VALUE AND PRICE: AN INTRODUCTION

At the end of 1998, Boeing's stock price was $ 32.25, giving its equity a total market value of $32.595 billion. The Home Depot' stock was trading at $57.94 giving it a market value of $85.668 billion. Boeing's market value of equity had stagnated from 1995 to 1998, whereas the Home Depot had doubled its value over the same period. Clearly, if the objective in decision making is maximizing stock price, the Home Depot has been much more effective than Boeing at accomplishing this objective. But why do these firms trade at the values that they do, and what makes these values change from time to time? To make effective corporate financial decisions, we need to understand the factors that determine firm value, and by extension, the value of the traded shares in the firm.

In this chapter, we examine the fundamental principles of valuation. In general, the value of any asset is the present value of the expected cash flows on the asset, and it is determined by the magnitude of the cash flows, the expected growth rate in these cash flows and the uncertainty associated with receiving these cash flows. We begin by looking at assets with guaranteed cash flows over a finite period, and then we extend the discussion to cover the valuation of assets when there is uncertainty about expected cash flows. As a final step, we consider the valuation of a firm, with the potential, at least, for an infinite life and uncertainty in the cash flows. We also introduce a second component of value that arises when an asset's cash flows are contingent on the occurrence of a specific event. These assets, which are options, cannot be valued using discounted cash flow models. We discuss the broad principles underlying the valuation of these assets.

In the final part of this chapter, we examine the relationship between the value and market price of an asset. In particular, we describe how market prices are set, why they might differ from asset values and what the differences tell us about the efficiency of markets.

Why do we need valuation?

Almost everything we do in corporate finance relies on valuation in one form or the other. When analyzing whether to invest in an asset or project, we assess the value of the asset and compare it to the cost of acquiring the asset. Thus, the question of whether Boeing should invest in a Super Jumbo jet can be re-framed as a question about the value of the Super Jumbo project. In the decision about how to finance investments – whether to use debt or equity – the argument that one financing mix is better than another is based upon the effect it has on the value of the firm and the stock. Similarly, the decision about whether a firm should return cash to its owners or reinvest it back into the business should be based upon the effect that such actions will have on the value of the firm. In fact, the objective in corporate finance is stated, most broadly, as the maximization of firm value, and more narrowly, as the maximization of the stock price. To make good investment, financing and dividend decisions, managers at firms need to understand what determines firm value, and how markets assess that value. In this chapter, we develop the valuation approaches that we will draw on as we approach each of these decisions.

There is another more self-serving reason why we need to understand what drives value. As investors in the stock of a firm, we need to understand what creates its the value and, by extension, what causes the stock price to change from period to period. On the other side of the equation, as managers in these firms are increasingly rewarded using stock or options on the stock of the firms, their compensation is tied to how well or badly the stock price does. Here again, an understanding of what drives value and stock prices may help managers maximize their own compensation.

In chapter 3, we considered the time value of money and the way cash flows in the future can be discounted to yield equivalent cash flows today. Extending this concept, we can estimate the value of an asset by taking the present value of the expected cash flows on that asset. Consequently, the value of any asset is a function of the cash flows generated by that asset, the life of the asset, the expected growth in the cash flows and the riskiness associated with the cash flows. We will begin this section by looking at valuing assets that have finite lives (at the end of which they cease to generate cash flows) and conclude by looking at the more difficult case of assets with infinite lives. We will also start the process by looking at firms whose cash flows are known with certainty and conclude by looking at how we can consider uncertainty in valuation.

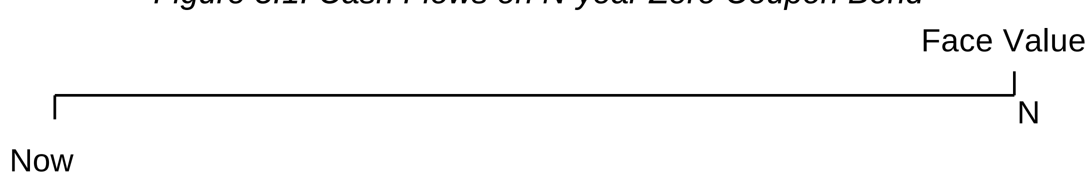
Valuing an Asset with Guaranteed Cash Flows

The simplest assets to value have cash flows that are guaranteed, i.e, assets whose promised cash flows are always delivered. Such assets are riskless, and the interest rate earned on them is called a **riskless rate**. The value of such an asset is the present value of the cash flows, discounted back at the riskless rate. Generally speaking, riskless investments are issued by governments that have the power to print money to meet any obligations they otherwise cannot cover. Not all government obligations are not riskless, though, since some governments have defaulted on promised obligations.

Default-free Zero-coupon Bond

The simplest asset to value is a bond that pays no coupon but has a face value that is guaranteed at maturity; this bond is a default-free zero coupon bond. Using the time lines we developed in the chapter on time value, we can show the cash flow on this bond as in Figure 5.1.

Figure 5.1: Cash Flows on N-year Zero Coupon Bond



The value of this bond can be written as the present value of a single cash flow discounted back at the riskless rate.

$$\text{Value of Zero Coupon Bond} = \frac{\text{Face Value of Bond}}{(1+r)^N}$$

where r is the riskless rate on the zero-coupon and N is the maturity of the zero-coupon bond. Since the cash flow on this bond is fixed, the value of the bond will vary inversely with the riskless rate. As the riskless rate increases, the value of the bond will decrease.

Illustration 5.1: Valuing a Zero Coupon Treasury Bond

Assume that the default-free ten-year interest rate on riskless investments is 4.55%, and that you are pricing a zero-coupon treasury bond, with a maturity of ten years and a face value of $ 1000. The price of the bond can be estimated as follows:

$$\text{Price of the Bond} = \frac{\$1,000}{(1.0455)^{10}} = \$\,640.85$$

Note that the face value is the only cash flow, and that this bond will be priced well below the face value of $ 1,000. Such a bond is said to be trading below par.

Conversely, we could estimate a default-free interest rate from the price of a zero-coupon treasury bond. For instance, if the 10-year zero coupon treasury were trading at $ 593.82, the default-free ten-year spot rate can be estimated as follows:

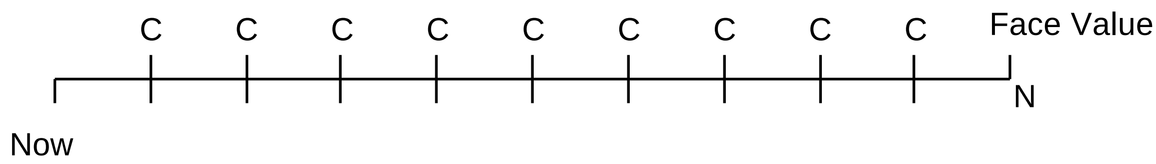
$$\text{Default-free Spot Rate} = \left(\frac{\text{Face Value of Bond}}{\text{Market Value of Bond}}\right)^{1/t} - 1 = \left(\frac{1000}{593.82}\right)^{1/10} - 1 = .0535$$

The ten-year default free rate is 5.35%.

Default-free Coupon Bond

Consider, now, a default-free coupon bond, which has fixed cash flows (coupons) that occur at regular intervals (usually semi annually) and a final cash flow (face value) at maturity. The time line for this bond is shown in Figure 5.2 (with C representing the coupon each period and N being the maturity of the bond).

Figure 5.2: Cash Flows on N-year Coupon Bond



This bond can actually be viewed as a series of zero-coupon bonds, and each can be valued using the riskless rate that corresponds to when the cash flow comes due:

$$\text{Value of Coupon Bond} = \sum_{t=1}^{t=N} \frac{\text{Coupon}}{(1+r_t)^t} + \frac{\text{Face Value of the Bond}}{(1+r_N)^N}$$

where r_t is the interest rate that corresponds to a t-period zero coupon bond and the bond has a life of N periods.

It is, of course, possible to arrive at the same value using some weighted average of the period-specific riskless rates used above; the weighting will depend upon how large each cash flow is and when it comes due. This weighted average rate is called the yield to maturity, and it can be used to value the same coupon bond:

$$\text{Value of Coupon Bond} = \sum_{t=1}^{t=N} \frac{\text{Coupon}}{(1+r)^t} + \frac{\text{Face Value of the Bond}}{(1+r)^N}$$

where r is the yield to maturity on the bond. Like the zero-coupon bond, the default-free coupon bond should have a value that varies inversely with the yield to maturity. As we will see shortly, since the coupon bond has cash flows that occur earlier in time (the coupons) it should be less sensitive to a given change in interest rates than a zero-coupon bond with the same maturity.

Illustration 5.2: Valuing a Default-free Coupon Bond

Consider now a five-year treasury bond with a coupon rate of 5.50%, with coupons paid every 6 months. We will price this bond initially using default-free spot rates for each cash flow in Table 5.1.

Table 5.1: Value of 5-year default-free bond

Time	*Coupon*	*Default-free Rate*	*Present Value*
0.5	$ 27.50	4.15%	$ 26.95
1	$ 27.50	4.30%	$ 26.37
1.5	$ 27.50	4.43%	$ 25.77
2	$ 27.50	4.55%	$ 25.16
2.5	$ 27.50	4.65%	$ 24.55
3	$ 27.50	4.74%	$ 23.93
3.5	$ 27.50	4.82%	$ 23.32
4	$ 27.50	4.90%	$ 22.71
4.5	$ 27.50	4.97%	$ 22.11
5	$ 1,027.50	5.03%	$ 803.92
			$ 1,024.78

The default-free spot interest rates reflect the market interest rates for zero coupon bonds for each maturity. The bond price can be used to estimate a weighted-average interest rate for this bond:

$$\$1,024.78 = \sum_{t=0.5}^{t=5} \frac{\$27.50}{(1+r)^t} + \frac{\$1,000}{(1+r)^5}$$

Solving for r, we obtain a rate of 4.99%, which is the yield to maturity on this bond.

☞*CC 5.1*: This bond trades at above its face value of $1,000 and has a yield to maturity below the coupon rate. What would the yield to maturity on this bond have to be for the bond to trade at par?

Bond Value and Interest Rate Sensitivity and Duration

As market interest rates change, the market value of a bond will change. Consider, for instance, the 10-year zero coupon bond and the 5-year coupon bond described in the last two illustrations. Figure 5.3 shows the market value of each of these bonds as market interest rates vary from 3% to 10%.

Note that the price of the 10-year zero-coupon bond is much more sensitive to interest rate changes than is the 5-year coupon bond to a given change in market interest rates. The 10-year zero coupon bond loses about half its value as interest rates increase from 3% to 10%; in contrast, the 5-year 5.5% coupon bond loses about 30% of its value. This should not be surprising since the present value effect of that interest rate increases the larger the cash flow, and the further in the future it occurs. Thus longer-term bonds will be more sensitive to interest rate changes than shorter-term bonds, with similar coupons. Furthermore, low-coupon or no-coupon bonds will be more sensitive to interest rate changes than high-coupon bonds.

The interest rate sensitivity of a bond, which is a function of both the coupon rate and the maturity of the bond, can be captured in one measure called the duration. The greater the duration of a bond, the more sensitive its price is to interest rate movements.. The simplest measure of duration, called Macaulay duration, can be viewed as a weighted maturity of the different cash flows on the bond.

$$\text{Duration of a Bond} = \frac{\sum_{t=1}^{t=N} t \frac{CF_t}{(1+r)^t}}{\sum_{t=1}^{t=N} \frac{CF_t}{(1+r)^t}}$$

where r is the yield to maturity on the bond.

For a zero-coupon bond, which has only one cash flow, due at maturity, the duration is equal to the maturity.

