta Estimation: The Index Effect
Solutions to the Regression Beta Problem

- Modify the regression beta by
  - changing the index used to estimate the beta
  - adjusting the regression beta estimate, by bringing in information about the fundamentals of the company

- Estimate the beta for the firm using
  - the standard deviation in stock prices instead of a regression against an index
  - accounting earnings or revenues, which are less noisy than market prices.

- Estimate the beta for the firm from the bottom up without employing the regression technique. This will require
  - understanding the business mix of the firm
  - estimating the financial leverage of the firm

- Use an alternative measure of market risk not based upon a regression.
The Index Game…

\[ \text{Aracruz ADR} = 2.80\% + 1.00 \text{ S&P} \]

\[ \text{Aracruz} = 2.62\% + 0.22 \text{ Bovespa} \]
Determinants of Betas

Beta of Equity

Beta of Firm

Nature of product or service offered by company:
Other things remaining equal, the more discretionary the product or service, the higher the beta.

Implications
1. Cyclical companies should have higher betas than non-cyclical companies.
2. Luxury goods firms should have higher betas than basic goods.
3. High priced goods/service firms should have higher betas than low priced goods/services firms.
4. Growth firms should have higher betas.

Operating Leverage (Fixed Costs as percent of total costs):
Other things remaining equal the greater the proportion of the costs that are fixed, the higher the beta of the company.

Implications
1. Firms with high infrastructure needs and rigid cost structures should have higher betas than firms with flexible cost structures.
2. Smaller firms should have higher betas than larger firms.
3. Young firms should have

Financial Leverage:
Other things remaining equal, the greater the proportion of capital that a firm raises from debt, the higher its equity beta will be.

Implications
Highly levered firms should have higher betas than firms with less debt.
In a perfect world... we would estimate the beta of a firm by doing the following:

1. Start with the beta of the business that the firm is in.
2. Adjust the business beta for the operating leverage of the firm to arrive at the unlevered beta for the firm.
3. Use the financial leverage of the firm to estimate the equity beta for the firm:
   \[
   \text{Levered Beta} = \text{Unlevered Beta} \times \left(1 + (1 - \text{tax rate}) \times \frac{\text{Debt}}{\text{Equity}}\right)
   \]
Adjusting for operating leverage…

- Within any business, firms with lower fixed costs (as a percentage of total costs) should have lower unlevered betas. If you can compute fixed and variable costs for each firm in a sector, you can break down the unlevered beta into business and operating leverage components.
  - Unlevered beta = Pure business beta * (1 + (Fixed costs/ Variable costs))
- The biggest problem with doing this is informational. It is difficult to get information on fixed and variable costs for individual firms.
- In practice, we tend to assume that the operating leverage of firms within a business are similar and use the same unlevered beta for every firm.
Conventional approach: If we assume that debt carries no market risk (has a beta of zero), the beta of equity alone can be written as a function of the unlevered beta and the debt-equity ratio

\[ \beta_L = \beta_u (1 + ((1-t)D/E)) \]

In some versions, the tax effect is ignored and there is no (1-t) in the equation.

Debt Adjusted Approach: If beta carries market risk and you can estimate the beta of debt, you can estimate the levered beta as follows:

\[ \beta_L = \beta_u (1 + ((1-t)D/E)) - \beta_{\text{debt}} (1-t) (D/E) \]

While the latter is more realistic, estimating betas for debt can be difficult to do.
Bottom-up Betas

Step 1: Find the business or businesses that your firm operates in.

Step 2: Find publicly traded firms in each of these businesses and obtain their regression betas. Compute the simple average across these regression betas to arrive at an average beta for these publicly traded firms. Unlevered beta for business = Average beta across publicly traded firms / (1 + (1 - t) (Average Debt to Equity ratio across firms))

Step 3: Estimate how much value your firm derives from each of the different businesses it is in.

