Extra Office Hours

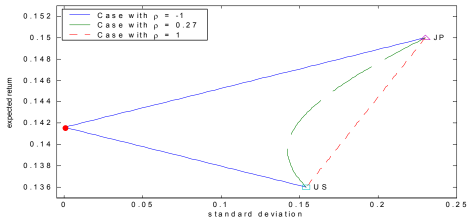
Midterm (June 5) is one week from today

Extra office hours:

- June 1 (Friday) 10 am to noon
- June 4 (Monday) 2 pm to 4 pm

1

Review: diversification



- There is benefit of diversification as long as $\rho < 1$ (not perfectly positively correlated)
- If $\rho = -1$(blue line), you might reduce the standard deviation of portfolio to zero. Then the return of that portfolio (red dot) must be equal to risk-free rate (otherwise, arbitrage opportunity)

2

Review: asset allocation

- 1 risky, 1 riskless asset
  - Investment opportunity set is a straight line on the expected return-standard deviation diagram
  - Called CAL
- 2 risky, no riskless asset
  - Benefits of diversification

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Review: asset allocation

- 2 (or many) risky assets and one riskless asset
  - Step 1: construct efficient frontier of risky assets only
  - Step 2: find the CAL that is tangent to this frontier (the steepest CAL or largest Sharpe Ratio)
  - Step 3: Investor's preference determines how much to invest between this Optimal Risky Portfolio (Tangency Portfolio) and the risk-free asset.

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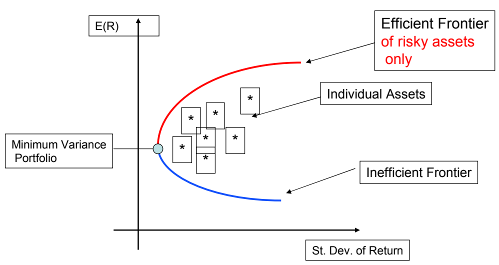
Step 1



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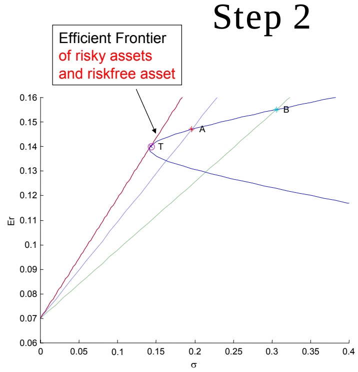
Step 2



Find a risky portfolio on efficient frontier (of risky assets only) that maximizes the Sharpe ratio or slope of CAL: tangency portfolio T

$$SR_i = \frac{E[R_i] - R_f}{\sigma_i}$$

Q: Where are the efficient portfolios after we include riskfree asset?

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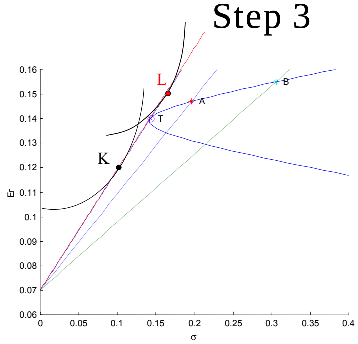
Step 3



People determine their portfolio on the efficient frontier based on their risk aversion

Fund Separation

- People's complete portfolio is always a combination of tangency portfolio and risk-free asset
- People hold the same risky portfolio (i.e. tangency portfolio)
- Therefore, people holds any two risky assets in the same ratio

  (e.g. tangency portfolio is 60% in US and 40% in Japan. Then in everyone's portfolio, the ratio of investment in US and in Japan is 3/2.)

