The Cost of Illiquidity

Aswath Damodaran
What is illiquidity?

- The simplest way to think about illiquidity is to consider it the cost of buyer’s remorse: it is the cost of reversing an asset trade almost instantaneously after you make the trade.
- Defined thus, all assets are illiquid. The difference is really a continuum, with some assets being more liquid than others.
- The notion that publicly traded firms are liquid and private businesses are not is too simplistic.

<table>
<thead>
<tr>
<th>Most liquid</th>
<th>Least liquid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treasury bonds and bills</td>
<td>Private business without control</td>
</tr>
<tr>
<td>Highly rated corporate bonds</td>
<td>Private business with control</td>
</tr>
<tr>
<td>Liquid, widely held stock in developed market</td>
<td>Real assets</td>
</tr>
<tr>
<td>Stock in traded company with small float</td>
<td>Stock in lightly traded, OTC or emerging market stock</td>
</tr>
</tbody>
</table>

Which is more illiquid?
The Components of Trading Costs for an asset

- **Brokerage Cost**: This is the most explicit of the costs that any investor pays but it is by far the smallest component.

- **Bid-Ask Spread**: The spread between the price at which you can buy an asset (the dealer’s ask price) and the price at which you can sell the same asset at the same point in time (the dealer’s bid price).

- **Price Impact**: The price impact that an investor can create by trading on an asset, pushing the price up when buying the asset and pushing it down while selling.

- **Opportunity Cost**: There is the opportunity cost associated with waiting to trade. While being a patient trader may reduce the previous two components of trading cost, the waiting can cost profits both on trades that are made and in terms of trades that would have been profitable if made instantaneously but which became unprofitable as a result of the waiting.
Why is there a bid-ask spread?

- In most markets, there is a dealer or market maker who sets the bid-ask spread, and there are three types of costs that the dealer faces that the spread is designed to cover.
  - The first is the risk cost of holding inventory;
  - the second is the cost of processing orders and
  - the final cost is the cost of trading with more informed investors.
- The spread has to be large enough to cover these costs and yield a reasonable profit to the market maker on his or her investment in the profession.
The Magnitude of the Spread

Figure 5.2: Prices and Spreads by Market Cap
More Evidence of Bid-Ask Spreads

- The spreads in U.S. government securities are much lower than the spreads on traded stocks in the United States. For instance, the typical bid-ask spread on a Treasury bill is less than 0.1% of the price.

- The spreads on corporate bonds tend to be larger than the spreads on government bonds, with safer (higher rated) and more liquid corporate bonds having lower spreads than riskier (lower rated) and less liquid corporate bonds.

- The spreads in non-U.S. equity markets are generally much higher than the spreads on U.S. markets, reflecting the lower liquidity in those markets and the smaller market capitalization of the traded firms.

- While the spreads in the traded commodity markets are similar to those in the financial asset markets, the spreads in other real asset markets (real estate, art...) tend to be much larger.
The Determinants of the Bid-Ask Spread

- Studies by Tinic and West (1972), Stoll (1978) and Jegadeesh and Subrahmanyam (1993) find that spreads as a percentage of the price are correlated negatively with the price level, volume and the number of market makers, and positively with volatility. Each of these findings is consistent with the theory on the bid-ask spread.

- A study by Kothare and Laux, that looked at average spreads on the NASDAQ also looked at differences in bid-ask spreads across stocks on the NASDAQ. In addition to noting similar correlations between the bid-ask spreads, price level and trading volume, they uncovered an interesting new variable. They found that stocks where institutional activity increased significantly had the biggest increase in bid-ask spreads. It might also reflect the perception on the part of market makers that institutional investors tend to be informed investors with more or better information.
Why is there a price impact?

- The first is that markets are not completely liquid. A large trade can create an imbalance between buy and sell orders, and the only way in which this imbalance can be resolved is with a price change. This price change, that arises from lack of liquidity, will generally be temporary and will be reversed as liquidity returns to the market.