Duration of 10-year zero-coupon bond = 10 years

The duration of the 5-year coupon bond requires a few more calculations, is calculated in the Table 5.2:

Table 5.2: Value of a 5-year Coupon Bond

Time (t)	*Coupon*	*Present Value (at 4.99%)*	*t *Present Value*
0.5	$27.50	$26.84	$13.42
1	$27.50	$26.19	$26.19
1.5	$27.50	$25.56	$38.34
2	$27.50	$24.95	$49.90
2.5	$27.50	$24.35	$60.87
3	$27.50	$23.76	$71.29
3.5	$27.50	$23.19	$81.17
4	$27.50	$22.63	$90.53
4.5	$27.50	$22.09	$99.40
5	$1,027.50	$805.46	$4,027.28
Sum		$1,025.02	$4,558.39

Duration of 5-year 5.5% coupon bond = $4,558/$1,025 = 4.45

The longer the duration of a bond, the more sensitive it is to interest rate changes. In our illustrations above, the ten-year coupon bond has a higher duration and will therefore be more sensitive to interest rate changes than the five-year coupon bond.

🖋 CT 5.1: The duration of a bond generally increases as the maturity of the bond increases. Estimate the duration of a perpetual bond.

Introducing Uncertainty into Valuation

We have to grapple with two different types of uncertainty in valuation. The first arises in the context of securities like bonds, where there is a promised cash flow to the holder of the bonds in future periods. The risk that these cash flows will not be delivered is called **default risk**; the greater the default risk in a bond, given its cash flows, the less valuable the bond will become.

The second type of risk is more complicated. When we make equity investments in assets, we are generally not promised a fixed cash flow but are entitled, instead, to whatever cash flows are left over after other claim holders (like debt) are paid; these cash flows are called residual cash flows. Here, the uncertainty revolves around what these residual cash flows will be, relative to expectations. In contrast to default risk, where the risk can only result in negative consequences (the cash flows delivered will be less than promised), uncertainty in the context of equity investments can cut both ways. The actual cash flows can be much lower than expected, but they can also be much higher. For the moment, we will label this risk **equity risk** and consider, at least in general terms, how best to deal with it in the context of valuing an equity investment.

Valuing an Asset with Default Risk

We will begin a section on how we assess default risk and adjust interest rates for default risk, and then consider how best to value assets with default risk.

Measuring Default Risk and Estimating Default-risk adjusted Rates

When valuing investments where the cash flows are promised, but there is a risk that they might not be delivered, it is no longer appropriate to use the riskless rate as the discount rate. The appropriate discount rate here will include the riskless rate and an appropriate premium for the default risk called a default spread. There are two parts to estimating this spread. The first part is assessing the default risk of an entity. While banks do this routinely when making loans to individuals and businesses, investors buying bonds in firms get some help, at least in the United States, from independent ratings agencies like Standard and Poor's and Moody's. These agencies measure the default risk and give the bonds a rating that measures the default risk. Table 5.3 summarizes the ratings used by Standard and Poor's and Moody's to rate US companies.

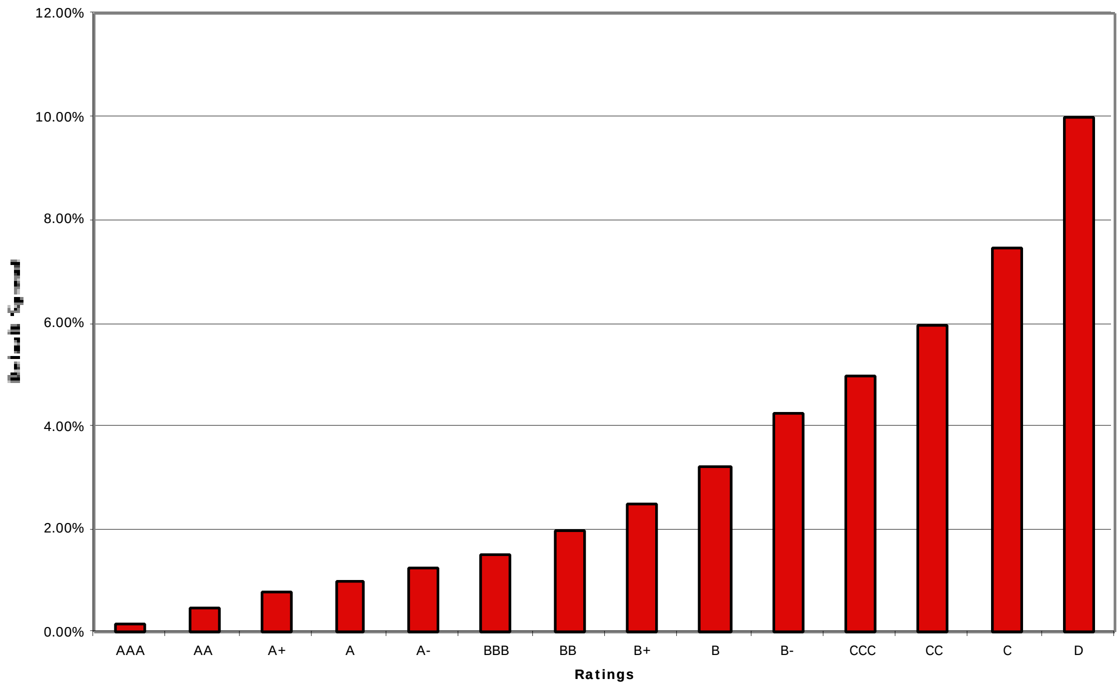
Table 5.3: Ratings Description

Standard and Poor's		Moody's	
AAA	The highest debt rating assigned. The borrower's capacity to repay debt is extremely strong.	Aaa	Judged to be of the best quality with a small degree of risk.
AA	Capacity to repay is strong and differs from the highest quality only by a small amount.	Aa	High quality but rated lower than Aaa because margin of protection may not be as large or because there may be other elements of long-term risk.
A	Has strong capacity to repay; Borrower is susceptible to adverse effects of changes in circumstances and economic conditions.	A	Bonds possess favorable investment attributes but may be susceptible to risk in the future.
BBB	Has adequate capacity to repay, but adverse economic conditions or circumstances are more likely to lead to risk.	Baa	Neither highly protected nor poorly secured; adequate payment capacity.
BB,B, CCC, CC	Regarded as predominantly speculative, BB being the least speculative andd CC the most.	Ba	Judged to have some speculative risk.
		B	Generally lacking characteristics of a desirable investment; probability of payment small.
D	In default or with payments in arrears.	Caa	Poor standing and perhaps in default.
		Ca	Very speculative; often in default.
		C	Highly speculative; in default.

While ratings agencies do make mistakes, the rating system saves investors a significant amount of cost that otherwise would have been expended doing research on the default risk of issuing firms.

The second part of the risk-adjusted discount rate assessment is coming up with the default spread. The demand and supply for bonds within each ratings class determines the appropriate interest rate for that rating. Lowly rated firms have more default risk and generally have to pay much higher interest rates on their bonds than highly rated firms. The spread itself will change over time, tending to increase for all ratings classes in economic recessions and to narrow for all ratings classes in economic recoveries. Figure 5.4 summarizes default spreads for bonds in S&P's different rating classes as of December 31, 1998:

Figure 5.4: Default Spreads and Ratings



These default spreads, when added to the riskless rate, yield the interest rates for bonds with the specified ratings. For instance, a D rated bond has an interest rate about 10% higher than the riskless rate.

Valuing an Asset with Default Risk

The most common example of an asset with just default risk is a corporate bond, since even the largest, safest companies still have some risk of default. While the basic structure of the valuation remains the same -- that is, expected cash flows are discounted at a discount rate -- the discount rate used for a bond with default risk will be higher than that used for default-free bond. Furthermore, as the default risk increases, so will the discount rate used:

$$\text{Value of Corporate Coupon Bond} = \sum_{t=1}^{t=N} \frac{\text{Coupon}}{(1+k_d)^t} + \frac{\text{Face Value of the Bond}}{(1+k_d)^N}$$

where k_d is the market interest rate given the default risk.

Illustration 5.3: Valuing a Coupon Bond with Default Risk

Boeing has a bond outstanding with a coupon rate of 8.75%, maturing in 35 years. Based upon its default risk (measured by a bond rating assigned to Boeing by Standard and Poor's at the time of this analysis), the market interest rate on Boeing's debt is 0.5% higher than the treasury bond rate of 5.5% for default-free bonds of similar maturity. The price of the bond can be estimated as follows:

$$\text{Price of Boeing bond} = \sum_{t=0.5}^{t=35} \frac{43.875}{(1.06)^t} + \frac{1{,}000}{(1.06)^{35}} = \$1{,}404.25$$

The coupons were assumed to be semi-annual and the present value was estimated using the annuity equation. Note that the default risk on the bond is reflected in the interest rate used to discount the expected cash flows on the bond. If Boeing's default risk increases, the price of the bond will drop to reflect the higher market interest rate.

☞*CC 5.2*: Assume now that Boeing is viewed as a riskier firm, and that its rating drops. If the market interest rate increases to 7.5%, estimate how much you would lose as the holder of this bond.

Valuing an Asset with Equity Risk

Having valued assets with guaranteed cash flows and those with only default risk, let us now consider the valuation of assets with equity risk. We will begin with the introduction to the way we estimate cash flows and consider equity risk in investments with equity risk, and then we look at how best to value these assets.

Measuring Cash Flows for an Asset with Equity Risk

Unlike the bonds that we have valued so far in this chapter, the cash flows on assets with equity risk are not promised cash flows. Instead, the valuation is based upon the expected cash flows on these assets over their lives. We will consider two basic questions: the first relates to how we measure these cash flows, and the second to how to come up with expectations for these cash flows.

To estimate cash flows on an asset with equity risk, let us first consider the perspective of the owner of the asset, i.e. the equity investor in the asset. Assume that the owner borrowed some of the funds needed to buy the asset. The cash flows to the owner will therefore be the cash flows generated by the asset after all expenses and taxes, and also after payments due on the debt. This cash flow, which is after debt payments, operating expenses and taxes, is called the **cash flow to equity investors**. There is also a broader definition of cash flow that we can use, where we look at not just the equity investor in the asset, but at the total cash flows generated by the asset for both the equity investor and the lender. This cash flow, which is before debt payments but after operating expenses and taxes, is called the **cash flow to the firm** (where the firm is considered to include both debt and equity investors).

Note that, since this is a risky asset, the cash flows are likely to vary across a broad range of outcomes, some good and some not so positive. To estimate the expected cash flow, we consider all possible outcomes in each period, weight them by their relative probabilities[1] and arrive at an expected cash flow for that period.

Measuring Equity Risk and Estimate Risk-Adjusted Discount Rates

When we analyzed bonds with default risk, we argued that the interest rate has to be adjusted to reflect the default risk. This default-risk adjusted interest rate can be considered

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[1] Note that in many cases, though we might not explicitly state probabilities and outcomes, we are implicitly doing so, when we use expected cash flows.

the **cost of debt** to the investor or business borrowing the money. When analyzing investments with equity risk, we have to make an adjustment to the riskless rate to arrive at a discount rate, but the adjustment will be to reflect the equity risk rather than the default risk. Furthermore, since there is no longer a promised interest payment, we will term this rate a risk-adjusted discount rate rather than an interest rate. We label this adjusted discount rate the **cost of equity**.

We argued earlier that a firm can be viewed as a collection of assets, financed partly with debt and partly with equity. The composite cost of financing, which comes from both debt and equity, is a weighted average of the costs of debt and equity, with the weights depending upon how much of each financing is used. This cost is labeled the cost of capital.

For instance, assume that Boeing has a cost of equity of 10.54% and a cost of debt of 3.58%. Assume also that it raised 80% of its financing from equity and 20% from debt. Its cost of capital would then be

Cost of Capital = 10.58% (.80) + 3.58% (.20) = 9.17%

Thus, for Boeing, the cost of equity is 10.54% while the cost of capital is only 9.17%.