Diversifiable vs. Nondiversifiable risk

- Adding independent risky assets into a portfolio is a good thing
  - As we add more, in the limit, the portfolio risk → 0
- In reality, security returns are correlated.
  - Adding correlated security can eliminate nonsystematic risk, but not systematic risk
  - Can use index model to separate the two risks

Single index models

Suppose $R_i = \alpha_i + \beta_i R_M + e_i$

- Just says

  "All return comes from three parts: a constant, a part common to many securities, and a part particular to this security"

- What is the source of the common part?
  - We don't know, but we can use a broad-based index as a proxy

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Measuring beta

- How?
  - Use formula:
    - $\beta_i = Cov(R_M, R_i) / \sigma^2_M$
  - Run a regression

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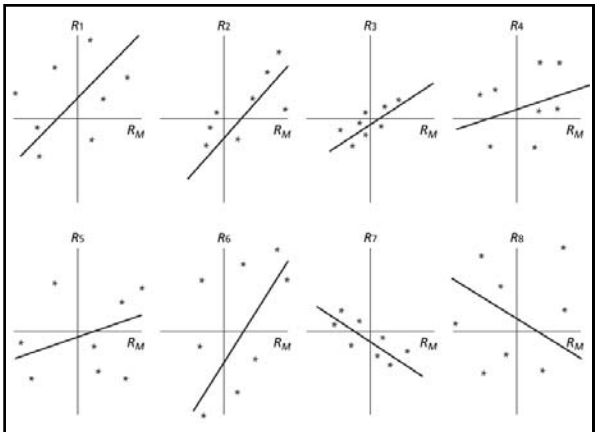


What do we do once we have beta?

- Beta tells us the sensitivity of the return on the asset to the factor
  - Here, broad-based index return
- The regression allows us to divide up movement
  - How much movement in the stock ‘caused’ by the market
  - How much by individual stock effects
- What is our measure of movement?
  - Variance

Dividing variances up

$R_i = \alpha_i + \beta_i R_M + e_i$

- Variance(LHS)=Variance(RHS)
- Or Variance(R_i)=0+Variance($\beta_i R_M$) +Variance(e_i)
- Because
  - in all regressions the error term and the X variable are uncorrelated
  - Constants have zero variance

Dividing variances up

- Variance(R_i)=Variance($\beta_i R_M$) +Variance(e_i)

  = $\beta_i^2 \sigma^2_M$+ Variance(e_i)

  = Systematic part + Idiosyncratic part

  = Part explained by the common factor+ part specific to the asset.

Proportion of Variance unexplained

- ABC Internet stock has a volatility of 90% and a beta of 3. The S&P500 has a volatility of 15%.
- What is the proportion of ABC Internet's variance which can be diversified away?

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_e{}^2$$

$$(0.9)^2 = 3^2 \times 0.15^2 + \sigma_e^2$$

$$\sigma_e^2 = 0.6075 \qquad (\sigma_e = 0.779)$$

$$\text{Hence } \frac{0.6075}{(0.9)^2} = 75\% \text{ of variance is diversified away}$$

Proportion of Variance explained

- Is just the R^2
  - $R^2=1$: all variance explained by the market
  - $R^2=0$: no variance explained by the market

The Capital Asset Pricing Model (CAPM)

Andre de Souza

Outline

- So far, we know, as individual investors, how to select a portfolio given E(R) and standard deviation.
- Now we turn around and ask: If everyone chose portfolios the way we said they should, what would the economy look like?

What do we mean by "look like"?

- Key questions:
  - What is the *equilibrium required return, E(R),* of a stock?
    - What is the *equilibrium price* of a stock?
  - Which *portfolios* should investors hold in equilibrium?
- Answer: Capital Asset Pricing Model (CAPM)
  - Assumptions
  - Results:
    - Identify the tangency portfolio in equilibrium
    - Hence, identify investors' portfolios
    - Derive equilibrium returns (and hence prices)

CAPM: Introduction

- CAPM is an equilibrium model that
  - predicts the relationship between expected return and risk
  - predicts optimal portfolio choices
  - underlies much of modern finance theory
  - underlies most of real-world financial decision making
- Derived using Markowitz's principles of portfolio theory, with additional simplifying assumptions.
- Sharpe, Lintner and Mossin are researchers credited with its development.
- William Sharpe won the Nobel Prize in 1990.

