
THE TIME VALUE OF MONEY

A dollar today is worth more than a dollar in the future, because we can invest the dollar elsewhere and earn a return on it. Most people can grasp this argument without the use of models and mathematics. In this chapter, we use the concept of time value of money to calculate exactly how much a dollar received or paid some time in the future is worth today, or vice versa.

What makes the time value of money compelling is the fact that it has applicability in a range of personal decisions, from saving for retirement or tuition to buying a house or a car. We will consider a variety of such examples in this chapter. The measurement of the time value of money is also central to corporate finance. In investment analysis, we are often called upon to analyze investments spread out over time. Thus, the managers at Boeing, when analyzing the Super Jumbo investment, have to consider not only what they will have to spend today but also what they will have to in the future, and measure this against what they expect to earn today and far into the future. The principles that we learn in this chapter also become crucial when we value assets or entire businesses, whose earnings will be generated over extended time periods. Given that our objective in corporate finance is to maximize value, it is clear that we cannot do so without an understanding of how to compare dollars at different points in time.

The Intuitive Basis for the Time Value of Money

Why is a dollar today worth more to you than a dollar a year from now? The simplest way to explain the intuition is to note that you could have invested the dollar elsewhere and earned a return on it, in the form of interest, dividends or price appreciation. Thus, if you could earn 5% in a savings account in a bank, the dollar today would be worth a \$1.05 a year from today.

While we often take the interest rate we can earn on our savings as a given, it is worth considering what goes into this interest rate. Assuming that you are guaranteed this return by the borrower, there are two reasons why you need to earn the interest rate to save. The first is that the presence of inflation means that the dollar today will buy more in terms of real goods than the same dollar a year from now. Consequently, you would demand an interest rate to compensate for the loss in purchasing power that comes with inflation. The second reason is that like most individuals, you prefer present consumption to future consumption. Thus, even if there were no inflation and the dollar today and the dollar a year from now purchased exactly the same quantity of goods, you would prefer to spend the dollar and consume the goods today. Therefore, to get you to postpone the consumption, the lender must offer you some compensation in the form of an interest rate on your savings; this is called a real interest rate. How much would you need to be offered? That will depend upon how strong your preference for current consumption is, with stronger preferences leading to higher real interest rates. The interest rate that includes the expected inflation in addition to the real interest rate is called a nominal interest rate.

Thus far, we have assumed you are guaranteed the return on your savings. If there is uncertainty about whether you will earn the return, there is a third component to the return that you would need to make on your investment. This third component is compensation for the uncertainty that you are exposed to and it should be greater as the uncertainty increases. When there is no certainty about what you will make on your investment, we measure the return not as an interest rate but as an expected return.

In summary then, when we talk about the return you can make by investing a dollar today elsewhere, there are three components of this return – the expected inflation rate, a real interest rate and a premium for uncertainty.

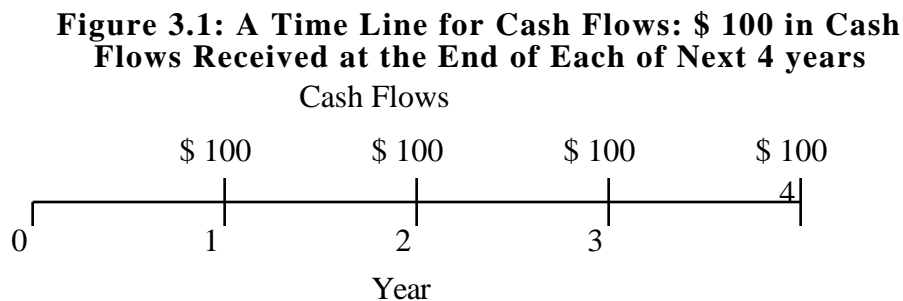
Central to the notion of the time value of money is the idea that the money can be invested elsewhere to earn a return. This return is what we call a discount rate. Note that the interest rate you can make on a guaranteed investment, say a government security, can

be used as the discount rate when your investment is expected to yield a guaranteed return. When there is uncertainty about whether you the dollar in the future will be received, the discount rate is the rate of return that you can expect to make on an investment with similar amount of uncertainty, in which case it will have to incorporate a premium for uncertainty. The “discount rate” is therefore a more general term than “interest rates” when it comes to time value, and that is the term that we will use through the rest of this chapter. In some of our examples, where what we will receive or pay out is known with a fair degree of certainty, the interest rate will be the discount rate. In other examples, where there is uncertainty about the future, we will use the expected return on investments of similar risk as the discount rate.

✍ CT 3.1: Economists and government officials have been wringing their hands over the desire for current consumption that has led American families to save less and consume more of their income. What are the implications for discount rates?

Cash Flows and Time Lines

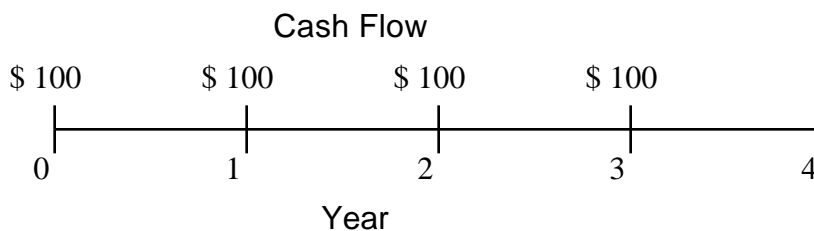
In addition to discount rates, the other variable that we will talk about in this chapter is cash flows. A cash flow is either cash that we expect to receive (a cash inflow) or cash that we expect to pay out (a cash outflow). Since this chapter is all about the significance of comparing cash flows across time, we will present cash flows on a time line that shows both the timing and the amount of each cash flow. Thus, cash flows of \$100 received at the end of each of the next 4 years can be depicted on a time line like the one depicted in Figure 3.1.



In the figure, time 0 refers to the present. A cash flow that occurs at time 0 does not need to be adjusted for time value. In this case, we have no cash flows at time 0 , but we have \$ 400 in cash flows over the next 4 years. However, the fact that they occur at different points in time means that the cash flows really cannot be compared to each other. That is, \$ 100 in one year should be worth less than \$ 100 today but more than \$ 100 in two years. In sum, \$ 400 over the next 4 years should be worth less than \$ 400 today.

Note the difference between a period of time and a point in time in Figure 3.1. The portion of the time line between 0 and 1 refers to period 1, which, in this example, is the first year. The cash flow that we receive at the point in time “1” refers to the cash flow that occurs at the end of period 1. Had the cash flows occurred at the beginning of each year instead of at the end of each year, the time line would have been redrawn as it appears in Figure 3.2.

Figure 3.2: A Time Line for Cash Flows: \$ 100 in Cash Received at the Beginning of Each Year for Next 4 years



Note that in time value terms, a cash flow that occurs at the beginning of year 2 is the equivalent of a cash flow that occurs at the end of year 1. Again, it is worth noting that while we receive \$ 400 in this case, as in the previous one, these cash flows should be worth more because we get each \$ 100 one year earlier than in the previous case.

In this chapter, we will examine ways to convert cash flows in the future into cash flows today. This process is called discounting, and the cash flows, once converted into cash flows today, yield a present value (PV). We will also reverse this process and ask a different question. How much would \$ 100 in year 1 be worth in year 4? This process of

converting cash flows today or in the future into cash flows even further into the future is called compounding, and the resulting value is called a future value (FV).

Time Value of Money: Compounding and Discounting

In this section, we will consider how to discount and compound a simple cash flow, and why we do it.

Compounding

Assume that you are the owner of InfoSoft, a private business that manufactures software, and that you have \$ 50,000 in the bank earning 6% interest for the foreseeable future. Over time, that investment will increase in value. Thus, at the end of 1 year, the \$ 50,000 will be worth \$ 53,000 (\$ 50,000 + interest of 6% on \$ 50,000). This is the future value at the end of the first year. We can write this value more formally as:

$$\text{Future Value at end of year 1} = \$ 50,000 (1.06) = \$ 53,000$$

At the end of year 2, the deposit would have grown further to \$56,180 (\$ 53,000 + Interest of 6% on \$ 53,000). This can also be written more formally as:

$$\text{Future Value at end of year 2} = \$ 50,000 (1.06) (1.06) = \$ 50,000 (1.06)^2 = \$ 53,000$$

Note that the future value at the end of 10 years would then be:

$$\text{Future Value at end of 10 years} = \$ 50,000 (1.06)^{10} = \$ 89,542$$

Note that in addition to the initial investment of \$ 50,000 earning interest, the interest earned in each year itself earns interest in future years.

