

The Minimum Value of a Call Option

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Assume the stock pays no dividends. Then the minimum value of the call equals:

$$C \geq \max(0, S - Xe^{-rT}).$$

This relationship holds at *or before* expiration. The minimum value is greater than the intrinsic value $\max(0, S - X)$.

1. Suppose that $S = \$101$, $X = \$100$, $r = .06$, $T = 1$. Then

$$C \geq \max(0, 101 - 94.18) = 6.82.$$

Note: \$5.82 of the \$6.82 minimum value comes from the time value of money, and \$1.00 comes from the intrinsic value.

2. Assume $C = \$1$, i.e. calls are selling at their intrinsic value and below their minimum value (they do not reflect the interest saved by the delayed payment of X). Then there is an arbitrage opportunity. Construct the following portfolio today:

	Cash flow
Buy 1 call	-\$1
Sell short one share	\$101
Invest proceeds at .06	<u>-\$100</u>
Net	0

3. At the end of the year, you have $\$100e^{.06} = \106.18 from your investment. You own a call, and you are short one share. Evaluate what your payoff is if $S \geq X$ or if $S < X$.

If $S \geq X$:

	Cash flow
Exercise call	-\$100.00
Deliver against short S	-
Receive proceeds of investment	<u>\$106.18</u>
Net	\$6.18

If $S < X$ (say $S = \$99$):

	Cash flow
Leave call unexercised	-
Buy S in the market	-\$99.00
Deliver against short S	-
Receive proceeds of investment	<u>\$106.18</u>
Net	\$7.18

4. Conclusions:

- (a) This is an arbitrage strategy: the portfolio established “today” is zero-cost and earns a profit next year whether $S \geq X$ or $S < X$.
- (b) As you exploit this arbitrage, you will drive up the price of C , or down the price of S until C is above its minimum value.
- (c) The minimum value of a call reflects the interest that you save by not having to pay X until expiration. The minimum arbitrage profit is the difference between having to pay $X = \$100$ now, or X at expiration: $100e^{.06} - 100 = 6.18$. If the time value of money is not embedded into the call, the arbitrageur can capture it by selling short one share and investing the proceeds.

Question: Can the minimum value be used to show why you would never exercise an American call option on a non-dividend paying stock prior to expiration?