Step 4: Compute a weighted average of the unlevered betas of the different businesses (from step 2) using the weights from step 3. Bottom-up Unlevered beta for your firm = \( \sum \text{Weights} \times \text{Average unlevered betas of the individual business} \)

Step 5: Compute a levered beta (equity beta) for your firm using the market debt to equity ratio for your firm. Levered bottom-up beta = Unlevered beta (1 + (1-t) (Debt/Equity))

Possible Refinements

If you can, adjust this beta for differences between your firm and the comparable firms on operating leverage and product characteristics.

While revenues or operating income are often used as weights, it is better to try to estimate the value of each business.

If you expect the business mix of your firm to change over time, you can change the weights on a year-to-year basis.

If you expect your debt to equity ratio to change over time, the levered beta will change over time.
Why bottom-up betas?

- The standard error in a bottom-up beta will be significantly lower than the standard error in a single regression beta. Roughly speaking, the standard error of a bottom-up beta estimate can be written as follows:
  \[
  \text{Std error of bottom-up beta} = \frac{\text{Average Std Error across Betas}}{\sqrt{\text{Number of firms in sample}}}
  \]

- The bottom-up beta can be adjusted to reflect changes in the firm’s business mix and financial leverage. Regression betas reflect the past.

- You can estimate bottom-up betas even when you do not have historical stock prices. This is the case with initial public offerings, private businesses or divisions of companies.
Bottom-up Beta: Firm in Multiple Businesses
Disney in 2003

Start with the unlevered betas for the businesses

<table>
<thead>
<tr>
<th>Business</th>
<th>Comparable firms</th>
<th>Number of firms</th>
<th>Average levered beta</th>
<th>Median D/E</th>
<th>Unlevered beta</th>
<th>Cash/Firm Value</th>
<th>Corrected for cash</th>
</tr>
</thead>
<tbody>
<tr>
<td>Media Networks</td>
<td>Radio and TV broadcasting companies</td>
<td>24</td>
<td>1.23</td>
<td>20.45%</td>
<td>1.08</td>
<td>0.75%</td>
<td>1.0932</td>
</tr>
<tr>
<td>Parks and Resorts</td>
<td>Theme park &amp; Entertainment firms</td>
<td>9</td>
<td>1.63</td>
<td>120.76%</td>
<td>0.91</td>
<td>2.77%</td>
<td>0.9364</td>
</tr>
<tr>
<td>Studio Entertainment</td>
<td>Movie companies</td>
<td>11</td>
<td>1.35</td>
<td>27.96%</td>
<td>1.14</td>
<td>14.08%</td>
<td>1.3310</td>
</tr>
<tr>
<td>Consumer Products</td>
<td>Toy and apparel retailers; Entertainment software</td>
<td>77</td>
<td>1.14</td>
<td>9.18%</td>
<td>1.07</td>
<td>12.08%</td>
<td>1.2186</td>
</tr>
</tbody>
</table>

Estimate the unlevered beta for Disney’s businesses

<table>
<thead>
<tr>
<th>Business</th>
<th>Revenues in 2002</th>
<th>EV/Sales</th>
<th>Estimated Value</th>
<th>Firm Value Proportion</th>
<th>Unlevered beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Media Networks</td>
<td>$9,733</td>
<td>3.41</td>
<td>$33,162.67</td>
<td>47.32%</td>
<td>1.0932</td>
</tr>
<tr>
<td>Parks and Resorts</td>
<td>$6,465</td>
<td>2.37</td>
<td>$15,334.08</td>
<td>21.88%</td>
<td>0.9364</td>
</tr>
<tr>
<td>Studio Entertainment</td>
<td>$6,691</td>
<td>2.63</td>
<td>$17,618.07</td>
<td>25.14%</td>
<td>1.3310</td>
</tr>
<tr>
<td>Consumer Products</td>
<td>$2,440</td>
<td>1.63</td>
<td>$3,970.60</td>
<td>5.67%</td>
<td>1.2186</td>
</tr>
<tr>
<td>Disney</td>
<td>$25,329</td>
<td></td>
<td>$70,085.42</td>
<td>100.00%</td>
<td>1.1258</td>
</tr>
</tbody>
</table>