If the cash flows that we are discounting are cash flows to equity investors, as defined in the previous section, the appropriate discount rate is the cost of equity. If the cash flows are prior to debt payments and therefore to the firm, the appropriate discount rate is the cost of capital.

Valuing an Asset with Equity Risk and Finite Life

Most assets that firms acquire have finite lives. At the end of that life, the assets are assumed to lose their operating capacity, though they might still preserve some value. To illustrate, assume that you buy an apartment building and plan to rent the apartments out to earn income. The building will have a finite life, say 30 to 40 years, at the end of which it will have to be torn down and a new building constructed, but the land will continue to have value even if this occurs.

This building can be valued using the cash flows that it will generate, prior to any debt payments, and discounting them at the composite cost of the financing used to buy the building, i.e. , the cost of capital. At the end of the expected life of the building, we estimate what the building (and the land it sits on) will be worth and discount this value back to the present, as well. In summary, the value of a finite life asset can be written as:

$$\text{Value of Finite-Life Asset} = \sum_{t=1}^{t=N} \frac{\text{E(Cash flow on Asset}_t)}{(1+k_c)^t} + \frac{\text{Value of Asset at End of Life}}{(1+k_c)^N}$$

where k_c is the cost of capital.

This entire analysis can also be done from your perspective as the sole equity investor in this building. In this case, the cash flows will be defined more narrowly as cash flows after debt payments, and the appropriate discount rate becomes the cost of equity. At the end of the building's life, we still look at how much it will be worth but consider only the cash that will be left over after any remaining debt is paid off. Thus, the value of the equity investment in an asset with a fixed life of N years, say an office building, can be written as follows:

$$\text{Value of Equity in Finite-Life Asset} = \sum_{t=1}^{t=N} \frac{\text{E(Cash Flow to Equity}_t)}{(1+k_e)^t} + \frac{\text{Value of Equity in Asset at End of Life}}{(1+k_e)^N}$$

where k_e is the rate of return that the equity investor in this asset would demand given the riskiness of the cash flows and the value of equity at the end of the asset's life is the value of the asset net of the debt outstanding on it.

Can you extend the life of the building by reinvesting more in maintaining it? Possibly. If you choose this course of action, however, the life of the building will be longer, but the cash flows to equity and to the firm each period have to be reduced[2] by the amount of the reinvestment needed for maintenance.

Illustration 5.4: Value of a Finite-Lived Asset: A Home Depot Store

Consider the Home Depot's investment in a proposed store. The store is assumed to have a finite life of 12 years and is expected to have cash flows before debt payments and after reinvestment needs of $ 1 million, growing at 5% a year for the next 12 years. The store is also expected to have a value of $ 2.5 million at the end of the 12th year (called the salvage value). The Home Depot's cost of capital is 9.51%. The value of the store can be estimated in Table 5.4:

Table 5.4: Value of Home Depot Store

Year	Expected Cash Flows	Value at End	PV at 9.51%
1	$ 1,050,000		$ 958,817
2	$ 1,102,500		$ 919,329
3	$ 1,157,625		$ 881,468
4	$ 1,215,506		$ 845,166
5	$ 1,276,282		$ 810,359
6	$ 1,340,096		$ 776,986
7	$ 1,407,100		$ 744,987
8	$ 1,477,455		$ 714,306
9	$ 1,551,328		$ 684,888
10	$ 1,628,895		$ 656,682
11	$ 1,710,339		$ 629,638
12	$ 1,795,856	$ 2,500,000	$ 1,444,124
		Value of Store =	$ 10,066,749

Note that the cash flows over the next 12 years represent what we called in chapter 3 a growing annuity, and the present value could have been computed with a simple present value equation, as well.

$$\text{Value of Store} = \frac{1{,}000{,}000\,(1.05)\left(1-\frac{(1.05)^{12}}{(1.0951)^{12}}\right)}{(.0951-.05)} + \frac{2{,}500{,}000}{(1.0951)^{12}} = \$10{,}066{,}749$$

[2] By maintaining the building better, you might also be able to charge higher rents, which may provide an offsetting increase in the cash flows.

This store has a value of $10.07 million to the Home Depot. The implication is that the company should open the store if it costs t less than $10.07 million. This value will increase as the expected growth rate increases and the cost of capital decreases.

Illustration 5.5: Valuing Equity in a Finite-Life Asset

Consider the Home Depot's equity investment in the store described in illustration 5.4. Assume that the cash flows from the store after debt payments and reinvestment needs are expected will be $ 850,000 a year, growing at 5% a year for the next 12 years. In addition, assume that the salvage value of the store, after repaying remaining debt will be $ 1 million. Finally, assume that the cost of equity is 9.78%. The value of equity in this store can be estimated as follows:

$$\text{Value of Equity in Store} = \frac{850{,}000\,(1.05)\left(1 - \frac{(1.05)^{12}}{(1.0978)^{12}}\right)}{(.0978-.05)} + \frac{1{,}000{,}000}{(1.0978)^{12}} = \$8{,}053{,}999$$

Note that the value of equity in the store is also an increasing function of expected growth and the store life, and a decreasing function of the cost of equity.

🖋 CT 5.2: A significant portion of the value of the store comes from the estimated value of the land at the end of the store’s life. If the Home Depot had leased the store rather than buying it, the land would have reverted back to the lessor at the end of the store’s life. Does this imply that the net present value of the store will decline if the store is leased?

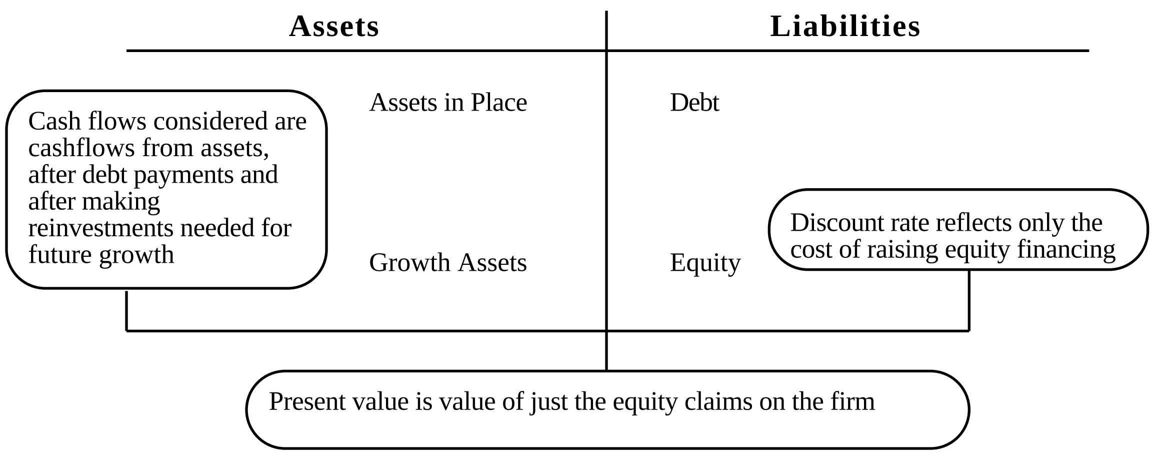
Valuing an Asset with an Infinite Life

When we value businesses and firms, as opposed to individual assets, we are often looking at entities that have no finite life. If they reinvest sufficient amounts in new assets each period, firms could keep generating cash flows forever. In this section, we value assets that have infinite lives and uncertain cash flows.

Equity and Firm Valuation

In the section on valuing assets with equity risk, we introduced the notions of cash flows to equity and cash flows to the firm. We argued that cash flows to equity are cash flows after debt payments, all expenses and reinvestment needs have been met. In the context of a business, we will use the same definition to measure the cash flows to its equity investors. These cash flows, when discounted back at the cost of equity for the business, yields the value of the equity in the business. This is illustrated in Figure 5.5:

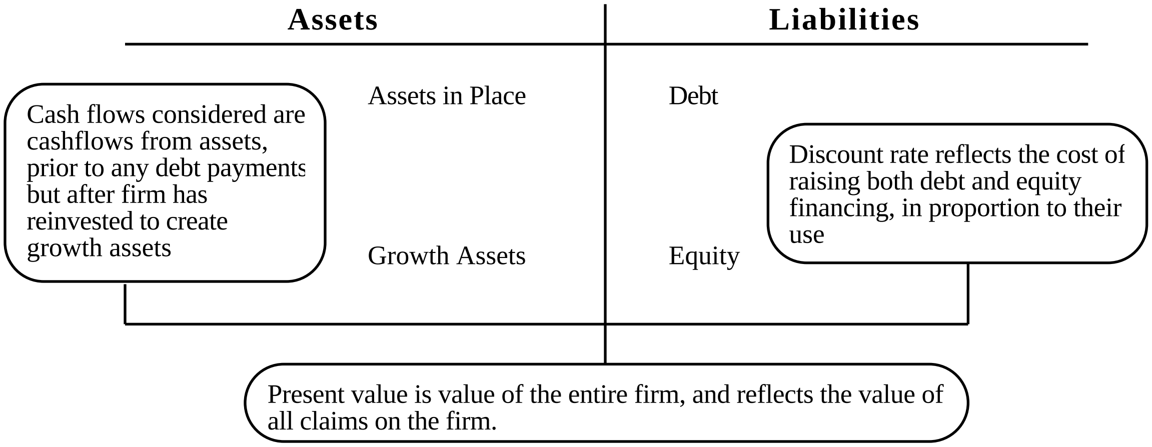
Figure 5.5: Equity Valuation



Note that our definition of both cash flows and discount rates is consistent – they are both defined in terms of the equity investor in the business.

There is an alternative approach in which, instead of valuing the equity stake in the asset or business, we look at the value of the entire business. To do this, we look at the collective cash flows not just to equity investors but also to lenders (or bondholders in the firm). The appropriate discount rate is the cost of capital, since it reflects both the cost of equity and the cost of debt. The process is illustrated in Figure 5.6.

Figure 5.6: Firm Valuation



Note again that we are defining both cash flows and discount rates consistently, to reflect the fact that we are valuing not just the equity portion of the investment but the investment itself.

Dividends and Equity Valuation

When valuing equity investments in publicly traded companies, we could argue that the only cash flows investors in these investments get from the firm are dividends. Therefore, the value of the equity in these investments can be computed as the present value of expected dividend payments on the equity.

$$\text{Value of Equity (Only Dividends)} = \sum_{t=1}^{t=\infty} \frac{E(\text{Dividend}_t)}{(1+k_e)^t}$$

The mechanics are similar to those involved in pricing a bond, with dividend payments replacing coupon payments, and the cost of equity replacing the interest rate on the bond. The fact that equity in a publicly traded firm has an infinite life, however, indicates that we cannot arrive at closure on the valuation without making additional assumptions.

One way in which we might be able to estimate the value of the equity in a firm is by assuming that the dividends, starting today, will grow at a constant rate forever. If we do that, we can estimate the value of the equity using the present value formula for a perpetually growing cash flow in chapter 3. In fact, the value of equity will be

$$\text{Value of Equity (Dividends growing at a constant rate forever)} = \frac{\text{E(Dividend next period)}}{(k_e - g_n)}$$

This model, which is called the **Gordon growth model**, is simple but limited, since it can value only companies that pay dividends, and only if these dividends are expected to grow at a constant rate forever. The reason this is a restrictive assumption is that no asset or firm's cash flows can grow forever at a rate higher than the growth rate of the economy. If it did, the firm would become the economy. Therefore, the constant growth rate is constrained to be less than or equal to the economy's growth rate. For valuations of firms in US dollars, this puts an upper limit on the growth rate of approximately 5-6%[3]. This constraint will also ensure that the growth rate used in the model will be less than the discount rate.