- The second reason for the price impact is informational. A large trade attracts the attention of other investors in that asset market because it might be motivated by new information that the trader possesses. This price effect will generally not be temporary, especially when we look at a large number of stocks where such large trades are made. While investors are likely to be wrong a fair proportion of the time on the informational value of large block trades, there is reason to believe that they will be right almost as often.
How large is the price impact? Evidence from Studies of Block Trades

Figure 5.3: Annualized Returns from Buying after Block Trades

Aswath Damodaran
Limitations of the Block Trade Studies

- These and similar studies suffer from a sampling bias - they tend to look at large block trades in liquid stocks on the exchange floor – they also suffer from another selection bias, insofar as they look only at actual executions.
- The true cost of market impact arises from those trades that would have been done in the absence of a market impact but were not because of the perception that it would be large.
Round-Trip Costs (including Price Impact) as a Function of Market Cap and Trade Size

<table>
<thead>
<tr>
<th>Sector</th>
<th>5</th>
<th>25</th>
<th>250</th>
<th>500</th>
<th>1000</th>
<th>2500</th>
<th>5000</th>
<th>10000</th>
<th>20000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smallest</td>
<td>17.30%</td>
<td>27.30%</td>
<td>43.80%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8.90%</td>
<td>12.00%</td>
<td>23.80%</td>
<td>33.40%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5.00%</td>
<td>7.60%</td>
<td>18.80%</td>
<td>25.90%</td>
<td>30.00%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4.30%</td>
<td>5.80%</td>
<td>9.60%</td>
<td>16.90%</td>
<td>25.40%</td>
<td>31.50%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2.80%</td>
<td>3.90%</td>
<td>5.90%</td>
<td>8.10%</td>
<td>11.50%</td>
<td>15.70%</td>
<td>25.70%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.80%</td>
<td>2.10%</td>
<td>3.20%</td>
<td>4.40%</td>
<td>5.60%</td>
<td>7.90%</td>
<td>11.00%</td>
<td>16.20%</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1.90%</td>
<td>2.00%</td>
<td>3.10%</td>
<td>4.00%</td>
<td>5.60%</td>
<td>7.70%</td>
<td>10.40%</td>
<td>14.30%</td>
<td>20.00%</td>
</tr>
<tr>
<td>8</td>
<td>1.90%</td>
<td>1.90%</td>
<td>2.70%</td>
<td>3.30%</td>
<td>4.60%</td>
<td>6.20%</td>
<td>8.90%</td>
<td>13.60%</td>
<td>18.10%</td>
</tr>
<tr>
<td>Largest</td>
<td>1.10%</td>
<td>1.20%</td>
<td>1.30%</td>
<td>1.71%</td>
<td>2.10%</td>
<td>2.80%</td>
<td>4.10%</td>
<td>5.90%</td>
<td>8.00%</td>
</tr>
</tbody>
</table>
Determinants of Price Impact

- Looking at the evidence, the variables that determine that price impact of trading seem to be the same variables that drive the bid-ask spread. That should not be surprising. The price impact and the bid-ask spread are both a function of the liquidity of the market. The inventory costs and adverse selection problems are likely to be largest for stocks where small trades can move the market significantly.

- In many real asset markets, the difference between the price at which one can buy the asset and the price at which one can sell, at the same point in time, is a reflection of both the bid-ask spread and the expected price impact of the trade on the asset. Not surprisingly, this difference can be very large in markets where trading is infrequent; in the collectibles market, this cost can amount to more than 20% of the value of the asset.
The Theory on Illiquidity Discounts

- **Illiquidity discount on value**: You should reduce the value of an asset by the expected cost of trading that asset over its lifetime.
  - The illiquidity discount should be greater for assets with higher trading costs
  - The illiquidity discount should be decrease as the time horizon of the investor holding the asset increases

- **Illiquid assets should be valued using higher discount rates**
  - Risk-Return model: Some illiquidity risk is systematic. In other words, the illiquidity increases when the market is down. This risk should be built into the discount rate.
  - Empirical: Assets that are less liquid have historically earned higher returns. Relating returns to measures of illiquidity (turnover rates, spreads etc.) should allow us to estimate the discount rate for less liquid assets.

