Announcements

- Extra Office Hours: Wed 9:30-11:30 and 12:30-2:30. Regular office hours today
- Don't forget the teaching evaluation, otherwise Stern will give you your grade late.
  https://ais.stern.nyu.edu/
  or see the class web-page for details
  http://www.stern.nyu.edu/~adesouza

Project: due on Thursday *before* exam (no late submission)

Final Exam:
- 40% of final grade
- Comprehensive, but with a huge focus on post-midterm material
- 2 hours long
- 2 pages of notes, with possibly both sides written on
- No make up exam!

Last class

- Defined calls and puts
- Drew payoff and profit diagrams
- Talked about strategies
  - Call options for leverage
  - Protective puts
  - Straddles
- What does someone who buys a call think about the stock price? Sells a call?

Part 2: No-Arbitrage Bounds on Option Prices

- Intrinsic value and time value
- Lower bounds on call option prices
- Put-Call Parity

Intrinsic Value of a Call Option

- The value you get if you exercise *now*
  (recall: American options can be exercised at any time before or at maturity)
- Assume current price is S_0, Strike price is X
- **Intrinsic value** for ATM call option (S_0__X) is ______
- **Intrinsic value** for OTM call option (S_0__X) is ______
- **Intrinsic value** for ITM call option (S_0__X) is ______

➔ What is the intrinsic value of a call option now?

____

Intrinsic Value of a Call Option

- Which is greater: the intrinsic value or the option price C_0 ?

➔ $C_0 \geq \max(0, S_0 - X)$

Why? Heuristic argument:

- Suppose intrinsic value=0. How much are you willing to pay for this call?
- Suppose intrinsic value>0. How much are you willing to pay for this call?

- The difference between the option price and the intrinsic value is called the **time value** of the option. (Note: this is not related to time value of money.)

A tighter lower bound: the adjusted intrinsic value

- Think of two portfolios
  - Buying a call
  - Buying a stock of S_0, and going short (borrowing) the PV of X to be repaid at T (expiration of the option), if this is positive, otherwise doing nothing.
    - i.e., need to put up $\max(0, S_0 - PV(X))$ of your own money
- At T, how much is the call worth?
  - If S_T>X, then ...
- How much is the second portfolio worth?
- #1 always delivers at least as much value as #2, so it must have a higher price than #2.
- $C_0 \geq \max(0, S_0 - PV(X))$
- Why? By arbitrage

Put-call Parity

- Think of two portfolios
  - Long a call, short a put
  - Holding the stock, and borrowing PV(X) for repayment at T
- At expiration:
  - If S_T>X then ...
  - If S_T<X then ...
- Conclusion: both portfolios are worth the same at maturity, no matter what happens.
- Therefore must have the same price.
- What is this PV(X)?

Put-call Parity

- Nice thing: can get equivalence relationships
  - Can say “Long call, short put is equal to buying stock on margin”
  - Or “A long call is the same as a long put plus buying stock on margin”
- Arbitrage opportunities:
  - Suppose $C_0 - P_0 > S_0 - Xe^{-rT}$
  - What do you do?
  - Suppose $C_0 - P_0 < S_0 - Xe^{-rT}$

Part 3: Option Pricing

Option pricing

- Thus far we've only spoken about options at expiration
  - Except for put-call parity, which gave us a link between call and put prices
- Now we find out how to value options before expiration
  - Binomial pricing
  - Black-Scholes model

Binomial Option Pricing: Two-state option pricing

Step 1: Assume that the stock price at expiration can have 2 possible values (2 scenarios). Compute the option payoff in each scenario

Step 2: Create a portfolio of the stock and a bond (risk-free loan/deposit) that has the same payoff as the option in both cases

Step 3: Find the price of this *replicating portfolio*. This will be the option price (again, because of no arbitrage) !

Binomial Option Pricing Example

Suppose stock price is 100 today and can either be $200 or $50 1 year later. Consider a call option with an exercise price of X=125 with 1 year maturity. Assume the risk-free rate is 8% and compounding is annual.

