CHAPTER 34

VALUING FUTURES AND FORWARD CONTRACTS

A futures contract is a contract between two parties to exchange assets or services at a specified time in the future at a price agreed upon at the time of the contract. In most conventionally traded futures contracts, one party agrees to deliver a commodity or security at some time in the future, in return for an agreement from the other party to pay an agreed upon price on delivery. The former is the seller of the futures contract, while the latter is the buyer.

This chapter explores the pricing of futures contracts on a number of different assets - perishable commodities, storable commodities and financial assets - by setting up the basic arbitrage relationship between the futures contract and the underlying asset. It also examines the effects of transactions costs and trading restrictions on this relationship and on futures prices. Finally, the chapter reviews some of the evidence on the pricing of futures contracts.

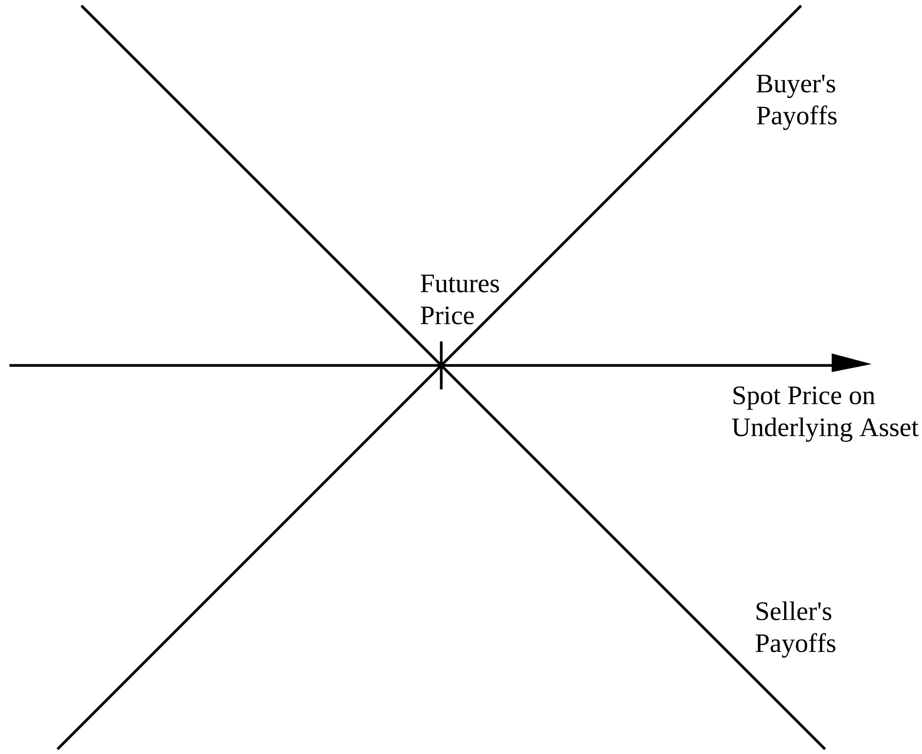
Futures, Forward and Option Contracts

Futures, forward and option contracts are all viewed as derivative contracts because they derive their value from an underlying asset. There are however some key differences in the workings of these contracts.

How a Futures Contract works

There are two parties to every futures contract - the seller of the contract, who agrees to deliver the asset at the specified time in the future, and the buyer of the contract, who agrees to pay a fixed price and take delivery of the asset.

Figure 34.1: Cash Flows on Futures Contracts



While a futures contract may be used by a buyer or seller to hedge other positions in the same asset, price changes in the asset after the futures contract agreement is made provide gains to one party at the expense of the other. If the price of the underlying asset increases after the agreement is made, the buyer gains at the expense of the seller. If the price of the asset drops, the seller gains at the expense of the buyer.

Futures versus Forward Contracts

While futures and forward contracts are similar in terms of their final results, a forward contract does not require that the parties to the contract settle up until the expiration of the contract. Settling up usually involves the loser (i.e., the party that guessed wrong on the direction of the price) paying the winner the difference between the contract price and the actual price. In a futures contract, the differences is settled every period, with the winner's account being credited with the difference, while the loser's account is reduced. This process is called *marking to the market*. While the net settlement is the same under the two approaches, the timing of the settlements is different and can lead to different prices for the two types of contracts. The difference is illustrated in the following example, using a futures contract in gold.

Illustration 34.1: Futures versus Forward Contracts - Gold Futures Contract

Assume that the spot price of gold is $400, and that a three-period futures contract on gold has a price of $415. The following table summarizes the cash flow to the buyer and seller of this contract on a futures and forward contract over the next 3 time periods, as the price of the gold futures contract changes.

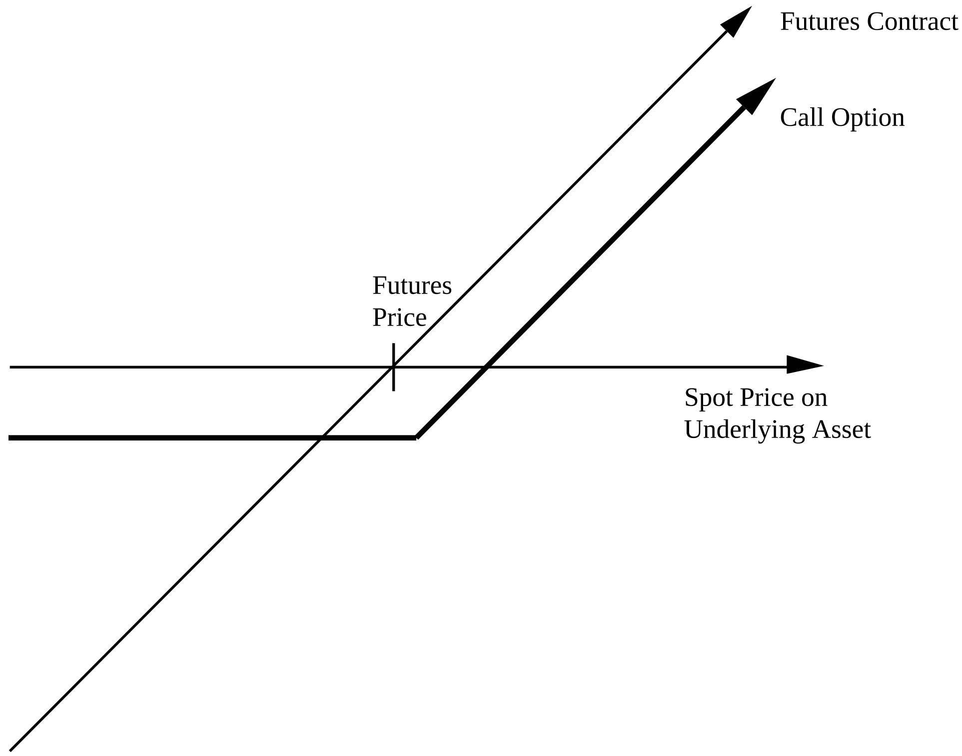
Time Period	Gold Futures Contract	Buyer's CF: Forward	Seller's CF: Forward	Buyer's CF: Futures	Seller's CF: Futures
1	$420	$0	$0	$5	-$5
2	$430	$0	$0	$10	-$10
3	$425	$10	-$10	-$5	$5
Net		$10	-$10	$10	-$10

