

## Arbitrage Example Demonstrating Put-Call Parity

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1. Put-call parity is an arbitrage relation between the price of a put, the price of a call, and the stock price. It holds both at expiration and prior to expiration. Assume that the stock pays no dividend. Put-call parity states that

$$C = S - Xe^{-rT} + P \quad (1)$$

2. Assume  $S = \$110$ ,  $X = \$100$ ,  $r = 0$ ,  $t = \text{anything}$  (because  $r = 0$ ). Then (1) implies  $C = \$110 - \$100 + P$ . Therefore:

If  $P = \$2$ , then  $C = \$12$

If  $P = \$5$  then  $C = \$15$

3. Assume  $S = \$110$ ,  $X = \$100$ ,  $T = 1$ . Also assume  $C = \$12$  and  $P = \$5$ . Thus the call is undervalued and/or the put is overvalued, given  $S = \$110$  and  $X = \$100$ .

Construct an arbitrage by buying the “cheap” call at \$12 and selling the “expensive” put at \$5. Recall that long a call and short a put both profit when  $S$  rises. Therefore, we complete the arbitrage by selling  $S$  short at \$110. Our portfolio today:

Buy 1 call	- \$ 12
Short (write) 1 put	+ \$ 5
Sell short 1 share	+ \$110
Invest proceeds	<u>-\$103</u>
Net	0

Examine what happens on expiration (1 year from now) if  $S > X$  and if  $S < X$ .

Suppose  $S > X$ :

Exercise call	-\$100
Deliver against short	-
Put expires worthless	0
Receive proceeds from investment	\$103
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/> \$3

Suppose  $S < X$  :

Call expires worthless	0
Put is exercised against you	-\$100
Take stock just received from put and return against short	-
Receive proceeds from investment	\$103
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There is a profit of \$3 is guaranteed no matter what happens. This profit equals the “mispricing” of  $C - P$  versus  $S - X$

$$\begin{aligned}
 C - P &= \$12 - \$5 = \$7 \\
 S - X &= \$110 - \$100 = \$10
 \end{aligned}$$

Another way of thinking about this is: Buying the call for \$12 and selling the put for \$5 allows you to be long the underlying stock (including exercise cost of \$100) for \$107. Since you sold the stock for \$110, you are entitled to \$3.

4. Since our position in part 4 is profitable and riskless, we do it as often as we can, driving up the value of  $C$  and driving down the value of  $P$  until they differ by exactly \$10. Thus  $C$  is driven up from \$12 and  $P$  is driven down from \$5. Whether  $C$  is \$14 and  $P$  is \$4 or  $C$  is \$15 and  $P$  is \$5 or whether  $C$  is \$12 and  $P$  is \$2 depends on the valuation of the call (or the put) from a model such as Black-Scholes. Put call parity just gives the relative value of  $C$  versus  $P$  given  $S$  and  $X$ .