Step 1: What is the call option payoff 1 year later in each scenario?

Step 2: Can we create a replicating portfolio? Suppose we buy X shares and Y units of the zero (risk free asset, Face Value $100).➔

Scenario 1: 200*X+100*Y=______

Scenario 2: 50*X+100*Y =______

➔ X=________, Y=_________

Step 3: price of call option = price of portfolio=X*___+Y*_____ =______

Q: what is the price of a put option with X=125?

Another way

- Instead of replicating payoffs, we can form a risk-free asset.
- How?
  - Hold 1 share, short 2 calls
  - What is the payoff to this asset?
  - What is the present value of this payoff?

Hedge Ratio or "Delta"

- The number of shares per unit of call in the replicating portfolio (also called *hedge portfolio*) is called the hedge ratio or delta, Δ
  - Also the number of shares to hold per unit of call shorted in forming a riskless portfolio
- The hedge ratio tells you how much the option price changes per dollar change in the stock price:

$$\Delta = \frac{C^{+} - C^{-}}{S^{+} - S^{-}}$$

- Where C+ is the Call price in upper state, S+ is the stock price in upper state, C- is the Call price in lower state, S- is the stock price in lower state.

Binomial Option Pricing: Option Pricing in a "Tree"

We considered a single period; the stock price can only go up or down. More realistic:

Step 1: Assume that the stock price evolves over time in a "tree": Every sub-period the stock price can go up or down. (Therefore, many scenarios at expiration.) Compute the option payoff at expiration in each scenario

Step 2: *Replicate* the option payoff with a *dynamic hedging* strategy using stocks and risk-free securities

Step 3: Compute the initial price of the replicating strategy. This is the option price (again, because of no arbitrage)

Example of Trees

- S=100, X=125 for a call
- Stock price can go up to 200, down to 50 next period
- In the period after next, it can double or halve
- Draw tree
- Price option, starting from the next-to-last period

Trees

- More flexible than single period
  - Many possible scenarios
  - Can incorporate early exercise
  - Can incorporate changing interest rates, etc
- This is dynamic replication
  - At each node, the portfolio is *rebalanced*, ie, money is moved between stocks and bonds, but *no money is put in from outside*.
  - That's why we can price the option in this way

Black-Scholes-Merton Formula

- Assumptions
- "The" formula
- Intuition
- Determinants

Assumptions

1. The risk-free interest rate is:
   - constant
   - continuously compounded
2. The stock price
   - is log-normally distributed
   - has no jumps
   - has constant volatility
3. The stock and risk-free security *can be traded all the time at no cost*

Black-Scholes-Merton Formula

- Binomial model
  - More and more scenarios,
  - Smaller and smaller up and down jumps
- In the limit, the price of a European call option on a non-dividend-paying stock is

$$C_0 = S_0 N(d_1) - Xe^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln(S_0 / X) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Determinants of Call Price

- Stock price (today), S_0 (+)
- Exercise price, X (-)
- Volatility of stock, σ (+)
- Time to expiration, T (+)
- Interest rate, r (+)

Determinants of Put Price

- Stock price (today), S_0 (-)
- Exercise price, X (+)
- Volatility of stock, σ (+)
- Time to expiration, T (?)
- Interest rate, r (-)

Intuition for BSM

- If the option is almost certain to be in-the-money at maturity, then
  - $N(d_1) \cong N(d_2) \cong 1$, and
  - the option price is adjusted intrinsic value, $S_0 - X e^{-rT}$
  - Under what circumstances does this happen?
- If the option is almost certain to be out-of-the-money at maturity, then
  - $N(d_1) \cong N(d_2) \cong 0$, and
  - the option price is close to 0
- In general, the price is the risk-adjusted expected payment at maturity

Final Exam Review

Andre de Souza

Final exam review

- Arbitrage
- Equity Valuation
- Bonds and Bond Valuation
- Options

Arbitrage

- Construct replicating portfolios: that is, portfolios with the same payoffs
- Then say, these must have the same price
- Why?