The net cash flow from the seller to the buyer is $10 in both cases, but the timing of the cash flows is different. On the forward contract, the settlement occurs at maturity. On the futures contract, the profits or losses are recorded each period.

Futures and Forward Contracts versus Option Contracts

While the difference between a futures and a forward contract may be subtle, the difference between these contracts and option contracts is much greater. In an options contract, the buyer is not obligated to fulfill his side of the bargain, which is to buy the asset at the agreed upon strike price in the case of a call option and to sell the asset at the strike price in the case of a put option. Consequently the buyer of an option will exercise the option only if it is in his or her best interests to do so, i.e., if the asset price exceeds the strike price in a call option and vice versa in a put option. The buyer of the option, of course, pays for this privilege up front. In a futures contract, both the buyer and the seller are obligated to fulfill their sides of the agreement. Consequently, the buyer does not gain an advantage over the seller and should not have to pay an up front price for the futures contract itself. Figure 34.2 summarizes the differences in payoffs on the two types of contracts in a payoff diagram.

Figure 34.2: Buying a Futures Contract versus Buying a Call Option



Traded Futures Contracts - Institutional Details

A futures contract is an agreement between two parties. In a traded futures contract, an exchange acts as an intermediary and guarantor, and also standardizes and regulates how the contract is created and traded.

Buyer of Contract ------------>Futures Exchange <---------- Seller of Contract

In this section, we will examine some of the institutional features of traded futures contracts.

1. Standardization

Traded futures contracts are standardized to ensure that contracts can be easily traded and priced. The standardization occurs at a number of levels.

(a) Asset Quality and Description: The type of asset that can be covered by the contract is clearly defined. For instance, a lumber futures contract traded on the Chicago Mercantile Exchange allows for the delivery of 110,000 board feet of lumber per contract. A treasury bond futures contract traded on the Chicago

Board of Trade requires the delivery of bonds with a face value of $100,000 with a maturity of greater than 15 years[1].

(b) Asset Quantity: Each traded futures contract on an asset provides for the delivery of a specified quantity of the asset. For instance, a gold futures contract traded on the Chicago Board of trade requires the delivery of 100 ounces of gold at the contracts expiration.

The purpose of the standardization is to ensure that the futures contracts on an asset are perfect substitutes for each other. This allows for liquidity and also allows parties to a futures contract to get out of positions easily.

2. Price Limits

Futures exchanges generally impose 'price movement limits' on most futures contracts. For instance, the daily price movement limit on orange juice futures contract on the New York Board of Trade is 5 cents per pound or $750 per contract (which covers 15,000 pounds). If the price of the contract drops or increases by the amount of the price limit, trading is generally suspended for the day, though the exchange reserves the discretion to reopen trading in the contract later in the day. The rationale for introducing price limits is to prevent panic buying and selling on an asset, based upon faulty information or rumors, and to prevent overreaction to real information. By allowing investors more time to react to extreme information, it is argued, the price reaction will be more rational and reasoned.

3. Marking to Market

One of the unique features of futures contracts is that the positions of both buyers and sellers of the contracts are adjusted every day for the change in the market price that day. In other words, the profits or losses associated with price movements are credited or debited from an investor's account even if he or she does not trade. This process is called marking to market.

4. Margin Requirements for Trading

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[1] The reason the exchange allows equivalents is to prevent investors from buying a significant portion of

In a futures agreement, there is no payment made by the buyer to the seller, nor does the seller have to show proof of physical ownership of the asset at the time of the agreement. In order to ensure, however, that the parties to the futures contract fulfill their sides of the agreement, they are required to deposit funds in a margin account. The amount that has to be deposited at the time of the contract is called the *initial margin.* As prices move subsequently, the contracts are marked to market, and the profits or losses are posted to the investor's account. The investor is allowed to withdraw any funds in the margin account in excess of the initial margin. Table 34.1 summarizes price limits and contract specifications for many traded futures contracts as of June 2001.

Table 34.1: Futures Contracts: Description, Price Limits and Margins

Contract	*Exchange*	*Specifications*	*Tick Value*	*Initial Margin/Contract*	*Daily Limit/unit*
			Softs		
Coffee	NYBOT	37,500 lbs	$18.75/0.05¢	$2,450	none
Sugar	NYBOT	112,000 lbs	$11.20/0.01¢	$840	none
Cocoa	NYBOT	10 metric tons	$10/1¢	$980	none
Cotton	NYBOT	50,000 lbs	$5/0.01¢	$1,000	3¢
Orange Juice	NYBOT	15,000 lbs	$7.50/0.05¢	$700	5¢
			Metals		
Gold	NYMEX	100 troy ozs	$10/10¢	$1,350	$75
Kilo Gold	CBOT	1 gross kgm	$3.22/10¢	$473	$50
Silver	NYMEX	5000 troy ozs	$25/0.5¢	$1,350	$1.50
5000oz Silver	CBOT	5000 troy ozs	$5/0.1¢	$270	$1
Copper	NYMEX	25,000 lbs	$12.50/0.05¢	$4,050	$0.20
Platinum	NYMEX	50 troy ozs	$5/10¢	$2,160	$25
Palladium	NYMEX	100 troy ozs	$5/5¢	$67,500	none
			Energy		
Crude	NYMEX	1,000 barrels	$10/1¢	$3,375	$7.50 first
Unleaded	NYMEX	42,000 gallons	$4.20/0.01¢	$3,375	20¢ first
Heating Oil	NYMEX	42,000 gallons	$4.20/0.01¢	$3,375	20¢ first
Natural Gas	NYMEX	10,000 mm	$10/0.01¢	$4,725	$1

the specified treasury bonds and cornering the market.