- **Illiquidity can be valued as an option**: When you are not allowed to trade an asset, you lose the option to sell it if the price goes up (and you want to get out).
a. Illiquidity Discount in Value

Amihud and Mendelson make the interesting argument that when you pay for an asset today will incorporate the present value of all expected future transactions costs on that asset. For instance, assume that the transactions costs are 2% of the price and that the average holding period is 1 year. The illiquidity discount can be computed as follows:

\[
\text{Illiquidity discount} = \frac{2\%}{(1.10)} + \frac{2\%}{(1.10)^2} + \frac{2\%}{(1.10)^3} \ldots = \frac{2\%}{.10} = 20\%
\]

With a holding period of 3 years, the illiquidity discount will be much smaller (about 6.67%)

It follows then that the illiquidity discount will be

An increasing function of transactions costs

A decreasing function of the average holding period
b. Adjusting discount rates for illiquidity

- Liquidity as a systematic risk factor
  - If liquidity is correlated with overall market conditions, less liquid stocks should have more market risk than more liquid stocks.
  - To estimate the cost of equity for stocks, we would then need to estimate a “liquidity beta” for every stock and multiply this liquidity beta by a liquidity risk premium.
  - The liquidity beta is not a measure of liquidity, per se, but a measure of liquidity that is correlated with market conditions.

- Liquidity premiums
  - You can always add liquidity premiums to conventional risk and return models to reflect the higher risk of less liquid stocks.
  - These premiums are usually based upon historical data and reflect what you would have earned on less liquid investments historically (usually smaller stocks with lower trading volume) relative to more liquid investments. Amihud and Mendelson estimate that the expected return increases about 0.25% for every 1% increase in the bid-ask spread.
c. Illiquidity as a lookback option

- Longstaff (1995) presents an upper bound for the option by considering an investor with perfect market timing abilities who owns an asset on which she is not allowed to trade for a period.
- In the absence of trading restrictions, this investor would sell at the maximum price that an asset reaches during the time period and the value of the lookback option estimated using this maximum price should be the outer bound for the value of illiquidity. Using this approach,
Valuing the Lookback Option

Figure 3: Upper bounds on Marketability Discount - Option Pricing Model

- Trade Restriction Period
- Illiquidity discount (as % of Value)

- s = 10%
- s = 20%
- s = 30%
The Cost of Illiquidity: Empirical Evidence
Bond Market

- **T.Bills versus T.Bonds**: The yield on the less liquid treasury bond was higher on an annualized basis than the yield on the more liquid treasury bill, a difference attributed to illiquidity.

- **Corporate Bonds**: A study compared over 4000 corporate bonds in both investment grade and speculative categories, and concluded that illiquid bonds had much higher yield spreads than liquid bonds. This study found that liquidity decreases as they moved from higher bond ratings to lower ones and increased as they move from short to long maturities.

- **Overall**: The consensus finding is that liquidity matters for all bonds, but that it matters more with risky bonds than with safer bonds.
The Cost of Illiquidity: Equity Markets - Cross Sectional Differences

- **Trading volume**: Brennan, Chordia and Subrahmanayam (1998) find that dollar trading volume and stock returns are negatively correlated, after adjusting for other sources of market risk. Datar,

- **Turnover Ratio**: Nair and Radcliffe (1998) use the turnover ratio as a proxy for liquidity. After controlling for size and the market to book ratio, they conclude that liquidity plays a significant role in explaining differences in returns, with more illiquid stocks (in the 90th percentile of the turnover ratio) having annual returns that are about 3.25% higher than liquid stocks (in the 10th percentile of the turnover ratio). In addition, they conclude that every 1% increase in the turnover ratio reduces annual returns by approximately 0.54%.

- **And it is not a size or price to book effect**: Nguyen, Mishra and Prakash (2005) conclude that stocks with higher turnover ratios do have lower expected returns. They also find that market capitalization and price to book ratios, two widely used proxies that have been shown to explain differences in stock returns, do not proxy for illiquidity
Controlled Studies

- All of the studies noted on the last page can be faulted because they cannot control for liquidity perfectly. Illiquid stocks are more likely to be in smaller companies that are not held by institutional investors. No matter how carefully a study is done, it will be difficult to categorically state that the observed return differences are due to liquidity.