No Arbitrage Condition

1. If two securities have the same payoffs, they must have the same price: *Law of One Price.* (see CATs vs. TIGRs example)
   (have a deep understanding about how trading costs determine the price range)
2. If a portfolio has the same payoff as a security, the price of the security must be equal to the price of the portfolio (see class example and Sun and Rain example in homework)
3. If a trading strategy has the same payoff as a security, the price of the security must be equal to the cost of the strategy (see example 3 in class), or options pricing (binomial model)

Example 3: Arbitrage Pricing with Dynamic Strategy

- Suppose
  - a zero-coupon bond that matures 1 year from today costs $98
  - 1 year from today, a zero-coupon bond that matures 2 years from today also costs $96
- Time line!!!
- What must be the price of a zero-coupon bond that matures 2 years from now? (consider a dynamic strategy: buying 1 year zero-coupon bond two years in a row)

Example 3: Arbitrage Pricing with Dynamic Strategy

- What must be the price of a zero-coupon bond that matures 2 years from now?

Q1: how many 1 year zero to buy in year 1 to get $100 in year 2? ➔ **1** unit, costs **$96**

Q2: how many 1 year zero to buy now to get $96 in year 1? ➔ **0.96** unit, costs **$94.08**

➔ If you spend **$94.08** on this dynamic strategy, you will get $100 in 2 years. Also if you buy a 2 year zero-coupon bond, you also will get $100 in 2 years.

Law of one price ➔ price= **$94.08 = $96*$98/100**

Arbitrage Pricing

- Now suppose given a 2 year ZCB, priced at 96, and a one-year ZCB priced at 94
- What is the price of a bond that pays $100 at the end of year one and $100 at the end of year two?
  - Can value any arbitrary cashflow combination

Equity Valuation

- Balance-sheet data
- Dividend discount models and the intrinsic value of a share
- Ratios

Review of Equity Valuation

- Gordon's Growth Model (GGM) (suppose that expected dividends grow at a rate *g)*

$$P_0 = \frac{D_0(1+g)}{E(R)-g} = \frac{E_0(1-b)(1+g)}{E(R)-g}$$
$$= \frac{E(D_1)}{E(R)-g} = \frac{E(E_1)(1-b)}{E(R)-g}$$

- Pay attention to the timing of the dividend (current D_0 or next year D_1)

Review of Equity Valuation

- Two-Stage DDM (assume expected dividends grow at a constant rate g after year t)

  - stage 1: use GGM to get the price at year t.

  - stage 2: use regular DDM formula to discount dividends before and at year t and the future price at year t to get the current value (or price)

  (see class example 3)

Fixed Income: Key Concepts

- YTM
- AHPR, and its relationship with YTM and the reinvestment rate
- Yield curves, forward rates, and arbitrage
- The Expectations hypothesis
- Duration

Fixed Income Securities: Relation between price and YTM

- For an annual-payment coupon bond

$$\text{price} = \sum_{t=1}^{T} \frac{\text{coupon}}{(1+YTM)^t} + \frac{\text{face value}}{(1+YTM)^T}$$

We can treat the coupon payment as an annuity ➔

$$\text{price} = \frac{Coupon * (1 - \frac{1}{(1+YTM)^T})}{YTM} + \frac{\text{face value}}{(1+YTM)^T}$$

- Special case: zero coupon bond

$$\text{price} = \frac{\text{face value}}{(1+YTM)^T} \implies YTM = (\frac{\text{face value}}{\text{price}})^{\frac{1}{T}} - 1$$

- Notice: here T is number of years till maturity

Fixed Income Securities: Relation between price and YTM

- For a semi-annual-pay coupon bond

$$\text{price} = \sum_{n=1}^{N} \frac{\text{coupon(per payment period)}}{(1+r)^n} + \frac{\text{face value}}{(1+r)^N}$$

where r=YTM/2

We can treat the coupon payment as an annuity

$$\text{price} = \frac{Coupon\ (per\_period) * (1 - \frac{1}{(1+r)^N})}{r} + \frac{\text{face value}}{(1+r)^N}$$

