1. Teaching evaluation available online today onwards: https://ais.stern.nyu.edu/ Go to the class webpage for instructions if you have trouble
2. Final exam next Thursday
3. Final is cumulative, but focuses on post mid-term
4. Project is due next Thursday (no late submission)

Plans for next 2 classes:

Today (Thursday):
1. Finish fixed income securities,
2. Start options

Tuesday:
1. finish options
2. final review, discuss problem sets and sample final

Last class:

- Started by defining the YTM
  - IRR of the bond: makes PV (coupons and face value) = price
  - YTM on a par bond is ___ the coupon rate
    - Discount bond
    - Premium bond

Realized return and YTM

- What is the realized return on a bond?
- Just because a bond has a YTM of 15% doesn't mean you'll make 15% holding it and reinvesting the coupon
- This is true if:
  - (condition 1) you can re-invest the coupons at the same rate (15%), and
  - (condition 2) you hold the bond until maturity, or the YTM at the time you sell the bond is still 15%
- If the rate at which you reinvest is > than 15% then...
- If the YTM at the time you sell the bond is > then 15% then..

Some terminology

- Short rate: the yield of the shortest maturity zero-coupon bond we're considering= annualized return on this bond if held to maturity, eg, $r_{0,1}$
- Long rate: the yield of a longer maturity zero coupon bond= annualized return on this bond if held to maturity, eg, $r_{0,2}$
- Future short rate: the short rate next period, eg, $r_{1,2}$

The yield curve and forward rates

- Yield curve: a plot of zero coupon yields for all maturities
- Forward rate: the interest rate on a loan/deposit to be made some time in the future, which is fixed today
- The 1 period ahead 1 period forward rate is f(1)

Forward rate determination

- Given a 1 year and a 2 year zero coupon bond, the forward rate one year ahead is given by:

$$(1+r_{0,1})(1+f(1))=(1+r_{0,2})^2$$

- Why? Arbitrage. You compare investing in one side versus investing in the other

Shapes of the yield curve

Theories

- Why is the curve generally upward sloping?
  - But is sometimes downward sloping?
- What does upward sloping mean anyway?
- We look at two explanations:
  - The expectations hypothesis
  - The liquidity-preference theory

The expectations hypothesis

- Assumption: Investors care only about expected returns (and not about ... ?)
- Suppose you have two zeros
  - 1 year, YTM 5%
  - 2 year, YTM 6%
- Suppose I want to invest for two years
- Can do two things
  - Invest in the 2 year ZCB, or
  - Invest in the 1 year bond, wait 1 year, and invest at the prevailing 1 year rate then
- Suppose everyone expects the one year rate one year from now to be 8%
- What are the (expected) payoffs from your two choices?
- Which way do you choose to invest?
- This is not an equilibrium

The expectations hypothesis

- Now suppose everyone expects the one year rate one year from now to be 4%.
- Now what are the two payoffs?
- Now in which way do you invest?
- This isn't an equilibrium either...
- What is the only possible choice for x that reflects equilibrium?
  - We're comparing $(1.05)(1+x)$ and $(1.06)^2$
- In other words ...

The expectations hypothesis

- That was for a two year investor
- Now think about a one year investor
- What are his choices?
  - Invest in the 1 year ZCB, make 1.05, or
  - Invest in the 2 year bond, wait 1 year, and sell it.
- For equilibrium, must have the implied (expected) 1 year rate on the 2 year bond be 5%
- What is the expected rate on the 1 year bond 1 year ahead that would mean that the current 1 year rate is 5%?
  - Make a wild guess

The expectations hypothesis (One result)

$$(1+r_{0,1})(1+E(r_{1,1}))=(1+r_{0,2})^2$$

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Implications of the EH

- The expected holding period return on all bonds is the same.
- The long term rate is a geometric average of current short rates and expected future short rates.
- The expected future 1-year interest rate is equal to the forward rate, compare :

$$(1+r_{0,1})(1+E(r_{1,1}))=(1+r_{0,2})^2$$

$$(1+r_{0,1})(1+f(1))=(1+r_{0,2})^2$$

- What does an upward-sloping term structure imply about the expected future short term interest rate?