- The studies that carry the most weight for measuring illiquidity, therefore, are studies where we can control for the difference. Usually, they involved shares issued by the same company, with the only difference being that one set of shares is liquid and the other is not. The difference in price can then be attributed entirely to illiquidity.
a. Restricted Stock Studies

- Restricted securities are securities issued by a company, but not registered with the SEC, that can be sold through private placements to investors, but cannot be resold in the open market for a one-year holding period, and limited amounts can be sold after that. Restricted securities trade at significant discounts on publicly traded shares in the same company.
  - Maher examined restricted stock purchases made by four mutual funds in the period 1969-73 and concluded that they traded an average discount of 35.43% on publicly traded stock in the same companies.
  - Moroney reported a mean discount of 35% for acquisitions of 146 restricted stock issues by 10 investment companies, using data from 1970.
  - In a recent study of this phenomenon, Silber finds that the median discount for restricted stock is 33.75%.

- Many of these older studies were done when the restriction stretched to two years. More recent studies since the change in the holding period come back with lower values for the discount (20-25%).
The problems with restricted stock

There are three statistical problems with extrapolating from restricted stock studies.

- First, these studies are based upon small sample sizes, spread out over long time periods, and the standard errors in the estimates are substantial.
- Second, most firms do not make restricted stock issues and the firms that do make these issues tend to be smaller, riskier and less healthy than the typical firm. This selection bias may be skewing the observed discount.
- Third, the investors with whom equity is privately placed may be providing other services to the firm, for which the discount is compensation.

Bajaj, Dennis, Ferris and Sarin compute a discount of 9.83% for private placements, where there is no illiquidity, and argue that controlling for differences across companies making restricted stock results in an illiquidity discount of 7.23% for restricted stock.
b. Initial Public Offerings.

Figure 4: Discount on IPO Price: Discounts on stocks sold in five months prior to the IPO

Number of transactions

Period


0% 10% 20% 30% 40% 50% 60%
There are two problems with the IPO studies that make us reluctant to conclude that it is illiquidity.

- The first is the sheer size of the discount suggests that there may be something else going on in these transactions. In particular, these might not be arms length transactions and the sellers of these shares may be getting compensating benefits elsewhere.
- The second is that there may be uncertainty about whether the IPO will go through and if it does, the price at which the company will go public. The discount may reflect how much the sellers are willing to pay to accept a certainty equivalent of a risky cash flow.
c. Companies with different share classes

- Some companies have multiple classes of shares in the same market, with some classes being more liquid than others. If there are no other differences (in voting rights or dividends, for instance) across the classes, the difference in prices can be attributed to liquidity.

- Chen and Xiong (2001) compare the market prices of the traded common stock in 258 Chinese companies with the auction and private placement prices of the RIS shares and conclude that the discount on the latter is 78% for auctions and almost 86% for private placements.

- There are companies in emerging markets with ADRs listed for their stock in the US. The ADRs historically have traded at significant premiums over the domestic listings and some of the difference can be attributed to the higher liquidity of the US market.
Dealing with illiquidity in valuation

- If we accept that illiquidity affects value, and both the theory and empirical evidence suggest that it does, the question becomes how best to bring it into the value.
- There are three choices:
  - Estimate the value of the asset as if it were a liquid asset and then discount that value for illiquidity
  - Adjust the discount rates and use a higher discount rate for illiquid companies
  - Estimate the illiquidity discount by looking at comparable companies and seeing how much their values are impacted by illiquidity
Illiquidity Discount  
The Rule of Thumb approach

- In private company valuation, illiquidity is a constant theme that analysts talk about.

- All the talk, though, seems to lead to a rule of thumb. The illiquidity discount for a private firm is between 20-30% and does not vary much across private firms.

- In our view, this reflects the objective of many appraisers of private companies which has been to get the largest discount that the courts will accept rather than the right illiquidity discount.
Determinants of the Illiquidity Discount

1. *Liquidity of assets owned by the firm*: The fact that a private firm is difficult to sell may be rendered moot if its assets are liquid and can be sold with no significant loss in value. A private firm with significant holdings of cash and marketable securities should have a lower illiquidity discount than one with factories or other assets for which there are relatively few buyers.

2. *Financial Health and Cash flows of the firm*: A private firm that is financially healthy should be easier to sell than one that is not healthy. In particular, a firm with strong earnings and positive cash flows should be subject to a smaller illiquidity discount than one with losses and negative cash flows.

