Last class: Equity valuation

- Need to get an idea of the "fair value" of stock
- How?
  - One way: Discount dividends (DDM)
    - Just like discounting cashflows to find present value
  - But dividends are random (risky)
    - So discount expected values
  - At rf?
    - No, at more than rf. Why?
    - Compensating for risk

Discounting expected dividends

- How much more than rf?
  - Need a rate that increases with risk. Why?
  - Use *k*, the required rate
    - From where? The CAPM, for e.g.
  - Why use *k*?
    - Compare (expected) returns across securities with same risk (beta).
    - Know that a "fairly priced" security will deliver *k*
    - So we wouldn't invest in this security unless it also promised us *k*
    - So if k=16%, then we say, for this level of risk, \$1 today == \$1.16 tomorrow. Therefore \$1 tomorrow == \$1/1.16 today. Work out for other time periods.

Dividend Discount Model (DDM):

$$V_0 = \frac{E(D_1)+E(V_1)}{1+k}$$

$$= \frac{E(D_1)}{1+k} + \frac{E(D_2)+E(V_2)}{(1+k)^2}$$

$$= \frac{E(D_1)}{1+k} + \frac{E(D_2)}{(1+k)^2} + \frac{E(D_3)}{(1+k)^3} + \frac{E(D_4)}{(1+k)^4} + \ldots$$

DDM: Special cases

- Constant expected dividends
  - Perpetuity
- Constant growth of expected dividends
  - The Gordon Growth Model (GGM)
- Two-stage GGM

Valuation by ratios

- Just say
  - " I know what the ratio of price to something (eg dividends) must be. So if I know the value of that something (eg dividends) for the company, I can tell you what the price is."
- Other possible ratios: P/E, P/Book, etc.
- Use per-share values

Third way: use balance sheet data

- Book value of stock
- Liquidation value
- Replacement cost

Fixed-Income Securities

Andre de Souza

Outline

- Main features of bonds
- Yield to maturity and holding period returns
- Forward rates
- Yield curve (or term structure of interest rates)
- Expectations Hypothesis
- Duration

Why “fixed income”?

- Cash flows known in advance, generally
- Very small probability of default

Main Features of Bonds

1. **Issuer:**
   - US Treasury/Government agencies
   - States, municipalities
   - Corporations
   - Foreign governments (sovereign debt)
2. **Term (number of years to maturity):**
   - Short (less than a year)
     - T-bills, CDs, Commercial papers
   - Long (more than a year)
     - T-bonds, corporate bonds
3. **Price vs. par value (= face value)**
   - par bond (issue price=face value)
   - discount bond (issue price < face value)
   - premium bond (issue price > face value)
     - Where do ZCBs fall?

Main Features of Bonds

4. **Coupon**
   - Coupon rate: total annual interest payment per dollar face value
   - Period (usually semi-annual)
   - Quotation
   - Possibly no coupons (zero-coupon bond)
   - Special types:
     - Floaters/ reverse floaters
     - Inflation indexed (TIPS)
5. **Credit risk**
   - Risk free (T-bill)
   - Defaultable (corporate bonds)

Main Features of Bonds

6. **Seniority and security**
   - Senior, subordinated senior, junior...
   - Covenants: Restrictions on additional issues, dividends, and other corporate actions.
7. **Option provisions**
   - Callability: After a certain period, issuer has the right to pay back the loan before it matures.
   - Puttability: After a certain period, bondholder has the right to demand payment of the loan before maturity
   - Convertibility: After a certain period, bondholder has the right to exchange the bond for shares in the issuer

Yield to maturity(YTM)

- Convenient single number to talk about bonds with
- Related to the HPR, but not the HPR
- More like an IRR

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Yield to Maturity (YTM) on Annual-Payment Coupon Bonds

- For an annual-payment coupon bond, the *YTM* is the same as the *IRR*.
- Hence, *YTM* is the rate that solves:

$$\text{price} = \sum_{t=1}^{T} \frac{\text{coupon}}{(1+YTM)^t} + \frac{\text{face value}}{(1+YTM)^T}$$

