Flippers in Housing Market Search*

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June 29, 2012

Abstract

We add arbitraging middlemen—investors who attempt to profit from buying low and selling high—to a canonical housing market search model. As expected, the opportunities offered to ordinary households to quickly dispose of their old houses by these middlemen are particularly welcome in a slack market in which it is most difficult for end-user households to sell. Less obvious is that the same opportunities are similarly welcome in a tight market in which houses can be sold quickly even without the aid of these intermediaries. To follow is the possibility of multiple equilibrium. In one equilibrium, most, if not all, transactions are intermediated, resulting in rapid turnover, a high vacancy rate, and high housing prices. In another equilibrium, few houses are bought and sold by middlemen. Turnover is sluggish, few houses are vacant, and prices are moderate. The housing market can then be intrinsically unstable even when all flippers are of the liquidity-providing variety in classical finance theory.

Key words: Search and matching, housing market, liquidity, flippers
JEL classifications: D83, R30, G12

*We thank Enrique Schroth as discussant of the paper in the 2011 NYU Economics PhD Alumni Conference, Angela Mak for excellent research assistance and Kuang Liang Chang, Vikas Kakkar, Fred Kwan, Edward Tang, Min Hwang, and Isabel Yan for discussions. Tse grateful acknowledges financial support from HK GRF grant HKU 751909H.

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1 Introduction

In many housing markets, the purchases of owner-occupied houses by individuals who attempt to profit from buying low and selling high rather than for occupation are commonplace. For a long time, anecdotal evidence abounds as to how the presence of these investors, who are popularly known as flippers in the U.S., in the housing market can be widespread.\(^1\) More recently, empirical studies have began to systematically document the extent to which transactions in the housing market are motivated by buying and selling for short-term gains and how these activities are correlated with the housing price cycle. Notable contributions include Depken et al. (2009), Bayer et al. (2011), and Haughwout et al. (2011). A common theme in the discussion is that housing market flippers can be of two types—the trend-chasing speculators versus the arbitraging middlemen. Whereas the speculators, as noise traders, inevitably destabilize the market, the middlemen, as liquidity providers in classical finance theory, help improve market efficiency. But is such a simple and clear-cut dichotomy warranted? To the extent that housing demand goes up and down along with the arbitraging middlemen entering and exiting the market, these agents may well contribute to market volatility to a certain extent. Moreover, would-be buyers can be worse off in a market in which middlemen abound. Without these intermediaries, buyers face less competition and prices should be lower.

In this paper, we study a housing market search model along the lines of Arnold (1989) and Wheaton (1990) in which houses are demanded by flippers in addition to end-user households. The flippers are of the liquidity-providing variety in classical finance theory. A role for these agents exist because ordinary households are assumed not able to hold more than one house at a time. The assumption, of course, can be justified by the usual liquidity constraint argument. In this case, a household which desires to move because the old house is no longer a good match must first sell it before the household can buy up a new house. In a buyer’s market—one in which sellers outnumber buyers by a significant margin, the wait can be lengthy. This opens up profitable opportunities for the flippers to just buy up the mismatched house at a discount in return for the time spent waiting for the eventual end-user buyer to arrive on behalf of the original owner. This is the usual reason for why flippers can improve market liquidity. The novelty in our analysis is that we find that mismatched homeowners could similarly prefer to sell quickly to flippers in a seller’s market to capitalize on the high prices in such a market sooner. In either case, transaction volume, vacancy, and housing price all increase with the extent of flippers’ presence in the market, whereas average Time-On-the-Market (TOM) declines in the interim.

\(^1\)Out of the five transactions in a large development in Hong Kong in August 2010, three were reported to involve investors who buy in anticipation of short-term gains (September 10, 2010, Hong Kong Economic Times). According to one industry insider, among all buyers of a new development in Hong Kong recently, only about 60% are buying for own occupation (November 20, 2010, Wenweipo).
Because flippers can survive in both slack and tight markets, multiple equilibrium is a distinct possibility in our model. With a multiplicity of equilibrium, wide swings in prices and transactions can happen without any underlying changes in technology, preference, and interest rate. Moreover, in our model, the market penetration of flippers can be rather sensitive to small interest rate shocks. In this case, the given interest rate shock will have an indirect impact on housing price through its influence on the entry and exit of flippers, in addition to the usual direct effect of interest rate on asset price. Then, a housing market populated by liquidity-providing middlemen can be prone to a substantial amount of volatility. In all, the presence of these agents in the housing market can be a double-edged sword—on the one hand, the flippers help improve liquidity; on the other hand, when the extent of their presence can be fickle, the housing market can become more volatile as a result.

It would be foolhardy to suggest that the volatility arising from the activities of liquidity-providing middlemen in our analysis is an important source of the housing market bubble in the U.S. in the early- to mid-2000s. Perhaps the trend-chasing speculators have played a deciding role. In any case, we should emphasis that our model is not meant to be a candidate explanation for any episodes of housing market bubble in the U.S. and beyond. Nevertheless, our quantitative analysis indicates that housing price can differ by up to 25 percent across steady-state equilibria and 30 percent from a seemingly unimportant interest rate shock when the model is calibrated to the turnovers characteristics of the U.S. housing market. Such price differences, of course, are nowhere near the observed 100-percent-plus price volatility in many episodes of housing market bubble around the world. Still, the 20-plus percent difference is a significant difference by any yardstick. Amid this substantial price difference, welfare, however, differs much less across the steady-state equilibria. Aggregate welfare in a “fully-intermediated” equilibrium is at most 9 percent higher than in a “no-intermediation” equilibrium. That any welfare increase from intermediation may be modest is because the increase is bounded by the losses suffered by the would-be buyers in a tighter market with higher prices.

Our model has a number of readily testable implications. First, it trivially predicts a positive cross-section relation between housing price and TOM—households can either sell to flippers at a discount or to wait for a better offer from an end-user buyer to arrive—which agrees with the evidence reported in Merlo and Ortalo-Magne (2004), Leung et al. (2002) and Genesove and Mayer (1997), among others.²

An important goal of the recent housing market search and matching literature is to understand the positive time-series correlation between housing price and transaction volume and the negative correlation between the two variables and average TOM.³ In Kranier (2001), for instance, a positive but temporary preference shock

²Albrecht et al. (2007) emphasis another aspect of the results reported in Merlo and Ortalo-Magne (2004), which is that downward price revisions are increasingly likely when a house spends more and more time on the market.

³Stein (1995), who explains how the down-payment requirement plays a crucial role in amplifying
can give rise to higher prices and a greater volume of transaction, whereas Diaz and Jerez’s (2009) analysis implies that an adverse shock to construction will shorten TOM, and may possibly lead to higher prices and a greater volume of transaction. In these papers, the increase in sales should be accompanied by a decline in vacancy—given that when a house is sold, it is sold to an end-user, who will immediately occupy it, vacancy must decline, or at least remains unchanged. Across steady-state equilibria, the same positive relation between price and transaction volume and negative relation between the two variables and average TOM also hold in our model. Specifically, with more houses sold to flippers, prices and sales both increase, whereas houses on average stay on the market for a shorter period of time. Unique to our model, however, is that vacancy tends to increase together with prices and transaction volume if the increase in transaction volume is due to more houses sold to flippers, who may then just leave them vacant for the time it takes for the end-user buyers to arrive.4

Figure 1 depicts the familiar positive housing price-transaction volume correlation for the U.S. for the 1991-I to 2010-IV time period.5 The usual housing market search model predicts that vacancy should decline in the housing market boom in the late-1990s to the mid-2000s and rise thereafter when the market collapses around 2007. Figures 2 and 3, however, show that any decline in vacancy is not apparent in the boom.6 In fact, if there is any co-movement between vacancy on the one hand and price and transaction on the other hand in the run-up to the peak of the housing market boom in 2006, vacancy appears to have risen along with price and transaction. True, vacancy does not appear to have fallen to follow the market collapse, as predicted by our analysis. But this probably is a combined result of the massive amounts of bank foreclosures and unsold new constructions in the market bust—two forces absent in our analysis.