CAPM Assumptions Outline

- Assumptions:
  1. The market is in a competitive equilibrium;
  2. Single-period investment horizon;
  3. All assets are tradable;
  4. No transaction costs, no taxes;
  5. Investors are rational mean-variance optimizers with
  6. homogeneous expectations
- Some assumptions can be relaxed, and CAPM still holds.
- If many assumptions are relaxed, generalized versions of CAPM apply. (Topic of ongoing research.)

1: The market is in a competitive equilibrium

- Equilibrium:
  - Prices are such that Supply = Demand
  - If Demand > Supply for a particular security, the excess demand drives up price and reduces expected return.
  - (Reverse if Demand < Supply)
- Competitive market:
  - Investors take prices as given
  - No investor can manipulate the market.
  - No monopolist

2: Single-period horizon

- All investors agree on a horizon.
- Ensures that all investors are facing the same investment problem.

3: All assets are tradable

- And available to everyone
- This includes in principle:
  - All financial assets (including international stocks)
  - Real estate
  - Human capital
- This ensures that every investor has the same assets to invest in:
  - Portfolio of all the risky assets in the world = "market portfolio"

4: No frictions

- No taxes
- No transaction costs (no commission and no bid-ask spread)
- Same interest rate for lending and borrowing
- All investors can borrow or lend unlimited amounts. (No margin requirements.)

5-6: Investors are rational mean-variance optimizers with homogeneous expectations

- Investors choose efficient portfolios consistent with their risk-return preferences
  - All construct the efficient portfolio of risky assets
  - All combine the risk-free asset with this portfolio depending on their own risk preferences
- Investors have the same views about expected returns, variances, and covariances (and hence correlations).

What is the Equilibrium Tangency Portfolio?

- Recall from portfolio theory:
  - All investors should have a (positive or negative) fraction of their wealth invested in the risk-free security, and
  - The rest of their wealth is invested in the tangency portfolio.
  - The *tangency portfolio* is the same for all investors (homogeneous beliefs).
- Everyone holds the same risky asset portfolio, just different amounts invested in it

- Tangency portfolio consists of 20% AMZN, 30% BA(Boeing), 50% CAT (Caterpillar)

Guy #	Invests in risky portfolio	AMZN	BA	CAT
1	$50	$6	$15	$25
2	$10,000	$2,000	$3,000	$5,000
3	$2,000	$400	$600	$1,000
Total	12,500	$2,406	$3,615	$6,025

- How much is the weight in each asset in the aggregate portfolio?
- Suppose I told you weights in the aggregate portfolio. Could you tell me weights in the tangency portfolio?

What are the weights of risky assets in the market portfolio?

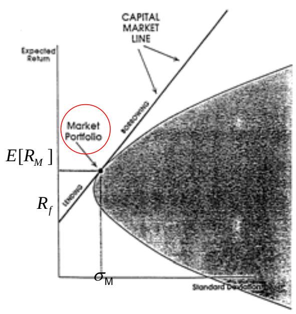
- Recall: Weight of asset=(amount invested in asset)/(total invested)
- Take a snapshot of the market
- How much wealth is invested in Dell?
  - Just (no of shares) times (price of shares)
- Total wealth invested in risky assets=$p_1*n_1+p_2*n_2+p_3*n_3+\ldots$
- Weight of asset:

$$= \frac{\text{market capitalization of security } i}{\text{total market capitalization}}$$

Now if I told you…

- The weights in the market portfolio, you could tell me what the optimal risky portfolio to invest in is
- Important! Now no need to form variances, covariances, or do our 3-step program
  - Just say: Market portfolio is the efficient risky-asset portfolio
- Important: All assets must be in the market(=tangency) portfolio
  - Otherwise what happens?

The E(R)-SD Frontier and the Capital Market Line (CML)



The Capital Market Line-CML

- In equilibrium, the market portfolio is the tangency portfolio, and vice versa.
- The market portfolio's CAL is called the Capital Market Line (CML).

➔ CML has the steepest slope.