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Problems with the EH strategies

- Predicts an upward sloping term structure just as often as a downward sloping one
  - But TS is almost always upward sloping
- Problem with my argument:
  - In one case (which case?), can't take money out after one year, at least, not without risk
  - People might want extra compensation for having their money locked up

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Liquidity Preference Theory

- Buyers of long-term bonds want to be compensated
  - for “tying up” money for a long time
  - for having a price risk if they need to sell before maturity
- Issuers of bonds are willing to pay a higher interest rate on long-term bonds because
  - they can lock in an interest rate for many years
- The associated risk premium is denoted the liquidity premium
- Based on this theory, what is the typical shape of the term-structure?

➔ upward sloping

Interest Rate Sensitivity and Duration

Some Observed Relations

- *First order effect:* Bond prices and interest rates are negatively related
- *Convexity:* An increase in the interest rate results in a smaller price decline than the price gain associated with a decrease of equal magnitude in the interest rate.
- *Maturity also matters:* Prices of long-term bonds are more sensitive to interest-rate changes than short-term bonds

How much do bond prices react to interest rate changes?

- We'd like a measure of the sensitivity of the bond price to interest rates
- Natural to think of taking derivatives
- What do we take derivatives with respect to?
  - Should be interest rates
  - But we don't have an expression linking bond prices and *interest rates*
  - But we do have one linking bond prices and *yields*
  - So let's differentiate with respect to yields.
- "Duration"

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Duration

- The *duration* (*D*) of a bond is defined as minus the **elasticity** of its price (*P*) with respect to (1 plus) its yield (*y*):

$$D = -\frac{dP/P}{dy/(1+y)}$$

- And if you actually do the differentiation, you find

$$= \sum_{1}^{T} w_t\, t\,, \quad \text{where} \quad w_t = \frac{Cashflow(t)}{(1+y)^t * P}$$

- Weights sum to 1. Why?
- Since duration is "weighted times", it's measured in years
- Natural to think of it as weighted-average-maturity: longer the life, more the sensitivity

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Duration

- The relative price-response to a yield change is therefore:

$$\frac{\Delta P}{P} \cong -\underbrace{\frac{D}{1+y}}_{\text{modified duration}} \Delta y$$

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Example 5: Duration of a Coupon Bond

- What is the duration of a 4-year coupon bond with an annual coupon rate of 8% and a YTM of 10% ?
  - What is the largest this duration could be?
- What if the coupon rate is 4% instead of 8%?
- If the YTM changes to 10.1%, what would be the (relative) change in price ?
- If the YTM changes to 11%, what would be the (relative) change in price ?

Duration Facts

- Duration can be used to capture the sensitivity of bond prices changes in response to interest rate changes.
- What is the duration of a zero-coupon bond?
- How does the duration of a coupon bond compare with its maturity?
- What happens to the duration of a coupon bond if (all else equal)
  - the maturity increases?
  - the coupon rate increases?
  - the yield increases?

Use of Duration: Interest-Rate Management

- Investors and financial institutions are subject to interest-rate risk, for instance,
  - homeowner: mortgage payments
  - bank: short-term deposits and long-term loans
  - pension fund: owns bonds and must pay retirees
- A change in the interest rate results in:
  1. price risk
  2. re-investment risk
- Want to construct a portfolio which is insensitive to interest-rate changes (immunization).