Estimate a levered beta for Disney
Market debt to equity ratio = 37.46%
Marginal tax rate = 37.60%
Levered beta = 1.1258 (1 + (1-.376)(.3746)) = 1.39
## Embraer’s Bottom-up Beta

<table>
<thead>
<tr>
<th>Business</th>
<th>Unlevered Beta</th>
<th>D/E Ratio</th>
<th>Levered Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aerospace</td>
<td>0.95</td>
<td>18.95%</td>
<td>1.07</td>
</tr>
</tbody>
</table>

Levered Beta = Unlevered Beta \( \times (1 + (1 - \text{tax rate}) \times \text{D/E Ratio}) \)

= 0.95 \( \times (1 + (1-.34) \times 0.1895) \) = 1.07
Comparable Firms?

Can an unlevered beta estimated using U.S. and European aerospace companies be used to estimate the beta for a Brazilian aerospace company?

☐ Yes

☐ No

What concerns would you have in making this assumption?
Gross Debt versus Net Debt Approaches

- Gross Debt Ratio for Embraer = $1953/11,042 = 18.95\%$
- Levered Beta using Gross Debt ratio = 1.07
- Net Debt Ratio for Embraer = (Debt - Cash)/ Market value of Equity
  $= (1953-2320)/11,042 = -3.32\%$
- Levered Beta using Net Debt Ratio = $0.95 (1 + (1-.34) (-.0332)) = 0.93$
- The cost of Equity using net debt levered beta for Embraer will be much lower than with the gross debt approach. The cost of capital for Embraer, though, will even out since the debt ratio used in the cost of capital equation will now be a net debt ratio rather than a gross debt ratio.
Small Firm and Other Premiums

- It is common practice to add premiums on to the cost of equity for firm-specific characteristics. For instance, many analysts add a small stock premium of 3-3.5% (historical premium for small stocks over the market) to the cost of equity for smaller companies.

- Adding arbitrary premiums to the cost of equity is always a dangerous exercise. If small stocks are riskier than larger stocks, we need to specify the reasons and try to quantify them rather than trust historical averages. (You could argue that smaller companies are more likely to serve niche (discretionary) markets or have higher operating leverage and adjust the beta to reflect this tendency).
The Cost of Equity: A Recap

Cost of Equity = Riskfree Rate + Beta * (Risk Premium)

Historical Premium
1. Mature Equity Market Premium: Average premium earned by stocks over T.Bonds in U.S.
2. Country risk premium = Country Default Spread* (\(\sigma_{\text{Equity}}/\sigma_{\text{Country bond}}\))

Implied Premium
Based on how equity market is priced today and a simple valuation model

Preferably, a bottom-up beta, based upon other firms in the business, and firm specific financial leverage

Has to be in the same currency as cash flows, and defined in same terms (real or nominal) as the cash flows
The cost of debt is the rate at which you can borrow at currently. It will reflect not only your default risk but also the level of interest rates in the market.

The two most widely used approaches to estimating cost of debt are:

- Looking up the yield to maturity on a straight bond outstanding from the firm. The limitation of this approach is that very few firms have long term straight bonds that are liquid and widely traded.
- Looking up the rating for the firm and estimating a default spread based upon the rating. While this approach is more robust, different bonds from the same firm can have different ratings. You have to use a median rating for the firm.