3. *Possibility of going public in the future*: The greater the likelihood that a private firm can go public in the future, the lower should be the illiquidity discount attached to its value. In effect, the probability of going public is built into the valuation of the private firm.

4. *Size of the Firm*: If we state the illiquidity discount as a percent of the value of the firm, it should become smaller as the size of the firm increases.

5. *Control Component*: Investing in a private firm is decidedly more attractive when you acquire a controlling stake with your investment. A reasonable argument can be made that a 51% stake in a private business should be more liquid than a 49% stake in the same business.
Illiquidity Discounts and Type of Business

- Rank the following assets (or private businesses) in terms of the liquidity discount you would apply to your valuation (from biggest discount to smallest)
  - A New York City Cab Medallion
  - A small privately owned five-and-dime store in your town
  - A large privately owned conglomerate, with significant cash balances and real estate holdings.
  - A large privately owned ski resort that is losing money
Illiquidity Discount
Firm-specific discounts

- Intuitively, it seems reasonable that illiquidity discounts should be different for different firms and assets.
- In practice, there are three ways in which we can adjust discounts for different businesses.
  - Look at differences in discounts across companies that make restricted stock issues or private placements
  - Estimate a synthetic bid-ask spread for a private business using data from publicly traded stocks
  - Estimate a discount based upon an option pricing model
1. Exploiting Cross Sectional Differences : Restricted Stock

- Silber (1991) develops the following relationship between the size of the discount and the characteristics of the firm issuing the registered stock –
  \[ \text{LN}(RPRS) = 4.33 + 0.036 \text{LN}(REV) - 0.142 \text{LN}(RBRT) + 0.174 \text{DERN} + 0.332 \text{DCUST} \]
  where,
  \( RPRS \) = Relative price of restricted stock (to publicly traded stock)
  \( REV \) = Revenues of the private firm (in millions of dollars)
  \( RBRT \) = Restricted Block relative to Total Common Stock in \%
  \( DERN \) = 1 if earnings are positive; 0 if earnings are negative;
  \( DCUST \) = 1 if there is a customer relationship with the investor; 0 otherwise;

- Interestingly, Silber finds no effect of introducing a control dummy - set equal to one if there is board representation for the investor and zero otherwise.
The Silber regression does provide us with a sense of how different the discount will be for a firm with small revenues versus one with large revenues. Consider, for example, two profitable firms that are equal in every respect except for revenues. Assume that the first firm has revenues of 10 million and the second firm has revenues of 100 million. The Silber regression predicts illiquidity discounts of the following:

- For firm with 100 million in revenues: 44.5%
- For firm with 10 million in revenues: 48.9%
- Difference in illiquidity discounts: 4.4%

If your base discount for a firm with 10 million in revenues is 25%, the illiquidity discount for a firm with 100 million in revenues would be 20.6%.
Liquidity Discount and Revenues

Figure 24.1: Illiquidity Discounts: Base Discount of 25% for profitable firms $10 million in revenues
Application to a private firm: Kristin Kandy

- Kristin Kandy is a profitable firm with $3 million in revenues.
- We computed the Silber regression discount using a base discount of 15% for a healthy firm with $10 million in revenues.
- The difference in illiquidity discount for a firm with $10 million in revenues and a firm with $3 million in revenues in the Silber regression is 2.17%.
- Adding this on to the base discount of 15% yields a total discount of 17.17%.
2. An Alternate Approach to the Illiquidity Discount: Bid Ask Spread

- As we noted earlier, the bid-ask spread is one very important component of the trading cost on a publicly traded asset. It can be loosely considered to be the illiquidity discount on a publicly traded stock.
- Studies have tied the bid-ask spread to:
  - the size of the firm
  - the trading volume on the stock
  - the degree

- Regressing the bid-ask spread against variables that can be measured for a private firm (such as revenues, cash flow generating capacity, type of assets, variance in operating income) and are also available for publicly traded firms offers promise.
A Bid-Ask Spread Regression

- Using data from the end of 2000, for instance, we regressed the bid-ask spread against annual revenues, a dummy variable for positive earnings (DERN: 0 if negative and 1 if positive), cash as a percent of firm value and trading volume.

\[
\text{Spread} = 0.145 - 0.0022 \ln (\text{Annual Revenues}) - 0.015 (\text{DERN}) - 0.016 (\text{Cash/Firm Value}) - 0.11 (\text{$/Monthly trading volume/Firm Value})
\]

- You could plug in the values for a private firm into this regression (with zero trading volume) and estimate the spread for the firm.