Notice: here N is the number of *periods* till maturity (=2*T)

Summary about YTM and Price

- There is an inverse relation between YTM and bond price.
- The relationship is not exactly linear (convexity to talk later)
- For par bonds (issue price=face value), YTM=coupon rate
- For discount bonds (issue price<face value), YTM>coupon rate
- For premium bonds (issue price>face value), YTM<coupon rate

Summary about AHPR and YTM

- AHPR=YTM if both condition 1 and condition 2 are satisfied.
- For a zero-coupon bond (no issue about reinvesting coupon), AHPR=YTM if bond is held to maturity or YTM doesn't change.
- For a zero-coupon bond, if you sell before maturity and the YTM changes, then the AHPR is not equal to the YTM.
- For a coupon bond, even if you hold to maturity, whether the AHPR is equal to the YTM still depends on whether you can reinvest the coupon at the YTM. If coupons are reinvested at a lower rate than the YTM, then AHPR<YTM. If coupon is reinvested at higher rate than the YTM, then AHPR>YTM.

Yield Curve

- What determines their shape?
  - Expectations Hypothesis
    - Investors only care about expected returns
    - Upward sloping <==> 1 year rate (short term rate) expected to increase in future
    - Downward sloping <==> 1 year rate (short term rate) expected to decrease in future
    - Flat <==> 1 year rate (short term rate) remains unchanged in future
  - Liquidity Preference
    - Risk premium on long term rates ➔ Upward sloping

Forward Rates

- A *forward rate* is an interest rate on a future loan that is fixed today.
- The forward rate for 1-year lending starting *t* years from now is denoted ***f(t).***
- The forward rate is determined by arbitrage to be:

$$(1+r_{0,n})^n(1+f(n))=(1+r_{0,n+1})^{n+1}$$

- Under the Expectations Hypothesis, the expected future interest rate=forward rate

  E(r(t))=f(t)

Example: Forward Rates

- Suppose that face values are $100 and
  - 2 year zero coupon bonds sell for 95.238
  - 3 year zero coupon bonds sell for 89
- What is the forward rate f(2)?
- According to the Expectations Hypothesis, what is the expected 1 year return starting at the end of year 2?

Duration and Interest Rate Sensitivity

- The *duration* (*D*) of a bond is defined as minus the **elasticity** of its price (*P*) with respect to (1 plus) its yield (*y*)
- When we take derivatives we find:

$$D=\sum_{1}^{T} w_t\, t\,, \quad \text{where} \quad w_t=\frac{Cashflow(t)}{(1+y)^t * P}$$

- Weights sum to 1
- The relative price-response to a yield change is therefore: $\frac{\Delta P}{P}\cong -\underbrace{\frac{D}{1+y}}_{\text{modified duration}}\Delta y$

Duration Facts

- Duration captures the sensitivity of bond prices to interest rate changes.
- What is the duration of a zero-coupon bond?
- How does the duration of a coupon bond compare with its maturity?
- What happens to the duration of a coupon bond if (all else equal) the coupon rate increases?

Course Project: what to report

1. Answers to the 5 questions
2. Regression output

If using LINEST, the 6 cells you highlight as follows:

0.947401703	0.001794758
0.028095339	0.001293278
0.910335648	0.013766474

If using “data analysis”, report the output as follows:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.954115112
R Square	0.910335648
Adjusted R Square	0.909535073
Standard Error	0.013766474
Observations	114

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.215498963	0.215498963	1137.102875	1.75023E-60
Residual	112	0.02122577	0.000189516		
Total	113	0.236724732			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.0017948	0.001	1.388	0.168	-0.001	0.004	-0.001	0.004
X Variable 1	0.9474017	0.028	33.721	0.000	0.892	1.003	0.892	1.003

beta

Jensen’s measure

That’s it.