Estimating Risk free Rates

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Models of risk and return in finance start off with the presumption that there exists a risk free asset, and that the expected return on that asset is known. The expected return on a risky asset is then estimated as the risk free rate (i.e., the expected return on the risk free asset) plus an expected risk premium. In practice, however, there are two major issue that we have to consider when estimating risk free rates. The first relates to the definition of a risk free security, and the characteristics such a security needs to possess. The second applies when there are no risk free assets, and examines how best to estimate a risk free rate under these conditions. We attempt to deal with both these issues in this paper.

The Risk free Rate

Most risk and return models in finance start off with an asset that is defined as risk free, and use the expected return on that asset as the risk free rate. The expected returns on risky investments are then measured relative to the risk free rate, with the risk creating an expected risk premium that is added on to the risk free rate. But what makes an asset risk free? And what do we do when we cannot find such an asset? These are the questions that we will deal with in this paper.

What is a risk free asset?

To understand what makes an asset risk free, let us go back to how risk is measured in finance. Investors who buys assets have a <u>return that they expect to make</u> over the time horizon that they will hold the asset. The <u>actual returns</u> that they make over this holding period may by very different from the expected returns, and this is where the risk comes in. Risk in finance is viewed in terms of the variance in actual returns around the expected return. For an investment to be risk free in this environment, then, the actual returns should always be equal to the expected return.

To illustrate, consider an investor with a 1-year time horizon buying a 1-year Treasury bill (or any other default-free one-year bond) with a 5% expected return. At the end of the 1-year holding period, the actual return that this investor would have on this investment will always be 5%, which is equal to the expected return. The return distribution for this investment is shown in Figure 1.

Figure 1: Returns on a Riskfree Investment

Probability = 1

Expected Return   Returns
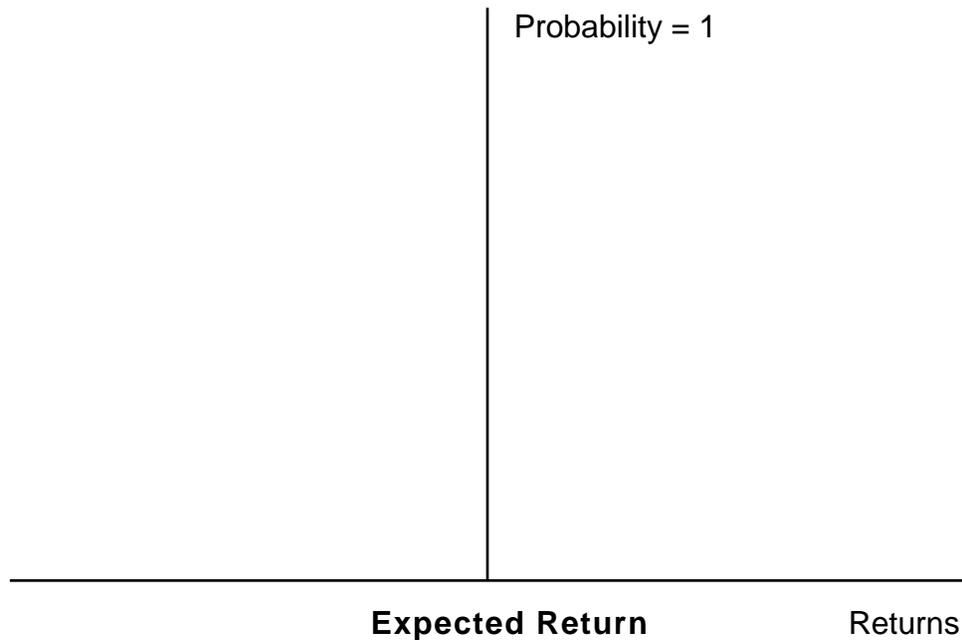
This investment is risk free because there is no variance around the expected return.

Requirements for an Asset to be Risk free

Under what conditions will the actual returns on an investment be equal to the expected returns? In our view, there are two basic conditions that have to be met. The first is that <u>there can be no default risk</u>. Essentially, this rules out any security issued by a private firm, since even the largest and safest firms have some measure of default risk. The only securities that have a chance of being risk free are government securities, not because governments are better run than corporations, but because they control the printing of currency. At least in nominal terms, they should be able to fulfil their promises. Even this assumption, straightforward though it might seem, does not always hold up, especially when governments refuse to honor claims made by previous regimes and when they borrow in currencies other than their own.