Options

Andre de Souza

Outline

- Option basics and option strategies
- No-arbitrage bounds on option prices
- Option pricing
  - Binomial tree pricing
  - Black-Scholes-Merton option pricing model

Part 1: Option basics and Option strategies

Derivatives

- Definition: A *derivative* is a financial security whose payoff depends directly on some observable value
  - This value is generally the price of some other asset
  - In this case, the other asset is called the *underlying*
- Examples:
  - Forwards, futures, options, swaps
- Examples of the underlying:
  - individual stocks, stock indices, foreign currencies, commodities (pork bellies, metal, sugar, coffee)
- Derivatives are used for
  - Risk management
  - Executive compensation
  - Speculation

Our focus

- We will focus on options
- For stock options, stock is the underlying security.

Options Basics

- Two main types
  - *call* options (or calls): the **right (but not the obligation)** to buy the underlying security at a predetermined exercise price (strike price)
  - *put* options (or puts): the **right (but not the obligation)** to sell the underlying security at a predetermined exercise price (strike price)
- Exercise time
  - European option: can only exercise at expiration

Options Basics: Call example

- Today the price of IBM is $70 ($S_0$)
- Consider an IBM *European call* option with 1 year to maturity, with a strike price of $80 (X)
- Price of the call option (C_0) is $3

- If A pays $3 now to buy the option from B, 1 year later A can , if he wants to, buy 1 share of IBM at $80 from B, but he doesn't have to.
- In this case, A *buys* a call, and B *writes or sells or goes short* a call
- Notice: if A decides to exercise the call option, B has the obligation to sell.

Options Basics: Call example

- If the price of IBM 1 year later is $65, will A exercise the option? What is his payoff 1 year later? What is his net profit? What is B's payoff and net profit?
- How about if it's $95?
- $81?
- Payoff table
- Profit table

Options Basics: Rule for call exercise

- Exercise if $S_T > X$
  - S_T: price of stock on expiration date
  - X: exercise price
- Payoff diagram
- Profit diagram
- When do you buy a call?
- When do you write a call?

Options Basics: Put example

- Today the price of IBM is $70 ($S_0$)
- Consider an IBM *European put* option with 1 year to maturity, with a strike price of $80 (X)
- Price of the put option (C_0) is $9
- If A pays $9 now to buy the option from B, 1 year later A can , if he wants to, sell 1 share of IBM at $80 from B, but he doesn't have to.
- In this case, A *buys* a put, and B *writes or sells or goes short* a put
- Notice: if A decides to exercise the call option, B has the obligation to buy

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Options Basics: Put example

- If the price of IBM 1 year later is $95, will A exercise the option? What is his payoff 1 year later? What is his net profit? What is B's payoff and net profit?
- How about if it's $65?
- $79?
- Payoff table
- Profit table

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Options Basics: Rule for put exercise

- Exercise if $S_T<X$
  - S_T: price of stock on expiration date
  - X: exercise price
- Payoff diagram
- Profit diagram
- When do you buy a put?
- When do you write a put?

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Moneyness of an option

- The buyer asks the question: If I were to exercise *now*, would my payoff be positive?
- Yes, it will: In the money (ITM)
  - Strike < current price for calls
  - Strike > current price for puts
- Payoff will be negative: Out of the money (OTM)
- Zero Payoff: At the money (ATM)
  - strike=current price

Options Basics Summary

1. **Key Option Terms**:

✓ Underlying security
✓ Strike price X
✓ Expiration (or Maturity)
✓ European vs. American
✓ Price of option or Cost of option (C_0 or P_0)

2 . Moneyness

- At The Money (ATM) option
- In The Money (ITM) option:
- Out of The Money (OTM) option

Options Basics Summary

3. Buyers and Sellers of options have different rights and obligations, and different payoff structures

	Buyer	Seller or Writer
Call Option	right to buy	obligation to sell, if buyer exercises
Put Option	right to sell	obligation to buy, if buyer exercises

Q: What do the writer of a call and the buyer of a put have in common?