		Btu			
Agriculture					
Live Cattle	CME	40,000 lbs	$10/2.5¢	$810	1.5¢
Feeder Cattle	CME	50,000 lbs	$12.50/2.5¢	$945	1.5¢
Lean Hogs	CME	40,000 lbs	$10/2.5¢	$999	2¢
Pork Bellies	CME	40,000 lbs	$10/2.5¢	$1,620	3¢
Lumber	CME	110,000 ft	$11/10¢	$1,013	$10
Currencies					
EuroCurrency	CME	125,000 Euros	$12.50/0.01¢	$2,349	400 ticks
Swiss Franc	CME	125,000 Sfr	$12.50/0.01¢	$1,755	400 ticks
Japanese Yen	CME	12,500,000 Yen	$12.50/0.0001¢	$2,835	400 ticks
British Pound	CME	62,500 Bp	$6.25/0.02¢	$1,418	800 ticks
Canadian Dlr	CME	100,000 C$	$10/0.01¢	$608	400 ticks
Australian Dlr	CME	100,000 A$	$10/0.01¢	$1,215	400 ticks
MexicanPeso	CME	500,000 pesos	$12.50/0.0025¢	$2,500	2000 ticks
Dollar Index	NYBOT	$1,000 times dollar index	$10/0.01¢	$1,995	2 pts
Interest Rate					
T-Bond	CBOT	$100,000 face value	$31.25/1/32	$2,363	None
T-Note (10)	CBOT	$100,000 face value	$31.25/1/32	$1,620	None
T-Note (5)	CBOT	$100,000 face value	$31.25/1/32	$1,080	None
Muni Bond	CBOT	$1,000 times the closing value of The Bond Buyer™ 40 Index	$31.25/1/32	$1,350	None
Midam Bond	MIDAM	$50,000 face value	$15.62/1/32	$878	3pts
T-Bills	CME	$1,000,000	$25/0.05¢	$540	None
Eurodollars	CME	$1,000,000	$25/0.05¢	$810	None
Indices					
S&P 500	CME	$250 times S&P 500 Index	$25/0.10 Pts.	$21,563	None
NYSE Index	NYBOT	$250 times S&P 500	$25/0.05 Pts.	$19,000	None

		Index			
Nasdaq 100	CME	$100 times NASDAQ	$5/0.05 Pts.	$33,750	None
Mini Nasdaq	CME	$20 times NASDAQ	$10/0.50 Pts.	$6,750	None
Mini S&P	CME	$50 times S&P 500 Index	$12.50/0.25 Pts	$4,313	None
DowJones Fut	CBOT	$10 times DJ Index	$10/1 Pt.	$6,750	None
Value Line	KCBT	$100 times VL Index	$25/0.05 Pts.	$3,500	None
Nikkei	CME	$ 5 times Nikkei Index	$25/5 Pts.	$6,750	None
GSCI	CME	$250 times GSCI	$12.50/0.05 Pts.	$3,750	None
CRB	NYBOT		$25/0.05 Pts.	$1,500	None
Grains					
Soybeans	CBOT	5000 bushels	$12.50/0.25¢	$945	50¢
Soymeal	CBOT	100 tons	$10/10¢	$810	$20
Bean Oil	CBOT	60,000 lbs	$6/0.01¢	$473	2¢
Wheat	CBOT	5000 bushels	$12.50/0.25¢	$743	30¢
Corn	CBOT	5000 bushels	$12.50/0.25¢	$473	20¢
Oats	CBOT	5000 bushels	$12.50/0.25¢	$270	20¢

CBOT: Chicago Board of Trade
KCBT: Kansas City Board of Trade
NYBOT: New York Board of Trade
NYNEX: New York Mercantile Exchange
CME: Chicago Mercantile Exchange
MIDAM: Mid American Exchange

If the investor has a string of losses, because of adverse price movements, his margin will decrease. To ensure that there are always funds in the account, the investor is expected to maintain a *maintenance margin*, which is generally lower than the initial margin. If the funds in the margin account fall below the maintenance margin, the investor will receive a *margin call* to replenish the funds in the account. These extra funds that have to be brought in is known as a *variation margin*. Maintenance margins can vary across contracts and even across different customers. Table 34.2, for instance, shows the relationship between maintenance and initial margins for a sampling of futures contracts from the Chicago Mercantile Exchange.

Table 34.2: Initial versus Maintenance Margins

Agricultural Group	*Maintenance Margin (per contract)*	*Initial Margin Mark Up Percentage*	*Initial Margin (per contract)*
Corn	$350	135%	$473
Oats	$200	135%	$270
Rough Rice	$500	135%	$675
Soybeans	$700	135%	$945
Soybean Meal	$600	135%	$810
Soybean Oil	$350	135%	$473
Wheat	$550	135%	$743

Illustration 34.2: Calculating Equity and Maintenance Margins

Assume that you buy 100 wheat futures contracts on the CME and that the spot price of wheat today is $3.15. Your initial margin can be computed based upon the $743 per contract specified by the exchange.

Initial margin = $743 * 100 contracts = $74,300

Assume that the price of wheat drops to $3.14 per bushel tomorrow. [NOTE: The solution was incorrect. It implied a drop of only 1 cent, not 10 cents. It was easier to change the question than the solution.] The contract will be marked to market, resulting in a loss to you.

$$\begin{aligned}\text{Loss from marking to market} &= (\text{change in price})(\text{Bushels/contract})(\text{Number of contracts})\\ &= (\$3.15 - \$3.14)(5000)(100)\\ &= \$5{,}000\end{aligned}$$

The equity in your account is now $69,300.

Equity after marking to market = $74,300 - $ 5,000 = $69,300

You are still safely above the maintenance margin requirement, but a series of price drops can cause your equity to drop below the maintenance margin.

Maintenance margin = $ 550 * 100 = $ 55,000

If you drop below this level, you will get a margin call. Failure to meet the margin call will result in the position being liquidated.

Price Limits: Effects on Liquidity

The logic of price limits is that they act as a brake on the market and prevent panic buying or selling. Implicit in their use is the assumption that trading can sometimes exacerbate volatility and cause prices to swing to unjustifiably high or low levels. The problem with price limits, however, is that they do not discriminate between rational price movements (caused by shifts in the underlying demand or supply of a commodity) and irrational ones. Consequently, price limits can limit liquidity when investors need it the most and slow down the process of price adjustment.

An interesting way to frame the question on price limits is to ask whether you would be willing to pay more or less for an asset that has price limits associated with trading than for an asset without those price limits. The trade off between lower volatility (from restrictions on trading) and less liquidity will determine how you answer the question.