- The synthetic bid-ask spread was computed using the spread regression presented earlier and the inputs for Kristin Kandy (revenues = $3 million, positive earnings, cash/ firm value = 6.56% and no trading)

\[
\text{Spread} = 0.145 - 0.0022 \ln (3) - 0.015 (1) - 0.016 (0.0696) - 0.11 (0) = 0.1265 \text{or 12.65%}
\]
3. Option Based Discount

- Liquidity is sometimes modeled as a put option for the period when an investor is restricted from trading. Thus, the illiquidity discount on value for an asset where the owner is restricted from trading for 2 years will be modeled as a 2-year at-the-money put option.

- The problem with this is that liquidity does not give you the right to sell a stock at today’s market price anytime over the next 2 years. What it does give you is the right to sell at the prevailing market price anytime over the next 2 years.

- One variation that will work is to Assume that you have a disciplined investor who always sells investments, when the price rises 25% above the original buying price. Not being able to trade on this investment for a period (say, 2 years) undercuts this discipline and it can be argued that the value of illiquidity is the product of the value of the put option (estimated using a strike price set 25% above the purchase price and a 2 year life) and the probability that the stock price will rise 25% or more over the next 2 years.
An option based discount for Kristin Kandy

- To value illiquidity as an option, we chose arbitrary values for illustrative purposes of an upper limit on the price (at which you would have sold) of 20% above the current value, an industry average standard deviation of 25% and a 1-year trading restriction. The resulting option has the following parameters:
  - \( S = \text{Estimated value of equity} = 1,796 \text{ million}; \ K = 1,796 \times (1.20) = 2,155 \text{ million}; \ t = 1; \ \text{Riskless rate} = 4.5\% \text{ and } \sigma = 25\% \)
  - Put Option value = $354 million

- The probability that the stock price will increase more than 20% over the next year was computed from a normal distribution with the average = 16.26% (cost of equity) and standard deviation = 25%.
  \[ Z = \frac{20-16.26}{25} = 0.15 \quad N(Z) = 0.5595 \]

- Value of liquidity = Value of option to sell at 20% above the current stock price * Probability that stock price will increase by more than 20% over next year = $354 million * 0.4405 = $156 million
## A Comparison of Illiquidity Discounts

<table>
<thead>
<tr>
<th>Approach</th>
<th>Estimated Discount</th>
<th>Liquidity Adjusted Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Discount- Restricted Stock</td>
<td>25.00%</td>
<td>$1,347.00</td>
</tr>
<tr>
<td>Fixed Discount- Restricted Stock vs Registered Placements</td>
<td>15.00%</td>
<td>$1,526.60</td>
</tr>
<tr>
<td>15% base discount adjusted for Revenues/Health (Silber)</td>
<td>17.17%</td>
<td>$1,487.63</td>
</tr>
<tr>
<td>Synthetic Spread</td>
<td>12.65%</td>
<td>$1,570.42</td>
</tr>
<tr>
<td>Option Based approach (20% upside; Industry variance of 25%; 1 year trading restriction)</td>
<td>8.67%</td>
<td>$1,640.24</td>
</tr>
</tbody>
</table>
Current Cashflow to Firm
EBIT(1-t) : 300,000
- Nt CpX : 100,000
- Chg WC : 40,000
= FCFF : 160,000
Reinvestment Rate = 46.67%

Expected Growth in EBIT (1-t)
.4667*.1364= .0636
6.36%

Return on Capital
13.64%

Stable Growth
\( g = 4\% ; \text{Beta} = 3.00 \%
\)
ROC = 12.54%
Reinvestment Rate = 31.90%

Terminal Value
\( \frac{10}{289/(.1254-.04)} = 3,403 \)

Cost of Equity
16.26%

Cost of Debt
\( \frac{(4.5\%+1.00)(1-.40)}{1.00} = 3.30\% \)

