

# Arbitrage

Andre de Souza

1

---

---

---

---

---

---

---

---

## Start out by asking...

- Why do all one-year riskless securities have the same price?
- Examples
  - One year treasuries
  - Ten year treasuries issued nine years ago
- Extend to other instruments

2

---

---

---

---

---

---

---

---

## Outline

- Arbitrage definitions
- Arbitrage pricing
- Arbitrage pricing with transactions costs
- Real-world arbitrage trading

3

---

---

---

---

---

---

---

---

## Arbitrage Definitions

- In finance *theory* an “arbitrage” is defined as:
  - a trading strategy that generates a **completely riskless** profit, that is,
  - a positive cash flow at some time and non-negative cash flows at all times.
- On Wall Street, “arbitrage” also often refers to
  - a trading strategy that is **expected** to make a profit
    - So it is risky
- In this class, we follow the finance-theory tradition: **an arbitrage generates a riskless profit**

4

---

---

---

---

---

---

---

---

## Arbitrage Pricing

- The “no arbitrage condition”:
  - Important insight in finance: there cannot be arbitrage opportunities
    - Because if there were, arbitrageurs would trade aggressively to exploit the arbitrage
    - This will remove the arbitrage opportunities.

5

---

---

---

---

---

---

---

---

## Preliminary example of arbitrage

- PHLX/NYSE
  - IBM at \$100 on NYSE, \$95 on PHLX
  - What do you do?
- What *can* you do?
  - Physically sell the same share on the NYSE that you buy on the PHLX, or,
  - Hold the share you bought on the PHLX forever, and keep a short position open on the NYSE forever

6

---

---

---

---

---

---

---

---

### Short selling is as though I'm issuing the security myself

- When I short sell, I
  - Get an upfront cash flow equal to the price of the security
  - Have to pay all dividends / other cash flows that the holders of the security get (to the person I borrowed the security from)

7

---

---

---

---

---

---

---

---

### But we don't want to keep short positions open. Why?

- Expensive to short
- Horizon is infinite
  - But this problem is reduced with finite-horizon instruments
    - E.g. Most derivatives, bonds.
- Note that it's almost costless to short futures and options

8

---

---

---

---

---

---

---

---

### Uses of the no-arbitrage condition

- Perhaps surprisingly, using the "no-arbitrage condition" alone we can:
  - compute restrictions on security prices
  - compute explicitly prices of derivatives
    - Talk about it in excruciating detail when we discuss options pricing

9

---

---

---

---

---

---

---

---

### Implications of the No Arbitrage Condition

1. If two securities have the same payoffs, they must have the same price: *Law of One Price*. (see example 1 later)
2. If a portfolio has the same payoff as a security, the price of the security must be equal to the price of the portfolio (see example 2 later)
  - That portfolio is called a *replicating portfolio*
3. If a trading strategy has the same payoff as a security, the price of the security must be equal to the cost of the strategy (see example 3 later)
  - This is called a *dynamic trading strategy*

10

---

---

---

---

---

---

---

---

### Example 1: Law of One Price

- Suppose there are two kinds of zero-coupon bonds, "CATs" and "TIGRs," both paying their face value of \$100 in 1 year.
  - Identical in all but name (and possibly price)
- Suppose today's prices are
  - CATs: \$94.34 (6% yield),
  - TIGRs: \$95.24 (5% yield).
- Same cash flow (or payoffs), but different prices → arbitrage opportunity

11

---

---

---

---

---

---

---

---

### Example 1: Law of one price

- ✓ How can you make money out of it?  
(general principle in arbitrage: buy "cheap" and sell "expensive")
- Today:**
  - Buy 1 CAT → pay \$ 94.34
  - Short Sell 1 TIGR from Chase → get \$ 95.24
  - net cash flow: \$ 0.9
- One year later:**
  - Get \$ 100 from the 1 CAT you buy
  - Deliver \$ 100 to Chase
  - net cash flow: \$ 0
- total net profit: \$ 0.9
- ✓ How many times do you want to do this?
- ✓ How will this affect the prices?

12

---

---

---

---

---

---

---

---

## Arbitrage Pricing with Transactions Costs

- If there are transactions costs, it is more difficult to make an arbitrage trading strategy
- Therefore, we cannot determine prices exactly using the No-Arbitrage Condition
- But, we can find an *upper- and lower-bound* for the price.

13

---

---

---

---

---

---

---

---

## Example 1: CATs vs. TIGRs with Transactions Costs

- Same as before, suppose there are two zero-coupon bonds, CATs and TIGRs, both paying a face value of \$100 in 1 year.
- Suppose the cost of shorting is \$1 for one bond (of whichever type), and the cost of buying is 0.
- Is arbitrage still profitable?  
Profit we have before without cost: \$ 0.9  
Cost of the arbitrage: \$ 1  
→ not profitable anymore.
- Suppose we did it anyway?