Pricing of Futures Contracts

Most futures contracts can be priced on the basis of arbitrage, i.e., a price or range of prices can be derived at which investors will not be able to create positions involving the futures contract and the underlying asset that make riskless profits with no initial investment. The following sections examine the pricing relationships for a number of futures contracts.

a. Perishable Commodities

Perishable commodities offer the exception to the rule that futures contracts are priced on the basis of arbitrage, since the commodity has to be storable for arbitrage to be feasible. On a perishable futures contract, the futures price will be influenced by:

(a) the expected spot price of the underlying commodity: If the spot price on the underlying commodity is expected to increase before the expiration of the futures contract, the futures prices will be greater than the current spot price of the commodity. If the spot price is expected to decrease, the futures price will be lower than the spot price.

(b) any risk premium associated taking the futures position: Since there is a buyer and a seller on a futures contract, the size and the direction of the risk premium will be vary from case to case and will depend upon whether the buyer is viewed as providing a service to the seller or vice versa. In an agricultural futures contract, where farmers or producers are the primary sellers of futures contracts and individual investors are the buyers, it can be argued that the latter are providing a service to the former and thus should be rewarded. In this scenario, the futures price will be lower than the expected spot price.

Futures price = Spot Price - Expected Risk Premium

In this type of relationship between futures and spot prices, prices are said to exhibit 'normal backwardation'.

In a futures contract, where buyers of the futures contract are industrial users (a good example would be Hershey's, a chocolate manufacturer, buying sugar futures to lock in favorable prices) and the sellers are individual investors, the buyers are being provided the service and the sellers could demand a reward, leading to a risk premium that is positive. In this case, the futures price will be greater than the expected spot price (assuming flat expectations) and futures prices are said to exhibit 'normal contango'.

In most modern commodity futures markets, neither sellers nor buyers are likely to be dominated by users or producers, and the net benefit can accrue to either buyers or sellers and there is no a priori reason to believe that risk premiums have to be positive or negative. In fact, if buyers and sellers are both speculating on the price, rather than hedging output or input needs, the net benefit can be zero, leading to a zero risk premium. In such a case the futures price should be equal to the expected spot price.

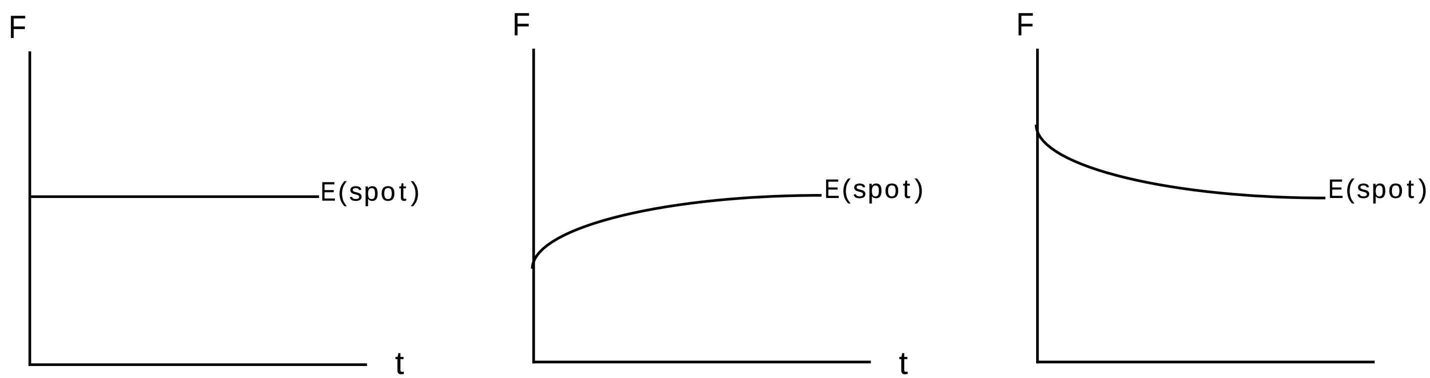
These three possible scenarios for the futures price, relative to the expected spot price, are graphed in Figure 34.3. The empirical evidence from commodity futures markets is mixed. An early study by Houthaker found that futures prices for commodities were generally lower than the expected spot prices, a finding that is consistent with a ‘normal backwardation’. Telser and Gray, however, report contradictory evidence from the wheat and corn futures markets.

Figure 34.3:Futures on Perishable Commodities

$F = E(S_t) + \text{Risk premium}$

	Expectations Hypothesis	Normal Backwardation	Normal Contango
Assumptions	1. Investors are risk-neutral	1. Hedgers are net short 2. Speculators are net long	1. Hedgers are net long 2. Speculators are net short
Futures price	$F = E(S_t)$	$F < E(S_t)$	$F > E(S_t)$

F vs S



b. Storable Commodities

The distinction between storable and perishable goods is that storable goods can be acquired at the spot price and stored till the expiration of the futures contract, which is the practical equivalent of buying a futures contract and taking delivery at expiration. Since the two approaches provide the same result, in terms of having possession of the commodity at expiration, the futures contract, if priced right, should cost the same as a strategy of buying and storing the commodity. The two additional costs of the latter strategy are as follows.

(a) Since the commodity has to be acquired now, rather than at expiration, there is an added financing cost associated with borrowing the funds needed for the acquisition now.

$$\text{Added Interest Cost} = (\text{Spot price})\left((1 + \text{Interest Rate})^{\text{Life of Futures contract}} - 1\right)$$

(b) If there is a storage cost associated with storing the commodity until the expiration of the futures contract, this cost has to be reflected in the strategy as well. In addition, there may be a benefit to having physical ownership of the commodity. This benefit is called the convenience yield and will reduce the futures price. The net storage cost is defined to be the difference between the total storage cost and the convenience yield.

If F is the futures contract price, S is the spot price, r is the annualized interest rate, t is the life of the futures contract and k is the net annual storage costs (as a percentage of the spot price) for the commodity, the two equivalent strategies and their costs can be written as follows.

Strategy 1: Buy the futures contract. Take delivery at expiration. Pay $F.

Strategy 2: Borrow the spot price (S) of the commodity and buy the commodity. Pay the additional costs.