4. General rule for exercise at expiration:

Call: When $S_T>X$

Put: When $S_T<X$

How do you use options: option strategies

- Use call options for leverage
- Protective puts
- Straddles

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Using call options for leverage

- Example
  - MSFT is now S_0=$70
  - A call option with X=$70 and 6-month maturity costs C_0=$10
- Suppose you have $7000. You think MSFT will go up in 6 months.
- Consider two strategies
  - Buy ____ shares of Microsoft
  - Buy ____ call options
- Six months later, Microsoft price rises to $85. What is the HPR on both strategies?
- What if Microsoft price rises to $100?
- What if Microsoft price drops to $60?

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Using call options for leverage

Share price	60	65	70	75	80	85	90
Payoff on A:	6000	6500	7000	7500	8000	8500	9000
Payoff on B:	0	0	0	3500	7000	10500	14000
Return on A:	-14.29%	-7.10%	0%	7.10%	14.30%	21.40%	28.60%
Return on B:	-100%	-100%	-100%	-50%	0%	50%	100%

Point: If I think that stock prices will rise, I can make much larger profits by buying calls instead of buying stock. But of course I can make much larger losses.

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Protective Put

- You own a share in Microsoft, bought at S_0=$70
- You are afraid that the stock price will drop.
- How do you limit your possible losses by trading options?

➔Long a put.

- How can a put protect you?
  - Payoff and profit diagrams at expiration.
- Why not always do this?
- What does the payoff to this strategy remind you of?

Straddle

- You have private information that a particular stock's price will change dramatically soon, but you do not know if it will go up or down.
- Which option strategy could you use to profit from this information?
- Straddle: buying one call and one put
  - Same expiration
  - Same exercise price
- Why straddle? Check the payoff diagram
- What does opposite side bet on?

Summary of Option Strategies

- Long Call: bet on price increase (profit can be unlimited)
- Long Put: bet on price decrease (profit is limited, since price can't go negative)
- Protective Put: long stock + long put (protect against the price decrease)
- Long Straddle: long call + long put (bet on price change dramatically or volatility increase)
- Short Straddle: short call + short put (bet on price not change much)

Part 2: No-Arbitrage Bounds on Option Prices

- Intrinsic value and time value
- Lower bounds on call option prices
- Put-Call Parity

Intrinsic Value of a Call Option

- The **intrinsic value** is:
  - the value you get if you exercise *now*
    (recall: American option can be exercised at any time before or at maturity)
- Assume current price is S_0, Strike price is X
- **Intrinsic value** for ATM call option (S_0__X) is ______
- **Intrinsic value** for OTM call option (S_0__X) is ______
- **Intrinsic value** for ITM call option (S_0__X) is ______

➔ What is the intrinsic value of a call option now?

Intrinsic Value of a Call Option

- Which is greater:
  - the intrinsic value or the option price C_0 ?

    ____

➔ $C_0 \geq \max(0, S_0 - X)$

Why? (otherwise, arbitrage)

- The difference between the option price and the intrinsic value is called the **time value** of the option. (Note: this is not related to time value of money.)

A tighter lower bound: the adjusted intrinsic value

- Think of two portfolios
  - Buying a call
  - Buying a stock of S_0 going short (borrowing) the PV of X to be repaid at T (expiration of the option)
- At T, how much is the call worth?
  - If $S_T > X$, then ...
- How much is the second portfolio worth?
- #1 always delivers at least as much value as #2, so it must have a higher price than #2.
- $C_0 \geq \max(0, S_0 - PV(X))$

Put-call Parity

- Think of two portfolios
  - Long a call, short a put
  - Holding the stock, and borrowing PV(X) for repayment at T
- At expiration:
  - If $S_T > X$ then ...
  - If $S_T < X$ then ...
- Conclusion: both portfolios are worth the same at maturity, no matter what happens.
- Therefore must have the same price.

Put call parity

- Nice thing: can get equivalence relationships
- Can say “Long call, short put is equal to buying stock on margin”
- Or “A long call is the same as a long put plus buying stock on margin”
- And so on.