- Higher YTM ←→ Lower Price
- Special case: zero coupon bond

$$\text{price} = \frac{\text{face value}}{(1+YTM)^T} \Longrightarrow YTM = \left(\frac{\text{face value}}{\text{price}}\right)^{\frac{1}{T}} - 1$$

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Interpreting the YTM

- The rate of return you get when you invest $1 in the bond, providing that
  - you reinvest all the coupons you get at the same rate, and
  - you hold the bond to maturity.
- A rough measure of the average rate of return

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Example 1

Suppose a 3-year bond has a face value of 100 and annual coupon rate is 8%.

- If YTM=8% (same as coupon rate), what is the current price?
- If YTM=6% (lower than coupon rate), what is the price?
- If YTM=10% (higher than coupon rate), what is the price?
- Then what seems to be true about the YTM:
  - for par bonds?
  - for discount bonds?
  - for premium bonds?

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YTMs on semiannual payment coupon bonds

- YTMs are quoted as APRs
- So, first find the "usual" (effective annual) IRR
- Then convert to an APR with semiannual compounding
- How?

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APRs ⇔ EARs

- *EAR = {(1+ (quoted rate* ***/ m*** *))* ***m*** *} – 1*
- *Quoted rate=m*((1+EAR)^(1/m) – 1)*
- Here, m=2
- The EAR in this case is called the *effective annual yield*

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Example 2: Semi-Annual-Pay Coupon Bond

- Suppose that a 2-year bond has a face value of 100 and pays semi-annual coupons and the annual coupon rate is 8%.
- If the YTM is 8%
  - what is the price?
  - what is the *effective annual yield*?
- If the YTM is 6%, what is the price?
- If the YTM is 10%, what is the price?

Summary about YTM and Price

- There is a negative relation between YTM and bond price.
- The relationship is not exactly linear (See convexity later)
- For par bonds (issue price=face value), YTM=coupon rate
- For discount bonds (issue price<face value), YTM>coupon rate
- For premium bonds (issue price>face value), YTM<coupon rate

Realized Return vs. YTM

- Suppose you buy a bond and hold it for a certain period.
- What is the relation between the return on your investment and the *YTM* ?
- The return on your investment is equal to the *YTM* of the bond **if:**
  - (condition 1) you can re-invest the coupons at the same rate (YTM), and
  - (condition 2) you hold the bond until maturity
- The return on your investment is different from the *YTM* if:
  - you re-invest the coupons at a different rate (e.g. interest rate changes after you buy the bond), or
  - you sell the bond before maturity at a price that corresponds to a different yield-to-maturity (e.g. market interest changes.)

Realized Holding Period Return

- Suppose that
  - at time 0, you buy a bond for V*(0)*
  - you collect the coupon payments and re-invest all coupons until date t *(interest rate might change, so you might not be able to re-invest the coupons at the rate same as original YTM)*
  - at time t, you sell the bond at P(t) and get the payment from all the re-invested coupons. Total value is V(t) (No intermediate cashflow)
- Hence, the annual *HPR* is:

$$AHPR = \left(\frac{V(t)}{V(0)}\right)^{1/t} - 1$$

Example 3: Realized Return on Zero-Coupon Bond

- Suppose you bought a 3-year zero-coupon bond with a YTM of 5% and hold it for 1 year.

  (no issue with condition 1, but related to condition 2)
- What is the bond's current price?
- Next year, the YTM changes to 7%
  - What is the price in that year?
  - What is the realized (holding period) return over the one year period?
- What if the YTM in year 1 had remained 5%?
  - What would be the price that year?
  - What would be the realized return over the one year period?
- See Q1 of Problem Set 5 for more examples.