(a) Interest cost $= S\left((1+r)^t - 1\right)$

(b) Cost of storage, net of convenience yield = S k t

If the two strategies have the same costs,

$$F^* \begin{aligned} &= S\left((1+r)^t - 1\right) + Skt \\ &= S\left((1+r)^t + kt\right) \end{aligned}$$

This is the basic arbitrage relationship between futures and spot prices. Any deviation from this arbitrage relationship should provide an opportunity for arbitrage, i.e., a strategy with no risk and no initial investment, and for positive profits. These arbitrage opportunities are described in Figure 34.4.

This arbitrage is based upon several assumptions. First, investors are assumed to borrow and lend at the same rate, which is the riskless rate. Second, when the futures contract is over priced, it is assumed that the seller of the futures contract (the arbitrageur) can sell short on the commodity and that he can recover, from the owner of the commodity, the storage costs that are saved as a consequence. To the extent that these assumptions are unrealistic, the bounds on prices within which arbitrage is not feasible expand. Assume, for instance, that the rate of borrowing is r_b and the rate of lending is r_a, and that short seller cannot recover any of the saved storage costs and has to pay a transactions cost of t_s. The futures price will then fall within a bound.

$$(S - t_s)(1 + r_a)^t < F^* < S((1 + r_b)^t + kt)$$

If the futures price falls outside this bound, there is a possibility of arbitrage and this is illustrated in Figure 34.5.

Figure 34.4: Storable Commodity Futures: Pricing and Arbitrage

$F^* = S((1+r)^t + k\,t)$

	If F > F*		If F < F*	
Time	Action	Cashflows	Action	**Cashflows**
Now:	1. Sell futures contract	0	1. Buy futures contract	0
	2. Borrow spot price at riskfree r	S	2. Sell short on commodity	S
	3. Buy spot commodity	-S	3. Lend money at riskfree rate	-S
At t:	1. Collect commodity; Pay storage cost.	-Skt	1. Collect on loan	$S(1+r)^t$
	2. Deliver on futures contract	F	2. Take delivery of futures contract	-F
	3. Pay back loan	$-S(1+r)^t$	3. Return borrowed commodity; Collect storage costs	+Skt
NCF=		**$F-S((1+r)^t - kt) > 0$**		**$S((1+r)^t + kt) - F > 0$**

Key inputs:

F* = Theoretical futures price
F = Actual futures price
S = Spot price of commodity
r= Riskless rate of interest (annualized)
t = Time to expiration on the futures contract
k = Annualized carrying cost, net of convenience yield (as % of spot price)

Key assumptions

1. The investor can lend and borrow at the riskless rate.
2. There are no transactions costs associated with buying or selling short the commodity.
3. The short seller can collect all storage costs saved because of the short selling.

Figure 34.5: storable commodity futures: pricing and arbitrage with modified assumptions

Modified Assumptions

1. Investor can borrow at r_b ($r_b > r$) and lend at r_a ($r_a < r$).
2. The transactions cost associated with selling short is t_s (where t_s is the dollar transactions cost).
3. The short seller does not collect any of the storage costs saved by the short selling.

$$F_h^* = S\left((1+r_b)^t + k\,t\right)$$

$$F_l^* = (S-t_s)\,(1+r_a)^t$$

If $F > F_h^*$ | If $F < F_l^*$

Time	Action	Cashflows	Action	Cashflows
Now:	1. Sell futures contract	0	1. Buy futures contract	0
	2. Borrow spot price at r_b	S	2. Sell short on commodity	$S - t_s$
	3. Buy spot commodity	-S	3. Lend money at r_a	$-(S - t_s)$
At t:	1. Collect commodity from storage	-Skt	1. Collect on loan	$(S-t_s)(1+r_a)^t$
	2. Delivery on futures contract	F	2. Take delivery of futures contract	-F
	3. Pay back loan	$-S(1+r_b)^t$	3. Return borrowed commodity; Collect storage costs	0
NCF=		$\mathbf{F-S((1+r_b)^t - kt) > 0}$		$\mathbf{(S-t_s)\,(1+r_a)^t - F > 0}$

F_h = Upper limit for arbitrage bound on futures prices

F_l = Lower limit for arbitrage bound on futures prices

c. Stock Index Futures

Futures on stock indices have become an important and growing part of most financial markets. Today, you can buy or sell futures on the Dow Jones, the S&P 500, the NASDAQ and the Value Line indices.

An index future entitles the buyer to any appreciation in the index over and above the index futures price and the seller to any depreciation in the index from the same benchmark. To evaluate the arbitrage pricing of an index future, consider the following strategies.

Strategy 1: Sell short on the stocks in the index for the duration of the index futures contract. Invest the proceeds at the riskless rate. (This strategy requires that the owners of the index be compensated for the dividends they would have received on the stocks.)

Strategy 2: Sell the index futures contract.

Both strategies require the same initial investment, have the same risk and should provide the same proceeds. Again, if S is the spot price of the index, F is the futures prices, y is the annualized dividend yield on the stock and r is the riskless rate, the cash flows from the two contracts at expiration can be written.

$$F^* = S\,(1 + r - y)^t$$

If the futures price deviates from this arbitrage price, there should be an opportunity from arbitrage. This is illustrated in Figure 34.6.

This arbitrage is conditioned on several assumptions. First, it, like the commodity futures arbitrage, assumes that investors can lend and borrow at the riskless rate. Second, it ignores transactions costs on both buying stock and selling short on stocks. Third, it assumes that the dividends paid on the stocks in the index are known with certainty at the start of the period. If these assumptions are unrealistic, the index futures arbitrage will be feasible only if prices fall outside a band, the size of which will depend upon the seriousness of the violations in the assumptions.