Synthetic rating = A-

Firm Value: 2,571
+ Cash: 125
- Debt: 900
= Equity: 1,796

Discount at Cost of Capital (WACC) = 16.26\% (.70) + 3.30\% (.30) = 12.37%

Riskfree Rate: Riskfree rate = 4.50\% (10-year T.Bond rate)

Beta / Correlation
0.98 / 0.33

Total Beta
2.94

Risk Premium
4.00\%

Unlevered Beta for Sectors: 0.82
Firm’s D/E Ratio: 1.69\%
Mature risk
Country Risk
premium
premum
4%
0%
b. Illiquidity Adjustments to the Discount Rate

1. Add a constant illiquidity premium to the discount rate for all illiquid assets to reflect the higher returns earned historically by less liquid (but still traded) investments, relative to the rest of the market.
   - Practitioners attribute all or a significant portion of the small stock premium of 3-4% reported by Ibbotson Associates to illiquidity and add it on as an illiquidity premium. Note, though, that even the smallest stocks listed in their sample are several magnitudes larger than the typical private company and perhaps more liquid.
   - An alternative estimate of the premium emerges from studies that look at venture capital returns over long period. Using data from 1984-2004, Venture Economics, estimated that the returns to venture capital investors have been about 4% higher than the returns on traded stocks. We could attribute this difference to illiquidity and add it on as the “illiquidity premium” for all private companies.

2. Add a firm-specific illiquidity premium, reflecting the illiquidity of the asset being valued: For liquidity premiums that vary across companies, we have to estimate a measure of how exposed companies are to liquidity risk. In other words, we need liquidity betas or their equivalent for individual companies.

3. Relate the observed illiquidity premium on traded assets to specific characteristics of those assets. Thus healthier firms with more liquid holdings should have a smaller liquidity premium added on to the discount rate than distressed firms with non-marketable assets.
Illiquidity Discount Rate adjustments for Kristin Kandy

- Adding an illiquidity premium of 4% (based upon the premium earned across all venture capital investments) to the cost of equity yields a cost of equity of 20.26% and a cost of capital of 15.17%. Using this higher cost of capital lowers the value of equity in the firm to $1.531 million, about 15.78% lower than the original estimated.

- Allowing for the fact that Kristin Kandy is an established business that is profitable would allow us to lower the illiquidity premium to 2% (based upon late stage venture capital investments). This will lower the cost of equity to 18.26%, the cost of capital to 13.77% and result in a value of equity of $1.658 million. The resulting illiquidity discount is 7.66%.
c. Relative Valuation adjustment to value

- You can value an illiquid company by finding out the market prices of other companies that were similarly illiquid.
- There are two variations that can be used
  - Use data on private company transactions to estimate the multiple of earnings, book value or revenues that this company should trade for
  - Use data on publicly traded firms and adjust the resulting multiple for illiquidity of a private business
Private Company Transactions Approach: Requirements for Success

- There are a number of private businesses that are similar in their fundamental characteristics (growth, risk and cashflows) to the private business being valued.
- There are a large enough number of transactions involving these private businesses (assets) and information on transactions prices is widely available.
- The transactions prices can be related to some fundamental measure of company performance (like earnings, book value and sales) and these measures are computed with uniformity across the different companies.
- Other information encapsulating the risk and growth characteristics of the businesses that were bought is also easily available.
Publicly Traded Company Approach: Variations

- Use an illiquidity discount, estimated using the same approaches described earlier, to adjust the multiple: For instance, an analyst who believes that a fixed illiquidity discount of 25% is appropriate for all private businesses would then reduce the public multiple by 25% for private company valuations. An analyst who believes that multiples should be different for different firms would adjust the discount to reflect the firm’s size and financial health and apply this discount to public multiples.