There is a second condition that riskless securities need to fulfil that is often forgotten. For an investment to have an actual return equal to its expected return, <u>there can be no reinvestment risk</u>. To illustrate this point, assume that you are trying to estimate the expected return over a five-year period, and that you want a risk free rate. A six-month treasury bill rate, while default free, will not be risk free, because there is the reinvestment risk of not knowing what the treasury bill rate will be in six months. Even a 5-year treasury bond is not risk free, since the coupons on the bond will be reinvested at rates that cannot be predicted today. The risk free rate for a five-year time horizon has to be the expected return on a default-free (government) five-year zero coupon bond. This clearly has painful implications for anyone doing corporate finance or valuation, where expected returns often have to be estimated for periods ranging from one to ten years. A purist's view of risk free rates would then require different risk free rates for each period, and different expected returns.

As a practical compromise, however, it is worth noting that the present value effect of using year-specific risk free rates tends to be small for most well-behaved[1] term structures. In these cases, we could use a duration matching strategy, where the duration of the default-free security used as the risk free asset is matched up to the duration[2] of the cash flows in the analysis. If, however, there are very large differences, in either

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[1] By well behaved term structures, I would include a normal upward sloping yield curve, where long term rates are at most 2-3% higher than short term rates.
[2] In investment analysis, where we look at projects, these durations are usually between 3 and 10 years. In valuation, the durations tend to be much longer, since firms are assumed to have infinite lives. The duration

direction, between short term and long term rates, it does pay to stick with year-specific risk free rates in computing expected returns.

The Practical Implications when a Default-free Entity exists

In most developed markets, where the government can be viewed as a default free entity, at least when it comes to borrowing in the local currency, the implications are simple. When doing investment analysis on longer term projects or valuation, the risk free rate should be the long term government bond rate. If the analysis is shorter term, the short term government security rate can be used as the risk free rate. The choice of a risk free rate also has implications for how risk premiums are estimated. If, as is often the case, historical risk premiums are used, where the excess return earned by stocks over and above a government security rate over a past period is used as the risk premium, the government security chosen has to be same one as that used for the risk free rate. Thus, the historical risk premium used in the US should be the excess return earned by stocks over treasury bonds, and not treasury bills, for purposes of long term analysis.

Cash Flows and Risk free Rates: The Consistency Principle

The risk free rate used to come up with expected returns should be measured consistently with the cash flows are measured. Thus, if cash flows are estimated in nominal US dollar terms, the risk free rate will be the US treasury bond rate. This also implies that it is not where a project or firm is domiciled that determines the choice of a

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in these cases is often well in excess of ten years, and increase with the expected growth potential of the firm.

risk free rate, but the currency in which the cash flows on the project or firm are estimated. Thus, Nestle can be valued using cash flows estimated in Swiss Francs, discounted back at an expected return estimated using a Swiss long term government bond rate, or it can be valued in British pounds, with both the cash flows and the risk free rate being British pound rates. Given that the same project or firm can be valued in different currencies, will the final results always be consistent? If we assume purchasing power parity, then differences in interest rates reflect differences in expected inflation. Both the cash flows and the discount rate are affected by expected inflation; thus, a low discount rate arising from a low risk free rate will be exactly offset by a decline in expected nominal growth rates for cash flows, and the value will remain unchanged.

If the difference in interest rates across two currencies does not adequately reflect the difference in expected inflation in these currencies, the values obtained using the different currencies can be different. In particular, projects and assets will be valued more highly when the currency used is the one with low interest rates relative to inflation. The risk, however, is that the interest rates will have to rise at some point to correct for this divergence, at which point the values will also converge.

Real versus Nominal Risk free Rates

Under conditions of high and unstable inflation, valuation is often done in real terms. Effectively, this means that cash flows are estimated using real growth rates and without allowing for the growth that comes from price inflation. To be consistent, the discount rates used in these cases have to be real discount rates. To get a real expected rate of return, we need to start with a real risk free rate. While government bills and

bonds offer returns that are risk free in nominal terms, they are not risk free in real terms, since expected inflation can be volatile. The standard approach of subtracting an expected inflation rate from the nominal interest rate to arrive at a real risk free rate provides at best an estimate of the real risk free rate.

Until recently, there were few traded default-free securities that could be used to estimate real risk free rates, but the introduction of inflation-indexed treasuries has filled this void. An inflation-indexed treasury security does not offer a guaranteed nominal return to buyers, but instead provides a guaranteed real return. Thus, an inflation-indexed treasury that offers a 3% real return, will yield approximately 7% in nominal terms if inflation is 4% and only 5% in nominal terms if inflation is only 2%.