What happens if we have to value a stock whose dividends are growing at 15% a year? The solution is simple. We value the stock in two parts. In the first part, we estimate the expected dividends each period for as long as the growth rate of this firm's dividends remains higher than the growth rate of the economy, and sum up the present value of the dividends. In the second part, we assume that the growth rate in dividends will drop to a stable or constant rate forever sometime in the future. Once we make this assumption, we can apply the Gordon growth model to estimate the present value of all dividends in stable growth. This present value is called the **terminal price** and represents the expected value of the stock in the future, when the firm becomes a stable growth firm. The present value of this terminal price is added to the present value of the dividends to obtain the value of the stock today.

$$\text{Value of Equity with high-growth dividends} = \sum_{t=1}^{t=N} \frac{\text{E(Dividends}_t)}{(1+k_e)^t} + \frac{\text{Terminal Price}_N}{(1+k_e)^N}$$

[3] The nominal growth rate of the US economy through the nineties has been about 5%. The growth rate of the global economy, in nominal US dollar terms, has been about 6% over that period.

where N is the number of years of high growth and the terminal price is based upon the assumption of stable growth beyond year N.

$$\text{Terminal Price} = \frac{E(\text{Dividend}_{N+1})}{(k_e - g_n)}$$

Illustration 5.6: Valuing a Stable Growth Stock with Dividends

Consolidated Edison, the utility that produces power for much of New York city, paid dividends per share of $ 2.12 in 1998. The dividends are expected to grow 5% a year in the long term, and the company has a cost of equity of 9.40%. The value per share can be estimated as follows:

Value of Equity per share = $2.12 (1.05) / (.094 - .05) = $ 50.59

The stock was trading at $ 54 per share at the time of this valuation. We could argue that based upon this valuation, the stock was mildly overvalued.

☞*CC 5.3*: If the stock is actually trading at $ 45, estimate the growth rate the market expects in Con Ed's dividends.

Illustration 5.7: Valuing a Dividend-paying Stock with High Growth in Dividends

Assume that you were trying to value Coca Cola. The company paid $0.69 as dividends per share during 1998, and these dividends are expected to grow 25% a year for the next 10 years. Beyond that, the expected growth rate is expected to be 6% a year forever. Assuming a cost of equity of 11% for Coca Cola, we can estimate the value of the stock in two parts and then estimate its value today.

I. Estimate the value of expected dividends during the next 10 years

The expected dividends during the high growth phase are estimated in the table 5.5. The present values of the dividends are estimated using the cost of equity of 11% in the last column.

Table 5.5: Value of Expected Dividends during High-Growth Phase

Year	Dividends per Share	Present Value

1	$ 0.86	$ 0.78
2	$ 1.08	$ 0.88
3	$ 1.35	$ 0.99
4	$ 1.68	$ 1.11
5	$ 2.11	$ 1.25
6	$ 2.63	$ 1.41
7	$ 3.29	$ 1.58
8	$ 4.11	$ 1.78
9	$ 5.14	$ 2.01
10	$ 6.43	$ 2.26
	PV of Dividends	$ 14.05

II. Estimate the terminal value of the stock at the end of the high growth phase

To estimate the terminal price, we first estimate the dividends per share one year past the high growth phase and use the perpetual growth equation to compute present value. For Coca Cola, the estimates are as follows:

Expected Dividends per share in year 11 = $ 6.43 *1.06 = $ 6.81

Expected Terminal Price = $ 6.81 / (.11 - .06) = $ 136.24

III. Estimate the value of the stock today

To estimate the value of the stock today, we add the present value of the terminal price estimated in the previous step to the present value of the dividends during the high growth period:

Value of Stock today = PV of Dividends in high growth + PV of Terminal Price

$$= \$ 14.05 + \$ 136.24/(1.11)^{10} = \$62.03$$

☞*CC 5.4*: Estimate Coca Cola's value per share, if the period of high growth is 5 years instead of 10 (the growth rate remains 25%).

A Broader Measure of Cash Flows to Equity

There are two significant problems with the use of just dividends to value equity. The first is that it works only cash flows to the equity investors take the form of dividends. It will not work for valuing equity in private businesses, where the owners often withdraw cash from the business but may not call it dividends, and it may not even work for publicly traded companies if they return cash to the equity investors by buying back stock, for instance. The second problem is that the use of dividends is based upon the assumption that firms pay out what they can afford to in dividends. When this is not true, the dividend discount models will mis-estimate the value of equity.

To counter this problem, we consider a broader definition of cash flow to which we call **free cash flow to equity**, defined as the cash left over after all operating expenses, net debt payments and reinvestment needs have been met. By **net debt payments**, we are referring to the difference between new debt issued and repayments of old debt. If the new debt issued exceeds debt repayments, the free cash flow to equity will be higher.

Free Cash Flow to Equity (FCFE) = Net Income – Reinvestment Needs – (Debt Repaid – New Debt Issued)

Think of this as potential dividends, or what the company could have paid out in dividend. To illustrate, in 1998, the Home Depot’s free cash flow to equity using this definition was:

$FCFE_{Boeing}$ = Net Income – Reinvestment Needs – (Debt Repaid – New Debt Issued)

= \$ 1,614 million - \$1,876 million – (8 – 246 million) = - \$ 24 million

Clearly, the Home Depot did not generate positive cash flows after reinvesment needs and net debt payments. Surprisingly, the firm did pay a dividend, albeit a small one. Any dividends paid by the Home Depot during 1998 had to be financed with existing cash balances, since the free cash flow to equity is negative.

Once the free cash flows to equity have been estimated, the process of estimating value parallels the dividend discount model. To value equity in a firm where the free cash flows to equity are growing at a constant rate forever, we use the present value equation to estimate the value of cash flows in perpetual growth:

$$\text{Value of Equity in Infinite -Life Asset} = \frac{E(FCFE_t)}{(k_e - g_n)}$$

All the constraints relating to the magnitude of the constant growth rate used that we discussed in the context of the dividend discount model, continue to apply here.

In the more general case, where free cash flows to equity are growing at a rate higher than the growth rate of the economy, the value of the equity can be estimated again in two parts. The first part is the present value of the free cash flows to equity during the high growth phase, and the second part is the present value of the terminal value of equity, estimated based on the assumption that the firm will reach stable growth sometime in the future.

$$\text{Value of Equity with high growth FCFE} = \sum_{t=1}^{t=N} \frac{E(FCFE_t)}{(1+k_e)^t} + \frac{\text{Terminal Value of Equity}_N}{(1+k_e)^N}$$

With the FCFE approach, we have the flexibility we need to value equity in any type of business or publicly traded company.

Illustration 5.8: Valuing Equity using FCFE

Consider the case of the Home Depot. Assume that we expect the free cash flows to equity at the firm to become positive next period and to grow for the next 10 years at rates much higher than the growth rate for the economy. To estimate the free cash flows to equity for the next 10 years, we make the following assumptions:

- The net income of $1,614 million will grow 15% a year each year for the next 10 years.
- The firm will reinvest 75% of the net income back into new investments each year, and its net debt issued each year will be 10% of the reinvestment.

Table 5.6 summarizes the free cash flows to equity at the firm for this period and computes the present value of these cash flows at the Home Depot's cost of equity of 9.78%.

Table 5.6: Value of FCFE

Year	*Net Income*	*Reinvestment Needs*	*Net Debt Issued*	*FCFE*	*PV of FCFE*
1	$ 1,856	$ 1,392	$ (139)	$ 603	$ 549
2	$ 2,135	$ 1,601	$ (160)	$ 694	$ 576
3	$ 2,455	$ 1,841	$ (184)	$ 798	$ 603
4	$ 2,823	$ 2,117	$ (212)	$ 917	$ 632
5	$ 3,246	$ 2,435	$ (243)	$ 1,055	$ 662
6	$ 3,733	$ 2,800	$ (280)	$ 1,213	$ 693
7	$ 4,293	$ 3,220	$ (322)	$ 1,395	$ 726

8	$ 4,937	$ 3,703	$ (370)	$ 1,605	$ 761
9	$ 5,678	$ 4,258	$ (426)	$ 1,845	$ 797
10	$ 6,530	$ 4,897	$ (490)	$ 2,122	$ 835
Sum of PV of Dividends =					$6,833

Note that since more debt is issued than paid, net debt issued increases the free cash flows to equity each year. To estimate the terminal price, we assume that net income will grow 6% a year forever after year 10. Since lower growth will require less reinvestment, we will assume that the reinvestment rate after year 10 will be 40% of net income; net debt issued will remain 10% of reinvestment.

$FCFE_{11}$ = Net Income$_{11}$ – Reinvestment$_{11}$ – Net Debt Paid (Issued)$_{11}$

= $6,530 (1.06) – $6,530 (1.06) (0.40) – (-277) = $ 4,430 million

Terminal Price$_{10}$ = $FCFE_{11}/(k_e – g)$ = $ 4,430 / (.0978 - .06) = $117,186 million

The value per share today can be computed as the sum of the present values of the free cash flows to equity during the next 10 years and the present value of the terminal value at the end of the 10th year.

Value of the Stock today = $ 6,833 million + $ 117,186/(1.0978)^{10} = $52,927 million

On a free cash flow to equity basis, we would value the equity at the Home Depot at $ 52.93 billion.

From Valuing Equity to Valuing the Firm

A firm is more than just its equity investors. It has other claim holders, including bondholders and banks. When we value the firm, therefore, we consider cash flows to all of these claim holders. We define the cash flow to the firm as being the cash flow left over after operating expenses, taxes and reinvestment needs, but before any debt payments (interest or principal payments).

Free Cash Flow to Firm (FCFF) = After-tax Operating Income – Reinvestment Needs

The two differences between FCFE and FCFF become clearer when we compare their definitions. The free cash flow to equity begins with net income, which is after interest expenses and taxes, whereas the free cash flow to the firm begins with after-tax operating income, which is before interest expenses. Another difference is that the FCFE is after net debt payments, whereas the FCFF is before net debt.

What exactly does the free cash flow to the firm measure? On the one hand, it measures the cash flows generated by the assets before any financing costs are considered and thus is a measure of operating cash flow. On the other, the free cash flow to the firm is the cash flow used to service all claim holders' needs for cash – interest and principal to debt holders and dividends and stock buybacks to equity investors.

To illustrate the estimation of free cash flow to the firm, consider Boeing in 1998. In that year, Boeing had adjusted operating income of $ 2,736 million, a tax rate of 35% and reinvested $1,719 million in new investments. The free cash flow to the firm for Boeing in 1998 is then:

$FCFF_{Boeing}$ = Operating Income (1- Tax Rate) – Reinvestment Needs

= $ 2,736 (1-.35) - $ 1,719 million = $ 59 million

Once the free cash flows to the firm have been estimated, the process of computing value follows a familiar path. If valuing a firm or business with free cash flows growing at a constant rate forever, we can use the perpetual growth equation:

$$\text{Value of Firm with FCFF growing at constant rate} = \frac{E(FCFF_1)}{(k_c - g_n)}$$

There are two key distinctions between this model and the constant-growth FCFE model used earlier. The first is that we consider cash flows before debt payments in this model, whereas we used cash flows after debt payments when valuing equity. The second is that we then discount these cash flows back at a composite cost of financing, i.e., the cost of capital to arrive at the value of the firm, while we used the cost of equity as the discount rate when valuing equiy.