Why do we care about the future value of an investment? It provides us with a measure that we can use to compare alternatives to leaving the money in the bank. For instance, if you could invest the \$ 50,000 elsewhere and end up with more than \$89,542 at the end of 10 years, you could argue for taking this investment, assuming this investment is just as safe as leaving your money in the bank. If it were riskier, you would need to end up with more than \$89,542 to compensate you for the uncertainty, which is equivalent to saying that you would need a higher rate of return than 6%.

In general, the value of a cash flow today (CF_0) at the end of a future period (t), when the discount rate is given (as r), can be written as:

$$\text{Future Value of Cash Flow} = CF_0 (1 + r)^t$$

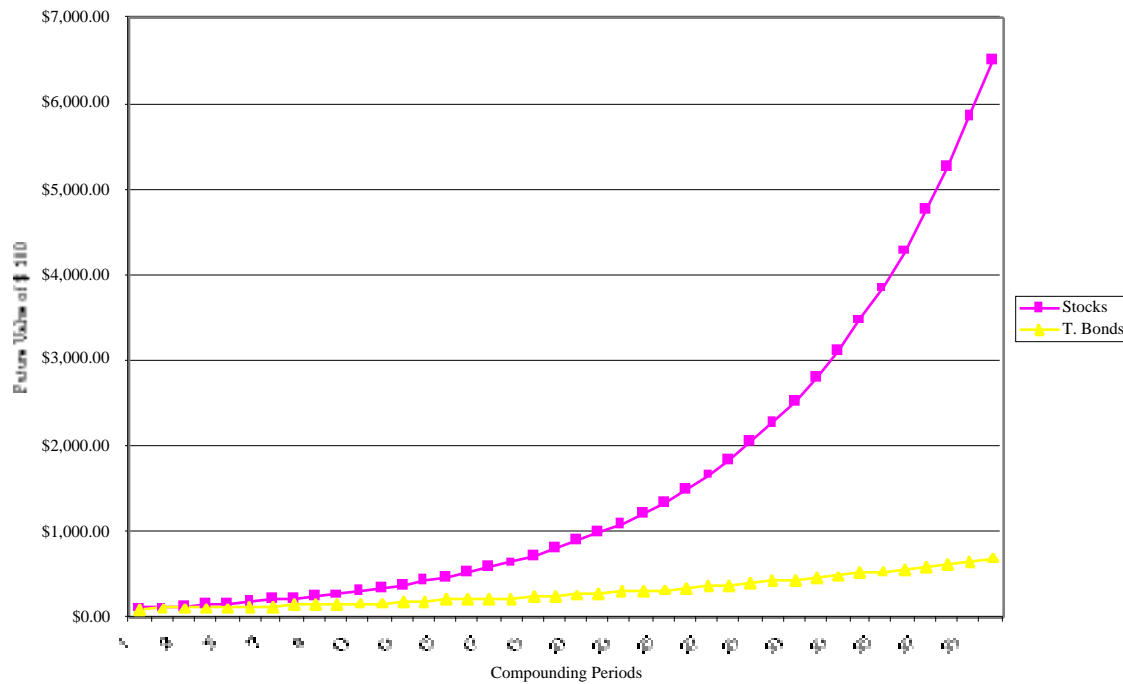
The future value will increase the further into the future we go and the higher the discount rate.

☞ *CC 3.1*: In computing the future value of \$ 50,000 in the example above, we assumed that interest earned was allowed to remain in the account and earn more interest. What would the future value be if you intended to withdraw all the interest income from the account each year?

Illustration 3.1: The Power of Compounding - Stocks, Bonds and Bills

In the example above, the future value increased as we increased the number of years for which we invested our money; we call this the compounding period. As the length of the compounding period is extended, small differences in discount rates can lead to large differences in future value. In a study of returns on stocks and bonds between 1926 and 1998, Ibbotson and Sinquefeld found that stocks on the average made about 11% a year, while government bonds on average made about 5% a year. Assuming that these returns continue into the future, figure 3.4 provides the future values of \$ 100 invested in stocks and bonds for periods extending up to 40 years.

Figure 3.4: Effect of Compounding Periods: Stocks versus T.Bonds



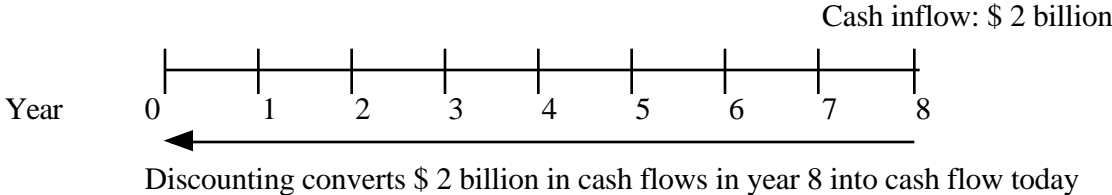
The differences in future value from investing at these different rates of return are small for short compounding periods (such as 1 year) but become larger as the compounding period is extended. For instance, with a 40-year time horizon, the future value of investing in stocks, at an average return of 11%, is more than 9 times larger than the future value of investing in treasury bonds at an average return of 5%.

Discounting

Discounting operates in the opposite direction from compounding. Instead of looking at how much a dollar invested today will be worth in the future, we ask what a dollar received or paid in the future will be worth today. Our measure of how much a future cash flow is worth today will depend upon our preferences for current consumption over future consumption, our views about inflation and the perceived uncertainty associated with the anticipated cash flow.

Consider the investment in the Super Jumbo that Boeing is considering making. Assume that Singapore Airlines is willing to place an order to buy \$ 2 billion worth of the Super Jumbo 8 years from now. While this will weigh in positively on whether Boeing will make this investment, it is worth less than an order on which Boeing would receive \$ 2 billion today. The time line for \$ 2 billion received in 8 years can be shown in Figure 3.3:

Figure 3.3: Present Value of a Cash Flow



One way of answering the question of what \$ 2 billion in 8 years is worth today is to reverse the question and ask how much would Boeing would need to invest today to end up with \$ 2 billion at the end of year 8. Assuming that Boeing’s discount rate for this investment is 10%, we could consider how much we would need to invest today to have \$ 2 billion in 8 years:

$$CF_t = \$ 2 \text{ billion} = CF_0 (1.10)^8$$

Solving for the cash flow today, we get:

$$CF_0 = \$ 2 \text{ billion} / (1.10)^8 = \$ 933 \text{ million}$$

Thus, \$ 2 billion in 8 years is worth \$ 933 million in present value terms. What exactly does that mean? With a 10% discount rate, Boeing would be indifferent between receiving \$ 933 million today and \$ 2 billion in 8 years.

Generalizing, if CF_t is the cash flow at the end of some future year t and r is the discount rate, the present value of a cash flow can be written as follows:

$$\text{Present Value of a Cash Flow} = \frac{CF_t}{(1 + r)^t}$$

The present value will decrease the further into the future a cash flow is expected to be received and as the discount rate increases.

Why do we discount? Discounting allows us to convert cash flows in the future into cash flows today, so that we can compare them and aggregate them for purposes of analysis. To illustrate, the Boeing Super Jumbo investment might have cash inflows and outflows occurring each year for the next 30 years. While these cash flows, by themselves, cannot be compared to each other, the present value of each of these cash flows can be aggregated or cumulated. We could, for instance, answer the question of whether the present value of the cash inflows on this project will exceed the present value of the cash outflows.