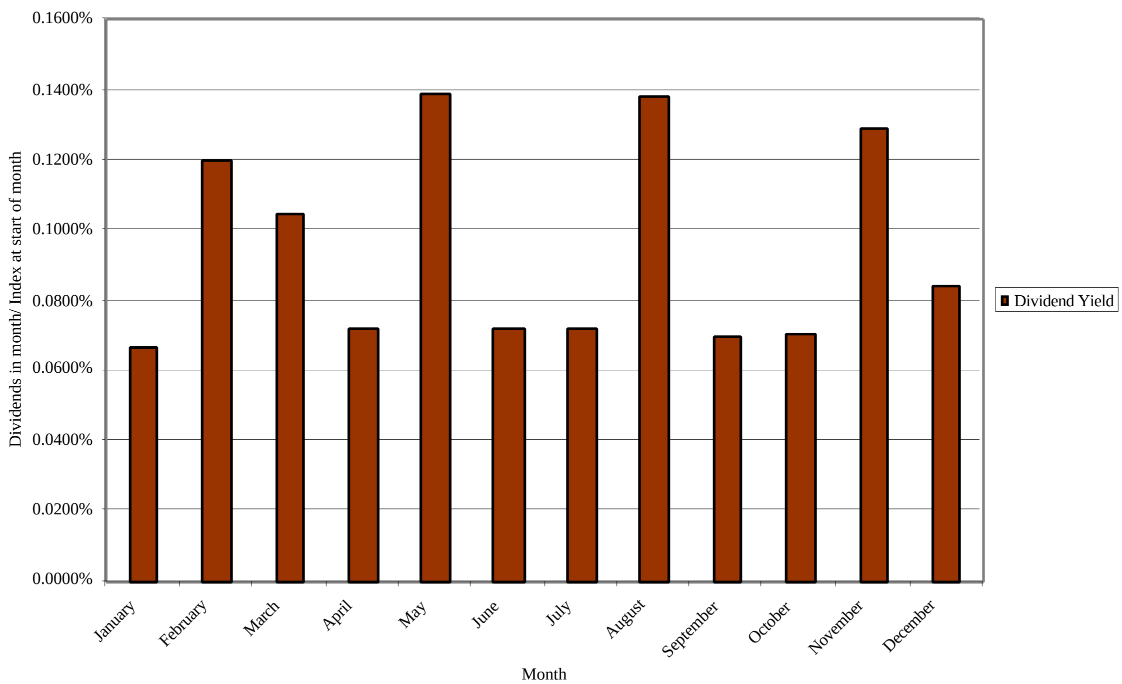
Assume that investors can borrow money at r_b and lend money at r_a and that the transactions costs of buying stock is t_c and selling short is t_s. The band within which the futures price must stay can be written as:

$$(S - t_s)(1 + ra - y)^t < F^* < (S + t_c)(1 + rb - y)^t$$

The arbitrage that is possible if the futures price strays outside this band is illustrated in Figure 34.7.

In practice, one of the issues that you have to factor in is the seasonality of dividends since the dividends paid by stocks tend to be higher in some months than others. Figure 34.8 graphs out dividends paid as a percent of the S&P 500 index on U.S. stocks in 2000 by month of the year.

Figure 34.8: Dividend Yields by Month of Year- 2000



Thus, dividend yields seem to peak in February, May, August and November.

Figure 34.6: Stock Index Futures: Pricing and Arbitrage

$F^* = S(1+r-y)^t$

If $F > F^*$		If $F < F^*$	
Action	Cashflows	Action	**Cashflows**
1. Sell futures contract	0	1. Buy futures contract	0
2. Borrow spot price of index at riskfree r	S	2. Sell short stocks in the index	S
3. Buy stocks in index	-S	3. Lend money at riskfree rate	-S
1. Collect dividends on stocks	$S((1+y)^t-1)$	1. Collect on loan	$S(1+r)^t$
2. Delivery on futures contract	F	2. Take delivery of futures contract	-F
3. Pay back loan	$-S(1+r)^t$	3. Return borrowed stocks; Pay foregone dividends	$-S((1+y)^t-1)$
	$F-S(1+r-y)^t > 0$		**$S(1+r-y)^t - F > 0$**

uts:

eoretical futures price — r= Riskless rate of interest (annualized)

ıal futures price — t = Time to expiration on the futures contract

: level of index — y = Dividend yield over lifetime of futures contract as % of current index level

umptions

ıvestor can lend and borrow at the riskless rate.

are no transactions costs associated with buying or selling short stocks.

ends are known with certainty.

Figure 34.7: Stock Index Futures: Pricing and Arbitrage with modified assumptions

·d Assumptions

tor can borrow at r_b ($r_b > r$) and lend at r_a ($r_a < r$).

·ansactions cost associated with selling short is t_s (where t_s is the dollar transactions cost) and the transactions cost associated with buying the stocl t_c.

$$F_h^* = (S+t_c)(1+r_b-y)^t$$

$$F_l^* = (S-t_s)(1+r_a-y)^t$$

If $F > F_h^*$		If $F < F_l^*$	
Action	Cashflows	Action	Cashflows
1. Sell futures contract	0	1. Buy futures contract	0
2. Borrow spot price at r_b	$S+t_c$	2. Sell short stocks in the index	$S - t_s$
3. Buy stocks in the index	$-S-t_c$	3. Lend money at r_a	$-(S - t_s)$
1. Collect dividends on stocks	$S((1+y)^t-1)$	1. Collect on loan	$(S-t_s)(1+r_a)^t$
2. Delivery on futures contract	F	2. Take delivery of futures contract	$-F$
3. Pay back loan	$-(S+t_c)(1+r_b)^t$	3. Return borrowed stocks; Pay foregone dividends	$-S((1+y)^t-1)$
	$\mathbf{F-(S+t_c)(1+r_b-y)^t > 0}$		$\mathbf{(S-t_s)(1+r_a-y)^t - F > 0}$

·per limit for arbitrage bound on futures prices F_l = Lower limit for arbitrage bound on futures prices

d. Treasury Bond Futures

The treasury bond futures traded on the CBOT require the delivery of any government bond with a maturity greater than fifteen years, with a no-call feature for at least the first fifteen years. Since bonds of different maturities and coupons will have different prices, the CBOT has a procedure for adjusting the price of the bond for its characteristics. The conversion factor itself is fairly simple to compute and is based upon the value of the bond on the first day of the delivery month, with the assumption that the interest rate for all maturities equals 8% per annum (with semi-annual compounding). The following example calculates the conversion factor for a 9% coupon bond with 18 years to maturity.

Illustration 34.3: Calculation Conversion Factors for T.Bond futures

Consider a 9% coupon bond with 20 years to maturity. Working in terms of a $100 face value of the bond, the value of the bond can be written as follows, using the interest rate of 8%.

$$\text{PV of Bond} = \sum_{t=0.5}^{t=20} \frac{4.50}{(1.08)^t} + \frac{100}{(1.08)^{20}} = \$111.55$$

The conversion factor for this bond is 109.90. Generally speaking, the conversion factor will increase as the coupon rate increases and with the maturity of the delivered bond.

The Delivery Option and the Wild Card Play

This feature of treasury bond futures, i.e., that any one of a menu of treasury bonds can be delivered to fulfill the obligation on the bond, provides an advantage to the seller of the futures contract. Naturally, the cheapest bond on the menu, after adjusting for the conversion factor, will be delivered. This *delivery option* has to be priced into the futures contract.