To value firms where free cash flows to the firm are growing at a rate higher than that of the economy, we can modify this equation to consider the present value of the cash flows until the firm is in stable growth. To this present value, we add the present value of the terminal value, which captures all cash flows in stable growth.

$$\text{Value of high-growth business} = \sum_{t=1}^{t=N} \frac{E(FCFF_t)}{(1+k_c)^t} + \frac{\text{Terminal Value of Business}_N}{(1+k_c)^N}$$

Illustration 5.9: Valuing an Asset with Stable Growth

Assume now that Boeing is interested in selling its information, space and defense systems division. The division reported cash flows before debt payments but after reinvestment needs of $ 393 million in 1998, and the cash flows are expected to grow 5% a year in the long term. The cost of capital for the division is 9%. The division can be valued as follows:

Value of Division = $ 393 (1.05) / (.09 - .05) = $ 10,318 million

Illustration 5.10: Valuing a Firm in High Growth

Assume that you are valuing Boeing as a firm, and that Boeing has cash flows before debt payments but after reinvestment needs and taxes of $ 850 million in the current year. Further, assume that these cash flows will grow at 15% a year for the next 5 years and at 5% thereafter. Boeing has a cost of capital of 9.17%. The value of Boeing as a firm can then be estimated in Table 5.7:

Table 5.7: Value of Boeing

Year	Cash Flow	Terminal Value	Present Value
1	$978		$895
2	$1,124		$943
3	$1,293		$994
4	$1,487		$1,047
5	$1,710	$43,049	$28,864
Value of Boeing as a firm =			$32,743

The terminal value is estimated using the free cash flow to the firm in year 6, the cost of capital of 9.17% and the expected constant growth rate of 5% as follows:

Terminal Value = $ 1710 (1.05)/(.0917-.05) = $ 43,049 million

It is then discounted back to the present to get the value of the firm today shown above as $32,743 million.

Note that this is not the value of the equity of the firm. To get to the value of the equity, we would need to subtract out from $32,743 million the value of all non-equity claims in the firm.

> ✐ CT 5.3: Assume that you value equity by discounting free cash flows to equity at the cost of equity. If you revalue the firm, using free cash flows to the firm and the cost of capital, and then subtract out the outstanding debt, would you get the same value for the equity? Should you?

Valuing an Asset with Contingent Cash Flows (Options)

In general, the value of any asset is the present value of the expected cash flows on that asset. In this section, we will consider an exception to that rule when we will look at assets with two specific characteristics:

- They derive their value from the values of other assets.
- The cash flows on the assets are contingent on the occurrence of specific events.

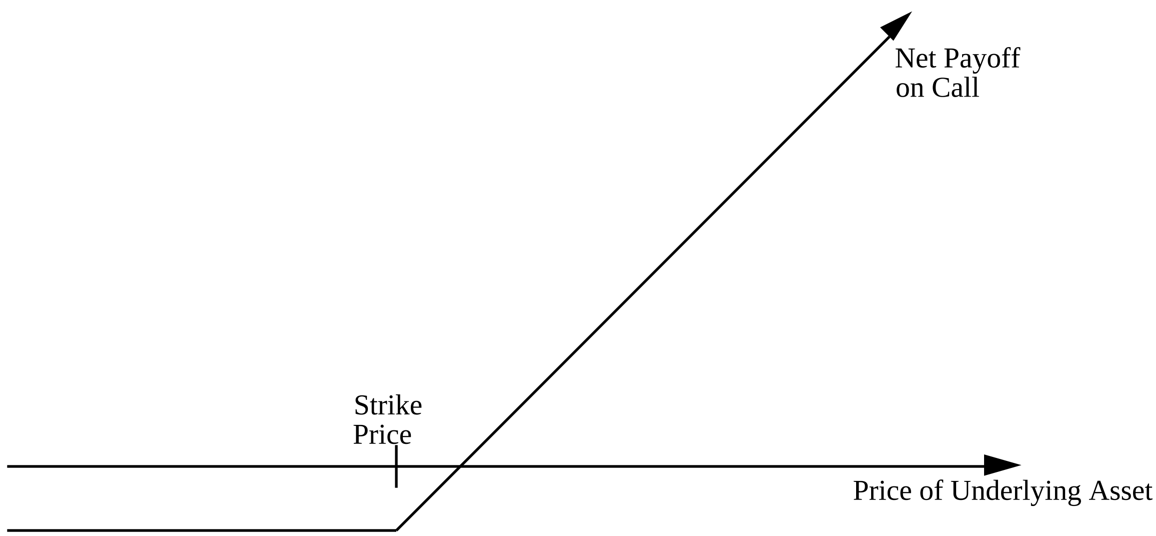
These assets are called options, and the present value of the expected cash flows on these assets will understate their true value. In this section, we will describe the cash flow characteristics of options, consider the factors that determine their value and examine how best to value them.

Cash Flows on Options

There are two types of options. A call option gives the buyer of the option the right to buy the underlying asset at a fixed price, whereas a put option gives the buyer the right to sell the underlying asset at a fixed price. In both cases, the fixed price at which the underlying asset can be bought or sold is called the strike or exercise price.

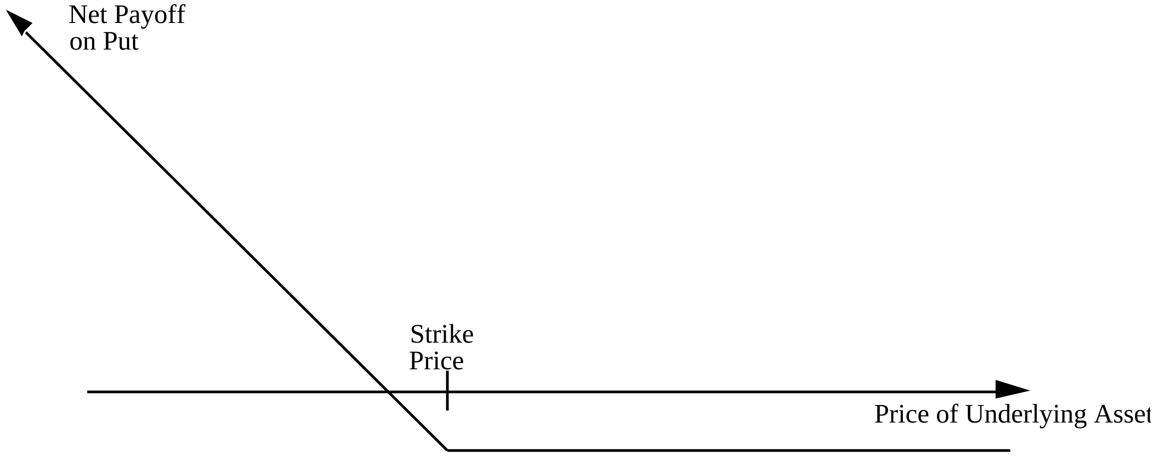
To look at the payoffs on an option, consider first the case of a call option. When you buy the right to sell an asset at a fixed price, you want the price of the asset to increase above that fixed price. If it does, you make a profit, since you can buy at the fixed price and then sell at the much higher price; this profit has to be netted against the cost initially paid for the option. However, if the price of the asset decreases below the strike price, it does not make sense to exercise your right to buy the asset at a higher price. In this scenario, you lose what you originally paid for the option. Figure 5.7 summarizes the cash payoff at expiration to the buyer of a call option.

Figure 27.1: Payoff on Call Option



With a put option, you get the right to sell at a fixed price, and you want the price of the asset to decrease below the exercise price. If it does, you buy the asset at the exercise price and then sell it back at the current price, claiming the difference as a gross profit. When the initial cost of buying the option is netted against the gross profit, you arrive at an estimate of the net profit. If the value of the asset rises above the exercise price, you will not exercise the right to sell at a lower price. Instead, the option will be allowed to expire without being exercised, resulting in a net loss of the original price paid for the put option. Figure 5.8 summarizes the net payoff on buying a put option.

Figure 27.2: Payoff on Put Option

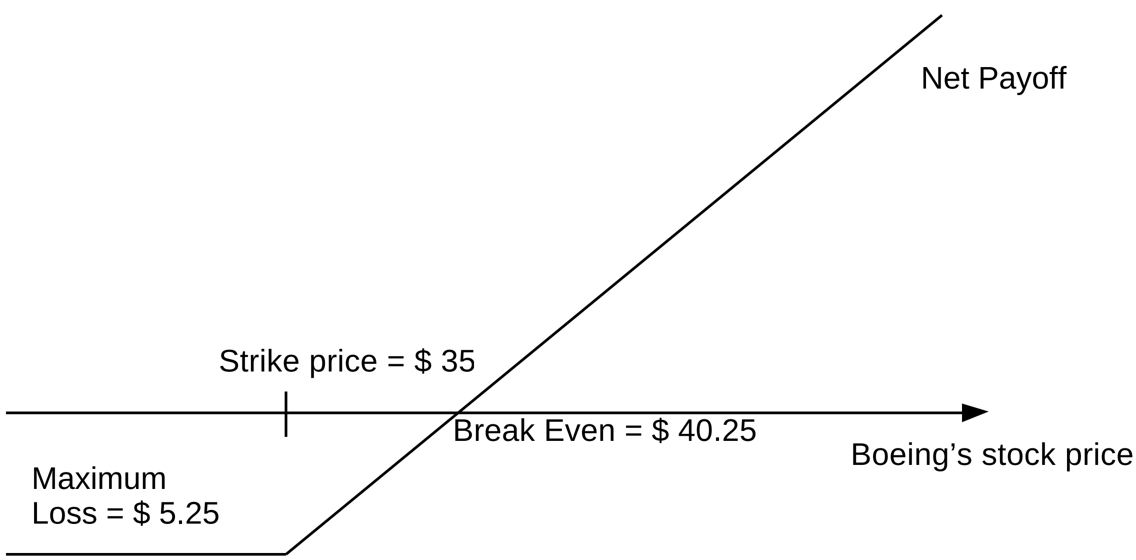


With both call and put options, the potential for profit to the buyer is significant, but the potential for loss is limited to the price paid for the option.

Illustration 5.11: Payoff Diagram for a Listed Option on Boeing

In late 1998, when Boeing stock was trading at $32.25, you could have bought a 3-month call option on the stock, with an exercise price of $ 35, for $ 5.25. If you had, you would have bought the right to buy Boeing at $ 35 anytime over those three months. The payoff diagram on this option is illustrated in Figure 5.9:

Figure 5.9: Payoff Diagram on Boeing Call Option



If Boeing's stock price ends up below $ 35, you lose the $ 5.25 that you paid for the option. If the stock price increases above $ 35, you will exercise, though you will not start making net profits until the stock price increases to $ 40.25 ($ 35 + $ 5.25).

☞*CC 5.5:* Draw the payoff diagram on a put option on Boeing with an exercise price of $ 35.

Determinants of Option Value

What is it that determines the value of an option? At one level, options have expected cash flows just like all other assets, and that may seem like good candidates for discounted cash flow valuation. The two key characteristics of options -- that they derive their value from some other traded asset, and the fact that their cash flows are contingent on the occurrence of a specific event -- does suggest an easier alternative. We can create a portfolio that has the same cash flows as the option being valued, by combining a position in the underlying asset with borrowing or lending. This portfolio is called a **replicating portfolio** and should cost the same amount as the option. The principle that two assets (the option and the replicating portfolio) with identical cash flows cannot sell at different prices is called the arbitrage principle.

Options are assets that derive value from an underlying asset; increases in the value of the underlying asset will increase the value of the right to buy at a fixed price and reduce the value to sell that asset at a fixed price. On the other hand, increasing the strike price will reduce the value of calls and increase the value of puts.

