Solution to Sample Final
Foundations of Financial Markets
Summer 2007, Section 1

1. **a.**

   A price of a zero coupon bond with t years to maturity is

$$p = \frac{FV}{(1 + YTM)^t}$$

   As the time to maturity t decreases over time, price increases. Note that zero coupon bonds always sell at discount, and if yield to maturity remains constant over time, then the bonds' price will increase gradually toward par. The closer to maturity, the closer the price is to par. This is known as "pull to par".

2. **c.**

   For the annual-pay coupon bonds, we have the following relationship between YTM and the coupon rate:

   a bond selling at premium ⇔YTM < coupon rate

   a bond selling at par ⇔ YTM = coupon rate

   a bond selling at discount ⇔ YTM > coupon rate

   In a, the bond selling at 103>100, therefore its YTM must be less than 8%; in b, the bond's YTM is 8%, and the YTM for c is $(\frac{1000}{665})^{\frac{1}{5}} - 1 = 8.5\%$.

3. **c**.

   The put option is in the money if and only if the price for the underlying stock is below 100, and the payoff for the put will be the strike price minus the underlying's market price. The best senario for a person who bought a put is that the underlying's price drops to zero, so that the option's intrinsic value is maximized. Therefore, the maximum payoff will be $100. At the same time, you need to pay $10 to buy the option, which makes the net profit $90.

4. **e**.

   According to the CAPM, the security's expected return is less than risk-free rate when β is less than 0.