- Instead of estimating a mean or median multiples for publicly traded firms, relate the multiples of these firms to the fundamentals of the firms (including size, growth, risk and a measure of illiquidity). The resulting regression can then be used to estimate the multiple for a private business.
### Kristin Kandy: Comparable publicly traded firms

<table>
<thead>
<tr>
<th>Company Name</th>
<th>Ticker Symbol</th>
<th>P/E/Sales</th>
<th>Operating Margin</th>
<th>Turnover Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gardenburger Inc</td>
<td>GBUR</td>
<td>0.62</td>
<td>0.03</td>
<td>0.63</td>
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<td>Paradise Inc</td>
<td>FMRF</td>
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<td>0.05</td>
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<td>Armanino Foods Dist</td>
<td>ARM F</td>
<td>0.59</td>
<td>0.06</td>
<td>0.57</td>
</tr>
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<td>Viva Food Prod s</td>
<td>VSF</td>
<td>0.57</td>
<td>0.02</td>
<td>0.14</td>
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<tr>
<td>Toceam Intl Inc</td>
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<td>0.07</td>
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<td>Allergy Research Group Inc.</td>
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<tr>
<td>Unitmark Group Inc</td>
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<tr>
<td>Tofutti Brand s</td>
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<td>0.05</td>
<td>0.10</td>
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<tr>
<td>Advanced Nutraceuticals Inc.</td>
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<tr>
<td>Sterling Sugars Inc</td>
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<td>Spectrum Organic Products Inc.</td>
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<td>0.02</td>
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<td>Northland Cranberries Inc.</td>
<td>NRCNA</td>
<td>0.66</td>
<td>0.10</td>
<td>0.07</td>
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<tr>
<td>Scheid Vineyard s</td>
<td>SVIN</td>
<td>1.77</td>
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<tr>
<td>Medifast Inc</td>
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<tr>
<td>Natrol Inc</td>
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<td>Monterey Gourmet Foods Inc.</td>
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<td>Risa Foods Inc</td>
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<tr>
<td>Tasty Baking g</td>
<td>TBC</td>
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<td>0.06</td>
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<tr>
<td>Scope Industries</td>
<td>SCPJ</td>
<td>0.75</td>
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<tr>
<td>Poore Brothers s</td>
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<td>0.10</td>
<td>0.70</td>
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<td>High Liner Foods Inc.</td>
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<td>0.23</td>
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<td>0.05</td>
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<td>LWAY</td>
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<td>0.22</td>
<td>1.95</td>
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<tr>
<td>Seneca Foods 'B'</td>
<td>SENE B</td>
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<td>0.07</td>
<td>0.13</td>
</tr>
<tr>
<td>PFI Limited</td>
<td>PFL TO</td>
<td>0.40</td>
<td>0.04</td>
<td>0.14</td>
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<tr>
<td>Rocky Mountain Choc Factory</td>
<td>RMC F</td>
<td>6.24</td>
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<td>Calivo Growers Inc.</td>
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<td>0.04</td>
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<tr>
<td>MG Ingredients Inc.</td>
<td>MGPI</td>
<td>0.55</td>
<td>0.07</td>
<td>1.80</td>
</tr>
</tbody>
</table>
Estimating Kristin Kandy’s value

- Regressing EV/Sales ratios for these firms against operating margins and turnover ratios yields the following:
  \[ EV/Sales = 0.11 + 10.78 \times \text{EBIT/Sales} + 0.89 \times \text{Turnover Ratio} - 0.67 \times \text{Beta} \]
  \[ R^2 = 45.04\% \]
  \[ (0.27) (3.81) (2.81) (1.06) \]

- Kristin Kandy has a pre-tax operating margin of 25%, a zero turnover ratio (to reflect its status as a private company) and a beta (total) of 2.94. This generates an expected EV/Sales ratio of 0.296.

- \[ EV/Sales = 0.11 + 10.78 \times (.25) + 0.89 \times (0) - 0.67 \times (2.94) = 0.835 \]

- Multiplying this by Kristin Kandy’s revenues of $3 million in the most recent financial year generates an estimated value for the firm of $2.51 million. This value is already adjusted for illiquidity.
Conclusion

- All assets are illiquid, but there are differences in the degree of illiquidity.
- Illiquidity matters to investors. They pay lower prices and demand higher returns from less liquid assets than from otherwise similar more liquid assets.
- The effect of illiquidity on value can be estimated in one of three ways:
  - The value of the asset can be computed as if it were liquid, and then adjusted for illiquidity at the end (as a discount).
  - The discount rate used for illiquid assets can be set higher than that used for liquid assets.
  - The illiquidity effect can be built into value by looking at how similar illiquid companies have been priced in transactions or by adjusting publicly traded company multiples for illiquidity.