Insofar as the flippers in our model act as middlemen between the original home-shocks, is an early non-search-theoretic explanation for the positive relation between price and sales. Hort (2000), Leung et al. (2003), among others, provides recent evidence. Ho and Tse (2006) show that the same relation holds in the cross section.

4In Ngai and Tenreyro (2010), households are assumed to move out of their old houses and into rental housing immediately when they become mismatched. In this way, vacancy is also positively correlated with price and transaction in their model, given the assumed increasing-returns-to-scale matching technology. But the correlation is arguably more of a simplifying assumption than a natural outcome of their model. A mismatched household in the model could well have stayed in the old house and avoid rental housing until it has successfully sold the old house.

5Housing Price is defined as the nominal house price, which is the transaction-based, seasonally-adjusted house price index from OFHEO (http://www.fhfa.gov), divided by the CPI, from the Federal Reserve Bank at St. Louis, seasonally-adjusted. We set Housing Price equal to 100 at 1991-I. Transaction is measured by the total sales in single-family homes, apartment condos, and co-ops, normed by the stock of such units. The sales data are from Moody’s Analytics, whereas the stock data are from the Bureau of Census’s CPS/HVS.

6Vacancy rate is obtained by dividing the number of vacant and for-sale-only housing units by the stock of such units. The data are from the Bureau of Census’s CPS/HVS.
Figure 1: Price and Transaction

Figure 2: Price and Vacancy
owners and the eventual end-user buyers, this paper contributes to the literature on middlemen in search and matching pioneered by Rubinstein and Wolinsky (1987). Previously, it was argued that middlemen could survive by developing reputations as sellers of high quality goods (Li, 1998), by holding a large inventory of differentiated products to make shopping less costly for others (Johri and Leach, 2002; Shevchenko, 2004; Smith, 2004), by raising the matching rate in case matching is subject to increasing returns (Masters, 2007), and by lowering distance-related trade costs for others (Tse, 2011). This paper studies the role of middlemen in the provision of market liquidity.

A simple model of housing market flippers as middlemen is also in Bayer et al. (2011). The model though is partial equilibrium in nature and cannot be used to answer many of the questions we ask in this paper. Intermediaries who serve to improve liquidity in the housing market are also present in the model of the interaction of the frictional housing and labor markets of Head and Lloyd-Ellis (2011), which is fully general equilibrium in nature. Analyzes of how middlemen may serve to improve liquidity in a search market also include Gavazza (2012) and Lagos et al. (2011). However, none of these studies allows end-user households a choice of whether to deal with the middlemen and for the multiplicity of equilibrium and how the market share of these middlemen may vary across the equilibria. Multiple equilibrium in a search and matching model with middlemen can also exist in Watanabe (2010). The
multiplicity in that model, however, is due to the assumption that the intermediation technology is subject to increasing returns to scale. Moreover, only one of the two steady-state equilibria in the model is stable, whereas there can be two stable steady-state equilibria in our model.

The next section presents the model. Section 3 contains the detailed analysis. Section 4 takes a more systematic look at the patterns shown in Figures 1-3 with reference to the model’s implications. In Section 5, we calibrate the model to the turnovers characteristics of the U.S. housing market to assess the amount of volatility that the model implies. Section 5 concludes. All proofs are relegated to the Appendix. For brevity, we restrict attention to analyzing steady-state equilibrium in this paper. A companion technical note (Leung and Tse, 2011) covers the analysis of the dynamics.\footnote{Not for publication, available for download in http://www.sef.hku.hk/~tsechung/index.htm}

2 Model

2.1 Basics

The city is populated by a continuum of measure one risk-neutral households, each of whom discounts the future at the same rate $r_H$. There are two types of housing in the city: owner-occupied, the supply of which is perfectly inelastic at $H < 1$ and rental, which is supplied perfectly elastically for a rental payment of $q$ per time unit. A household staying in a matched owner-occupied house enjoys a flow utility of $\nu > 0$, whereas a household either in a mismatched house or in rental housing none. A household-house match breaks up exogenously at a Poisson arrival rate $\delta$, after which the household may continue to stay in the house but it no longer enjoys the flow utility $\nu$. In the mean time, the household may choose to sell the old house and search out a new match. An important assumption is that a household cannot hold more than one house at a time. Then a mismatched homeowner must first sell the old house before she can buy a new one. The qualitative nature of our results should hold as long as there is a limit, not necessarily one, on the number of houses a household can own at a time. The one-house-limit assumption simplifies considerably.

The search market The flow of matches in the search market is governed by a concave and CRS matching function $M(B, S)$, where $B$ and $S$ denote, respectively, the measures of buyers and sellers in the market. Let $\theta = B/S$ denote market tightness. Then, the rate at which a seller finds a buyer is

$$\eta = \frac{M(B, S)}{S} = M(\theta, 1),$$

\footnote{Not for publication, available for download in http://www.sef.hku.hk/~tsechung/index.htm}
whereas the buyer’s matching rate is \( \mu = \eta / \theta \). As usual, increased market tightness raises the seller’s but lowers the buyer’s matching rate; i.e.,

\[
\frac{\partial \eta}{\partial \theta} > 0, \quad \frac{\partial \mu}{\partial \theta} < 0.
\]

We impose the usual regularity conditions on \( M \) to ensure that

\[
\lim_{\theta \to 0} \eta = \lim_{\theta \to \infty} \mu = 0, \quad \lim_{\theta \to \infty} \eta = \lim_{\theta \to 0} \mu = \infty.
\]

Prices in the search market fall out of the Nash bargaining between each pair of matched buyer and seller.

The Walrasian investment market Instead of waiting out a buyer to arrive in the search market, a mismatched homeowner may sell her old house right away in a Walrasian market populated by specialist investors—agents who do not live in the houses they have bought but rather attempt to profit from buying low and selling high. Because homogeneous flippers do not gain by selling and buying houses to and from one another, the risk-neutral flippers may only sell in the end-user search market and will succeed in doing so at the same rate \( \eta \) that any household-seller does in the market. We allow for flippers to discount the future at a possibly different rate \( r_F \) than the households in the city.\(^8\) In the competitive investment market, prices adjust to eliminate any excess returns on real estate investment.

We recognize that the assumption of a Walrasian investment market seemingly completely contradicts the motivations for applying the search and matching framework to the study of the housing market. What is needed in the analysis, however, is not an investment market altogether free of search frictions of any kind, but one in which the frictions are less severe than in the end-user market.\(^9\) If flippers are entirely motivated by arbitrage considerations and do not care if the houses to be purchased are good matches for their own occupation, search should not a particularly serious problem. In reality, we imagine that households who intend to sell quickly and are willing to accept a lower price will convey their intentions to real estate agents, who in turn will alert any specialist investors the availability of such deals. The competition among flippers should then drive prices up to just enough to eliminate any excess returns on investment. A Walrasian market assumption captures the favor of such arrangements in the simplest possible manner.