It is interesting to also look at present value from the perspective of Singapore Airlines. Singapore Airlines, after entering into a contract to pay \$ 2 billion in 8 years, will have an expected cash outflow of that amount at the end of the eighth year. Assume that it wants to set aside the money today to ensure that it will have \$ 2 billion at the end of the eighth year, and that it can earn 7% on its investments. The present value of \$ 2 billion in 8 years can be written as:

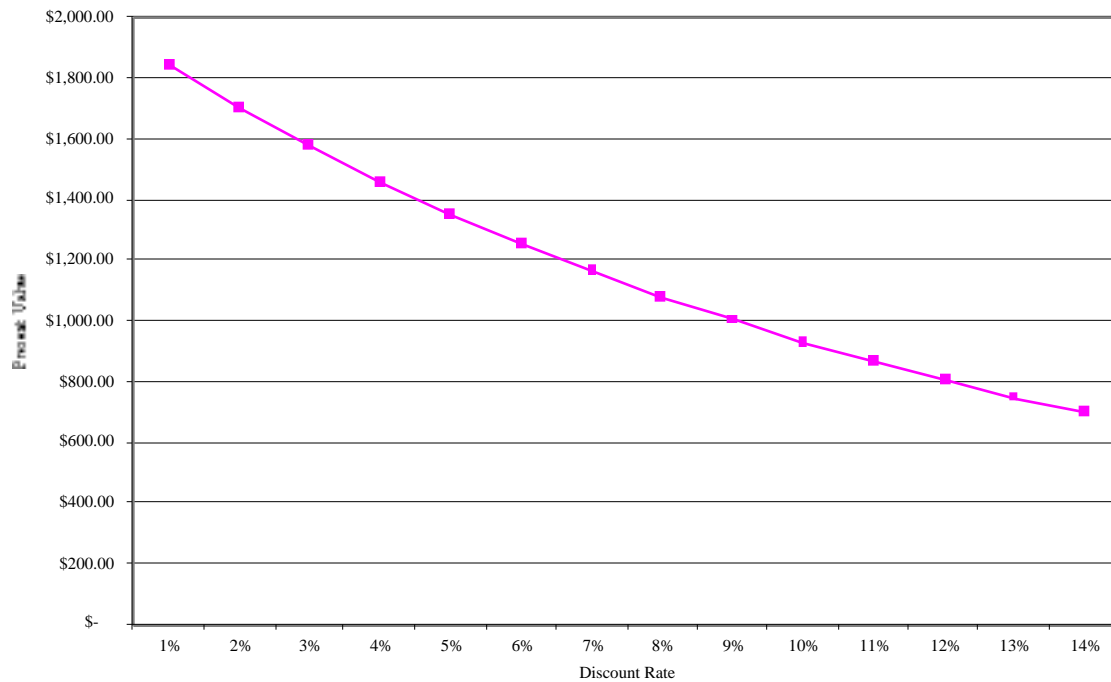
$$\text{Present Value of Payment} = \frac{\$2,000}{(1.07)^8} = \$1,164 \text{ million}$$

Singapore Airlines would have to set aside \$ 1,164 million today, earning 7% a year, to ensure that it had \$ 2 billion at the end of 8 years.

Illustration 3.2: The Present Value Effects of Higher Discount Rates

In the example above, we assumed that Singapore Airlines could make 7% on its investments and calculated that it would need to set aside \$ 1,164 million today to arrive at a value of \$ 2 billion in 8 years. What if it could earn a rate higher than 7%? Or, what if the rate of return were much lower? The higher the return that Singapore Airlines can earn on its investments, the lower is the present value, and the less is the amount that the firm would need to set aside to get \$ 2 billion at the end of 8 years. This is illustrated in Figure 3.5.

Figure 3.5: Present Value of \$ 2 billion in 8 years



At a 14% rate of return, for instance, Singapore Airlines would need to set aside only \$ 701 million to reach \$ 2 billion at the end of year 8.

The Frequency of Discounting and Compounding

In the examples above, the cash flows were discounted and compounded annually — i.e., interest payments and income were computed at the end of each year, based on the balance at the beginning of the year. In some cases, however, the interest may be computed more frequently, such as on a monthly or semi-annual basis. In these cases, the present and future values may be very different from those computed on an annual basis.

To illustrate, consider the investment of \$ 50,000 in the bank earning 6% a year that we considered in the section on compounding. The future value of \$ 89,542 was based upon the assumption that the bank computed interest income on the income at the end of each of the next 10 years. Assume instead that the bank computed interest every 6 months. We can then compute the future value of the investment at the end of 10 years as follows:

$$\text{Future Value at end of year 10} = \$ 50,000 (1+.06/2)^{20} = \$90,306$$

The interest rate every six months is now 3% (6%/2), but there are 20 six-month compounding periods in 10 years. Where does the increase in value from \$ 89,542 to \$90,306 come from? It arises from the fact that the interest income is now computed at the end of six months to be \$1,500; this interest income now earns interest over the remaining six months of the first year. A similar compounding benefit occurs with each interest payment from the bank. If the compounding were done every month instead of every six months, the future value would be:

$$\text{Future Value at end of year 10 (monthly compounding)} = \$ 50,000 (1+.06/12)^{20} = \$90,970$$

In general, then, the future value of a cash flow, where there are t compounding periods each year, can be written as follows:

$$\text{Future Value of Cash Flow} = \text{Cash Flow today} \left(1 + \frac{\text{Stated Annual Interest Rate}}{t} \right)^{n/t}$$

This analysis can also be reframed in terms of the interest rate that you earn on your investment. While the stated annual rate on the investment in our example is 6%, the effective annual rate is much higher when compounding occurs every 6 months. In fact, it can be computed as follows:

$$\text{Effective Interest Rate} = \left(1 + \frac{.06}{2} \right)^2 - 1 = 6.09\%$$

In general, the effective annual interest rate, given that there are t compounding periods every year, can be computed as follows

$$\text{Effective Interest Rate} = \left(1 + \frac{\text{Stated Annual Interest Rate}}{t} \right)^t - 1$$

The bank can compute interest on a weekly or daily basis, in which case the future value and the effective interest rate would be even higher. At the limit, though, the bank could compound at every instant in time, in which case it is called continuous compounding. As compounding becomes continuous, the effective interest rate can be computed as follows:

$$\text{Effective Interest rate} = \left(1 + \frac{\text{Stated Annual Interest Rate}}{t} \right)^t - 1 = \exp^{\text{stated rate}} - 1$$

In our example above, for instance, the effective interest rate when a 6% annual rate is compounded continuously would be:

$$\text{Effective Interest rate} = \exp^{0.06} - 1 = 6.18\%$$

The future value of \$ 50,000 at the end of 10 years with continuous compounding would then be:

$$\text{Future Value of } \$ 50,000 \text{ with continuous compounding} = \$ 50,000 \exp^{(0.06)(10)} = \$91,106$$

In the context of discounting, more frequent compounding, by increasing the effective discount rate, reduces the present value. It should come as no surprise then that loan sharks use daily compounding to keep track of the amounts owed to them.

Illustration 3.3: Estimating Effective Interest Rates on Mortgage Loans

Most home mortgage loans in the United States require monthly payments and consequently have monthly compounding. Thus, the annual interest rates quoted on loans can be deceptive because they are actually too low. A loan with an annual interest rate of 8.00%, for example, when adjusted for monthly compounding, will have an effective interest rate of

$$\text{Effective Interest Rate} = 1 + \frac{.08}{12}^{12} - 1 = 8.3\%$$

Illustration 3.4: APR Legislation

Prior to 1968, banks in the United States were allowed to advertise using any interest rate they chose on both deposits and mortgage loans. Consequently, consumers were faced with a blizzard of rates, some stated, some effective, and some adjusted, which could not be compared across institutions. In 1968, Congress passed a law called the Truth-in-Lending Act, requiring that more information be provided on the true cost of borrowing to enable consumers to compare interest rates on loans. Under this law, which has been amended several times since its passage, financial institutions must provide an annual percentage rate (APR) in conjunction with any offer they might be making. The annual percentage rate is computed by multiplying the periodic rate by the number of

periods per year. Thus, a monthly rate of 1% will result in an annual percentage rate of 12%. Since this does not allow for the compounding effect, some lenders may get higher effective annual interest rates by changing the compounding periods on their loans. The APR include an amortization of any fixed charges that have to be paid up front for the initiation of the loan. For instance, on a mortgage loan, these fixed charges would include the closing costs that are normally paid at the time that the loan is taken.

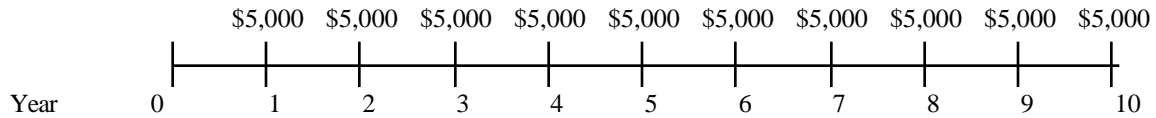
✍ CT 3.2: Assume that you are comparing interest rates on several loans, with different approaches to computing interest. The first loan has a stated interest rate of 8%, with compounding occurring every month. The second loan has a stated interest rate of 7.8%, with compounding occurring every week. The third loan has a stated interest rate of 7.5%, with continuous compounding. Which is the cheapest loan?

