Problem Set 5

Foundations of Financial Markets

Summer 2007, Section 1

Due date: June 19

*** Required problems:** 1 2(a) 4 5 6

Others: suggested problems. I'd recommend doing these before the exam, though.

1. * Suppose you buy a five-year zero-coupon Treasury bond for $800 per $1000 face value. Answer the following questions:
   (a) What is the yield to maturity (annual compounding) on the bond?
   (b) Suppose you buy the bond. Immediately after you buy it, the yield to maturity on comparable zeros increases to 7% and remains there. Calculate your annual return (holding period yield) if you sell the bond after one year.
   (c) Assume yields to maturity on comparable bonds remain at 7%. Instead of selling the bond after 1 year, as in (b), now you sell the bond after two years. Calculate your annual return.
   (d) Suppose after 3 years, the yield to maturity on similar zeros declines to 3%. Calculate the annual return if you sell the bond at that time.
   (e) If yield remains at 3%, calculate your annual return if you sell the bond after four years.
   (f) Calculate the annual return if you sell the bond after five years.
   (g) What explains the relationship between annual returns calculated in (b) through (f) and the yield to maturity in (a)?
2. Assume the government issues a semi-annual pay bond that matures in 5 years with a face value of $1,000 and a coupon rate of 10 percent.
   (a) *What price would you be willing to pay for such a bond if the yield to maturity (semi-annual compounding) on similar 5-year governments were 8%?
   (b) What would be the price if the yield to maturity (semi-annual compounding) on similar governments were 12%?
   (c) If the price of the bond is 103 19/32 per $100 of face value, what is the yield to maturity?
   (d) Suppose you held the bond in (c) for 6 months, at which time you received a coupon payment and then sold the bond for a price of 102 (per $100 of face value). What would be the annualized holding period return?
3. For each of the bonds and reinvestment rates listed below calculate the amount of money accumulated at the end from a $1000 initial investment:
   (a) Invest $1000 in a 5-year zero coupon bond with a yield to maturity of 9 percent.
   (b) Buy a 5-year 9% coupon annual pay bond at par ($1000) and reinvest the annual coupons at 9% (annual compounding).
   (c) Same as (b) but reinvest the annual coupons at 12%.
   (d) Same as (b) but reinvest the annual coupons at 6%.

(e) For (a) through (d) calculate the annual holding period return. What can you conclude about the relationship between the reinvestment rate, yield to maturity and holding period returns?

4. * Suppose the yield to maturity on a one-year zero-coupon bond is 8%. The yield to maturity on a two-year zero-coupon bond is 10%. Answer the following questions (use annual compounding):

   (a) According to the Expectations Hypothesis, what is the expected one-year rate in the marketplace for year 2?

   (b) Consider an investor who only wants to invest for a year. She expects the yield to maturity on a one-year zero to be 6% next year. In which one will she choose to invest for a year: the one-year zero-coupon bond or the two-year zero-coupon bond? [Hint: compare the holding period return for both bonds]

   (c) If all investors behave like the investor in (b), what will happen to the equilibrium term structure according to the Expectations Hypothesis?

5. *A zero coupon bond with 2.5 years to maturity has a yield to maturity of 25% per annum. A 3-year maturity annual-pay coupon bond has a face value of $1000 and a 25% coupon rate. The coupon bond also has a yield to maturity of 25%.

   Calculate for each bond the percentage price change associated with a change of yield to maturity from 25% to 26%. Does the longer maturity bond have a larger interest rate sensitivity? Why or why not?

6. * Construct profit diagrams or profit tables on expiration to show what position in IBM puts, calls and/or underlying stock best expresses the investor's objectives described below. Assume IBM currently sells for $150 so that profit diagrams/ tables between $100 and $200 (in $10 increments) are appropriate. Also assume that "at the money" puts and calls cost $15 each. (As usual, the profit calculations ignore dividends and interest.)

   (a) An investor wants upside potential if IBM increases but wants (net) losses no greater than $15 if prices decline.

   (b) An investor wants to capture profits if IBM declines in price but wants a guaranteed limited loss if prices increase.

   (c) An investor wants to capture profits if IBM declines in price and is ready to accept unlimited losses if prices increase. Further, the investor wants to break even if the stock price does not change between now and the maturity of the options.

   (d) An investor wants to profit if IBM's upcoming earnings announcement is either unexpectedly good or disappointingly bad.

   (e) An investor already owns IBM (at a price of $150) and wants to protect against price declines but wants to retain the possibility of upside if prices rise. Only one transaction is permitted here.

   (f) Suppose the NYSE suspended trading in IBM pending a news announcement. You already own a share of IBM, which you want to sell before the announcement (but you can't, since trading in IBM stock is halted). Options trading in IBM, however, continues uninterrupted on the CBOE. How do you neutralize your exposure to stock price changes by trading options? (You can use the put-call parity.)

7. Suppose that the risk-free rate is 5%, that Lego's stock is currently at \$100, and that, the next two years, the stock price movements are well approximated by the following tree:

$t = 0$	$t = 1$	$t = 2$
		144
	120	
100		108
	90	
		81

We see that every year Lego will either increase by 20% or decrease by 10% (with equal probability). Suppose that a 2-year (European-style) "binary" option is traded on Lego. This option pays the option-holder \$10 if the stock price is greater than \$100 at maturity, $t = 2$.

To compute the initial value of the option, you should make a tree for the development of the option price. Fill out the tree, starting from the last date, by answering the following questions:

(a) Compute the value of the option at maturity ($t = 2$) at each of the 3 scenarios.

(b) Suppose at time 1, the stock price is 120. Create a portfolio of stocks and risk-free securities that replicates the option's payoff at time 2.

(*Hint:* Recall that the number of stocks in the replicating portfolio is always

$$\Delta = \frac{C^+ - C^-}{S^+ - S^-}$$

where S^+ and S^- are the two possible stock prices in the next period, and C^+ and C^- are the corresponding option values next period. The amount invested in the riskfree security should make the total value of the portfolio equal to the value of the option in both scenarios next period.)

What is the price of this replicating portfolio? This is the value of the option at time 1 in this upper-branch scenario. Put this value in your option tree.

(c) Suppose at time 1, the stock price is 90. Create a portfolio of stocks and risk-free securities that replicates the option's payoff at time 2. What is the price of this portfolio? This is the value of the option at time 1 in this lower-branch scenario. Put this value in your option tree. (Use hint from (7b).)

(d) At time 0, create a portfolio of stocks and risk-free securities that replicates the option values at time 1. (The time-1 option values were computed in (7b)–(7c).) What is the price of this portfolio? This is the value of the option at time 0. Put this value in your option tree. (Use hint from (7b).)

8. Suppose a European call option has an exercise price of \$100 and the underlying stock has a price of \$100. The stock will pay no dividends over the next year. The option expires in 1 year and the continuously compounded interest rate is 6%.

(a) What is the intrinsic value of this option?

(b) What will the option be worth on expiration if the stock price in 1 year is \$110? What if the stock price is \$90?

(c) What is the lower bound on the price of this option today?

(d) Will the value of the option be larger or smaller if the volatility of the underlying asset is higher than otherwise?

(e) Will the value be larger or smaller if the option has 3 months rather than 6 months to expiration?

(f) Will the value be larger or smaller if the interest rate is larger?