\(^{8}\)While institutional and cash-rich investors may be able to finance investment at a lower interest rate, banks in many places charge higher mortgage interest rates for those who are buying a second home and for those who report that they are not buying a house for occupation. We leave it as an open question as to whether \( r_F < r_H \) is the more empirically relevant case.

\(^{9}\)Let’s say, for example, the meetings in the investment market are governed by another CRS matching function \( M_F(B, S) \), whereby \( M_F(B, S) > M(B, S) \) for any \( \{B, S\} \) pair.
2.2 Accounting identities and housing market flows

**Accounting identities** At any one time, a household can either be staying in a matched house, in a mismatched house, or in rental housing. Let $n_M$, $n_U$, and $n_R$ denote the measures of households in the respective states. Given a unit mass of households residing in the city,

$$n_M + n_U + n_R = 1. \quad (1)$$

Each owner-occupied house in the city must be held either by a household in the city or by a flipper. Hence,

$$n_M + n_U + n_F = H, \quad (2)$$

where $n_F$ denotes both the measures of active flippers and houses held by these individuals.

If each household can hold no more than one house at any moment, the only buyers in the search market are households in rental housing; i.e.,

$$B = n_R. \quad (3)$$

On the other hand, sellers in the search market include mismatched homeowners and flippers, so that

$$S = n_U + n_F. \quad (4)$$

**Housing market flows** In each unit of time, households in rental housing who just manage to buy up a house ($\mu n_R$) make up the flows into matched owner-occupied housing, whereas the outflows are comprised of those who become mismatched ($\delta n_M$) in the interim. In the steady state,

$$\mu n_R = \delta n_M. \quad (5)$$

Households’ whose matches just break up may choose to sell their old houses right away to flippers in the investment market or to wait out a buyer to arrive in the search market. Let $\alpha$ denote the fraction of mismatched households who choose to sell in the investment market and $1 - \alpha$ the fraction who choose to sell in the search market. In each time unit then, the measure of mismatched homeowners increase by $(1 - \alpha) \delta n_M$. The exits are comprised of mismatched homeowners who just manage to dispose of their properties in the search market ($\eta n_U$). In the steady state,

$$(1 - \alpha) \delta n_M = \eta n_U. \quad (6)$$

Households moving into rental housing are mismatched households who just sell their properties to flippers ($\alpha \delta n_M$) and to end users ($\eta n_U$), respectively. The exits are comprised of households who just find a match in owner-occupied housing ($\mu n_R$). In the steady state,

$$\alpha \delta n_M + \eta n_U = \mu n_R, \quad (7)$$
In each time unit, the measure of houses held by flippers increases by the measure of houses recently mismatched households decide to dispose right away in the investment market ($\alpha \delta n_M$) and declines by the measure of houses flippers manage to sell to end-users ($\eta n_F$). In the steady state,\(^{10}\)

$$\alpha \delta n_M = \eta n_F.$$  \hfill (8)

Insofar as $\alpha$ measures the fraction of mismatched households selling in the investment market, it should also measure the market share of flippers in the search market since houses bought by flippers in the investment market will next be put up for sale in the search market. In the steady state, the equivalence exactly obtains.

**Lemma 1** In the steady state,

$$\alpha = \frac{n_F}{n_U + n_F};$$

i.e., the fraction of houses held by flippers among all houses offered for sale in the search market.

### 2.3 Flippers’ market share, market tightness, and turnovers

Equations (1)-(8) can be combined to yield a single equation,

$$\delta + \eta (1 - H) - (1 - \alpha + \theta) H \delta = 0,$$  \hfill (9)

in $\theta$ and $\alpha$.\(^{11}\)

**Lemma 2** An implicit function $\theta = \tilde{\theta} (\alpha)$, for $\alpha \in [0, 1]$, defined by (9), is guaranteed single-valued, and that $\partial \tilde{\theta} / \partial \alpha > 0$. Both the lower and upper bounds, given by, respectively, $\tilde{\theta}_L = \tilde{\theta} (0)$ and $\tilde{\theta}_U = \tilde{\theta} (1)$, are strictly positive and finite. Furthermore, $\tilde{\theta}_U > 1/H > 1$.

A priori, one would expect that when no houses are sold to flippers ($\alpha = 0$), $n_F$ can only be equal to zero in the steady state. On the other hand, when all mismatched houses are sold to flippers in the first instance, $n_U$, in the steady state, should just be equal to zero. And then in general, as $\alpha$ increases from 0 toward 1, $n_F$ should increase and $n_U$ decline. Lemma 3 confirms these intuitions.

\(^{10}\)Where (1) and (2) are two equations in four unknowns, once any two of the four variables are given, the other two are uniquely determined. In this connection, it is straightforward to verify that only two of the four steady-state flow equations (5)-(8) constitute independent restrictions.

\(^{11}\)See the proof of Lemma 2 in the Appendix for the derivation of the equation.
Lemma 3

a. At $\alpha = 0$, 
\[ n_F = 0, \quad n_R = 1 - H, \quad n_M = H - n_U, \]
whereas $n_U$ is given by the solution to (57) in the Appendix.

b. As $\alpha$ increases from 0 toward 1,
\[ \frac{\partial n_F}{\partial \alpha} > 0, \quad \frac{\partial n_R}{\partial \alpha} > 0, \quad \frac{\partial n_M}{\partial \alpha} > 0, \quad \text{whereas} \quad \frac{\partial n_U}{\partial \alpha} < 0. \]

c. At $\alpha = 1$, 
\[ n_U = 0, \quad n_R = 1 - H + n_F, \quad n_M = H - n_F, \]
whereas $n_F$ is given by the solution to (58) in the Appendix.

What is less obvious in the Lemma is that both $n_R$ and $n_M$ increase along with $\alpha$. The first tendency follows from the fact that if both the city’s population and the housing stock are given, a unit increase in the measure of houses held by flippers must be matched by a unit decline in the measure of houses occupied by the households in the city. To follow then is the same unit increase in the city’s population in rental housing. For the second tendency, at a larger $\alpha$, fewer households spend any time at all selling their old houses in the search market before initiating search for a new match. In the mean time, the increase in $\theta$ (Lemma 2), through lowering $\mu$, lengthens the time a household spends on average in rental housing before a new match can be found. By Lemma 3, the first effect dominates, so that more households are in matched owner-occupied housing in the steady state.