- CML gives risk-return combinations of efficient portfolios (portfolios of the risk-free security and the market portfolio):

$$E(R_p) = R_f + \left(\frac{[E(R_M) - R_f]}{\sigma_M} \right) \sigma_p$$

Required return for efficient assets (on the CML)

- For efficient portfolios on CML, we know the relationship between return and risk as follows:

$$E(R_p) = R_f + \left(\frac{[E(R_M) - R_f]}{\sigma_M} \right) \sigma_p$$

- Just the return relationship for a CAL
- This holds only for efficient portfolios.
- Sharpe ratio is also called the Market Price of Risk

Required return for individual stocks

- CAPM is most famous for its prediction concerning the relationship between return and risk for **any security:**

$$E[R_i] = R_f + \beta_i \cdot (E[R_M] - R_f)$$

where $\beta_i = \dfrac{Cov(R_i, R_M)}{\sigma_M^2}$

- This holds for all securities.
- Don't confuse the two!

Understand CAPM

$$E[R_i] = R_f + \beta_i \cdot (E[R_M] - R_f)$$

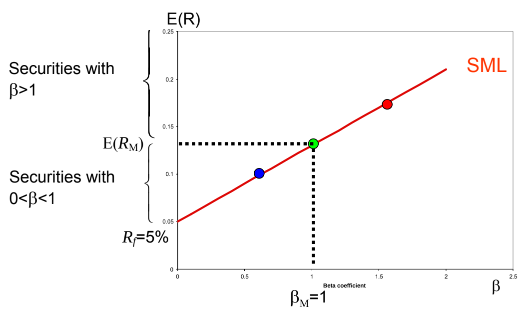
where $\beta_i = \dfrac{Cov(R_i, R_M)}{\sigma_M^2}$

- CAPM predicts that expected return of an asset is linear in its "beta" (hold for any asset).
- The beta measures the security's sensitivity to market movements.
- Higher beta stock is compensated with higher return. Lower beta (even negative) stocks require lower return.
- This linear relation is called Security Market Line (SML).

Understand CAPM

- We know that we can eliminate non-systematic risk. But we cannot eliminate market (systematic) risk.
- Think of adding a stock to a portfolio of 1,000 stocks
  - Which risk survives?
- Relevant measure of risk for an investor who holds portfolios is systematic risk. This risk is measured by beta.
- β_i measures security i's contribution to the total risk of a well-diversified portfolio, namely the market portfolio. So β_i measures the non-diversifiable risk of the stock.
- Investors must be compensated for holding non-diversifiable risk. Higher beta ➔ higher non-diversifiable risk ➔ higher required return. **This explains the CAPM equation.**
- In CAPM, a stock's risk is captured by beta, not standard deviation anymore.

Security Market Line (SML)



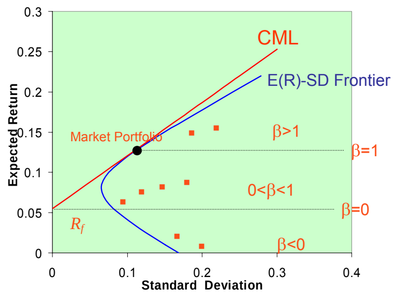
- Now let's draw a mean-standard deviation diagram
- Remember the CAPM relationship

$$E[R_i] = R_f + \beta_i \cdot (E[R_M] - R_f)$$

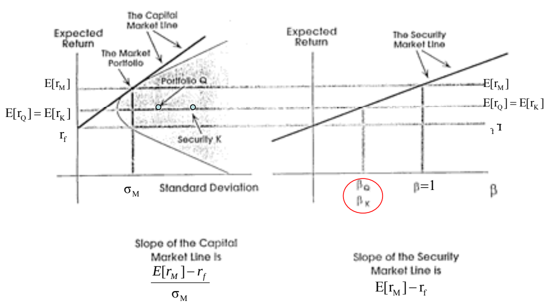
$$\beta_i = \frac{Cov(R_i, R_M)}{\sigma_M^2}$$

- So if I tell you expected return, you can tell me beta

Mean-Standard Deviation Frontier and betas



Capital Market Line (CML) and Security Market Line (SML)



Difference between CML and SML

- X-axis
- Slope
- SML applies to all securities, efficient or inefficient, but CML only apply to efficient.