Time Value of Money: Annuities and Perpetuities

The mechanics of time value of money described in the last section can be extended to compute the present value or future value of any set of cash flows. There are a couple of special types of cash flows, in which discounting and compounding can be simplified. We will look at those cash flows next.

Annuities

An annuity is a stream of constant cash flows that occur at regular intervals for a fixed period of time. Consider again the example we used earlier of investing \$ 50,000 today at 6%, and estimating how much it would be worth at the end of 10 years. Assume, instead, that you intend to set aside \$ 5,000 at the end of each year for the next 10 years, and you want to estimate how much you would have at the end of 10 years, assuming an interest rate of 6%. The amount set aside each year (\$5,000) is the annual cash flow on the annuity. It can be presented in a time line as follows:

Figure 3.6: Annuity of \$ 5,000 each year for 10 years

An annuity can occur at the end of each period, as in this time line, or at the beginning of each period.

Compounding an Annuity

To estimate how much \$ 5,000 set aside at the end of each year for the next 10 years would be worth, we could estimate the future value of each deposit at the end of the tenth year. Thus, \$ 5000 invested at the end of year 1 would earn interest at 6% for 9 years to be worth \$8,447 by the end of the tenth year.

Future Value of \$ 5000 invested in year 1 = \$ 5,000 $(1.06)^9$ = \$8,447

Future Value of \$ 5,000 invested in year 2 = \$ 5,000 $(1.06)^8$ = \$7,969

Computing the future values of all ten investments, and then adding them up, we obtain:

Cumulated Future Value = \$ 5000 $(1.06)^9$ + \$ 5000 $(1.06)^8$ + \$ 5000 $(1.06)^7$ + \$ 5000 $(1.06)^6$ + \$ 5000 $(1.06)^5$ + \$ 5000 $(1.06)^4$ + \$ 5000 $(1.06)^3$ + \$ 5000 $(1.06)^2$ + \$ 5000 $(1.06)^1$ + \$5000 = \$ 5000 $(1.06^9+1.06^8+1.06^7+1.06^6+1.06^5+1.06^4+1.06^3+1.06^2+1.06^1+1)$

This can be simplified to yield the following:

Cumulated Future Value = \$ 5000 $(1.06^{10}-1)/.06$ = \$65,904

In general, the future value of an annuity (A), received or paid, at the end of each year for n years with a discount rate r can be calculated as follows:

$$\text{FV of an Annuity} = \text{FV}(A,r,n) = A \frac{(1+r)^n - 1}{r}$$

Thus, the notation we will use throughout this book for the future value of an annuity will be $\text{FV}(A,r,n)$.

This analysis is based upon the assumption that the cash flows occur at the end of each year. If they occurred at the beginning of each year instead, each cash flow would earn an additional year of interest. This would result in a future value for the annuity that is greater by this factor:

$$\text{FV of a Beginning-of -the-Period Annuity} = A (1+r) \frac{(1+r)^n - 1}{r}$$

This future value will be higher than the future value of an equivalent annuity at the end of each period.

Illustration 3.5 : Individual Retirement Accounts (IRA)

Individual retirement accounts (IRAs) allow some tax payers to set aside \$2,000 a year for retirement, and the interest earned on these accounts is exempt from taxation. If an individual starts setting aside money in an IRA early in her working life, its value at retirement can be substantially higher than the amount actually put in. For instance, assume that this individual sets aside \$2,000 at the end of every year, starting when she is 25 years old, for an expected retirement at the age of 65, and that she expects to make 8% a year on her investments. The expected value of the account on her retirement date can be calculated as follows:

$$\text{Expected Value of IRA set -aside at 65} = \$2,000 \frac{(1.08)^{40} - 1}{.08} = \$518,113$$

The tax exemption adds substantially to the value because it allows the investor to keep the pre-tax return of 8% made on the IRA investment. If the income had been taxed at 40%, the after-tax return would have dropped to 4.8%, resulting in a much lower expected value:

$$\text{Expected Value of IRA set -aside at 65 if taxed} = \$2,000 \frac{(1.048)^{40} - 1}{.048} = \$230,127$$

As you can see, the available funds at retirement drop by more than 55% as a consequence of the loss of the tax exemption.

Consider also the effect of setting aside the savings at the beginning of each year instead of the end of each year, for the next 40 years. The future value of this annuity would be:

$$\text{Expected Value of IRA (beginning of year)} = \$2,000 (1.08) \frac{(1.08)^{40} - 1}{.08} = \$559,562$$

As you can see, the gains from making payments at the beginning of each period can be substantial.

As a final example, consider a different scenario, where an investor or a company is saving to meet a goal and wants to estimate how much to save in each period to reach it. The analysis can be modified fairly simply to answer this question. For instance, assume in the example just described that the individual can save money at the end of each year for the next 40 years in an IRA account, earning 8% a year, and wants to have accumulated savings of \$ 400,000 at the end of the 40th year. To estimate how much the annual savings would need to be, we can do the following:

$$\text{Expected Value of IRA set -aside at 65} = \$400,000 = \text{Annual Savings} \frac{(1.08)^{40} - 1}{.08}$$

Solving for the annual savings,

$$\text{Annual Savings} = \$400,000 * \frac{.08}{(1.08)^{40} - 1} = \$1544$$

Annual savings of \$ 1544 at the end of each year for the next 40 years, and an annual return of 8% a year, produce a future value of \$ 400,000. In general, the payment needed to arrive at a required future value can be calculated as follows:

$$\text{Annual Cashflow given Future Value} = A(FV,r,n) = FV \frac{r}{(1+r)^n - 1}$$

☞ *CC 3.2*: How much would you need to save each year for the next 40 years to arrive at a future value of \$ 400,000 if you saved at the beginning of each year instead of the end?

Discounting an Annuity

In 1997 and 1998, there was discussion of a comprehensive settlement between tobacco firms in the United States and the federal government, whereby the tobacco firms would pay approximately \$ 20 billion a year for 25 years in exchange for immunity from lawsuits on smoking-related deaths. While the agreement was never ratified by Congress, let us consider how much the cost of the agreement would have been to tobacco firms, in present value dollars, if it had been approved.

Assume that the tobacco firms collectively would have to guarantee the payments and that the discount rate is 6%. The present value of the payments can be computed by taking each payment and discounting it back to the present. Thus, the present value of \$ 20 billion in 1 year, at a 6% discount rate, is \$ 18.87 billion, computed as follows:

Present Value of \$ 20 billion in one year = \$ 20 billion/1.06

The present value of each of the remaining 24 payments can be computed similarly and then added up to yield the following:

$$\begin{aligned} \text{Cumulated Present Value} &= \$ 20 \text{ billion } (1/1.06 + 1/1.06^2 + 1/1.06^3 + \dots + 1/1.06^{25}) \\ &= \$ 255.67 \text{ billion} \end{aligned}$$

This can be simplified to yield a short cut to computing the present value of an annuity:

$$\text{PV of } \$ 20 \text{ billion for 25 years} = \$ 20 \text{ billion } \frac{1 - \frac{1}{(1.06)^{25}}}{.06} = \$255.67 \text{ billion}$$

In general, the present value of an annual cash flow (A) each year for n years, with a discount rate r, can be calculated as follows:

$$\text{PV of an Annuity} = \text{PV}(A,r,n) = A \frac{1 - \frac{1}{(1+r)^n}}{r}$$

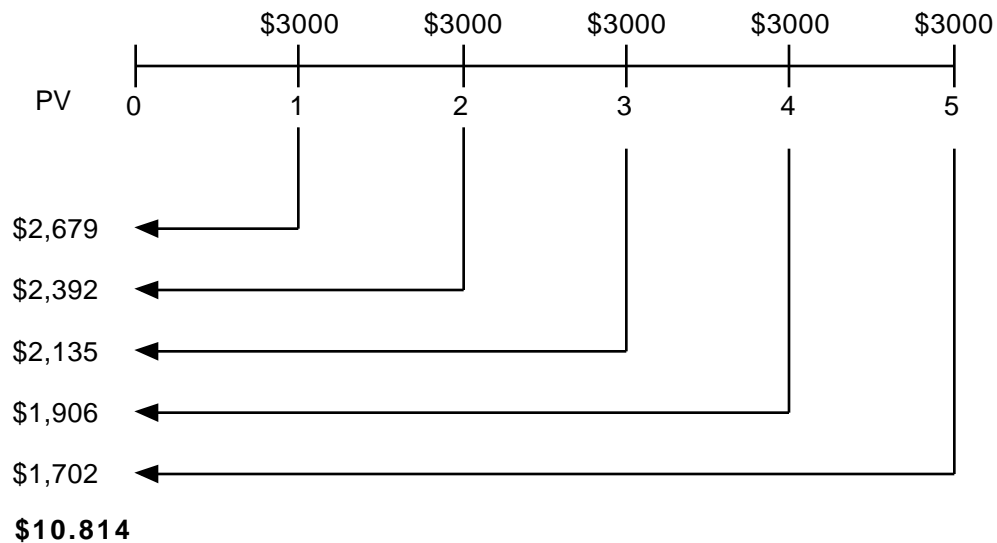
Accordingly, the notation we will use in the rest of this book for the present value of an annuity will be PV(A,r,n).