Example 4: Realized Return on an Annual-Pay Coupon Bond

- You buy a 4-year coupon bond, face value 1000, annual coupon rate 8%, and a YTM of 8% and hold till maturity (4 years later)

  (no issue related to condition 2 but problems with condition 1)
- What is the bond's current price?
- Scenario 1: Suppose you reinvest the coupon at 8%
  - What is the bond's future value at year 4?
  - What is your annual holding period return?
- Scenario 2: t=2,3, the coupon payment is re-invested at a yield of 10% (i.e. interest rates increase in year 2 and stay there) (Same questions)
- Scenario 3: t=2,3, the coupon payment is re-invested at a yield of 4% (i.e. interest rates decrease in year 2 and stay there) (Same questions)

Summary about HPR and YTM

- YTM is known at time 0, but HPR is uncertain. It depends on what the interest rate tomorrow turns out to be (or the price of the bond tomorrow).
- HPR=YTM if both condition 1 and condition 2 are satisfied.
- For zero-coupon bonds (no issue about reinvesting coupon), HPR=YTM if bond is held to maturity or YTM doesn't change.
- For zero-coupon bonds, if you sell before maturity and the YTM changes, then HPR is not equal to YTM.
- For coupon bonds, even if you hold to maturity, whether HPR is equal to YTM still depends on whether you can reinvest the coupon at YTM. If coupon is reinvested at lower rate than YTM, then HPR<YTM. If coupon is reinvested at higher rate than YTM, then HPR>YTM.

Yield Curve and Forward Rates

The yield curve

- A plot of ZCB yields of all maturities against maturities
- Shapes :
  - upward sloping (most typical)
  - downward sloping
  - flat
  - hump shaped
- Other names
  - the term structure of zero-coupon bond yields
  - the term structure of (spot) interest rates

Forward Rates

- A *forward rate* is an interest rate on a loan in the future that is fixed today.
- A firm foresees the need for a short-term loan one year from now but is worried about rising interest rates. Can they "lock in" a borrowing rate for a one-year loan *starting one year from now*?
- A company will receive cash next year and must make a payment two years from now. The company is worried about the re-investment risk related to the incoming payment. Can the company lock in a lending rate *starting one year from now?*
- The forward rate *f(t)* for 1-year lending/ borrowing starting *t* years from now

Example

- My rich aunt dies and leaves me $100, to be paid to me one year from now. Since I graduate in two years, I'd like to be able to invest this $100 somewhere so that when I graduate, I'll have the $100 plus interest to play around with
- I'm worried about interest rates falling in the next year
- What do I do?

Engineering Forward Rates

- Suppose that
  - a 1-year zero with face value 100 has a YTM of 6%
  - a 2-year zero with face value 100 has a YTM of 7%
- What are the prices of these bonds?
- How can you trade these bonds to replicate a loan between year 1 and year 2?

  (You create a "synthetic" loan.)
- What is the interest rate on that loan?
- This interest rate is 1-year forward rate, f(1).

Forward rates

- Now suppose those 1 year and 2 year ZCBs are trading
- Suppose someone offers you a 1-year loan starting one year from now
  - How much must the interest rate on that loan be?
- Must be that your return in both cases is the same
  - Else?

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Forward rates

- Or,

$$(1+r_{0,1})(1+f(1))=(1+r_{0,2})^2$$

- More generally,

$$(1+r_{0,n})^n(1+f(n))=(1+r_{0,n+1})^{n+1}$$

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Engineering Forward Rates

- In words:
- The return from investing in an n-period bond and the 1 year forward rate must be the same as investing in an n+1 period bond.

- Suppose I give you prices. How do you find forward rates?