Systematic and nonsystematic risk

- Remember the single-factor model?

  $R_i = \alpha_i + \beta_i R_M + e_i$
- Suppose instead of returns we imagine a model of excess returns

  $R_i - Rf = \alpha_i + \beta_i (R_M - Rf) + e_i$
- Now take expectations, and notice that E(e)=0

  $E[R_i - Rf] = \alpha_i + \beta_i E[R_M - Rf]$

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Systematic and nonsystematic risk

$E[R_i - Rf] = \alpha_i + \beta_i E[R_M - Rf]$

- Compare with the CAPM

  $E[R_i - Rf] = \beta_i E[R_M - Rf]$
- Although single index models and the CAPM are different, they yield equations that look almost the same
  - We can estimate quantities from the CAPM equation just as we would the Single Index Model equation
    - Beta
    - Divide variance in two parts

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Systematic and Non-Systematic Risk: Example

- ABC Internet stock has a volatility of 90% and a beta of 3. The market portfolio has an expected return of 14% and a volatility of 15%. The risk-free rate is 7%.
- What is the equilibrium expected return on ABC stock?
- What is the proportion of ABC Internet's variance which is diversified away in the market portfolio?

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_e^2$$

$$(0.9)^2 = 3^2 \times 0.15^2 + \sigma_e^2$$

$$\sigma_e^2 = 0.6075 \qquad (\sigma_e = 0.779)$$

$$\text{Hence } \frac{0.6075}{(0.9)^2} = 75\% \text{ of variance is diversified away}$$

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Systematic and Non-Systematic Risk: summary

- β_i measures security i's contribution of to the total risk of a well-diversified portfolio, namely the market portfolio.
- Hence, β_i measures the non-diversifiable risk of the stock
- Investors must be compensated for holding non-diversifiable risk. This explains the CAPM equation:

  $E(R_i) = R_f + \beta_i [E(R_M) - R_f], \quad i = 1,\ldots,N$

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How to estimate Beta

An Example:

1. Many institutions estimate betas, e.g.:
   - –Bloomberg
   - –Merrill Lynch
   - –Value Line
   - –Yahoo Financials

Some Betas, June 1996

Battle Mountain Gold Company	.40
Boeing Corporation	.90
Bristol-Myers Squibb	.95
California Water Company	.45
Caterpillar Inc.	1.20
Coca-Cola	.95
Dow Chemical	1.15
Exxon Corporation	.65
The Gap, Inc.	1.45
General Electric	1.15
Harley-Davidson	1.65
Idaho Power Company	.65
Intel Corporation	1.35
Kaufman & Broad Home	1.65
Kellogg	1.00
Merrill Lynch & Company	1.90
Oshkosh B'Gosh (clothing mfg.)	.60
Outback Steakhouse	2.10
Procter & Gamble	1.05
Ralston Purina	.90
Telefonos de Mexico	1.35
Tootsie Roll Industries	.75
Toys 'Я' Us	1.45
Western Digital	1.85

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Estimating Beta by Linear Regressions (OLS)

- CAPM SML: $E[R_i] - R_f = \beta_i (E[R_m] - R_f)$
- Get data on "excess returns":

  $R_i^e(t) = R_i(t) - R_f \qquad R_M^e(t) = R_M(t) - R_f$
- where R_f is the risk-free rate from time t-1 to time t.
- Estimate β_i by running the regression:

  $$R_i^e(t) = \alpha_i + \beta_i R_m^e(t) + error_i(t)$$
- Typically, more than 60 months of data are used.