When in trouble (either because you have no ratings or multiple ratings for a firm), estimate a synthetic rating for your firm and the cost of debt based upon that rating.
The rating for a firm can be estimated using the financial characteristics of the firm. In its simplest form, the rating can be estimated from the interest coverage ratio

\[
\text{Interest Coverage Ratio} = \frac{\text{EBIT}}{\text{Interest Expenses}}
\]

For Embraer’s interest coverage ratio, we used the interest expenses from 2003 and the average EBIT from 2001 to 2003. (The aircraft business was badly affected by 9/11 and its aftermath. In 2002 and 2003, Embraer reported significant drops in operating income)

- Interest Coverage Ratio \(= \frac{462.1}{129.70} = 3.56\)
### Interest Coverage Ratios, Ratings and Default Spreads

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 8.50 ( &gt;12.50)</td>
<td>AAA</td>
<td>0.75%</td>
<td>0.35%</td>
</tr>
<tr>
<td>6.50 - 8.50 (9.5-12.5)</td>
<td>AA</td>
<td>1.00%</td>
<td>0.50%</td>
</tr>
<tr>
<td>5.50 - 6.50 (7.5-9.5)</td>
<td>A+</td>
<td>1.50%</td>
<td>0.70%</td>
</tr>
<tr>
<td>4.25 - 5.50 (6-7.5)</td>
<td>A</td>
<td>1.80%</td>
<td>0.85%</td>
</tr>
<tr>
<td>3.00 - 4.25 (4.5-6)</td>
<td>A–</td>
<td>2.00%</td>
<td>1.00%</td>
</tr>
<tr>
<td>2.50 - 3.00 (4-4.5)</td>
<td>BBB</td>
<td>2.25%</td>
<td>1.50%</td>
</tr>
<tr>
<td>2.25- 2.50 (3.5-4)</td>
<td>BB+</td>
<td>2.75%</td>
<td>2.00%</td>
</tr>
<tr>
<td>2.00 - 2.25 (3-3.5)</td>
<td>BB</td>
<td>3.50%</td>
<td>2.50%</td>
</tr>
<tr>
<td>1.75 - 2.00 (2.5-3)</td>
<td>B+</td>
<td>4.75%</td>
<td>3.25%</td>
</tr>
<tr>
<td>1.50 - 1.75 (2-2.5)</td>
<td>B</td>
<td>6.50%</td>
<td>4.00%</td>
</tr>
<tr>
<td>1.25 - 1.50 (1.5-2)</td>
<td>B–</td>
<td>8.00%</td>
<td>6.00%</td>
</tr>
<tr>
<td>0.80 - 1.25 (1.25-1.5)</td>
<td>CCC</td>
<td>10.00%</td>
<td>8.00%</td>
</tr>
<tr>
<td>0.65 - 0.80 (0.8-1.25)</td>
<td>CC</td>
<td>11.50%</td>
<td>10.00%</td>
</tr>
<tr>
<td>0.20 - 0.65 (0.5-0.8)</td>
<td>C</td>
<td>12.70%</td>
<td>12.00%</td>
</tr>
<tr>
<td>&lt; 0.20 (&lt;0.5)</td>
<td>D</td>
<td>15.00%</td>
<td>20.00%</td>
</tr>
</tbody>
</table>

The first number under interest coverage ratios is for larger market cap companies and the second in brackets is for smaller market cap companies. For Embraer, I used the interest coverage ratio table for smaller/riskier firms (the numbers in brackets) which yields a lower rating for the same interest coverage ratio.
Cost of Debt computations

- Companies in countries with low bond ratings and high default risk might bear the burden of country default risk, especially if they are smaller or have all of their revenues within the country.

- Larger companies that derive a significant portion of their revenues in global markets may be less exposed to country default risk. In other words, they may be able to borrow at a rate lower than the government.