There is an additional option embedded in treasury bond futures contracts that arises from the fact that the T.Bond futures market closes at 2 p.m., whereas the bonds themselves continue trading until 4 p.m. The seller does not have to notify the clearing house until 8 p.m. about his intention to deliver. If bond prices decline after 2 p.m., the seller can notify the clearing house of intention to deliver the cheapest bond that day. If not, the seller can wait for the next day. This option is called the *wild card play*.

Valuing a T.Bond Futures Contract

The valuation of a treasury bond futures contract follows the same lines as the valuation of a stock index future, with the coupons of the treasury bond replacing the dividend yield of the stock index. The theoretical value of a futures contract should be –

$$F^* = (S - PVC)(1 + r)^t$$

where,

F* = Theoretical futures price for Treasury Bond futures contract

S = Spot price of Treasury bond

PVC = Present Value of coupons during life of futures contract

r = Riskfree interest rate corresponding to futures life

t = Life of the futures contract

If the futures price deviates from this theoretical price, there should be the opportunity for arbitrage. These arbitrage opportunities are illustrated in Figure 34.8.

This valuation ignores the two options described above - the option to deliver the cheapest-to-deliver bond and the option to have a wild card play. These give an advantage to the seller of the futures contract and should be priced into the futures contract. One way to build this into the valuation is to use the cheapest deliverable bond to calculate both the current spot price and the present value of the coupons. Once the futures price is estimated, it can be divided by the conversion factor to arrive at the standardized futures price.

Figure 34.8: Treasury Bond Futures: Pricing And Arbitrage

$F^* = (S - PVC)(1+r)^t$

	If F > F*		If F < F*	
Time	Action	Cashflows	Action	Cashflows
Now:	1. Sell futures contract	0	1. Buy futures contract	0
	2. Borrow spot price of bond at riskfree r	S	2. Sell short treasury bonds	S
	3. Buy treasury bonds	-S	3. Lend money at riskfree rate	-S
Till t:	1. Collect coupons on bonds; Invest	$PVC(1+r)^t$	1. Collect on loan	$S(1+r)^t$
	2. Deliver the cheapest bond on contract	F	2. Take delivery of futures contract	-F
	3. Pay back loan	$-S(1+r)^t$	3. Return borrowed bonds; Pay foregone coupons w/interest	$-PVC(1+r)^t$
NCF=		$\mathbf{F-(S-PVC)(1+r)^t > 0}$		$\mathbf{(S-PVC)(1+r)^t - F > 0}$

Key inputs:

F* = Theoretical futures price

F = Actual futures price

S = Spot level of treasury bond

r= Riskless rate of interest (annualized)

t = Time to expiration on the futures contract

PVC = Present Value of Coupons on Bond during life of futures contract

Key assumptions

1. The investor can lend and borrow at the riskless rate.
2. There are no transactions costs associated with buying or selling short bonds.

e. Currency Futures

In a currency futures contract, you enter into a contract to buy a foreign currency at a price fixed today. To see how spot and futures currency prices are related, note that holding the foreign currency enables the investor to earn the risk-free interest rate (R_f) prevailing in that country while the domestic currency earn the domestic riskfree rate (R_d). Since investors can buy currency at spot rates and assuming that there are no restrictions on investing at the riskfree rate, we can derive the relationship between the spot and futures prices. *Interest rate parity* relates the differential between futures and spot prices to interest rates in the domestic and foreign market.

$$\frac{\text{Futures Price}_{d,f}}{\text{Spot Price}_{d,f}} = \frac{(1+R_d)}{(1+R_f)}$$

where Futures $\text{Price}_{d,f}$ is the number of units of the domestic currency that will be received for a unit of the foreign currency in a forward contract and Spot $\text{Price}_{d,f}$ is the number of units of the domestic currency that will be received for a unit of the same foreign currency in a spot contract. For instance, assume that the one-year interest rate in the United States is 5% and the one-year interest rate in Germany is 4%. Furthermore, assume that the spot exchange rate is $0.65 per Deutsche Mark. The one-year futures price, based upon interest rate parity, should be as follows:

$$\frac{\text{Futures Price}_{d,f}}{\$\ 0.65} = \frac{(1.05)}{(1.04)}$$

resulting in a futures price of $0.65625 per Deutsche Mark.

Why does this have to be the futures price? If the futures price were greater than $0.65625, say $0.67, an investor could take advantage of the mispricing by selling the futures contract, completely hedging against risk and ending up with a return greater than the riskfree rate. When a riskless position yields a return that exceeds the riskfree rate, it is called an **arbitrage position**. The actions the investor would need to take are summarized in Table 34.3, with the cash flows associated with each action in brackets next to the action.

Table 34.3: Arbitrage when currency futures contracts are mispriced

Forward Rate Mispricing	*Actions to take today*	*Actions at expiration of* futures *contract*
If futures price > \$0.65625 e.g. \$0.67	1. Sell a futures contract at \$0.67 per Deutsche Mark. (\$0.00) 2. Borrow the spot price in the U.S. domestic markets @ 5%. (+\$0.65) 3. Convert the dollars into Deutsche Marks at spot price. (-\$0.65/+1 DM) 4. Invest Deustche Marks in the German market @ 4%. (-1 DM)	1. Collect on Deutsche Mark investment. (+1.04 DM) 2. Convert into dollars at futures price. (-1.04 DM/ +\$0.6968) 3. Repay dollar borrowing with interest. (-\$0.6825) **Profit = \$0.6968 - \$0.6825 = \$ 0.0143**
If futures price < \$0.65625 e.g. \$0.64	1. Buy a futures price at \$0.64 per Deutsche Mark. (\$0.00) 2. Borrow the spot rate in the German market @4%. (+1 DM) 3. Convert the Deutsche Marks into Dollars at spot rate. (-1 DM/+\$0.65) 4. Invest dollars in the U.S. market @ 5%. (-\$0.65)	1. Collect on Dollar investment. (+\$0.6825) 2. Convert into dollars at futures price. (-\$0.6825/1.0664 DM) 3. Repay DM borrowing with interest. (1.04 DM) **Profit = 1.0664-1.04 = 0.0264 DM**

The first arbitrage of Table 34.3 results in a riskless profit of \$0.0143, with no initial investment. The process of arbitrage will push down futures price towards the equilibrium price.

If the futures price were lower than \$0.65625, the actions would be reversed, with the same final conclusion. Investors would be able to take no risk, invest no money and still end up with a positive cash flow at expiration. In the second arbitrage of Table 34.3, we lay out the actions that would lead to a riskless profit of .0164 DM.