The only problem is that real valuations are seldom called for or done in the United States, which has stable and low expected inflation. The markets where we would most need to do real valuations, unfortunately, are markets without inflation-indexed default-free securities. The real risk free rates in these markets can be estimated by using one of two arguments:

- The first argument is that as long as capital can flow freely to those economies with the highest real returns, there can be no differences in real risk free rates across markets. Using this argument, the real risk free rate for the United States, estimated from the inflation-indexed treasury, can be used as the real risk free rate in any market.

- The second argument applies if there are frictions and constraints in capital flowing across markets. In that case, the expected real return on a economy, in the long term, should be equal to the expected real growth rate, again in the long term, of that economy, for equilibrium. Thus, the real risk free rate for a mature economy like Germany should be much lower than the real risk free rate for a economy with greater growth potential, such as Hungary.

Risk free Rates when there is no Default-free Entity

Our discussion, hitherto, has been predicated on the assumption that governments do not default, at least on local borrowing. There are many emerging market economies where this assumption might not be viewed as reasonable. Governments in these markets are perceived as capable of defaulting even on local borrowing. When this is coupled with the fact that many governments do not borrow long term locally, there are scenarios where obtaining a local risk free rate, especially for the long term, becomes difficult. Under these cases, there are compromises that give us reasonable estimates of the risk free rate:

- Look at the largest and safest firms in that market, and use the rate that they pay on their long term borrowings in the local currency as a base. Given that these firms, in spite of their size and stability, still have default risk, I would use a rate that is marginally lower[3] than the corporate borrowing rate.

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[3] I would use 0.50% less than the corporate borrowing rate as my risk free rate. This is roughly a AA default spread in the US.

- If there are long term dollar-denominated forward contracts on the currency, you can use interest rate parity and the dollar borrowing rate to arrive at an estimate of the local borrowing rate.

$$\text{Forward Rate}^t{}_{FC,\$} = \text{Spot Rate}_{FC,\$} \, (1+ \text{Interest Rate}_{FC})^t / (1+ \text{Interest Rate}_\$)^t$$

where,

Forward Rate$_{FC,\$}$ = Forward rate for foreign currency units/$

Spot Rate$_{FC,\$}$ = Spot rate for foreign currency units/$

Interest Rate$_{FC}$ = Interest rate in foreign currency

Interest Rate$_\$$ = Interest rate in US dollars

For instance, if the current spot rate is 38.10 Thai Baht per US dollar, the ten-year forward rate is 61.36 Baht per dollar, and the current ten-year US treasury bond rate is 5%, the ten-year Thai risk free rate (in nominal Baht) can be estimated as follows:

$$61.36 \ = 38.10 \, (1+ \text{Interest Rate}_{\text{Thai Baht}})^{10} / \, 1.05^{10}$$

Solving for the Thai interest rate yields a ten-year risk free rate of 10.12%. The biggest limitation of this approach, however, is that forward rates are difficult to come by for periods beyond a year[4] for many of the emerging markets, where we would be most interested in using them.

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[4] In cases where only a one-year forward rate exists, an approximation for the long term rate can be obtained by first backing out the one-year local currency borrowing rate, taking the spread over the one-year treasury bill rate, and then adding this spread on to the long term treasury bond rate. For instance, with a one-year forward rate of 39.95 on the Thai bond, we obtain a one-year Thai baht riskless rate of 9.04% (given a one-year T.Bill rate of 4%). Adding the spread of 5.04% to the ten-year treasury bond rate of 5% provides a ten-year Thai Baht rate of 10.04%.

Summing Up

The risk free rate is the starting point for all expected return models. For an asset to be risk free, it has to meet two conditions –

(1) there can be no risk of default associated with its cash flows and

(2) there can be no reinvestment risk

Using these criteria, the appropriate risk free rate to use to obtain expected returns should be a default-free (government) zero coupon rate that is matched up to when the cash flow or flows that are being discounted occur. In practice, however, it is usually appropriate to match up the duration of the risk free asset to the duration of the cash flows being analyzed. In corporate finance and valuation, this will lead us towards long term government bond rates as risk free rates.

It is also important that the risk free rate be consistent with the cash flows being discounted. In particular, the currency in which the risk free rate is denominated and whether it is a real or nominal risk free rate should be determined by the currency in which the cash flows are estimated and whether the estimation is done in real or nominal terms.