- The synthetic rating for Embraer is A-. Using the 2004 default spread of 1.00%, we estimate a cost of debt of 9.29% (using a riskfree rate of 4.29% and adding in two thirds of the country default spread of 6.01%):

Cost of debt

= Riskfree rate + 2/3(Brazil country default spread) + Company default spread = 4.29% + 4.00% + 1.00% = 9.29%
Synthetic Ratings: Some Caveats

- The relationship between interest coverage ratios and ratings, developed using US companies, tends to travel well, as long as we are analyzing large manufacturing firms in markets with interest rates close to the US interest rate.
- They are more problematic when looking at smaller companies in markets with higher interest rates than the US.
Weights for the Cost of Capital Computation

- The weights used to compute the cost of capital should be the market value weights for debt and equity.
- There is an element of circularity that is introduced into every valuation by doing this, since the values that we attach to the firm and equity at the end of the analysis are different from the values we gave them at the beginning.
- As a general rule, the debt that you should subtract from firm value to arrive at the value of equity should be the same debt that you used to compute the cost of capital.
Estimating Cost of Capital: Embraer

- **Equity**
  - Cost of Equity = 4.29% + 1.07 (4%) + 0.27 (7.89%) = 10.70%
  - Market Value of Equity = 11,042 million BR ($3,781 million)

- **Debt**
  - Cost of debt = 4.29% + 4.00% + 1.00% = 9.29%
  - Market Value of Debt = 2,083 million BR ($713 million)

- **Cost of Capital**
  \[
  \text{Cost of Capital} = 10.70\% \times 0.84 + 9.29\% \times (1 - 0.34) \times 0.16 = 9.97\%
  \]

The book value of equity at Embraer is 3,350 million BR.
The book value of debt at Embraer is 1,953 million BR; Interest expense is 222 mil BR;
  Average maturity of debt = 4 years

Estimated market value of debt = 222 million (PV of annuity, 4 years, 9.29%) + $1,953 million/1.0929^4 = 2,083 million BR
If you had to do it….Converting a Dollar Cost of Capital to a Nominal Real Cost of Capital

Approach 1: Use a BR riskfree rate in all of the calculations above. For instance, if the BR riskfree rate was 12%, the cost of capital would be computed as follows:
- Cost of Equity = 12% + 1.07(4%) + 0.27 (7.89%) = 18.41%
- Cost of Debt = 12% + 1% = 13%
- (This assumes the riskfree rate has no country risk premium embedded in it.)

Approach 2: Use the differential inflation rate to estimate the cost of capital. For instance, if the inflation rate in BR is 8% and the inflation rate in the U.S. is 2%

\[
\text{Cost of capital} = \left(1 + \text{Cost of Capital}_\$\right) \frac{1 + \text{Inflation}_\text{BR}}{1 + \text{Inflation}_\$} = 16.44\%
\]
Dealing with Hybrids and Preferred Stock

- When dealing with hybrids (convertible bonds, for instance), break the security down into debt and equity and allocate the amounts accordingly. Thus, if a firm has $125 million in convertible debt outstanding, break the $125 million into straight debt and conversion option components. The conversion option is equity.

- When dealing with preferred stock, it is better to keep it as a separate component. The cost of preferred stock is the preferred dividend yield. (As a rule of thumb, if the preferred stock is less than 5% of the outstanding market value of the firm, lumping it in with debt will make no significant impact on your valuation).
Decomposing a convertible bond...

Assume that the firm that you are analyzing has $125 million in face value of convertible debt with a stated interest rate of 4%, a 10 year maturity and a market value of $140 million. If the firm has a bond rating of A and the interest rate on A-rated straight bond is 8%, you can break down the value of the convertible bond into straight debt and equity portions.

- Straight debt = (4% of $125 million) (PV of annuity, 10 years, 8%) + 125 million/1.08^{10} = $91.45 million
- Equity portion = $140 million - $91.45 million = $48.55 million
Recapping the Cost of Capital

Cost of Capital = Cost of Equity \left( \frac{\text{Equity}}{\text{Debt} + \text{Equity}} \right) + \text{Cost of Borrowing} \left(1 - t\right) \left( \frac{\text{Debt}}{\text{Debt} + \text{Equity}} \right)

- Cost of equity based upon bottom-up beta
- Cost of borrowing should be based upon:
  1. synthetic or actual bond rating
  2. default spread
- Marginal tax rate, reflecting tax benefits of debt
- Weights should be market value weights