Now, if $\partial n_R/\partial \alpha > 0$ and given that $B = n_R$, there will be more buyers in the search market to follow an increase in $\alpha$. Second, given that by (2),
\[ n_U + n_F = H - n_M, \]
so that
\[ \frac{\partial [n_U + n_F]}{\partial \alpha} = -\frac{\partial n_M}{\partial \alpha} < 0. \]
Where the sum $n_U + n_F$ just happens to be the measure of sellers in the search market, there will also be fewer sellers to follow the same increase in $\alpha$. With more buyers and fewer sellers, market tightness, given by
\[ \theta = \frac{B}{S} = \frac{n_R}{n_U + n_F}, \]
should only increase when more transactions are intermediated by flippers. These tendencies, of course, are the forces behind the comparative statics in Lemma 2.
In the model housing market, the entire stock of vacant house comprises of houses held by flippers. With a given housing stock, the vacancy rate is simply equal to $n_F/H$. A direct corollary of Lemma 3b is that:

**Lemma 4** In the steady state, the vacancy rate for owner-occupied houses is increasing in $\alpha$.

Housing market transactions per time unit in the model are comprised of (i) $\alpha\delta n_M$ houses sold from households to flippers, (ii) $\eta n_F$ houses flippers sell to households, and (iii) $\eta n_U$ houses sold by one household to another, adding up to an aggregate transaction volume,

$$TV = \alpha\delta n_M + \eta n_F + \eta n_U.$$  \hspace{1cm} (10)

**Lemma 5** In the steady state, transaction volume is increasing in $\alpha$.

The usual measure of turnover in the housing market is the time it takes for a house to be sold, what is known as Time-On-the-Market (TOM). Given that houses sold in the investment market are on the market for a vanishingly small time interval and houses sold in the search market for a length of time equal to $1/\eta$ on average, we may define the model’s average TOM as

$$TOM = \frac{\alpha\delta n_M}{TV} \times 0 + \frac{\eta n_F + \eta n_U}{TV} \times \frac{1}{\eta}.$$  \hspace{1cm} (11)

**Lemma 6** In the steady state, on average, TOM is decreasing in $\alpha$.

TOM is a measure of the turnover of houses for sale, and as such it does not carry any direct welfare implications. A more household-centric measure of turnover is the length of time a household (rather than a house) has to stay unmatched. We define what we call Time-Between-Matches (TBM) as the sum of two spells: (1) the time it takes for a household to sell the old house, and (2) the time it takes to find a new match. While the first spell (TOM) on average is shorter with an increase in $\alpha$, the second is longer as the increase in $\theta$ to accompany the increase in $\alpha$ causes $\mu$ to fall. A priori then it is not clear what happens to the average length of the whole spell. The old house is sold more quickly. But it also takes longer on average to find a new match in a market with more buyers and fewer sellers. To examine which effect dominates, write the model’s average TBM as

$$TBM = \frac{1}{\mu} + (1 - \alpha) \left(\frac{1}{\eta} + \frac{1}{\mu}\right).$$

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where $1/\mu$ is the average TBM for households who sell in the investment market\textsuperscript{12} and $1/\eta + 1/\mu$ for households who sell in the search market.\textsuperscript{13}

**Lemma 7** In the steady state, on average, TBM is decreasing in $\alpha$.

Lemma 7 may be taken as the dual of Lemma 3a ($\partial n_M/\partial \alpha > 0$). When matched households are more numerous in the steady state, on average, people must be spending less time between matches.

Up to this point, the model is purely mechanical. Given $\alpha$, market tightness $\theta$ is completely isomorphic of the determination of housing prices in equilibrium. The same conclusion carries over to the determination of vacancy, turnover, and transaction volume. If not for the inclusion of flippers in the model housing market, $\alpha$ is identically equal to 0 and Lemma 2 would have completed the analysis of everything that seems to be of any interest. With the inclusion of flippers and their market share measured by $\alpha$, Lemmas 6 and 7 show how changes in the latter affect the turnovers of houses and households, which can have important consequences on welfare—a question we shall address in the following. But first $\alpha$ obviously should be made endogenous to which we next turn.

### 2.4 Asset values and housing prices

**Asset values for flippers** Let $V_F$ be the value of a vacant house to a flipper and $p_{FS}$ the price she expects to receive for selling it in the search market. In the steady state,

$$r_F V_F = \eta (p_{FS} - V_F). \tag{12}$$

Let $p_{FB}$ be the price the flipper has paid for the house in the competitive investment market in the first place. In equilibrium, where any excess returns on real estate investment are eliminated,

$$p_{FB} = V_F. \tag{13}$$

**Asset values for households** There are three (mutually exclusive) states to which a household can belong at any one time,

1. in a matched house; value $V_M$,
2. in a mismatched house; value $V_U$,
3. in rental housing; value $V_R$.

\textsuperscript{12}The household sells the old house instantaneously. Given a house-finding rate $\mu$, the average TBM is then $1/\mu$.

\textsuperscript{13}Let $t_1$ denote the time it takes the household to sell the old house in the search market and $t_2 - t_1$ the time it takes the household to find a new match after the old house is sold. Then the household’s TBM is just $t_2$. On average, $E[t_2] = \int_0^\infty \eta e^{-\eta t_1} \left( \int_{t_1}^{\infty} t_2 \mu e^{-\mu(t_2-t_1)} dt_2 \right) dt_1 = 1/\eta + 1/\mu$. 

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The flow payoff for a matched owner-occupier first of all includes the utility she derives from staying in a matched house \( u \). The match will be broken, however, with probability \( \delta \), after which the household may sell the house right away in the investment market at price \( p_{FB} \) and switch to rental housing immediately thereafter. Alternatively, the household can continue to stay in the house while trying to sell it in the search market. In all then,

\[
r_H V_M = u + \delta \left( \max \{ V_R + p_{FB}, V_U \} - V_M \right). \tag{14}
\]

Let \( p_H \) denote the price a household-seller expects to receive in the search market. Then the flow payoff of a mismatched owner-occupier is equal to

\[
r_H V_U = \eta \left( V_R + p_H - V_U \right). \tag{15}
\]

Two comments are in order. First, in (15), the mismatched owner-occupier is entirely preoccupied with disposing the old house while she makes no attempt to search for a new match. This is due to the assumption that a household cannot hold more than one house at a time and the search process is memoryless. Second, under (14) and (15), the household has only one chance to sell the house in the investment market—at the moment the match is broken. Those who forfeit this one-time opportunity must wait out a buyer in the search market to arrive. This restriction is without loss of generality in a steady-state equilibrium, in which the asset values and housing prices stay unchanging over time. No matter, after the old house is disposed of, the household moves to rental housing to start searching for a new match. Hence, with \( \alpha \) equal to the fraction of houses offered for sale in the search market held by flippers and \( 1 - \alpha \) the fraction held by ordinary households,

\[
r_H V_R = -q + \mu \left( V_M - \left( \alpha p_{FS} + (1 - \alpha) p_H \right) - V_R \right). \tag{16}
\]

**Bargaining** Prices in the search market fall out of Nash bargaining between a matched buyer-seller pair. There is only one buyer type in the search market—households in rental housing. The sellers can be either flippers or mismatched home-owners. When a household-buyer is matched with a flipper, the division of surplus in Nash Bargaining satisfies

\[
V_M - p_{FS} - V_R = p_{FS} - V_F, \tag{17}
\]

whereas when the household-buyer is matched with a household-seller, the division of surplus in Nash Bargaining satisfies\(^{14}\)

\[
V_M - p_H - V_R = V_R + p_H - V_U. \tag{18}
\]

\(^{14}\)With multiple types, the assumption of perfect information in bargaining is perhaps stretching a bit. We could have specified a bargaining game with imperfect information as in Harsanyi and Selten (1972), Chatterjee and Samuelson (1983), or Riddell (1981), for instance. It is not clear what may be the payoffs for the added complications.
2.5 Which market to sell?