Illustration 3.6 : Estimating the Present Value of Annuities

Assume again that you are the owner of Infosoft, and that you have a choice of buying a copier for \$11,000 cash down or paying \$ 3,000 a year for 5 years for the same copier. If the opportunity cost is 12%, which would you rather do?

Consider the present value of paying \$ 3,000 a year for 5 years. Each payment can be discounted back to the present to yield the values in figure 3.7:

Figure 3.7: Payment of \$ 3000 at the beginning of each of next 5 years



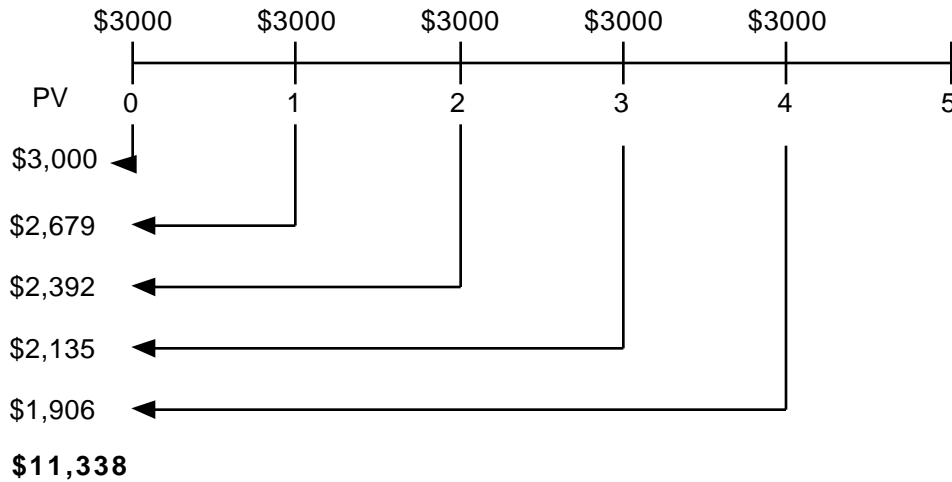
Alternatively, the present value of the payments can be calculated using the short cut described earlier.

$$\text{PV of \$3000 each year for next 5 years} = \$3000 \frac{1 - \frac{1}{(1.12)^5}}{.12} = \$10,814$$

The present value of the installment payments is less than the cash-down price; therefore, you would want to buy the copier on the installment plan.

In this case, we assumed that the payments were made at the end of each year. If, however, the payments were due at the beginning of each year, the present value would be much higher, since each payment will be discounted back one less year.

Figure 3.8: Payment of \$ 3000 at the beginning of each of next 5 years



In fact, the present value of \$ 3000 at the beginning of each of the next 5 years can be calculated to be:

$$\text{PV of } \$3000 \text{ each year for next 5 years} = \$3000 + \$3000 \frac{1 - \frac{1}{(1.12)^4}}{.12} = \$11,338$$

In general, the present value of an annuity where the cash flows are at the beginning of each period for the next n periods can be written as follows:

$$\text{PV of Beginning of Period Annuities over } n \text{ years} = A + A \frac{1 - \frac{1}{(1+r)^{n-1}}}{r}$$

Illustration 3.7 : Making Sense of Sports Contracts

Sports contracts for big-name players often involve mind-boggling amounts of money. While the contracts are undoubtedly large, the use of nominal dollars in estimating the size of these contracts is actually misleading because the contracts are generally multi-year contracts. Consider, for instance, the \$105 million contract signed by Kevin Brown to play baseball for the Los Angeles Dodgers on December 12, 1998. As the first player to crack the \$ 100 million barrier, he clearly will not be pleading poverty in the near future. The contract, however, requires the payment of approximately \$ 15 million a year for seven

years. In present value terms, assuming a discount rate of 6%, the contract is worth \$83.74 million.

$$PV \text{ of } \$15 \text{ m each year for next 7 years} = \$15 \text{ m} \frac{1 - \frac{1}{(1.06)^7}}{.06} = \$83.74 \text{ mil}$$

The use of nominal values for contracts does serve a useful purpose. Both the player and the team signing him can declare victory in terms of getting the best deal. The player's ego is satisfied by the size of the nominal contract, while the team's financial pain can be minimized by spreading the payments over more time, thus reducing the present value of the contract.

Illustration 3.8 : How Do They Do That? Lottery Prizes

State-run lotteries have proliferated in recent years, as states recognize their potential to create revenues for a variety of causes. The New York State lottery, for instance, was expected to generate funds for education — since 50% of the revenue generated from the lottery was supposed to go towards education. It is therefore surprising sometimes to see lottery prizes that exceed the revenues from ticket sales. How, for instance, can a lottery pay out \$40 million in prizes on ticket sales of \$35 million and still claim to generate revenues for education? The answer is that while the sales are in current dollars, the prizes are paid out as annuities over very long time periods, resulting in a present value that is much lower than the announced prize. The present value of \$2 million paid out each year for 20 years is significantly lower than the \$35 million that the state receives today.

☞ *CC 3.3:* Assume that you run the lottery and you want to ensure that 50% of ticket revenues go towards education, while preserving the nominal prizes at \$40 million. How much can you afford to pay out each year, assuming a discount rate of 10%?

Illustration 3.9: Present Value of Multiple Annuities

Suppose you are the pension fund consultant to The Home Depot, and that you are trying to estimate the present value of its pension obligations, which are expected to be the following:

Years	Annual Cash Flow
1 - 5	\$ 200.0 million
6 - 10	\$ 300.0 million
11 - 20	\$ 400.0 million

If the discount rate is 10%, the present value of these three annuities can be calculated as follows:

Present Value of first annuity = \$ 200 million * PV (A, 10%, 5) = \$ 758 million

Present Value of second annuity = \$ 300 million * PV (A,10%,5) / 1.10⁵ = \$ 706 million

Present Value of third annuity = \$ 400 million * PV (A,10%,10) / 1.10¹⁰ = \$ 948 million

The present values of the second and third annuities can be calculated in two steps. First, the standard present value of the annuity is computed over the period that the annuity is received. Second, that present value is brought back to the present. Thus, for the second annuity, the present value of \$ 300 million each year for 5 years is computed to be \$1,137 million; this present value is really as of the end of the fifth year.¹ It is discounted back 5 more years to arrive at today's present value, which is \$ 706 million.

Present Value = \$ 758 million+\$706 million+\$948 million = \$2,412 million

Estimating an Annual Cash Flow

In some cases, the present value of the cash flows is known and the annual cash flow needs to be estimated. This is often the case with home and automobile loans, for example, where the borrower receives the loan today (present value) and pays it back in

¹ A common error is to assume that since the first payment in this annuity is at the end of the sixth year, the present value is also at that point. The process of computing the present value, however, moves the cash flows back one year prior to the first cash flow, which in this case is the end of the fifth year.

equal monthly installments over an extended period of time. In these cases, the payment can be calculated from the equation we developed for estimating the present value of an annuity in the last section:

$$\text{PV of an Annuity} = \text{PV}(A,r,n) = A \frac{1 - \frac{1}{(1+r)^n}}{r}$$

If we know the present value, the discount rate r and the number of years for which the annuity is to be paid or received, the annual cash flow can be calculated as follows:

$$\text{Annual Cashflow given Present Value} = A(\text{PV},r,n) = \text{PV} \frac{r}{1 - \frac{1}{(1+r)^n}}$$

To illustrate, suppose you borrow \$200,000 to buy a house on a 30-year mortgage with monthly payments. The annual percentage rate on the loan is 8%. The monthly payments on this loan, with the payments occurring at the end of each month, can be calculated using this equation:

$$\text{Monthly interest rate on loan} = \text{APR}/12 = 0.08/12 = 0.0067$$

$$\text{Monthly Payment on Mortgage} = \$200,000 \frac{0.0067}{1 - \frac{1}{(1.0067)^{360}}} = \$1473.11$$

This monthly payment will be higher if the loan has a higher interest rate.