Effects of Special Features in Futures Contracts

The arbitrage relationship provides a measure of the determinants of futures prices on a wide range of assets. There are however some special features that affect futures prices. One is the fact that futures contracts require marking to the market, while forward contracts do not. Another is the existence of trading restrictions, such as price limits on futures contracts. The following section examines the pricing effects of each of these special features.

a. Futures versus Forward Contracts

As described earlier in this section, futures contracts require marking to market while forward contracts do not. If interest rates are constant and the same for all maturities, there should be no difference between the value of a futures contract and the value of an equivalent forward contract. When interest rates vary unpredictably, forward prices can be different from futures prices. This is because of the reinvestment assumptions that have to be made for intermediate profits and losses on a futures contract, and the borrowing and lending rates assumptions that have to be made for intermediate losses and profits, respectively. The effect of this interest rate induced volatility on futures prices will depend upon the relationship between spot prices and interest rates. If they move in opposite directions (as is the case with stock indices and treasury bonds), the interest rate risk will make futures prices greater than forward prices. If they move together (as is the case with some real assets), the interest rate risk can actually counter price risk and make futures prices less than forward prices. In most real world scenarios, and in empirical studies, the difference between futures and forward prices is fairly small and can be ignored.

There is another difference between futures and forward contracts that can cause their prices to deviate and it relates to credit risk. Since the futures exchange essentially guarantees traded futures contracts, there is relatively little credit risk. Essentially, the exchange has to default for buyers or sellers of contracts to not be paid. Forward contracts are between individual buyers and sellers. Consequently, there is potential for significant default risk which has to be taken into account when valuing a forward contract.

b. Trading Restrictions

The existence of price limits and margin requirements on futures contract are generally ignored in the valuation and arbitrage conditions described in this chapter. It is however possible that these restrictions on trading, if onerous enough, could impact value. The existence of price limits, for instance, has two effects. One is that it might reduce the volatility in prices, by protecting against market overreaction to information and thus make futures contracts more valuable. The other is that it makes futures contracts less liquid and this may make them less valuable. The net effect could be positive, negative or neutral.

Conclusion

The value of a futures contract is derived from the value of the underlying asset. The opportunity for arbitrage will create a strong linkage between the futures and spot prices; and the actual relationship will depend upon the level of interest rates, the cost of storing the underlying asset and any yield that can be made by holding the asset. In addition the institutional characteristics of the futures markets, such as price limits and 'marking to market', as well as delivery options, can affect the futures price.

Problems

1. The following is an excerpt from the Wall Street Journal futures page. It includes the futures prices of gold. The current cash (spot) price of gold is $403.25. Make your best estimates of the implied interest rates (from the arbitrage relationship) in the futures prices. (You can assume zero carrying costs for gold.)

Contract expiring in	*Trading at*
1 month	$404.62
2 months	$406.11
3 months	$407.70
6 months	$412.51
12 months	$422.62

2. You are a portfolio manager who has just been exposed to the possibilities of stock index futures. Respond to the following situations.

(a) Assume that you have the resources to buy and hold the stocks in the S&P 500. You are given the following data. (Assume that today is January 1.)

Level of the S&P 500 index = 258.90

June S&P 500 futures contract = 260.15

Annualized Rate on T.Bill expiring June 26 (expiration date) = 6%

Annualized Dividend yield on S&P 500 stocks = 3%

Assume that dividends are paid out continuously over the year. Is there potential for arbitrage? How would you go about setting up the arbitrage?

(b) Assume now that you are known for your stock selection skills. You have 10,000 shares of Texaco in your portfolio (now selling for 38) and are extremely worried about the direction of the market until June. You would like to protect yourself against market risk by using the December S&P 500 futures contract (which is at 260.15). If Texaco's beta is 0.8, how would you go about creating this protection?

3. Assume that you are a mutual fund manager with a total portfolio value of $100 million. You estimate the beta of the fund to be 1.25. You would like to hedge against market movements by using stock index futures. You observe that the S&P 500 June futures are selling for 260.15 and that the index is at 258.90. Answer the following questions.

(a) How many stock index futures would you have to sell to protect against market risk?

(b) If the riskfree rate is 6% and the market risk premium is 8%, what return would you expect to make on the mutual fund? (Assuming you don't hedge.)

(c) How much would you expect to make if you hedge away all market risk?

4. Given the following information on gold futures prices, the spot price of gold, the riskless interest rate and the carrying cost of gold, construct an arbitrage position. (Assume that it is December 1987 now.)

December 1988 futures contract price = 515.60/troy oz

Spot price of gold = 481.40/troy oz

Interest rate (annualized) = 6%

Carrying cost (annualized) = 2%

a. What would you have to do right now to set up the arbitrage?

b. What would you have to do in December to unwind the position? How much arbitrage profit would you expect to make?

c. Assume now that you can borrow at 8%, but you can lend at only 6%. Establish a price band for the futures contract, within which arbitrage is not feasible.

5. The following is a set of prices for stock index futures on the S&P 500.

Maturity	Futures price
March	246.25
June	247.75

The current level of the index is 245.82 and the current annualized T.Bill rate is 6%. The annualized dividend yield is 3%. (Today is January 14. The March futures expire on March 18 and the June futures on June 17.)

(a) Estimate the theoretical basis and actual basis in each of these contracts.

(b) Using one of the two contracts, set up an arbitrage. Also show how the arbitrage will be resolved at expiration. [You can assume that you can lend or borrow at the riskfree rate and that you have no transaction costs or margins.]

(c) Assume that a good economic report comes out on the wire. The stock index goes up to 247.82 and the T.Bill rate drops to 5%. Assuming arbitrage relationships hold and that the dollar dividends paid do not change, how much will the March future go up by?

6. You are provided the following information.

Current price of wheat = $19,000 for 5000 bushels

Riskless rate = 10 % (annualized)

Cost of storage = $200 a year for 5000 bushels

One-year futures contract price = $20,400 (for a contract for 5000 bushels)

a. What is F* (the theoretical price)?

b. How would you arbitrage the difference between F and F*? (Specify what you do now and at expiration and what your arbitrage profits will be.)

c. If you can sell short (Cost $100 for 5000 bushels) and cannot claim any of the storage cost for yourself on short sales[2], at what rate would you have to be able to lend for this arbitrage to be feasible?

[2] In theory, we make the unrealistic assumption that a person who sells short (i.e. borrows somebody else's property and sells it now) will be able to collect the storage costs saved by the short sales from the other party to the transaction.