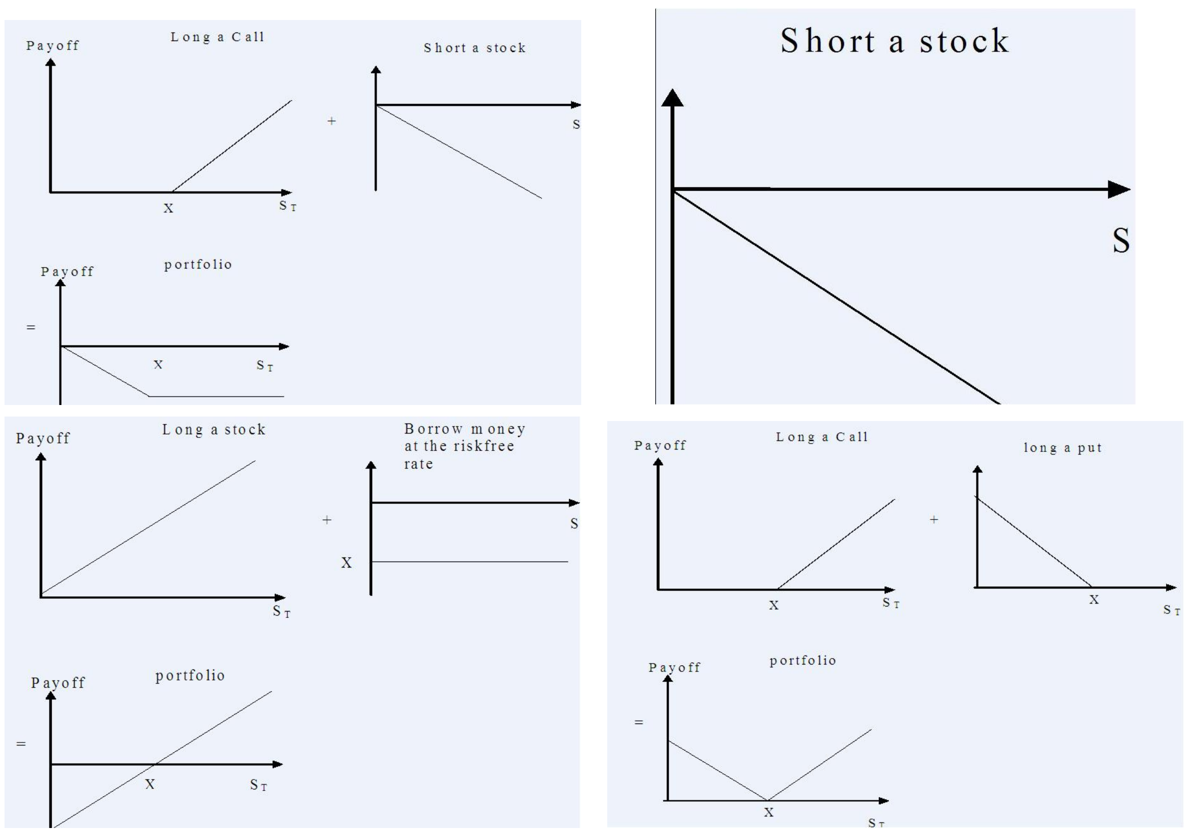

Figure 1: Payoff diagrams for question 5. Clockwise from top left: a, b, c and d.

5. **c**.The trick in answering this kind of question is to draw graphs or diagrams of the payoff of the securities.

6. **b**.

   The HPR for the one-year zero is just its yield, 7%. For the HPR for the 2-year zero sold after one year, we need to know how much the 2-year zero is worth after one year, V(1), as well as its current price, V(0). Since we know its current yield is 8%, we can compute its current price, which is $\frac{100}{(1+0.08)^2} = 85.73$. The expected one-year yield after year 1 is 7.5%, which means the price of the 2-year zero at the end of the first year is $\frac{100}{1+0.075} = 93.023$. Then the HPR for the 2-year zero during the first year is

$$HPR = \frac{V(1)}{V(0)} - 1 = \frac{93.023}{85.73} - 1 = 8.5\%$$

7. **d.**

   The no-arbitrage condition implies put-call parity must hold.

$$\begin{aligned} C_0 - P_0 &= S_0 - Xe^{-rT} \\ &= 110 - 100 \\ &= \$10 \end{aligned}$$

   If $C_0 - P_0 > \$10$, then put is relative cheaper than call, so you should buy put and sell call. If $C_0 - P_0 < \$10$,then call is relatively cheaper, you should buy the call and sell the put. In this problem, you buy the call and sell the put to create an arbitrage, so you must have that $C_0 - P_0 < \$10$.

8. **a.**

   Interest rate sensitivity is -$\frac{D}{1+y}$ In this question, both D and y (YTM) are the same for the two bonds. So the price change will be the same.

9. **a.**

   $\alpha = 0.14 - 0.03 - 1.45 \times (0.1 - 0.03) = 0.0085$

10. **b.**

11. **c.**

12. **c.**

13. **c.**

    For zero-coupon bonds, the condition for HPR equal to YTM is that you hold the zero till maturity.

    For coupon bonds, in order for the HPR be equal toYTM, you need not only hold the bond to maturity, but also be able to reinvest the coupon payments at the YTM.

14. **c.**

15. **c.**

    The buyer of a put will profit if the price of the underlying falls below the strike price, because he can buy the stock at the market price and sell it to whoever wrote the option to him at the strike price, which is higher than the market price.

    For a call option, exercising will not be worthwhile if the stock price at the time of exercise is below the strike. So the value of the call is 0. However, the seller of the call was paid the option price at the beginning, so his total profit is the price he received.

16. **d.**

    The bond is selling at par, so YTM=10%. If YTM next year is still 10%, the investor will get $121. But if the interest rate decreases at the end of year 1, the bond holder's final payoff will be less than $121 at the end of year 2, because he has to reinvest the coupon payment at a lower rate.

17. d.

    According to the Gordon growth model $\frac{P_0}{D_0} = \frac{1+g}{k-g}$ and $\frac{P_0}{E_0} = \frac{(1-b)(1+g)}{k-g}$.Therefore, higher g will have higher P/E and P/D ratio.

18. First, calcualte the required rate of return:

$$k = 0.03 + 1.25 \times .08 = .13$$

    Then the price of the stock is

$$\begin{aligned} P &= \frac{D_o(1+g)}{k-g} \\ &= \frac{20(1+.06)}{.13-.06} \\ &= 302.86 \end{aligned}$$

19. The payoff for the call is

$$C \begin{array}{l} 18 \\ \\ 0 \end{array}$$

We now form the replicating portfolio for the call using the underlying stock and riskfree asset. Let x be the number of shares for the underlying and y be the number of shares for the risk free asset (one year zero coupon bond).