In recent years, mortgage holders have been provided the option of making their payments at the beginning of each month, rather than the end of the month. As was the case with the future value of annuities, there is an effect on the required monthly payment. To estimate the monthly payment at the beginning of each month rather than the end, we modify the equation to allow for the fact that each payment, will be discounted back one less period (month).

$$\text{Monthly Payment on Mortgage} = \$200,000 \frac{0.0067}{1 - \frac{1}{(1.0067)^{360}}} \frac{1}{1.0067} = \$1463.31$$

The home-owner can save \$ 9.80 every month by making the payments at the beginning of the month rather than the end of the month.

Illustration 3.10 : Cash Discount versus a Lower Interest Rate - An Automobile Loan

Now suppose you are trying to buy a new car that has a sticker price of \$15,000. The dealer offers you two deals:

- you can borrow \$15,000 at a special annual percentage rate of 3%, for 36 months.
- you can reduce the sticker price by \$1,000 and borrow \$14,000 at the normal financing rate of 12% per annum, for 36 months

To examine which is the better deal, you must calculate the monthly payments on each one.

$$\text{Monthly Rate of Interest} = 3\%/12 = 0.25\%$$

$$\text{Monthly Payment on Special Financing Deal} = \$15,000 \frac{0.0025}{1 - \frac{1}{(1.0025)^{36}}} = \$436.22$$

$$\text{Monthly Rate of Interest} = 12\%/12 = 1\%$$

$$\text{Monthly Payment on Discount Deal} = \$14,000 \frac{0.01}{1 - \frac{1}{(1.01)^{36}}} = \$465.00$$

The monthly payments are lower on the special financing deal, making it the better one. Another way of looking at these choices is to compare the present value of the savings you get from the lower rate against the dollar value of the discount. In this case, for instance, the monthly payment on a \$15,000 loan at an annual rate of 3% is \$436.22, while the monthly payment on the same loan at an annual rate of 12% is \$498.21. The monthly savings is \$61.99, yielding a present value of savings of

$$\text{Present Value of Monthly Savings} = \$61.99 \frac{1 - \frac{1}{(1.01)^{36}}}{.01} = \$1866.34$$

The present value of the savings is greater than the price discount of \$1,000. The dealer would therefore have to offer a much larger discount (>\$1866.34) for you to take the second deal.

Options for Computing Time Value of Money

In the chapter so far, we have computed present value through equations. While these equations are not particularly complicated, there are two alternatives that are widely used when it comes to compounding and discounting. One is the use of time value tables. These tables summarize what are called present value and future value factors that can be used to compute the present or future value of a single cash flow or an annuity. To illustrate how such tables are constructed, consider the earlier example where we computed the present value of Kevin Brown's contract with the Los Angeles Dodgers for \$ 15 million a year for 7 years, at a 6% discount rate.

$$PV \text{ of } \$15 \text{ m each year for next 7 years} = \$ 15 \text{ m} \frac{1 - \frac{1}{(1.06)^7}}{.06} = \$83.74 \text{ mil}$$

If we had isolated only the second term in the equation, we would have obtained the present value factor for 6% and 7 years:

$$PV \text{ of } \$ 1 \text{ each year for next 7 years (at 6\%)} = \frac{1 - \frac{1}{(1.06)^7}}{.06} = 5.5824$$

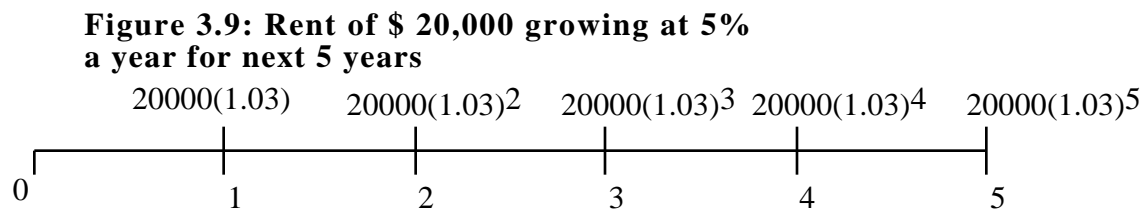
This factor, which would be available in the table summarizing present value factors for annuities under 6% and 7 years, could then be multiplied by the annuity of \$ 15 million to yield a value of \$ 83.74 million. While tables are convenient and available in the appendix to this book, they are restrictive because they usually report factors only for certain discount rates (6% but not 6.08%, for instance) and for specific periods (10 years but not 13 years, for instance).

The second alternative is to use a financial calculator. Financial calculators today are powerful enough to compute the time value of almost any type of cash flow. Many of them

have the present value and future value of annuity equations built into them and thus require the user to input only the key variables. For the example above, for instance, using my HP-17B calculator, we would have inputted the payment (\$ 15 million), the time period (7 years) and the discount rate (6%) and used the present value button on the calculator to arrive at the value of \$ 83.74 million. The only note of caution that we would add is that most financial calculators now allow for myriad options including whether the payment is at the beginning or end of each period and how many periods of compounding and discounting there are in each year. It is important to keep track of these options and ensure that they are correctly set.

Growing Annuities

In the last section, we looked at ways in which we can compound and discount an annuity. A growing annuity is a cash flow that grows at a constant rate for a specified period of time. As an example, assume that you rent your office space and that the rent currently is \$ 20,000 a year. Assume also that there is inflation clause in the agreement that allows the owner of the office building to increase your rent at the rate of inflation, which is expected to be 3% a year. The expected rental cost for the next 5 years, for instance, can then be written as follows:



To compute the present value of these rental payments, we can discount each cash flow back separately, and add up the discounted values. Thus, if the discount rate is 10%, the present value of the rental payment in year 1 can be written as:

$$\text{Present value of year 1 rental payment} = \$20,000 (1.03)/1.10$$

Summing up the values across all five years then, we obtain the following:

$$\begin{aligned} \text{Cumulated present value of rental payments} &= \$20,000 \left(\frac{1.03}{1.10} + \frac{1.03^2}{1.10^2} + \frac{1.03^3}{1.10^3} + \frac{1.03^4}{1.10^4} + \frac{1.03^5}{1.10^5} \right) \\ &= \$20,000 (1.03) \frac{1 - \frac{(1.03)^5}{(1.10)^5}}{(1.10 - 1.03)} \end{aligned}$$

In general, if A is the current cash flow, and g is the expected growth rate, the present value of a growing annuity can be calculated by using the following equation–

$$PV \text{ of a Growing Annuity} = A(1+g) \frac{1 - \frac{(1+g)^n}{(1+r)^n}}{r-g}$$

The present value of a growing annuity can be calculated in all cases but one - where the growth rate is equal to the discount rate. In that case, the present value is equal to the sum of the nominal annuities over the period. The growth effect is exactly offset by the discounting effect. Thus, in the above example, if both the growth rate in rental payments and the discount rate were 12%, the present value of \$20,000 a year for the next 5 years would be \$ 100,000 (\$20,000 *5).

$$PV \text{ of a Growing Annuity for } n \text{ years (when } r=g) = n A$$

Note also that this equation works even when the growth rate is greater than the discount rate.²


 This spreadsheet allows you to estimate the present value of a growing annuity

Illustration 3.11: The Value of A Gold Mine

Suppose you have the rights to a gold mine for the next 20 years, over which period you plan to extract 5,000 ounces of gold every year. The current price per ounce is