Write
\[ \Delta = V_R + p_{FB} - V_U \]  \tag{19} as the difference in payoff for a mismatched homeowner between selling in the investment market \((V_R + p_{FB})\) and in the search market \((V_U)\). By (12), (13), (17), and (18),
\[ \Delta = 2 \left( \frac{\eta + r_F}{\eta} p_{FB} - p_H \right) = 2 (p_{FS} - p_H). \]

That is, mismatched homeowners prefer to sell right away in the investment market if the given instantaneous reward \((p_{FB})\) dominates an appropriately-discounted reward of waiting out in the search market \(\left( \frac{\eta}{\eta + r_F} p_H \right)\). In turn, the condition is equivalent to whether the price at which flippers sell in the search market \((p_{FS})\) is greater than or less than the price at which ordinary households sell in the same market \((p_H)\). Lemma 9 in the Appendix presents the solutions of \(p_{FS}\) and \(p_H\), together with the various asset values from (12)-(18). It is then straightforward to show that \(p_{FS} - p_H\) has just the same sign as
\[ S_\Delta \equiv \left( \frac{r_H}{r_F} - 1 - z \right) \eta + \mu - 2 (\delta + r_H) z, \]  \tag{20}
where \(z = q/v\).\textsuperscript{15} If \(S_\Delta\) is thought of as measuring the incentives for mismatched homeowners to sell in the investment market, such incentives are weakened at larger \(r_F\) and \(q\). Intuitively, flippers may only offer a lower price when they face a higher cost of financing and it becomes less attractive for mismatched homeowners to switch to rental housing sooner by quickly selling in the investment market if rental housing is more costly. On the other hand, the incentives are strengthened at a larger \(v\). One interpretation is that it becomes more attractive to shorten Time-Between-Matches by quickly selling in the investment market when there is a higher reward for staying in a matched owner-occupied house. Most of all, however, \(S_\Delta\) can be thought of as a function of \(\theta\). In this connection, we can define a correspondence \(\hat{\alpha} : \mathcal{R}^+ \Rightarrow [0,1]\), whereby
\[
\hat{\alpha} (\theta) = \begin{cases} 
0 & S_\Delta (\theta) < 0 \\
[0,1] & S_\Delta (\theta) = 0 \\
1 & S_\Delta (\theta) > 0 
\end{cases} \tag{21}
\]
that gives the fraction of mismatched homeowners who choose to sell in the investment market.

\textsuperscript{15} Lemma 9 in the Appendix presents two sets of prices and asset values, one derived under the assumption that \(\Delta \leq 0\) and the other \(\Delta > 0\). In either case, \(p_{FS} - p_H\) is seen to have the same sign as \(S_\Delta\) in (20).
2.6 Equilibrium

We now have two steady-state relations between $\alpha$ and $\theta$: the $\tilde{\theta}(\alpha)$ function in (9) and the $\hat{\alpha}(\theta)$ correspondence in (21). A steady-state equilibrium is any $\{\alpha, \theta\}$ pair that simultaneously satisfies the two relations.

3 Analysis

3.1 Existence of equilibrium

To show the existence of equilibrium, it is useful to define $F(\alpha) \equiv \hat{\alpha} \left( \tilde{\theta}(\alpha) \right)$, a correspondence mapping $[0,1]$ into itself. Equilibrium then is any fixed point of $F$.

Proposition 1 Equilibrium exists for all positive $\{r_H, r_F, v, q, \delta, H\}$ tuple.

3.2 Multiplicity

To check for uniqueness and multiplicity, we begin with inverting the $\tilde{\theta}(\alpha)$ function in (9) to define $\tilde{\alpha} \equiv \tilde{\theta}^{-1}$, whereby $\tilde{\alpha} : [\tilde{\theta}_L, \tilde{\theta}_U] \to [0,1]$. Given that $\partial \tilde{\theta} / \partial \alpha > 0$, 

Figure 4: The $\tilde{\alpha}$ function
likewise, $\partial \tilde{\alpha} / \partial \theta > 0$, for $\theta \in \left[ \tilde{\theta}_L, \tilde{\theta}_U \right]$. That is, $\tilde{\alpha} (\theta)$ increases continuously from 0 at $\theta = \tilde{\theta}_L$ to 1 at $\theta = \tilde{\theta}_U$. Figure 4 depicts an example of the $\tilde{\alpha} (\theta)$ schedule. Now, for the $\tilde{\alpha} (\theta)$ correspondence:

**Lemma 8** As a function of $\theta$,

a. if $r_F < \hat{r}_F$, where

$$\hat{r}_F = \frac{r_H}{1 + z},$$

and assuming that

$$2 \frac{\partial \eta}{\partial \theta} \left( \eta - \theta \frac{\partial \eta}{\partial \theta} \right) + \theta \frac{\partial^2 \eta}{\partial \theta^2} \eta \leq 0, \quad \text{17}$$

then $S_\Delta$ is U-shaped, and that $\lim_{\theta \to 0} S_\Delta = \lim_{\theta \to \infty} S_\Delta = \infty$.

b. if $r_F \geq \hat{r}_F$, then $\partial S_\Delta / \partial \theta < 0$ throughout, and that $\lim_{\theta \to 0} S_\Delta > 0$ but $\lim_{\theta \to \infty} S_\Delta < 0$.

By Lemma 8a, for $r_F < \hat{r}_F$, $S_\Delta$ can stay positive throughout (the upper curve in the left panel of Figure 5). In this case, $\tilde{\alpha} (\theta) = 1$ for all $\theta \geq 0$, as depicted in Panel A of Figure 6. Alternatively, there can also be two roots to $S_\Delta = 0$ (the lower curve in the left panel of Figure 5), say $\hat{\theta}_1$ and $\hat{\theta}_2$, where $\hat{\theta}_1 < \hat{\theta}_2$. In this case,

$$\tilde{\alpha} (\theta) = \begin{cases} 
0 & \theta \in \left( \hat{\theta}_1, \hat{\theta}_2 \right) \\
[0, 1] & \theta = \hat{\theta}_1 \text{ and } \hat{\theta}_2 \\
1 & \theta \in \left[ 0, \hat{\theta}_1 \right) \cup \left( \hat{\theta}_2, \infty \right) 
\end{cases},$$

\[16\] Indeed, a closed-form solution for $\tilde{\alpha} (\theta)$ exists, given by (52) in the Appendix.

\[17\] The condition is guaranteed to hold if $\eta$ is isoelastic.
as depicted in Panels C and D of Figure 6. By Lemma 8b, for $r_F \geq \hat{r}_F$, there is a unique root to $S_\Delta = 0$, say $\tilde{\theta}$, as shown in the right panel of 5. The $\tilde{\alpha}(\theta)$ correspondence that follows is thus given by

$$\tilde{\alpha}(\theta) = \begin{cases} 
1 & \theta < \tilde{\theta} \\
[0, 1] & \theta = \tilde{\theta} \\
0 & \theta > \tilde{\theta} 
\end{cases},$$

as depicted in Panels E and F of Figure 6.

In all, for small $\theta = B/S$, $\tilde{\alpha}(\theta) = 1$ must hold, whether or not $r_F < \hat{r}_F$—whereas it can take a long time to sell in a slack search market in which there are relatively few buyers, mismatched homeowners are better off to just sell in the investment market at a discount. In a tight search market with a large $\theta$, houses are not just sold quickly but can be at a high price as well. Then, mismatched homeowners may prefer to just sell right away in the investment market to capitalize on the high housing price sooner. By Lemma 8a, for smaller $r_F$ ($r_F < \hat{r}_F$), under which flippers can afford to pay relatively high prices, it is indeed optimal for mismatched homeowners to also sell in the investment market for large $\theta$. For intermediate $\theta$, neither of the two incentives to sell in the investment market is strong enough to cause $S_\Delta \geq 0$, so that mismatched homeowners would prefer to wait it out in the search market.

