Understanding the Expectations Hypothesis

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What explains the difference between yields on short-term bonds and yields on long-term bonds? The most well-known equilibrium theory of the yield curve is the expectations hypothesis.

The expectations hypothesis states that, in equilibrium, long-term yields are geometric averages of current and expected future short-term yields. An equivalent statement is that the expected holding period return on short-term bonds equals the expected holding period return on long-term bonds.

We will see why these statements are equivalent, and in the process, prove the expectations hypothesis. For simplicity, we will assume that there are only two bonds, a one-year bond and a two-year bond. When there are more than two bonds, the notation becomes complicated but the concepts are the same.

Notation

$y_{1,t}$	Yield on a one-year bond at time t
$y_{2,t}$	Yield on a two-year bond at time t
$P_{1,t}$	Price of a one-year bond at time t
$P_{2,t}$	Price of a two-year bond at time t
$\bar{y}_{1,t+1}$	Expected yield on a one-year bond at time $t+1$ (one year later)

At time t, $y_{1,t}$ and $y_{2,t}$ are known to investors. However $y_{1,t+1}$ is not known because it is the yield next year. We assume that investors form expectations about it. The expected value is $\bar{y}_{1,t+1}$. The expectations hypothesis states that:

$$[(1+y_{1,t})(1+\bar{y}_{1,t+1})]^{1/2} - 1 = y_{2,t}. \tag{1}$$

In words, the two-year yield is a geometric average of the one-year yield this year and the expected one-year yield next year.

Examples

1. Suppose $y_{1,t} = .02$ and $\bar{y}_{1,t+1} = .04$. According to the expectations hypothesis, what must be the equilibrium value of the two-year yield?

   This question can be answered by applying (1) directly:

$$[(1 + .02)(1 + .04)]^{1/2} - 1 = .02995 \approx .03$$

2. Suppose that $y_{1,t} = .03$, and $y_{2,t} = .045$. According to the expectations hypothesis, what does the market expect the one-year yield to be next year?

   This question can be answered by rearranging (1):

$$\bar{y}_{1,t+1} = \frac{(1 + y_{2,t})^2}{1 + y_{1,t}} - 1.$$

   Substituting in the values:

$$\bar{y}_{1,t+1} = \frac{1.045^2}{1.03} - 1 = .06$$

Now we are ready to prove the expectations hypothesis.

Proof of the expectations hypothesis

Suppose that $y_{1,t} = .02$ and $\bar{y}_{1,t+1} = .04$. We want to prove that

$$y_{2,t} = [(1 + .02)(1 + .04)]^{1/2} - 1 \approx .03$$

is an equilibrium. Note that

$$P_{1,t} = \frac{100}{1.02} = 98.0392$$
$$P_{2,t} = \frac{100}{1.03^2} = 94.2596$$

1. Assumptions

   (a) Individuals maximize expected profit

   (b) There are no transaction costs

   (c) Bonds are zero-coupon and default free.

2. Overview

   Assume that $y_{2,t} = .03$ and examine investor holding period returns. If short-term (one-year) investors realize the same holding period return by buying the one-year bond compared with buying the two-year bond and selling after one year, they will be indifferent between investing in the one-year bond and the two-year bond.

   If long-term (two-year) investors realize the same holding period return by buying the two-year bond compared with buying the one-year bond this year, and using the proceeds to buy the one-year bond next year, they too will be indifferent between investing in the one-year bond and the two-year bond.

   If both sets of investors are indifferent, there would be no buying or selling in the marketplace, and prices and yields would be in equilibrium.

3. One-year investor alternative strategies.

   Strategy 1: Buy the one-year bond at 98.0392 and hold to maturity:

$$\text{HPR} = \frac{100}{98.0392} - 1 = .02$$

   Strategy 2: Buy two-year bond at 94.2596 and sell after one year. Because the two-year bond becomes a one-year bond after one year and because one-year bonds next year are expected to yield 4%, the expected price of the two-year bond next year is

$$\frac{100}{1.04} = \$96.154$$

Thus:

$$\text{HPR} = \frac{\$96.154}{\$94.2596} - 1 = .02$$

4. Two-year investor alternative strategies:

   Strategy 1: Buy the one-year bond and, after one year, invest the proceeds at $E(y_{1,t+1})$.

   $$\text{HPR} = [(1.02)(1.04)]^{1/2} - 1 = .03$$

   Strategy 2: Buy the two-year bond and hold to maturity.

   $$\text{HPR} = \left(\frac{100}{94.2596}\right)^{1/2} - 1 = .03$$

5. Conclusion:

   Because the holding period returns to both strategies are equal for all investors, the term structure is in equilibrium with $y_{2,t}$ a geometric average of $y_{1,t}$ and $\bar{y}_{1,t+1}$.

What happens when expectations change?

What happens to the equilibrium $y_{2,t}$ when expectations change? Suppose that $\bar{y}_{1,t+1} = .01$. What is the new equilibrium value of $y_{2,t}$, and how does it come about? Assume that $y_{1,t} = .02$ as before.

We will prove that the new equilibrium equals:

$$y_{2,t} = [(1.02)(1.01)]^{1/2} - 1 = .015$$

because portfolio adjustments by market participants make it so.

1. Overview

   Assume that $y_{2,t}$ remains at .03. Examine what portfolio adjustments "two-year" investors will undertake and see what impact these will have on $P_{2,t}$, and therefore on $y_{2,t}$. After determining the new equilibrium $P_{2,t}$ that makes two-year investors indifferent between both of their investment strategies, see if that $P_{2,t}$ also makes one-year investors indifferent. If so, that is the new equilibrium $P_{2,t}$ with associated new equilibrium $y_{2,t}$.

2. Two-year investor strategies:

   Strategy 1: Buy the one-year bond and, after one year, invest the proceeds at $\bar{y}_{1,t+1}$. This can be accomplished by buying the newly issued one-year bond.

$$\text{HPR} = [(1.02)(1.01)]^{1/2} - 1 = .015$$

   Strategy 2: Buy the two-year bond and hold to maturity.

$$\text{HPR} = \left(\frac{100}{94.2596}\right)^{1/2} - 1 = .03$$

   Strategy 2 is now more profitable. Two-year investors all buy two-year bonds, driving up $P_{2,t}$ (and down $y_{2,t}$), until the returns on the second strategy equals that of the first. That occurs when the HPR on the second strategy is .015.

   Therefore, the new equilibrium is

$$P_{2,t} = \frac{100}{1.015^2} = \$97.07$$

   At this new price

$$y_{2,t} = \left(\frac{100}{97.07}\right)^{1/2} = .015$$

   Therefore two-year investors are indifferent between the two strategies when $P_2 = \$97.07$.

3. Is this also an equilibrium for one-year investors? It is, as can be found using exactly the same approach as above.

Conclusion

Market forces in the form of portfolio adjustments by investors drive long-term yields into an average of current and expected future short-term yields.