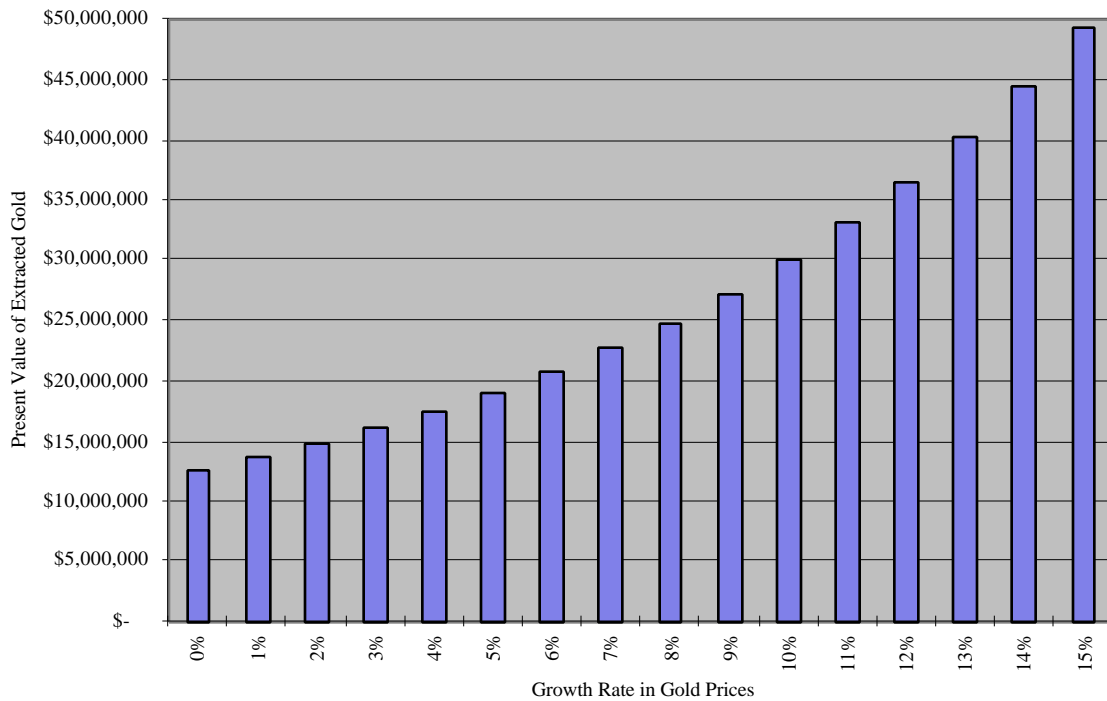
² When g is greater than r , the denominator becomes negative but so does the numerator. The net effect is that the present value can still be computed – it will not become negative.

\$300, but it is expected to increase 3% a year. Assume that the discount rate is 10%. The present value can be computed as follows:

$$PV \text{ of extracted gold} = \$300 * 5000 * (1.03) \frac{1 - \frac{(1.03)^{20}}{(1.10)^{20}}}{.10 - .03} = \$16,145,980$$

The present value of the gold expected to be extracted from this mine is \$16.146 million; it is an increasing function of the expected growth rate in gold prices. Figure 3.10 illustrates the present value as a function of the expected growth rate.

Figure 3.6: Present Value of Extracted Gold as a function of Growth Rate



👉 **CC 3.4:** If both the growth rate and the discount rate increase by 1%, will the present value of the gold to be extracted from this mine increase or decrease? Why?

Perpetuities

An annuity is a constant cash flow for a specific time period. What if you were able to receive a constant cash flow forever? An annuity that lasts forever is called a perpetuity. To compute the present value of a perpetuity, we can use the annuity equation

that we developed earlier and look at the present value as the number of periods approach infinity ().

$$\text{PV of a perpetuity} = A \frac{1 - \frac{1}{(1+r)^n}}{r} = \frac{A}{r}$$

As an example, consider an investment where you will make income of \$60 a year in perpetuity. The present value of this investment, assuming that the interest rate today is 9%, can be computed to be \$ 667.

$$\text{Present Value} = \$ 60/.09 = \$ 667$$

There are at least two cases where this approach is useful. One is the case of a consol bond. Unlike traditional bonds that repay the principal at the end of a specified period (called the maturity date), a consol never matures and pays a fixed coupon forever. In the late 1800s and the early part of the 1900s, the British and Canadian governments issued such bonds, and a few are still in existence. The other is preferred stock. The owner of preferred stock gets a fixed dollar payment, called a preferred dividend, every period, and preferred stock also has an infinite life.

Growing Perpetuities

In corporate finance, we are often called upon to value publicly traded firms, which at least in theory have infinite lives and could conceivably keep growing over these lives. In some cases, we have to analyze projects that could last for very long periods, if not forever, again with increasing earnings each period. In both these scenarios, we have to value not just an infinite stream of cash flows, but a stream of cash flows that grows over that period. A growing perpetuity is a cash flow that is expected to grow at a *constant rate* forever. To estimate the value of a growing perpetuity, we can draw on the equation for a growing annuity and set n to ∞ again.

$$\text{PV of a Growing Perpetuity} = A(1+g) \frac{1 - \frac{(1+g)^n}{(1+r)^n}}{r-g}$$

As long as g is less than r , this equation simplifies to the following:

$$PV \text{ of Growing Perpetuity} = \frac{A(1+g)}{(r-g)}$$

where the numerator is the expected cash flow next year, g is the constant growth rate and r is the discount rate.

When the growth rate is equal to or exceeds the discount rate, the present value of a growing perpetuity is infinite and thus cannot be computed. While there is a mathematical possibility of the growth rate exceeding the discount rate, we can rule it out by making sure that we are reasonable in our estimates of constant growth rate. Since any asset that grows at a rate higher than the growth rate of the economy forever will eventually become the economy, the constant growth rate in this equation has to be less than or equal to the growth rate of the economy. In the United States, for instance, when using this formula, it is almost never reasonable to assume constant growth rates that exceed 5-6%³.

Illustration 3.12: Valuing a Project with Growing Cash Flows

Assume that you have been called upon to compute the present value of the expected cash flows on a theme park (say, Disneyland) for Disney. Assuming that the park can be maintained with new investments each year, we would argue that Disney can not only keep generating cash flows for very long periods from this park, but can also increase ticket prices at about the inflation rate. If Disneyland generated \$ 100 million in cash flows for Disney last year, the discount rate that Disney would use to analyze these cash flows is 12% and the expected inflation rate is 3%, the present value of the cash flows from Disneyland can be calculated:

$$\text{Present Value of Cash Flows} = \$ 100 \text{ million} (1.03)/(.12-.03) = \$1,144 \text{ million}$$

³ This is approximately the nominal growth rate (including inflation) of the US economy. The global economy grows at a slightly higher rate (5.5%-6.5%).

Two points are worth repeating. First, the numerator is the expected cash flow in the next year and thus reflects the expected inflation of 3%. Second, this present value is a fairly good approximation for a long-lived investment, even though it might not last forever. For instance, if we assumed a 50-year life for the park, we would estimate the value of \$ 100 million growing at 3% a year for 50 years to be \$1,127 million:

$$\text{Value of Theme park (with 50-year life)} = \$100 (1.03) \frac{1 - \frac{(1.03)^{50}}{(1.10)^{50}}}{(.10 - .03)} = \$1,127 \text{ million}$$

Thus, the growing perpetuity formula provides a short cut to estimating the value of any long-term investment.

CT 3.3: Assume that you have a cash flow that is expected to grow at different rates each year over time, but the average growth rate forever is 5%. Can you use the growing perpetuity formula? Why or why not?

Summary

The time value of money is a central factor in corporate finance, since we are called upon to analyze projects that generate cash flows over multiple years and value assets with the same characteristics. In this chapter, we began with an intuitive rationale for why we prefer a dollar today to a dollar in the future. The first reason is the presence of inflation that reduces the purchasing power of the dollar over time; the second is the desire for consumption now over consumption in the future; the final component is the uncertainty associated with whether we will receive the dollar in the future. These three factors are measured in a discount rate.

We then looked at the two basic actions we take in computing the time value of money. We first looked at compounding, where we examined how much a dollar today would be worth in the future, when that dollar is invested to earn a rate of return. We then reversed the process and asked how much a dollar in the future would be worth today in a

process called discounting. By discounting cash flows to today, we are able to make cash flows that we receive at different points in time comparable.

As a final section, we looked at devising short cuts to estimating the present value of four types of cash flows. The first of these were annuities, which are constant cash flows each period for a certain number of periods. The second are growing annuities, which is a cash flow growing at a constant rate each period for a certain number of periods. The third are perpetuities, which are annuities that last forever. The fourth are growing perpetuities, which are growing annuities that last forever.