The replicating portfolio must have the same payoff as the call for both of the two states

$$\begin{aligned} x \times 120 + y \times 100 &= 18 \\ x \times 90 + y \times 100 &= 0 \end{aligned}$$

Solve for x and y. Note that the number of shares bought for the underlying is equal to $\frac{18-0}{120-90} = 0.6$,which is also called the hedge ratio. Therefore, the replicating portfolio for the call is to buy 0.6 share of the underlying and -0.54 shares of the riskfree asset (short sell 0.54 shares)

The price of the call must be equal to the cost of the replicating portfolio

$$\begin{aligned} c &= 0.6 \times 100 - 0.54 \times \frac{100}{1.03} \\ &= \$7.5728 \end{aligned}$$

20. To calculate the HPR over the 3 years, we need to first compute the total payoff of your investment at the end of the 3 year. It is helpful to use a diagram to keep track of the cash flows:

1	2	3		
\$80	$\longrightarrow$	$\longrightarrow$ \$80(1.08)^2	=	\$93.312
	\$80	$\longrightarrow$ \$80(1.08)	=	\$86.4
		\$80 + 1020	=	\$1100
			=	\$1279.712

The sum of the values at the end of year 3 is \$1279.712

Therefore the annualized HPR=$(\frac{1279.712}{1000})^{\frac{1}{3}} - 1$ =0.085686

21. Note the 20 coupon payments can be treated as annuity.

$$price = \frac{10 * (1 - \frac{1}{(1+0.08)^{20}})}{0.08} + \frac{100}{(1+0.08)^{20}} = 119.6362948$$

22. According to the Gordon growth model $P_0 = \frac{E(D_1)}{k-g}$,which implies

$$\begin{aligned} k &= \frac{E(D_1)}{P_0} + g \\ &= \frac{6}{80} + 0.04 \\ &= 0.115 \end{aligned}$$

Note there are two versions of the Gordon growth model depending on the timing of the first dividend payment. If the first dividend is the year one dividend, then the price is equal to $\frac{E(D_1)}{k-g}$; if the first dividend is the year 0 dividend, then the price is equal to $\frac{D_0(1+g)}{k-g}$. These two versions are equivalent since $E(D_1) = D_0(1+g)$ according to the constant-growth assumption of the GGM. **But you should take particular care to note the timing of the dividend in all problems of this kind.**

23. From the expectations hypothesis, the expected future interest rate equals the forward rate:

$E(R(2)) = f(2) = \frac{(1+YTM(3))^3}{(1+YTM(2))^2} - 1 = \frac{1.04^3}{1.03^2} - 1 = 0.0603$

Similarly, we can calculate the expected rate of interest on a 1 year loan starting 1 year from now: $E(R(1)) = f(1) = \frac{(1+YTM(2))^2}{1+YTM(1)} - 1 = \frac{1.03^2}{1.01} - 1 = 0.05040$

If you plot the yield curve, it will be upward sloping since the 3-year zero has a higher yield than the 2-year zero and the 2-year zero, in turn, has a higher yield than the 1-year zero. This implies that the one-year rate is expected to increase in the future, which is verified by our calculation above.

24. Duration D=2.44 years.

Percentage price change is captured by -$\frac{D}{1+Y} = -\frac{2.44}{1+0.25} = -1.952$. So when YTM change from 25% to 26%, price goes down 1.952%

25. The price of a TIGR must be between 95 and 99. Notice this question is different from the example in class. In class, we assume no cost for buying. Now we have trading costs for both buying and selling a zero coupon bond. So if you want to create an

arbitrage (i.e. buy low and sell high), the total cost will be \$2. So the maximum price difference between CATs and TIGRs will be \$2. You can show if the price of the TIGR goes out of this range, there is an arbitrage opportunity even considering the costs.