While calls and puts move in opposite directions when stock prices and strike prices are varied, they both increase in value as the life of the option and the variance in the underlying asset's value increases. The reason for this is the fact that options have limited losses. Unlike traditional assets that tend to get less valuable as risk is increased, options become more valuable as the underlying asset becomes more volatile. This is so because the added variance cannot worsen the downside risk (you still cannot lose more than what you paid for the option) while making potential profits much higher. In addition, a longer life for the options just allows more time for both call and put options to appreciate in value. Since calls provide the right to buy the underlying asset at a fixed price, an increase in the value of the asset will increase the value of the calls. Puts, on the other hand, become less valuable as the value of the asset increase.

The final two inputs that affect the value of the call and put options are the riskless interest rate and the expected dividends on the underlying asset. The buyers of call and put options usually pay the price of the option up front, and wait for the expiration day to exercise. There is a present value effect associated with the fact that the promise to buy an asset for $ 1 million in 10 years is less onerous than paying it now. Thus, higher interest rates will generally increase the value of call options (by reducing the present value of the price on exercise) and decrease the value of put options (by decreasing the present value of the price received on exercise). The expected dividends paid by assets make them less valuable; thus, the call option on a stock that does not pay a dividend should be worth more than a call option on a stock that does pay a dividend. The reverse should be true for put options.

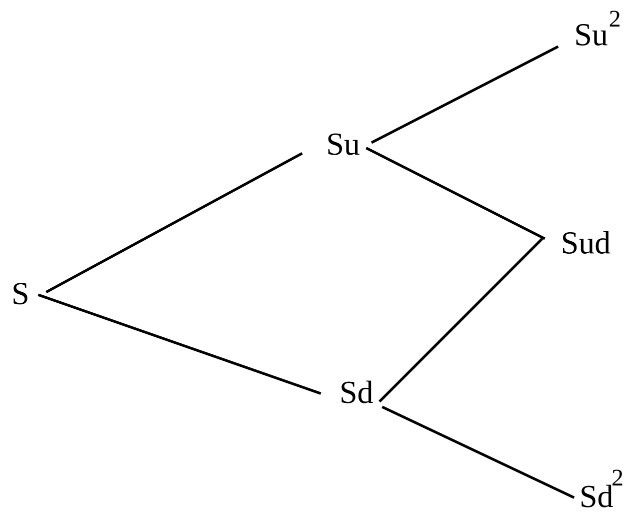
A Simple Model for Valuing Options

Almost all models developed to value options in the last three decades are based upon the notion of a replicating portfolio. The earliest derivation, by Black and Scholes, is mathematically complex, and we will return to it in chapter 27. In this chapter, we consider the simplest replication model for valuing options – the binomial model.

The Binomial Model

The binomial option pricing model is based upon a simple formulation for the asset price process in which the asset, in any time period, can move to one of two possible prices. The general formulation of a stock price process that follows the binomial is shown in Figure 5.10.

Figure 5.10: General Formulation for Binomial Price Path



In this figure, S is the current stock price; the price moves up to Su with probability p and down to Sd with probability 1-p in any time period. For instance, if the stock price today is $ 100, u is 1.1 and d is 0.9, the stock price in the next period can either be $ 110 (if u is the outcome) and $ 90 (if d is the outcome).

Creating A Replicating Portfolio

The objective in creating a replicating portfolio is to use a combination of risk-free borrowing/lending and the underlying asset to create the same cash flows as the option being valued. In the case of the general formulation above, where stock prices can either move up to Su or down to Sd in any time period, the replicating portfolio for a call with a given strike price will involve borrowing $B and acquiring Δ of the underlying asset. Of course, this formulation is of no use if we cannot determine how much we need to borrow and what Δ is. There is a way, however, of identifying both variables. To do this, note that the value of this position has to be same as the value of the call no matter what the stock price does. Let us assume that the value of the call is C_u if the stock price goes to Su, and C_d if the stock price goes down to Sd. If we had borrowed $B and bought Δ shares of stock with the money, the value of this position under the two scenarios would have been as follows:

	Value of Position	Value of Call
If stock price goes up to Su	Δ Su - \$ B (1+r)	C_u
If stock price goes down to Sd	Δ Sd - \$ B (1+r)	C_d

Note that, in either case, we have to pay back the borrowing with interest. Since the position has to have the same cash flows as the call, we get

Δ Su - \$ B (1+r) = C_u

Δ Sd - \$ B (1+r) = C_d

Solving for Δ, we get

Δ = Number of units of the underlying asset bought = $(C_u - C_d)/(Su - Sd)$

where,

C_u = Value of the call if the stock price is Su

C_d = Value of the call if the stock price is Sd

When there are multiple periods involved, we have to begin with the last period, where we know what the cash flows on the call will be, solve for the replicating portfolio and then estimate how much it would cost us to create this portfolio. We then use this value as the estimated value of the call and estimate the replicating portfolio in the previous period. We continue to do this until we get to the present. The replicating portfolio we obtain for the present can t be priced to yield a current value for the call.

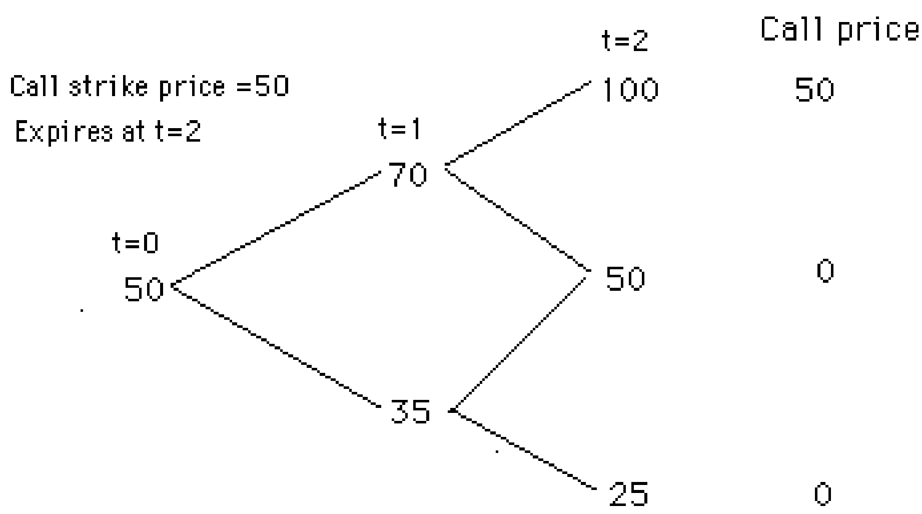
Value of the call = Current value of underlying asset * Option Delta

- Borrowing needed to replicate the option

Illustration 5.12: An Example of Binomial valuation

Assume that the objective is to value a call with a strike price of 50, which is expected to expire in two time periods, on an underlying asset whose price currently is 50 and is expected to follow a binomial process. Figure 5.11 illustrates the path of underlying asset prices and the value of the call (with a strike price of 50) at the expiration.

Figure 5.11: Binomial Price Path



Note that since the call has a strike price of \$ 50, the gross cash flows at expiration are as follows:

If the stock price moves to \$ 100: Cash flow on call = \$ 100 - \$ 50 = \$ 50

If the stock price moves to \$ 50: Cash flow on call = \$ 50 - \$ 50 = \$ 0

If the stock price moves to \$ 25: Cash flow on call = \$ 0 (Option is not exercised).

Now assume that the interest rate is 11%. In addition, define

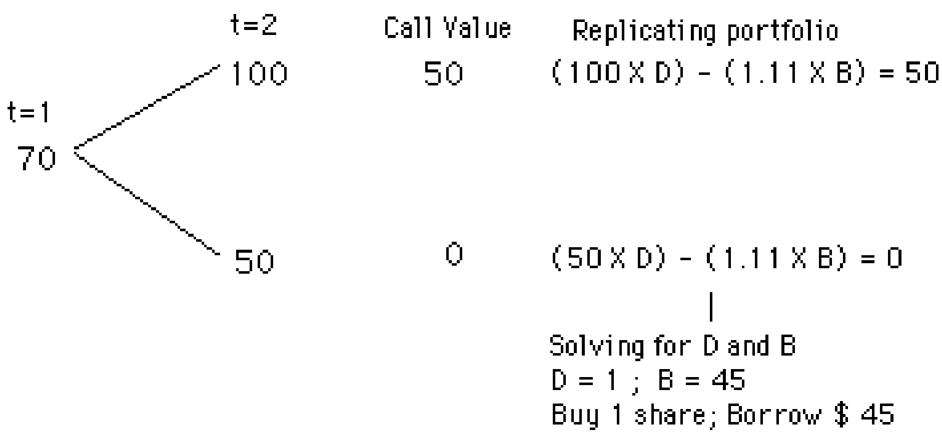
Δ = Number of shares in the replicating portfolio

B = Dollars of borrowing in replicating portfolio

The objective, in this analysis, is to combine Δ shares of stock and B dollars of borrowing to replicate the cash flows from the call with a strike price of \$ 50.

The first step in doing this, is to start with the last period and work backwards. Consider, for instance, one possible outcome at t =1. The stock price has jumped to \$ 70, and is poised to change again, either to \$ 100 or \$ 50. We know the cash flows on the call under either scenario, and we also have a replicating portfolio composed of Δ shares of the underlying stock and $ B of borrowing. Writing out the cash flows on the replicating portfolio under both scenarios (stock price of $ 100 and $ 50), we get the replicating portfolios in figure 5.12:

Figure 5.12: Replicating Portfolios when Price is $ 70



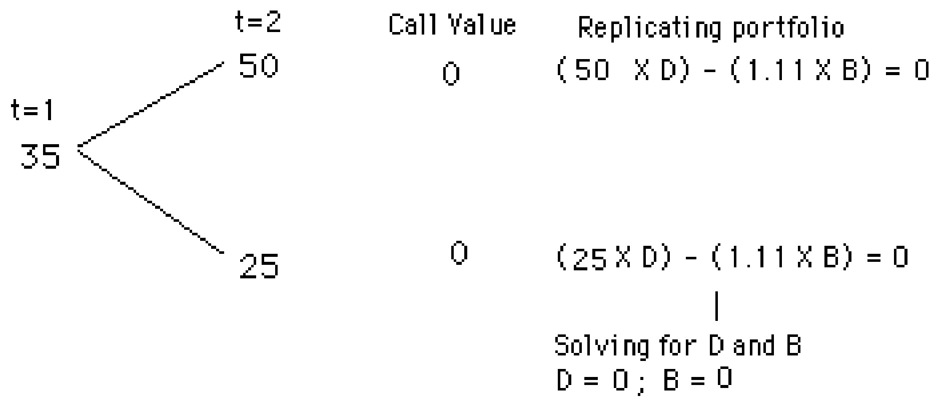
In other words, if the stock price is $70 at t=1, borrowing $45 and buying one share of the stock will give the same cash flows as buying the call. The value of the call at t=1, if the stock price is $70, should therefore be the cash flow associated with creating this replicating position and it can be estimated as follows:

$70\ \Delta - B = 70-45 = 25$

The cost of creating this position is only $ 25, since $ 45 of the $ 70 is borrowed. This should also be the price of the call at t=1, if the stock price is $ 70.