Given the $\tilde{\alpha}(\theta)$ schedule in Figure 4 and the $\tilde{\alpha}(\theta)$ graphs in Figure 6, equilibrium is any $\theta$ at which $\tilde{\alpha}(\theta) \subset \tilde{\alpha}(\theta)$. Now, if $0 \subset \tilde{\alpha}(\tilde{\theta}_L)$, then $\theta = \tilde{\theta}_L$ and $\alpha = 0$ is a steady-state equilibrium since by construction, $\tilde{\alpha}(\tilde{\theta}_L) = 0$. In this equilibrium, all sales and purchases are between two end-users while turnover is slowest. On the other hand, if $1 \subset \tilde{\alpha}(\tilde{\theta}_U)$, then $\theta = \tilde{\theta}_U$ and $\alpha = 1$ is a steady-state equilibrium since by construction, $\tilde{\alpha}(\tilde{\theta}_U) = 1$. In this equilibrium, all transactions are intermediated and turnover is fastest. In between, there can be equilibrium in which $\theta = \tilde{\theta}, \tilde{\theta}_1, \text{or} \tilde{\theta}_2$ and $\alpha = \tilde{\alpha}(\theta)$ if the given $\theta \in (\tilde{\theta}_L, \tilde{\theta}_U)$. In such an equilibrium, with mismatched homeowners indifferent between selling in the investment and search markets, a fraction, but only a fraction, of all transactions are intermediated.

**Proposition 2** For sufficiently small $r_F$, the unique equilibrium is $\theta = \tilde{\theta}_U$; for $r_F \geq \hat{r}_F$, there is a unique equilibrium at either $\theta = \tilde{\theta}_L$, $\tilde{\theta}$, or $\tilde{\theta}_U$. For $r_F < \hat{r}_F$ and where there are two $\theta$ that solves $S_\Delta = 0$ (Panels C and D of Figure 6), there exist at least two equilibria at $\theta = \tilde{\theta}_2$ and $\theta = \tilde{\theta}_1$ or $\tilde{\theta}_L$ if and only if $\tilde{\theta}_2 \in [\tilde{\theta}_L, \tilde{\theta}_U]$. A third equilibrium at a distinct $\tilde{\theta}_U$ exists if $\tilde{\theta}_U > \tilde{\theta}_2$.

Figures 7 and 8 illustrate the situations covered by the second part of the Proposition. In both figures, with $\tilde{\theta}_U > \tilde{\theta}_2$, there are three equilibria. Intuitively, in a tight market with a large $\theta = B/S$, under which houses are sold at relatively high
Figure 6: The $\hat{\alpha}(\theta)$ correspondence
prices, mismatched homeowners find it advantageous to sell their old houses quickly in the investment market. On the other hand, if all mismatched houses are sold in the investment market in the first instance, there will be rapid turnover and few houses are for sale, resulting in a tight market. In this way, $\theta = \theta_U$ and $\alpha = 1$ is equilibrium in Figures 7 and 8. For smaller $\theta$, households’ incentives to sell in the investment market are weakened. Meanwhile, if fewer or none at all mismatched houses are sold in the investment market, turnover slows down and more houses are for sale in the search market, resulting in a relatively slack market. As a result, a smaller $\alpha$ and a smaller $\theta$ is also equilibrium in Figures 7 and 8.

With multiple steady-state equilibrium, the presence of flippers’ in the market can be fickle, especially when the equilibrium the market happens to be in is unstable, a subject we should turn to shortly. In general, where there are multiple equilibrium, any seemingly unimportant shock can completely dislocate the market from one equilibrium and move it to another, causing catastrophic declines in flippers’ market share, turnover, and transaction volume. To accompany such discrete declines in the activities of flippers can be discrete declines in housing price, a subject we should follow up in Section 3.5. In sum, the model housing market can be consistent with a certain amount of volatility. In Section 5, we should calibrate the model to the turnovers characteristics of the U.S. housing market to assess quantitative how important such a channel of volatility can be.
3.3 Dynamics and stability

In Leung and Tse (2011), we show that, of the five types of steady-state equilibrium:

1. $\theta = \hat{\theta}_L$,
2. $\theta = \hat{\theta}_U$,
3. $\theta = \hat{\theta}_1$,
4. $\theta = \hat{\theta}_2$,
5. $\theta = \hat{\theta}$,

all except the $\hat{\theta}_2$ equilibrium are guaranteed to be locally stable, whereas the $\hat{\theta}_2$ equilibrium is almost always unstable.\(^{18}\) Intuitively, the $\hat{\theta}_2$ equilibrium is unstable because it lies at where the $\hat{\alpha}(\theta)$ correspondence is “increasing”. Consider for instance, a small negative perturbation from the $\hat{\theta}_2$ equilibrium in Figure 7. Then, $\Delta$ should turn negative, after which no mismatched homeowners should still sell in the investment market. When turnover slows down as a result, market tightness falls. To follow is a further decline in $\theta$. Eventually, the market should just settle at the $\hat{\theta}_1$ steady-state equilibrium in the Figure. Conversely, a positive perturbation from the $\hat{\theta}_2$ equilibrium should send the market to the $\hat{\theta}_U$ steady-state equilibrium.\(^{19}\) We also show in Leung and Tse (2011) that local stability notwithstanding, convergence to the $\hat{\theta}_L$, $\hat{\theta}_1$, and $\hat{\theta}$ steady-state equilibria is almost always oscillatory, with $\theta$ under- and overshooting the given steady state and $\Delta$ changing signs in the transition dynamics as long as initially $n_F > 0$. If both $\alpha$ and $n_F$ are forced to zero a priori,

\(^{18}\)We cannot completely rule out local stability for the $\hat{\theta}_2$ equilibrium but find the conditions for it to be the case highly improbable.

\(^{19}\)Granted that the $S_\Delta$ and $\hat{\theta}$ functions are merely steady-state relations, the arguments above no doubt involve a good dose of hand waving. The full-fetched dynamic analysis is in Leung and Tse (2011).
however, the only equilibrium is a $\tilde{\theta}_L$ equilibrium, the convergence of which must be direct.

3.4 Cost of financing and flippers’ market share

By Proposition 2, $\alpha = 1$ for small $r_F$ and then at larger $r_F$, $\alpha$ can fall below unity. This is intuitive. Flippers can afford to pay the highest price when they can finance investment at the least cost. As $r_F$ increases, their presence should only diminish. More precisely, by Lemma 10 in the Appendix:

(A) For the smallest $r_F$, $S_\Delta > 0$ for all $\theta$, so that the $\tilde{\alpha}(\theta) = 1$ throughout as shown in Panel A of Figure 6.

(B) As $r_F$ increases to some given level below $\hat{r}_F$, the minimum of the $S_\Delta$ curve in the left panel in Figure 5 would just be tangent to the horizontal axis at which $\tilde{\alpha}(\theta) = [0, 1]$, whereas $\tilde{\alpha}(\theta)$ remains equal to unity at all other $\theta$; the $\tilde{\alpha}(\theta)$ correspondence turns into the one in Panel B of Figure 6.