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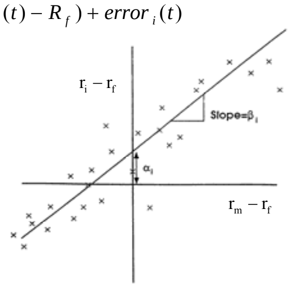
Security Characteristic Line (SCL)

The SCL is the "regression line":

$$R_i(t) - R_f = \alpha_i + \beta_i (R_M(t) - R_f) + error_i(t)$$

Note:
CAPM implies α_i=0

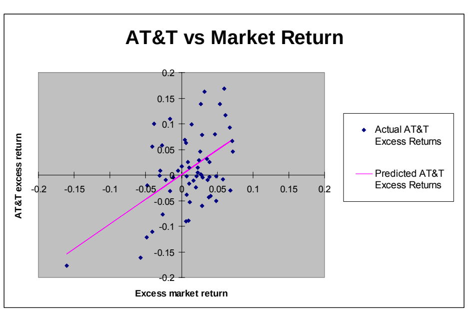


Estimating Beta: Real Life Example, AT&T

- Take 5 years (1994-1998) of monthly data on AT&T returns, S&P500 returns and 1 month US T-bills.
- Construct excess returns
- Run the regression, for instance using Excel:
  - apply *Tools, Add-ins, Analysis ToolPak*
  - use *Tools, Data Analysis, Regression*
- Excel Regression output:

	Coefficients	*SE*	*t Stat*	*P-value*
Intercept	0.0007	0.0091	0.0748	0.9406
X Variable 1	0.9637	0.2172	4.4366	0.0000

Estimating Beta: Real-Life SCL for AT&T



Applications of the CAPM

- Portfolio choice
- Shows what a “fair” security return is
- Gives benchmark for security analysis
- Required return used in capital budgeting

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Portfolio Choice: Active and Passive Strategies

- An “active” strategy tries to beat the market by stock picking, by timing, or other methods
- But, CAPM implies that
  - security analysis is not necessary
  - every investor should just buy a mix of the risk-free security and the market portfolio, a “passive” strategy.
  - What happens when nobody bothers?

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Deciding which stocks to pick

- Try and find stocks which make more than their “fair” return
- A security’s *alpha* is defined as:

$$\alpha_i = E[R_i] - R_f - \beta_i \cdot \left[E[R_M] - R_f\right]$$

$$\text{where } \beta_i = \frac{\text{cov}[R_i, R_M]}{\sigma_M^2}$$

- CAPM predicts that all alphas are zero.
- Some fund managers try to buy positive-alpha stocks and sell negative-alpha stocks.
  - Drives alphas to zero

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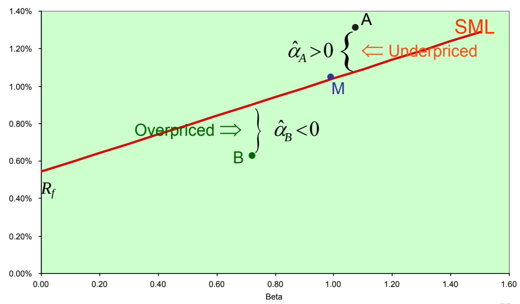
Stock Selection



Capital Budgeting

- Should firm undertake long-term risky project?
- Decide how much it is worth
- In Time Value of Money, we talked about valuing cash flows
- Mostly risk-free, or with a given interest rate
- When cash flows are risky, how do you compare?
- Increase the discount rates for riskier cash flows
  - Two penalties: time and risk
- How do you compute these discount rates?
- Use the CAPM!

Summary

- The CAPM comes from equilibrium conditions in a frictionless mean-variance economy with rational investors.
- Prediction 1: Everyone should hold a mix of the market portfolio and the risk-free asset. (That is, everyone should hold a portfolio on the CML.)
- Prediction 2: The expected return on a stock is a linear function of its beta. (That is, stocks should be on SML.)
- The beta is given by: $\beta_i = \dfrac{\text{cov}[R_i, R_M]}{\sigma_M^2}$
- A stock's beta can be estimated using historical data by linear regression. (That is, by estimating the SCL.)

Readings

- Today's class:
  BKM 6.5, 7.1, 7.2
- Next class
  BKM 8, 17.1