Consider now the other possible outcome at t=1, where the stock price is $ 35 and is poised to jump to either $ 50 or $ 25. Here again, the cash flows on the call can be estimated, as can the cash flows on the replicating portfolio composed of Δ shares of stock and $B of borrowing. Figure 5.13 illustrates the replicating portfolio:

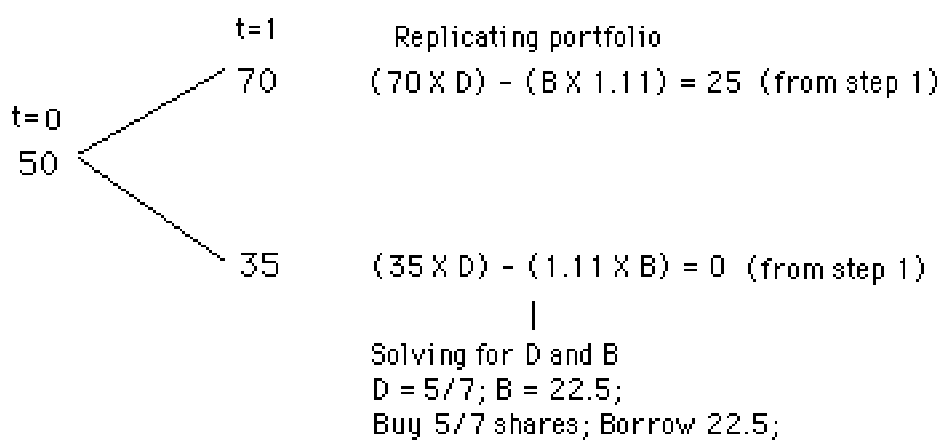
Figure 5.13: Replicating Portfolio when Price is $ 35



Since the call is worth nothing, under either scenario, the replicating portfolio also is empty. The cash flow associated with creating this position is obviously zero, which becomes the value of the call at t=1, if the stock price is $ 35.

We now have the value of the call under both outcomes at t=1; it is worth $ 25 if the stock price goes to $ 70 and $0 if it goes to $ 35. We now move back to today (t=0), and look at the cash flows on the replicating portfolio. Figure 5.14 summarizes the replicating portfolios as viewed from today:

Figure 5.14: Replicating Portfolios for Call Value



Using the same process that we used in the previous step, we find that borrowing $22.5 and buying 5/7 of a share will provide the same cash flows as a call with a strike price of $50. The cost, to the investor, of borrowing $ 22.5 and buying 5/7 of a share at the current stock price of $ 50 yields:

Cost of replicating position = 5/7 X $ 50 - $ 22.5 = $ 13.20

This should also be the value of the call.

The Determinants of Value

The binomial model provides insight into the determinants of option value. The value of an option is determined not by the expected price of the asset but by its current price, which, of course, reflects expectations about the future. In fact, the probabilities that we provided in the description of the binomial process of up and down movements do not enter the option valuation process, though they do affect the underlying asset's value. The reason for this is the fact that options derive their value from other assets, which are often traded. Consequently, the capacity investors possess to create positions that have the same cash flows as the call operates as a powerful mechanism controlling option prices. If the option value deviates from the value of the replicating portfolio, investors can create an arbitrage position, i.e., one that requires no investment, involves no risk, and delivers positive returns. The option value increases as the time to expiration is extended, as the price movements (u and d) increase, and as the interest rate increases.

The second insight is that the greater the variance in prices in the underlying asset in this example, the more valuable the option becomes. Thus, increasing the up and down movements, in the illustration above, makes options more valuable. This occurs because of the fact that options do not have to be exercised if it is not in the holder's best interests to do so. Thus, lowering the price in the worst case scenario to $ 10 from $ 25 does not, by itself, affect the gross cash flows on this call. On the other hand, increasing the price in the best case scenario to $ 150 from $ 100 benefits the call holder and makes the call more valuable.

The binomial model is a useful model for illustrating the replicating portfolio and the effect of the different variables on call value. It is, however, a restrictive model, since asset prices in the real world seldom follow a binomial process. Even if they did, estimating all possible outcomes and drawing a binomial tree, as we have, can be an extraordinarily tedious exercise. In a later chapter looking at option pricing models in more depth, we will consider more general option pricing models that may be of more practical use.

✍ CT 5.4: With a conventional asset, the value of the asset decreases as the riskiness of the asset increases. With a contingent claim asset or option, the value of the option increases as the riskiness of the asset increases. Explain the reason for the difference.

Market Prices and Value

We have spent most of this chapter talking about how to value an asset. We began the chapter, however, by talking about the market prices of Boeing and the Home Depot and how they had changed over the last few years. Is the value of the asset equal to its market price? If not, why are they different? The answers to these two questions are important not only for the way we define the objective of the firm (maximizing value versus maximizing prices) but also for the way we approach investment analysis and financing decisions. In this section, we will begin by looking at the process through which market prices are set, why these prices might be different from asset values and the implications for financial markets.

The Pricing Process

How do financial markets set market prices? While market prices are set by demand and supply, the pricing mechanism can vary across different markets. In general, markets can be either continuous or call markets. In a **continuous market**, prices are determined through the trading day as buyers and sellers submit their orders. The order flow determines the price; if buy orders exceed sell orders, the price goes up to equalize the two; if sell orders exceed buy orders, the price goes down. In a **call market**, an auctioneer (or a market maker) holds an auction at certain times in the trading day and sets a market-clearing price, based upon the orders grouped together at that time.

Some markets use both mechanisms. For instance, the New York Stock Exchange starts the trading day with an auction market but then becomes a continuous market for the rest of the day. The London Gold Bullion market, on the other hand, has two call auctions a day; the price from the morning auction is called a 'morning fix', and the price from the afternoon market is the 'afternoon fix'.

Finally, technology is also changing the pricing process. Historically, trading has occurred in a physical location, such as the New York Stock Exchange. But the market for traded assets is increasingly becoming an electronic market. The NASDAQ, where many of the smallest companies in the United States are traded, is an electronic exchange. Investors buy and sell stocks from dealers, who quote prices at which they are willing to trade.

Information, Expectations and Prices

No matter how markets are structured, the market price of an asset is an estimate of its value. Investors in the market make assessments of the price based upon their expectations for the future cash flows on the asset. They form these expectations using the information that is available to them and this information can arrive in different forms. It can be information about the past price performance of the asset, or public information available in annual reports or filings with the SEC, or information available to one or a few investors.

While the steps in this process – receive information, process the information to form expectations and trade on the asset – may be the same for all investors, there are wide variations across investors in how much information they have available, and how they process the information. Some investors have access to more information than others. For instance, an equity research analyst whose job it is to evaluate Boeing as an investment will have access to more information about the firm than a small investor making the same decision. These differences in information are compounded by the different ways in which investors use the information to form expectations. Some investors build complex quantitative models, converting the information into expected earnings and cash flows, and value investments. Other investors use the same information to make comparisons across traded investments. The net effect is that, at any point in time, investors will disagree on how much an asset is worth. Those who think that it is worth more will be the buyers of the asset, and those who think it is worth less will sell the asset. The market price represents the price at which the market clears, i.e, where demand (buying) is equal to supply (selling).

Let us now consider the relationship between price and value. In the earlier part of this chapter, we argued that the value of an asset is the present value of the expected cash flows over its lifetime. The price of that asset represents the product of a process in which investors use the information available on the asset to form expectations about the future. The price can and usually will deviate from the value for three reasons. First, the information available may be insufficient or incorrect; then expectations based upon this information will also be wrong. Second, investors may not do a good job of processing the information to arrive at expectations. Third, even if the information is correct and investors, on average, form expectations properly, there might still be investors who are willing to trade at prices that do not reflect these expectations. Thus, an investor who assesses the value of a stock to be $ 50 might still be willing to buy the stock for $ 60, because he or she believes that it can be sold to someone else for $ 75 later.

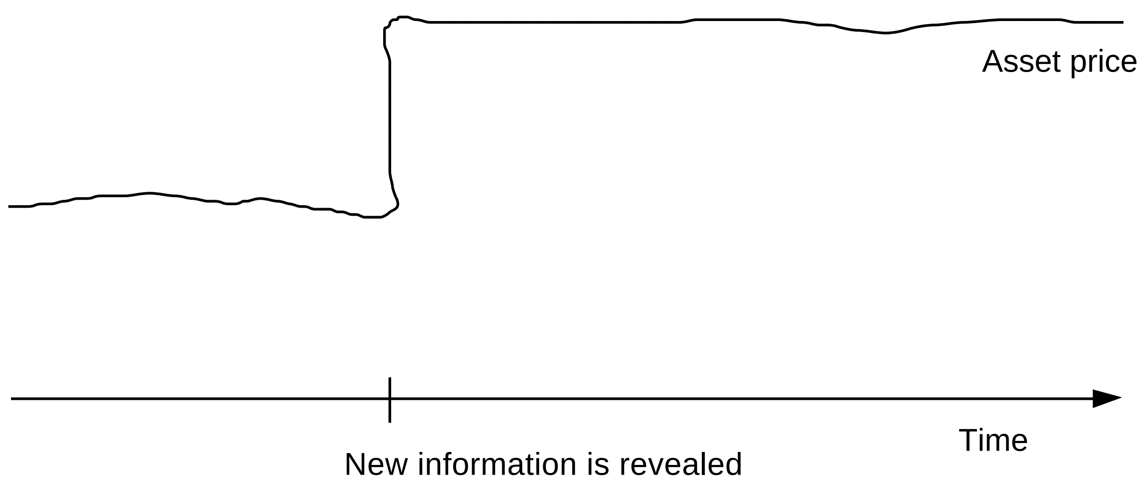
Market Efficiency

One of the key questions we need an answer to before we examine whether to invest in new projects or how to finance them is whether markets are efficient. There are three ways of measuring or defining market efficiency. One is to look at how much and for how long prices deviate from true value. The second is to measure how quickly and completely prices adjust to reflect new information. The third is to measure whether some investors in markets consistently earn higher returns than others who are exposed to the same amount of risk.

We define market efficiency in terms of how much the price of an asset deviates from a firm's true value. The smaller and less persistent the deviations are, the more efficient a market is. Market efficiency does not require that the market price be equal to true value at every point in time. All it requires is that errors in the market price be unbiased, i.e., prices can be greater than or less than true value, as long as these deviations are random.

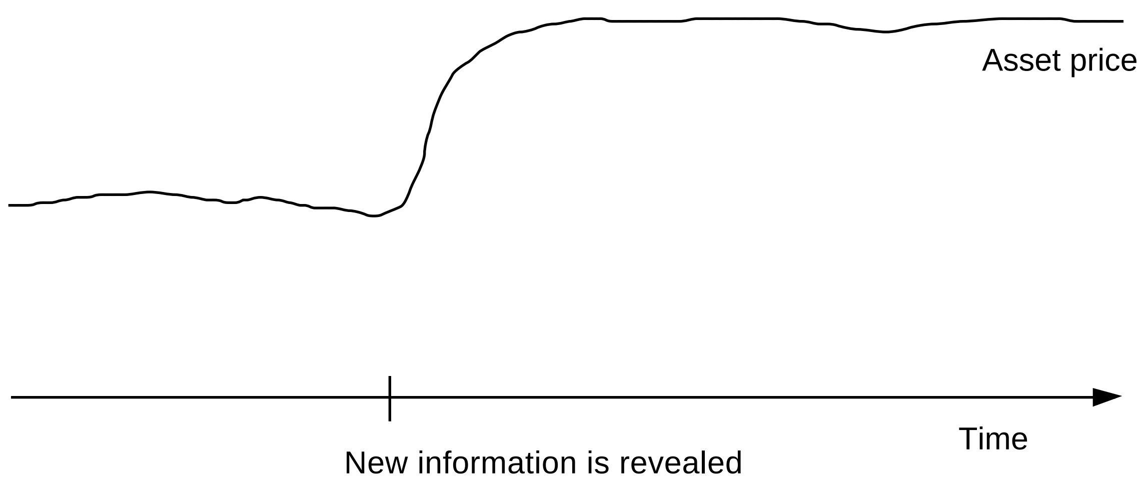
Another way of assessing market efficiency is to look at how quickly and how well markets react to new information. The value of an asset should increase when new information that affects any of the inputs into value – the cash flows, the growth or the risk – reaches the market. In an efficient market, the price of the asset will adjust instantaneously and, on average, correctly to the new information, as shown in figure 5.15.