(C) Thereafter, as $r_F$ continues to increase, the minimum of the $S_\Delta$ curve in the left panel in Figure 5 dips below zero. With two roots to $S_\Delta = 0$, the $\tilde{\alpha}(\theta)$ correspondence becomes like the one in Panel C of Figure 6.

(D) While $r_F$ remains below $\hat{r}_F$, $\partial \tilde{\theta}_1 / \partial r_F < 0$ and $\partial \tilde{\theta}_2 / \partial r_F > 0$. Panel C turns into D.

(E) In the limit as $r_F \rightarrow \hat{r}_F$, $\hat{\theta}_2 \rightarrow \infty$ and $\hat{\theta}_1 \rightarrow \hat{\theta}_U$—the unique root of $S_\Delta = 0$ at which $r_F = \hat{r}_F$ just holds. Panel D turns into E.

(F) Given that $r_F \geq \hat{r}_F$, $\partial \tilde{\theta} / \partial r_F < 0$, while the limiting value of $\tilde{\theta}$ as $r_F$ becomes arbitrarily large, $\tilde{\theta}_L$, is straightly positive. Panel E gradually evolves toward F.

Granted that $\tilde{\alpha}(\theta)$ is independent of $r_F$, the effects of an increase in $r_F$ on equilibrium $\{\alpha, \theta\}$ can then be read off by superimposing the same given $\tilde{\alpha}(\theta)$ successively into Panels A to F of Figure 6. We can conclude from this exercise the following.

Proposition 3

a. For sufficiently small $r_F$, equilibrium is $\theta = \tilde{\theta}_U$ and $\alpha = 1$.

b. For larger $r_F$, $\alpha$ must fall below unity and $\theta$ below $\tilde{\theta}_U$ in equilibrium.

i. If $\tilde{\theta}_L \geq \tilde{\theta}_L$, as $r_F$ becomes large enough, $\theta = \tilde{\theta}_L$ and $\alpha = 0$.

ii. Otherwise, $\alpha$ stays positive for arbitrarily large $r_F$.

c. If $\theta = \tilde{\theta}_U$ and $\alpha = 1$ is not equilibrium at a certain $r_F$, the pair is not equilibrium for any larger $r_F$. If $\theta = \tilde{\theta}_L$ and $\alpha = 0$ is equilibrium at a certain $r_F$, the pair remains equilibrium for any larger $r_F$.

d. In any $\tilde{\theta}_1$ or $\tilde{\theta}$ equilibrium, $\alpha$ is decreasing in $r_F$. 

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e. In a $\theta_2$ equilibrium, $\alpha$ is increasing in $r_F$.

Parts (a)-(d) of the Proposition conform to the intuitive notion that an increase (decrease) in $r_F$ should have a negative (positive) impact on flippers’s market share. Still, it is surprising that $\alpha$ can remain strictly positive even for arbitrarily large $r_F$, for which flippers can only finance investment at a huge disadvantage vis-a-vis ordinary households. The condition for this to be the case, $\theta_L < \theta_L$, holds for small $\theta_L$. By (50) in the proof of Lemma 2, $\partial \theta_L/\partial H < 0$, and that $\lim_{H \rightarrow 1} \theta_L = 0$. In general, we can show that $\partial \theta / \partial H < 0$ for each $\alpha \in [0,1]$. With a larger housing stock, there can only be more units for sale, other things being equal. In the mean time, if there are fewer households in rental housing, there can only be fewer buyers in the search market. Then, a role for flippers can remain no matter what, if the extreme slack in the end-user search market makes selling very difficult.

Part (e) is rather counterintuitive too for it says that an increase (decrease) in $r_F$ can have a positive (negative) impact on flippers’ market share in equilibrium. In Figures 7 and 8, $\theta_2$ is the smallest $\theta$ for which mismatched homeowners prefer to sell in the investment market to quickly capitalize on the high housing price. Now, flippers can only afford to pay lower prices at a larger $r_F$ at each $\theta$, pushing up the lower bound $\theta_2$ of the $[\theta_2, \infty)$ interval over which people prefer to sell in the investment market. If equilibrium is at $\theta_2$, the increase in $\theta_2$ is a movement up $\alpha(\theta)$, giving rise to a larger $\alpha$.

On the contrary, by Part (d), an increase (decrease) in $r_F$ will have the expected negative (positive) impact on flippers’ market share in a $\theta_1$ or $\theta$ equilibrium. In Figures 7, $\theta_1$ is the endpoint of the interval $[0, \theta_1]$, over which mismatched homeowners find it advantageous to sell in the investment market because it can take a long time to sell in the search market. An increase in $r_F$, by lowering the price flippers are able to offer to mismatched homeowners, shortens the interval. Where the decline in $\theta_1$ is a movement down the $\alpha(\theta)$ schedule, in equilibrium $\alpha$ falls. A similar analysis applies to a $\theta$ equilibrium.

On the whole, one can conclude that at a larger $r_F$, fewer transactions would be intermediated if one is willing to dismiss any $\theta_2$ equilibrium on stability grounds and the possibility that agents may coordinate to a larger $\alpha$ equilibrium in case there exist multiple equilibrium. However, we do not think that the analysis, strictly speaking, allows us to reach any such unambiguous conclusions. And we cannot rule out occasions, admittedly rare, in which an increase in $r_F$ can, rather perversely, be followed by a heightened presence of flippers in the model housing market.

### 3.5 Housing prices

**Housing prices in no-intermediation equilibrium** Absent flippers, all housing market transactions are between pairs of end-user households at price $p_H$, given by
(33) evaluated at $\alpha = 0$,

$$p_H = \frac{(\eta + r_H - \mu) v + (2\delta + \eta + 2r_H) q}{(2\delta + \eta + 2r_H) r_H}.$$  \hfill (23)

In Leung and Tse (2011), we find that, nearing the given steady state whereby $n_F \approx 0$, housing price and transaction volume move in opposite directions in the transition dynamics. Vacancy either stays equal to zero (when $n_F = 0$ in the transition) or declines monotonically (when $n_F > 0$ in the transition).

**Housing prices in fully-intermediated equilibrium** In a fully-intermediated equilibrium, all houses are first sold from mismatched homeowners to flippers at $p_{FB}$, given by (41), in the investment market and then at $p_{FS}$, given by (40), from flippers to end-user households in the search market, where $p_{FB} < p_{FS}$. Now, houses sold from households to flippers stay on the market for a vanishingly small time interval, whereas houses sold from flippers to households in the search market stay on the market for, on average, $1/\eta > 0$ units of time. There should then be a positive cross-section relation between price and TOM in the model housing market, as in the real-world housing market. Besides, with $p_{FB} < p_{FS}$, the model trivially predicts that houses bought by flippers are at lower prices than are houses bought by non-flippers. Both Depken et al. (2009) and Bayer et al. (2011) find the tendency to hold in their respective hedonic price regressions. In Leung and Tse (2011), we find that around the $\theta_U$ steady state, price and vacancy move in opposite directions, whereas there exists no definite relationship between the two variables and transaction volume in the transition dynamics.