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Shapes of the yield curve

Theories

- Why is the curve generally upward sloping?
  - But is sometimes downward sloping?
- What does upward sloping mean anyway?
- The expectations hypothesis
- The liquidity-preference theory

The Expectations Hypothesis

- Suppose we have two zero coupon bonds:
  - 1 year YTM 5%
  - 2 years YTM 8%
- Suppose I want to invest for two years
- Can do two things
  - Invest in the 2 year ZCB, or
  - Invest in the 1 year bond, wait 1 year, and invest at the prevailing 1 year rate then
- The EH says these two should *on average* give you the same return

$$(1+r_{0,1})(1+E(r_{1,1})) = (1+r_{0,2})^2$$

Implications of the EH

- The expected holding period return on all bonds is the same
- The long term rate is a geometric average of current short rates and expected future short rates.
- The expected future 1-year interest rate is equal to the forward rate, compare :

$$(1+r_{0,1})(1+E(r_{1,1}))=(1+r_{0,2})^2$$

$$(1+r_{0,1})(1+f(1))=(1+r_{0,2})^2$$

- What does an upward-sloping term structure imply about the expected future short term interest rate?

Problems with the EH strategies

- Predicts an upward sloping term structure just as often as a downward sloping one
  - But TS is almost always upward sloping
- Problem with my argument:
  - In one case (which case?), can't take money out after one year, at least, not without risk
  - People might want extra compensation for having their money locked up

Liquidity Preference Theory

- Buyers of long-term bonds want to be compensated
  - for "tying up" money for a long time
  - for having a price risk if they need to sell before maturity
- Issuers of bonds are willing to pay a higher interest rate on long-term bonds because
  - they can lock in an interest rate for many years
- The associated risk premium is denoted the liquidity premium
- Based on this theory, what is the typical shape of the term-structure?

  ➔ upward sloping

Interest Rate Sensitivity and Duration

Relation between bond price and interest rates (yield)

- *First order effect:* Bond prices and yields are negatively related
- *Convexity:* An increase in a bond's YTM results in a smaller price decline than the price gain associated with a decrease of equal magnitude in the YTM.
- *Maturity also matters:* Prices of long-term bonds are more sensitive to interest-rate changes than short-term bonds

How much do bond prices react to interest rate changes?

- We'd like a measure of the sensitivity of the bond price to interest rates (yields)
- Natural to think of taking derivatives
- "Duration"

Duration

- The *duration* (D) of a bond is defined as minus the **elasticity** of its price (P) with respect to (1 plus) its yield (y):

$$D = -\frac{dP/P}{dy/(1+y)}$$

- And if you actually do the differentiation, you find

$$= \sum_{1}^{T} w_t \, t, \quad \text{where} \quad w_t = \frac{Cashflow(t)}{(1+y)^t * P}$$

- Weights sum to 1. Why?
- Since duration is "weighted times", it's measured in years
- Natural to think of it as weighted-average-maturity: longer the life, more the sensitivity

Duration

- The relative price-response to a yield change is therefore:

$$\frac{\Delta P}{P} \cong -\underbrace{\frac{D}{1+y}}_{\substack{\text{modified}\\ \text{duration}}} \Delta y$$

Example 5: Duration of a Coupon Bond

- What is the duration of a 4-year coupon bond with an annual coupon rate of 8% and a YTM of 10% ?
  - What is the largest this duration could be?
- What if the coupon rate is 4% instead of 8%?
- If the YTM changes to 10.1%, what would be the (relative) change in price ?
- If the YTM changes to 11%, what would be the (relative) change in price ?

Duration Facts

- Duration can be used to capture the sensitivity of bond prices changes in response to interest rate changes.
- What is the duration of a zero-coupon bond?
- How does the duration of a coupon bond compare with its maturity?
- What happens to the duration of a coupon bond if (all else equal)
  - the maturity increases?
  - the coupon rate increases?
  - the yield increases?

Use of Duration: Interest-Rate Management

- Investors and financial institutions are subject to interest-rate risk, for instance,
  - homeowner: mortgage payments
  - bank: short-term deposits and long-term loans
  - pension fund: owns bonds and must pay retirees
- A change in the interest rate results in:
  1. price risk
  2. re-investment risk
- Want to construct a portfolio which is insensitive to interest-rate changes (immunization).