Figure 5.15: Price Adjustment in an Efficient Market



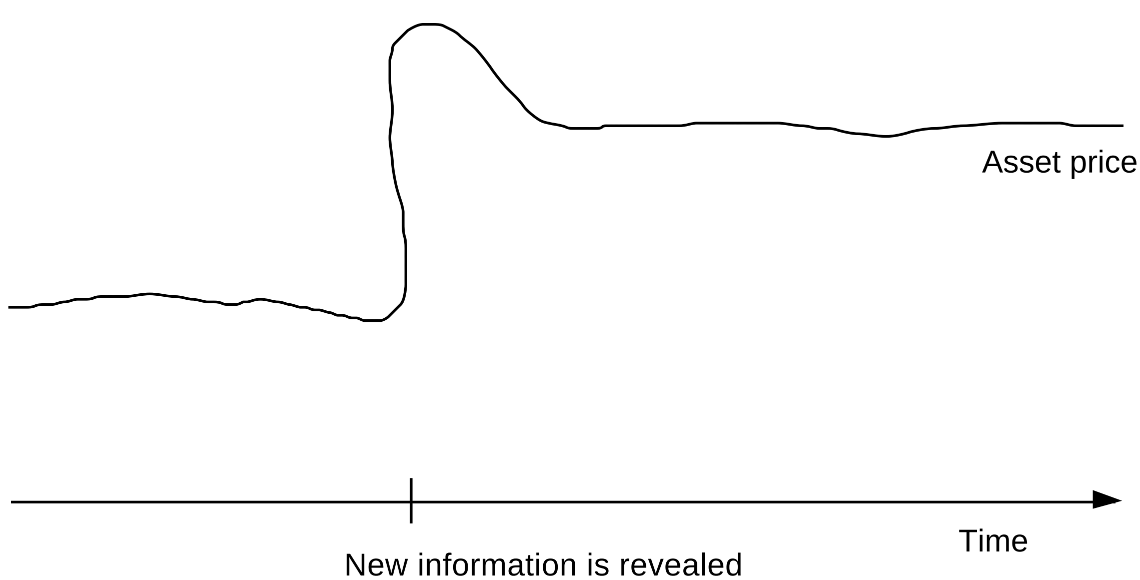
The adjustment will be slower if investors are slow in assessing the impact of the information on value. In figure 5.16, we show the price of an asset adjusting slowly to new information. The drift in prices that we observe after the information arrives is indicative of a slow learning market.

Figure 5.16: A Slow Learning Market



In contrast, the market could adjust instantaneously to the new information but overestimate the effect of the information on value. Then, the price of the asset will increase by more than it should, given the effect of the new positive information on value, or drop by more than it should, with negative information. Figure 5.17 shows the drift in prices in the opposite direction, after the initial reaction.

Figure 5.17: An Overreacting Market



There is one final dimension on which we can measure and define market efficiency. In an efficient market, some investors should not be able consistently to earn higher returns than other investors who are exposed to the same amount of risk. This, of course, requires that we define risk and determine how much return an investor exposed to that risk can expect to make. We will consider this question in detail in the next chapter, but the difference between the actual returns made by investors and the returns they could have expected to make is called an excess or abnormal return. If investors earn more than expected, they have positive excess returns; if they earn less, they earn negative excess returns. In an efficient market, investors can expect to make no excess returns, no matter how sophisticated their models.

☞*CC 5.6:* In an efficient market, would you expect stock prices to go up when firms announce good news. Why or why not?

Testing Market Efficiency

There is substantial evidence from financial markets about how efficient the market is, using all three measures of market efficiency. We could examine whether the market price of an asset reflects the expected cash flows and risk of that asset, how well the price adjusts to reflect new information that might change these expectations and whether investors earn excess returns.

The market price is affected by the fundamental inputs that determine value – cash flows, growth and risk. Assets with higher expected cash flows and higher growth are generally priced higher than assets with lower cash flows and lower growth. There does, however, seem to be a substantial amount of error in the process, with prices sometimes deviating significantly from value, especially for firms where there is limited information available to investors.

The market reacts quickly to new information about an asset. Thus, when a firm reports earnings that are higher than expected, the stock price tends to increase immediately. There is mixed evidence as to whether than initial assessment is appropriate. Some studies suggest that markets over react to very good or very bad news; in the case of earnings announcements, this would imply that the stock price goes up too much on good news and drops too much on bad news. Other studies indicate that, at least in the short term, markets under react to information announcements. Cosmetic changes made by firms that increase reported earnings, while not affecting expected cash flows, growth or risk, affect prices in the short term, but the price effect does not last.

The third way of testing market efficiency is to examine whether some investors in financial markets are able consistently to earn higher returns than the rest of the market. If they were, this would suggest that deviations from the market price were not random, and that markets were not efficient. There are numerous studies of this issue, as well. While there are strategies that seem to earn excess returns on paper, there seems to be little evidence that investors are able to earn these same excess returns in practice. With transactions costs and problems in execution, the overall conclusion of these studies seems to be that markets are efficient for most investors most of the time.

✎ CT 5.5: The efficiency of a market can be measured by the speed and accuracy of the price response to new information or by whether some investors can consistently earn higher returns than the rest of the market. Is there a link between the two measures? Which is the stronger test?

Summary

In this chapter, we have laid the foundations for the models that we will be using to value both assets and firms in the coming chapters. We have described two classes of valuation models. The more general of these models, discounted cash flow valuation, can be used to value any asset with expected cash flows over its life. The value is the present value of the expected cash flows at a discount rate that reflects the riskiness of the cash flows, and this principle applies whether one is looking at a zero-coupon government bond or equity in high risk firms. There are some assets that generate cash flows only in the event of a specified contingency, and these assets will not be valued accurately using discounted cash flow models. Instead, they should be viewed as options and valued using option pricing models.

Having discussed value, we turn to the issue of price. Since price is determined by demand and supply, and some investors do not have or want to use information that is available on the future of a firm, it can be different from value. The smaller the differences between market price and value, and the sooner the price adjusts to value when there is a difference, the more efficient markets are. The empirical evidence on financial markets is comforting, for the most part. Market prices reflect a firm's earnings, growth and risk characteristics, albeit with error, and adjust quickly to new information that change expectations about these variables.

Questions

1. Value a ten-year zero-coupon government bond, with a face value of $ 1,000, if the interest rate on the bond is 6%.

2. Value a ten-year, 5% coupon, bond, with a face value of $1,000, if the interest rate on the bond is 6%.

3. Value a 15-year corporate bond, with a coupon rate of 8% and a face value of $1,000, if the market interest rate on bonds of similar risk is 7%.

4. Estimate the default spread on a 15-year, 7% coupon, corporate bond, if the price of the bond is $ 977 and the face value is $ 1,000. The treasury bond rate is 6.5%.

5. Estimate the value of an asset that is expected to generate $ 100 million in cash flows, growing 10% a year for 10 years. The appropriate discount rate (cost of capital) is 10%.

6. Con Ed pays a dividend of $ 2.12. If you expect these dividends to grow 3% a year in perpetuity, and the cost of equity is 9%, estimate the value per share.

7 . In an efficient market, the market price is defined to be an 'unbiased estimate' of the true value. This implies that

(a) the market price is always equal to true value.

(b) the market price has nothing to do with true value

(c) markets make mistakes about true value, and investors can exploit these mistakes to make money

(d) market prices contain errors, but the errors are random and therefore cannot be exploited by investors.

(e) no one can beat the market.

Problems

1. You buy a 10-year zero-coupon bond, with a face value of $1000, for $300. What is the rate of return will you make on this bond?

2. What is the value of a 15-year corporate bond, with a coupon rate of 9%, if current interest rates on similar bonds is 8%? How much would the value change if interest rates increased to 10%? Under what conditions will this bond trade at par (face value)?

3. You are comparing the prices of bonds issued by two corporations. NV Technologies has a 8%, 15-year bond outstanding, trading at par. GEV Technologies has the same bond rating as NV Technologies and has a 7.5% bond, trading at 5% below par. What is the maturity of GEV's outstanding bond?

4. You have been asked to value a 40-year bond, issued by Boeing, with the following features. The coupon rate for the first 20 years will be 6% of the face value of $ 1,000. After 20 years, the coupon rate will increase to 7% for the remaining 20 years. Estimate the value of this bond, if Boeing is rated AA. (AA rated bonds are trading at a default spread of .50% over the treasury bond rate of 6.50%)

5. You have valued a British consol bond (perpetual) at $ 636. Assuming that the coupons are paid semi-annually, and that the interest rate on riskfree Government bonds is 6%, estimate the annual coupon on this bond.

6. Pacific Telesis currently pays out $1.50 per share in dividends and you expect these dividends to grow 5% a year forever. You can assume that investors require a 13% return on stocks of equivalent risk.

a. Estimate the value per share

b. Assume that the stock is currently trading at $ 15 per share. Assuming that the cost of equity of 13% is correct, estimate the growth rate that is implicit in this stock price.

7. ANC Bank is trading at $ 51.25 per share. The stock pays a dividend of $ 2.50 currently. Assuming that the expected growth in dividends will be 5% a year forever, estimate the return that you can expect to make as an equity investor in this stock.

8. What is the value of stock in a company that currently pays out $1.00 per share in dividends, and expects these dividends to grow 15% a year for the next 5 years, and 6% a year forever after that? (You can assume that investors require a 12.5% return on stocks of equivalent risk and that the dividend payout ratio will double after the fifth year.)

9. GTE Corporation paid out dividends per share of $ 1.88 in the current year. However, the firm had a free cash flow to equity of $ 2.40 per share in the same year. Assume that the firm is in stable growth, growing 5% a year forever, and that its cost of equity is 10%.

a. Estimate the value per share using a dividend discount model
b. Estimate the value per share using a FCFE discount model
c. Why is there a difference, and which value would you view as more accurate.

10. You are trying to value CVS Corporation, a leading chain of drugstores. You have estimated the free cash flows to equity for the firm this year to be $ 300 million in the current year. You anticipate that these cash flows will grow 15% a year for the next 5 years, and 5% thereafter (forever). The firm has a cost of equity of 11%, and 392 million shares outstanding.

a. Estimate the total value of equity in CVS.
b. Estimate the value of equity per share in CVS.
c. If the stock is trading at $ 36 per share, assuming that the estimated growth rate of 15% is correct, how long would growth have to continue for the price to be justified. (You can assume that the stable growth rate remains 5%)

11. Lear Corporation, a manufacturer of automotive supplies generated $ 650 million in free cash flow to the firm (prior to debt payments, but after reinvestment needs and taxes) last year. The firm has a cost of capital of 8.5%, and debt outstanding of $ 3.88 billion.

The firm has 66.5 million shares outstanding. The cash flows are expected to grow 4.5% a year in perpetutity.

a. Estimate the value of Lear Corporation (as a firm)
b. Estimate the value of equity in Lear Corporation
c. If the stock is trading at $ 32 per share, how under or over valued is the stock?

12. Nokia, the cellular phone giant, had $ 3 billion in free cash flows to the firm in the current year. These cash flows are expected to grow 15% a year for the next 10 years, and 5% thereafter. The firm has a cost of capital of 9%, and $ 1.2 billion in debt outstanding. If there are 1.15 billion shares outstanding, estimate the value per share.

13. A retailing firm announces that its sales and earnings in the fourth quarter of last year were 50% higher than sales and earnings in the third quarter of last year. Would you expect the stock price to increase on this announcement? Why or why not?

14. There are many who believe that markets are not efficient. As evidence, they point out that there are investors every year who beat the market (earn more than other investors in the market). Is this sufficient? Why or why not? What additional evidence would you need to conclude that markets are efficient?

15. Assume that you have a market where all investors receive the same information about assets. Will all investors agree on the value of all assets? If yes, what are the implications for how much trading you will observe in the market? If not, why would investors disagree?