Questions

1. You have a loan of \$ 100,000 coming due in five years. Assuming that you can earn 6% on your investments, how much would you need to set aside today to repay the loan, when it comes due?
2. You buy a new Porsche convertible, put no money down, and agree to make the payments in equal monthly installments over the next 60 months. If the car costs \$ 60,000 and the car dealer charges you 1% a month (as interest), estimate your monthly payments.
3. If you pay 1% a month in interest on a loan, what is your annual interest rate?
4. If you save \$ 15,000 each year for the next 40 years, and earn a 5% interest rate on your savings, how much would you expect to have at the end of the 40th year?
5. You are valuing real estate. The building that you are valuing is expected to generate \$ 25,000 in rental income, growing 3% a year for the next 20 years. With a discount rate of 8%, what is the present value of the rental income?
6. You have just been left an inheritance of \$ 1 million, which is currently earning an interest rate of 5%. If you quit your job, and plan to withdraw \$ 100,000 each year from the inheritance, how long will it last?
7. You have been offered a share of a new business. The business is expected to generate cash flows of \$ 1 million, growing at 3% a year forever. With a discount rate of 15%, what is the value of the business?

Problems

1. You have an expected liability (cash outflow) of \$500,000 in 10 years, and you use a discount rate of 10%.
 - a. How much would you need right now as savings to cover the expected liability?
 - b. How much would you need to set aside at the end of each year for the next 10 years to cover the expected liability?

2. You are examining whether your savings will be adequate to meeting your retirement needs. You saved \$1500 last year, and you expect your annual savings to grow 5% a year for the next 15 years. If you can invest your money at 8%, how much would you expect to have at the end of the fifteenth year?

3. You have just taken a 30-year mortgage loan for \$200,000. The annual percentage rate on the loan is 8%, and payments will be made monthly. Estimate your monthly payments.

4. You are planning to buy a car worth \$20,000. Which of the two deals described below would you choose:
 - the dealer offers to take 10% off the price and lend you the balance at the regular financing rate (which is an annual percentage rate of 9%)
 - the dealer offers to lend you \$20,000 (with no discount) at a special financing rate of 3%

5. A company is planning to set aside money to repay \$100 million in bonds that will be coming due in 10 years. If the appropriate discount rate is 9%,
 - a. how much money would the company need to set aside at the end of each year for the next 10 years to be able to repay the bonds when they come due?
 - b. how would your answer change if the money were set aside at the beginning of each year?

6. You are reviewing an advertisement by a finance company offering loans at an annual percentage rate of 9%. If the interest is compounded weekly, what is the effective interest rate on this loan?
7. You have an relative who has accumulated savings of \$ 250,000 over his working lifetime and now plans to retire. Assuming that he wishes to withdraw equal installments from these savings for the next 25 years of this life, how much will each installment amount to if he is earning 5% on his savings?
8. You are offered a special set of annuities by your insurance company, whereby you will receive \$20,000 a year for the next 10 years and \$30,000 a year for the following 10 years. How much would you be willing to pay for these annuities, if your discount rate is 9% and the annuities are paid at the end of each year? How much would you be willing to pay if they were at the beginning of each year?
9. A bill that is designed to reduce the nation's budget deficit passes both houses of legislature. Congress tells us that the bill will reduce the deficit by \$500 billion over 10 years. What it does not tell us is the timing of the reductions.

<i>Year</i>	<i>Deficit Reduction</i>
1	\$ 25 Billion
2	\$ 30 Billion
3	\$ 35 Billion
4	\$ 40 Billion
5	\$ 45 Billion
6	\$ 55 Billion
7	\$ 60 Billion
8	\$ 65 Billion
9	\$ 70 Billion
10	\$ 75 Billion

If the federal government can borrow at 8%, what is the true deficit reduction in the bill?

10. New York State has a pension fund liability of \$25 billion, due in 10 years. Each year the legislature is supposed to set aside an annuity to arrive at this future value. This annuity is based on what the legislature believes it can earn on this money.

a. Estimate the annuity needed each year for the next 10 years, assuming that the interest rate that can be earned on the money is 6%.

b. The legislature changes the investment rate to 8% and recalculates the annuity needed to arrive at the future value. It claims the difference as budget savings this year. Do you agree?

11. Poor Bobby Bonilla! The newspapers claim that he is making \$5.7 million a year. He claims that this is not true in a present value sense and that he will really be making the following amounts for the next 5 years:

<i>Year</i>	<i>Amount</i>
0 (now)	\$ 5.5 million (Sign up Bonus)
1	\$ 4 million
2	\$ 4 million
3	\$ 4 million
4	\$ 4 million
5	\$ 7 million

a. Assuming that Bonilla can make 7% on his investments, what is the present value of his contract?

b. If you wanted to raise the nominal value of his contract to \$30 million, while preserving the present value, how would you do it? (You can adjust only the sign up bonus and the final year's cash flow.)

12. You are comparing houses in two towns in New Jersey. You have \$100,000 to put as a down payment, and 30-year mortgage rates are at 8% -

	Chatham	South Orange
Price of the house	\$ 400,000	\$ 300,000
Annual Property Tax	\$ 6,000	\$ 12,000

The houses are roughly equivalent.

- Estimate the total payments (mortgage and property taxes) you would have on each house. Which one is less expensive?
- Are mortgage payments and property taxes directly comparable? Why or why not?
- If property taxes are expected to grow 3% a year forever, which house is less expensive?

13. You bought a house a year ago for \$250,000, borrowing \$200,000 at 10% on a 30-year term loan (with monthly payments). Interest rates have since come down to 9%. You can refinance your mortgage at this rate, with a closing cost that will be 3% of the loan. Your opportunity cost is 8%. Ignore tax effects.

- How much are your monthly payments on your current loan (at 10%)?
- How would your monthly payments be if you could refinance your mortgage at 9% (with a 30-year term loan)?
- You plan to stay in this house for the next 5 years. Given the refinancing cost (3% of the loan), would you refinance this loan?
- How much would interest rates have to go down before it would make sense to refinance this loan (assuming that you are going to stay in the house for five years)?

14. You are 35 years old today and are considering your retirement needs. You expect to retire at age 65 and your actuarial tables suggest that you will live to be 100. You want to move to the Bahamas when you retire. You estimate that it will cost you \$ 300,000 to make the move (on your 65th birthday) and that your living expenses will be \$30,000 a year (starting at the end of year 66 and continuing through the end of year 100) after that.

- How much will you need to have saved by your retirement date to be able to afford this course of action?

b. You already have \$50,000 in savings. If you can invest money, tax-free, at 8% a year, how much would you need to save each year for the next 30 years to be able to afford this retirement plan?

c. If you did not have any current savings and do not expect to be able to start saving money for the next 5 years, how much would you have to set aside each year after that to be able to afford this retirement plan?

15. Assume that you the manager of a professional soccer team, and that you are negotiating a contract with your team's star player. You can afford to pay the player only \$1.5 million a year over 3 years (the remaining life of his contract). The player's agent insists that the player will not accept a contract with a nominal value less than \$ 5 million. Can you meet the agent's demand without relaxing your financial constraint on how much you can afford to pay him?

16. You have been hired to run a pension fund for TelDet Inc, a small manufacturing firm. The firm currently has \$5 million in the fund and expects to have cash inflows of \$2 million a year for the first 5 years followed by cash outflows of \$ 3 million a year for the next 5 years. Assume that interest rates are at 8%.

a. How much money will be left in the fund at the end of the tenth year?

b. If you were required to pay a perpetuity after the tenth year (starting in year 11 and going through infinity) out of the balance left in the pension fund, how much could you afford to pay?

17. You are an investment advisor who has been approached by a client for help on his financial strategy. He has \$250,000 in savings in the bank. He is 55 years old and expects to work for 10 more years, making \$100,000 a year. (He expects to make a return of 5% on his investments for the foreseeable future. You can ignore taxes)

- a. Once he retires 10 years from now, he would like to be able to withdraw \$80,000 a year for the following 25 years (his actuary tells him he will live to be ninety years old.). How much would he need in the bank 10 years from now to be able to do this?
- b. How much of his income would he need to save each year for the next 10 years to be able to afford these planned withdrawals (\$80,000 a year) after the tenth year?
- c. Assume that interest rates decline to 4% 10 years from now. How much, if any, would you client have to lower his annual withdrawal by, assuming that he still plans to withdraw cash each year for the next 25 years?

18. You are trying to assess the value of a small retail store that is up for sale. The store generated a cash flow to its owner of \$ 100,000 in the most year of operation, and is expected to have growth of about 5% a year in perpetuity.

- a. If the rate of return required on this store is 10%, what would your assessment be of the value of the store?
- b. What would the growth rate need to be to justify a price of \$ 2.5 million for this store?