14

---

---

---

---

---

---

---

---

## Example 1: CATs vs. TIGRs with Transactions Costs

- Arbitrage not profitable
- But can prices go *very far* out of sync?
- Suppose today's prices are
  - CATs: \$94.34
  - TIGRs: \$99.00
- Do I make money?
- What is the most the TIGR can be valued more than the CAT?

15

---

---

---

---

---

---

---

---

### Example 1: CATs vs. TIGRs with Transactions Costs

- How about the other way?
- Suppose today's prices are
  - CATs: \$94.34
  - TIGRs: \$90.00
- Do I make money?
- What is the most the TIGR can be valued less than the CAT?
- What if the cost of shorting is \$0.50 and the cost of buying is \$0.50?

16

---

---

---

---

---

---

---

---

### Example 2: Arbitrage Pricing with a Replicating Portfolio

- Suppose the following securities are traded:
  - a zero-coupon bond with face value \$100 that matures 1 year from today costs \$98
  - a zero-coupon bond with face value \$100 that matures 2 years from today costs \$96
  - a zero-coupon bond with face value \$100 that matures 3 years from today costs \$93
- Timeline?

17

---

---

---

---

---

---

---

---

### Example 2: Arbitrage Pricing with a Replicating Portfolio

- Suppose a 3-year coupon bond with face value \$1000 and 10% annual coupon rate is also traded
- Payment timeline?
- Can we construct the coupon bond payment timeline by holding some combination of the zero coupon bonds?

18

---

---

---

---

---

---

---

---

### Example 2: Arbitrage Pricing with a Replicating Portfolio

- Price of the coupon bond is \$1000. Can you create an arbitrage?
  - Again, buy “cheap” and sell “expensive”

Today: net cash flow:  $\$+217$   
1 year later: net cash flow:  $\$0$   
2 years later: net cash flow:  $\$0$   
3 years later: net cash flow:  $\$0$   
→ total profit:  $\$+217$

- Since everyone will do this arbitrage strategy, this will push the price up to its correct price.

19

---

---

---

---

---

---

---

---

---

---

### Example 2: Arbitrage Pricing with a Replicating Portfolio

- Recall: get PV or price of a multiple payment security by breaking it into many single payment securities
- Price of this coupon bond=
  - price of coupon of year 1
  - + price of coupon of year 2
  - + price of coupon and principal of year 3
- price=  $\$98+\$96+\$93*11= \$1,217$

20

---

---

---

---

---

---

---

---

---

---

### Example 3: Arbitrage Pricing with Dynamic Strategies

- Upto now, only bought/sold at time 0
- “Dynamic” means buying and selling on days other than today: at some time in the future

21

---

---

---

---

---

---

---

---

---

---

### Example 3: Arbitrage Pricing with Dynamic Strategies

- Suppose
  - a zero-coupon bond that matures 1 year from today costs \$98
  - 1 year from today, a 1-year zero-coupon bond that matures 2 years from today costs \$96
- Timeline?
- What must be the price of a zero-coupon bond that matures 2 years from now?
- Consider buying 1 year zero-coupon bond two years in a row

22

---

---

---

---

---

---

---

---

### Example 3: Arbitrage Pricing with Dynamic Strategies

- What must be the price of a zero-coupon bond that matures 2 years from now?
- Q1: How many 1-year zeros do you buy at the end of year 1 to get \$100 at year 2? → one: costs \$96
- Q2: how many 1 year zeros do you buy now to get \$96 in year 1? → \_\_\_\_\_

23

---

---

---

---

---

---

---

---

### Example 3: Arbitrage Pricing with Dynamic Strategies

- If you spend \$ \_\_\_\_\_ on this strategy today, you will get \$100 in 2 years. On the other hand, if you buy a 2 year zero-coupon bond, you also will get \$100 in 2 years.
- Law of one price →
- Price of 2-year ZCB today = \$ \_\_\_\_\_

24

---

---

---

---

---

---

---

---

### Example 3: Arbitrage Pricing with Dynamic Strategies

- Price of this 2-year zero is \$95, how do you create an arbitrage?

– Again, buy “cheap” and sell “expensive”

Today: \_\_\_\_\_ net cash flow: \$ \_\_\_\_\_

1 year later: \_\_\_\_\_ net cash  
flow: \$ \_\_\_\_\_

2 years later: \_\_\_\_\_ net cash  
flow: \$ \_\_\_\_\_

→ total profit: \$ \_\_\_\_\_

25

---

---

---

---

---

---

---

---

### Real-World “Arbitrage” Strategies

- Relative mispricing and convergence trades
  - index arbitrage
  - fixed-income securities,  
e.g. on-the-run vs. off-the-run treasuries
- Special situations
  - mergers and acquisitions (“risk arbitrage”)
  - devaluations of currency
  - IPOs
  - announcements  
(e.g. of earnings or macro news)
- NOT ARBITRAGES!!!

26

---

---

---

---

---

---

---

---

### Readings

- Today
  - CP: 13\*
- Next Class:
  - BKM: 12\*, CP: 12\*
- Due by Saturday, by email
  - Excel sheet with all your project data; see  
webpage for details

27

---

---

---

---

---

---

---

---