**Housing prices in partially-intermediated equilibrium** In a steady-state equilibrium in which mismatched homeowners sell in both the investment and search markets, in addition to the two prices $p_{FB}$ and $p_{FS}$ for transactions between a flipper and an end-user household, there will also be transactions between two end-user households, carried out at price $p_H$. In any partially-intermediated equilibrium, $p_H = p_{FS}$, however, so that all transactions in the search market are at the same price after all. In this case, $p_H = p_{FS}$ is given by either (33) or (34) evaluated at $S_{\Delta} = 0$,

$$p_H = p_{FS} = \frac{(\eta + r_F) v}{r_F (2\delta + \eta + 2r_H)}.$$  \hfill (24)

whereas in the investment market, $p_{FB}$ is given by (35), similarly evaluated at $S_{\Delta} = 0$,

$$p_{FB} = \frac{\eta v}{r_F (2\delta + \eta + 2r_H)}.$$  \hfill (25)

Just as in the fully-intermediated equilibrium, a positive relation between TOM and price holds in the cross section and houses bought by flippers are at lower prices.

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20The equations for the housing prices and asset values referred to hereinafter can be found in Lemma 9 in the Appendix.
Prices across equilibria  Recall that across steady-state equilibria, $\theta = B/S$ is largest in the equilibrium where flippers are most numerous. A priori, one would expect that prices are also highest in such an equilibrium where the competition among buyers is most intense.

Proposition 4  Across steady-state equilibria in case there exist multiple equilibrium, housing prices in both the search and investment markets are highest in the equilibrium with the tightest market and lowest in the equilibrium with the slackest market.

Now, a direct corollary of the Proposition and Lemmas 4-7 is that:

Proposition 5  Across steady-state equilibria in case multiple equilibrium exist, price, vacancy, and transaction volume increase or decrease together from one to another equilibrium, whereas average TOM and TBM move with the former set of variables in the opposite direction.

Interest rate shocks  As usual, in the present model, interest rates can play an important role in determining housing prices. First, in a no-intermediation equilibrium, a decline in $r_H$ will lead to higher prices, as can be verified by differentiating (23), but market tightness, vacancy, turnover, and transaction volume will just stay at the given levels entirely determined by the housing stock, the rate matched households becomes mismatched, and the matching technology in the search market, as described in Lemmas 2-7 with $\alpha = 0$. In the entire absence of flippers, not surprisingly, $r_F$ plays no role at all.

In a fully-intermediated equilibrium, $r_F$ should certainly be an important factor in determining housing prices. Indeed, both $p_{FB}$ and $p_{FS}$ are decreasing in $r_F$, as can be verified by differentiating (41) and (40) in the Appendix. Just as in the no-intermediation equilibrium, such interest rate shocks will leave no impact on market tightness, transaction volume, turnover, and vacancy, if all transactions were already intermediated in the first place.

In a partially-intermediated equilibrium where $S_\Delta = 0$, prices in the search market $p_{FS}$ ($= p_H$), as well as in the investment market, $p_{FB}$, are decreasing in $r_F$, just as they are in a fully-intermediated equilibrium. But where $\theta$ was not already fixed at the boundary of $\tilde{\theta}_U$, housing prices can also change to follow any movements in $\theta$ triggered by the given interest rate shock. Differentiating (24) and (25) with respect to $\theta$ confirms that prices in both markets are increasing in $\theta$, so that prices are higher in a tighter market. Hence, if a given positive (negative) interest rate shock should cause $\theta$ to decrease (increase), there will be lower (higher) housing prices to follow because of a direct negative (positive) impact and of an indirect effect through dampening (raising) flippers’ presence in the market. In this case, when the two effects work in the same direction, a given interest rate shock can cause substantially more housing price volatility than in a model that only allows for the usual effect of interest rate on asset price.
However, a positive interest rate shock need not cause θ and α to fall. By Proposition 3e, along any $\hat{\theta}_2$ equilibrium, the given interest rate shock will be followed by increases in θ and α. Furthermore, in case there exist multiple equilibrium, the shock can dislocate the market from a given equilibrium type and send it to another equilibrium type. In what direction housing prices will move then cannot be unambiguously read off from (24) and (25) as the direct effect of any interest rate shock and the indirect effect through the changes in θ can affect housing prices differently. To proceed, we solve $S_\Delta = 0$ for $r_F$ and substitute the result into (24) and (25), respectively,

$$p_{FS} = p_H = \frac{(\eta + r_H - \mu) v + (2\delta + \eta + 2r_H) q}{r_H(2\delta + \eta + 2r_H)},$$  \hspace{1cm} (26)$$

$$p_{FB} = \frac{(\eta - \mu) v + (2\delta + \eta + 2r_H) q}{r_H(2\delta + \eta + 2r_H)}. \hspace{1cm} (27)$$

The two equations are independent of $r_F$; whatever effects a given change in $r_F$ will have on housing prices are subsumed through the effects of changes in θ that follow the change in $r_F$ obtained from holding $S_\Delta = 0$. To evaluate the the effects of $r_F$ on housing prices then is to differentiate these two expressions just with respect to $\theta$.

**Proposition 6** Across steady-state equilibria and holding $S_\Delta = 0$, a shock to $r_F$, whether positive or negative, will cause housing price to increase (decrease), as long as to follow the interest rate shock are increases (decreases) in $\theta$ and $\alpha$.

By Proposition 6, the indirect effect of an interest rate shock on housing prices through the changes in flippers’ presence and then in market tightness always dominates the direct effect shall the two be of opposite tendencies. A surprising implication then is that a given increase in flippers’ cost of financing can actually lead to an increase in housing prices, if to follow the higher interest rate is also a heightened presence of flippers’ in the market. No matter, a direct corollary of Lemmas 4-7 and Proposition 6 is that:

**Proposition 7** Across steady-state equilibria and holding $S_\Delta = 0$, a shock to $r_F$ will cause housing price, transaction volume, and vacancy to move in the same direction, whereas average TOM and TBM will move in the opposite direction.

### 3.6 A general interest rate shock

So far, we have restricted attention to analyzing how an increase in $r_F$ alone affects the extent of intermediation and the consequent effects on housing prices. Many of the implications, however, survive for a general change in interest rate that affects both flippers and ordinary households alike. To begin, write $R$ for $r_H/r_F$ in (20),

$$S_\Delta = (R - 1 - z) \eta + \mu - 2(\delta + r_H) z. \hspace{1cm} (28)$$
Then equiproportionate increases in $r_H$ and $r_F$, while leaving $R$ unchanged, lower $S_\Delta$. A general increase in interest rate thus weakens mismatched homeowners’ incentives to sell in the investment market, just as an increase in $r_F$, holding fixed $r_H$, does. Analogous to Proposition 3 is that:

**Proposition 8** Holding constant $R$ at some given level,

a. for sufficiently large $r_H$, in equilibrium, $\theta < \tilde{\theta}_U$ and $\alpha < 1$. Eventually, as $r_H$ rises above a certain level, $\theta = \tilde{\theta}_L$ and $\alpha = 0$ must obtain.

b. if $\theta = \tilde{\theta}_U$ and $\alpha = 1$ is not equilibrium at some $r_H$, the pair is not equilibrium for larger $r_H$. If $\theta = \tilde{\theta}_L$ and $\alpha = 0$ is equilibrium at some $r_H$, the pair remains equilibrium for larger $r_H$.

c. In any $\hat{\theta}_1$ or $\hat{\theta}$ equilibrium, $\alpha$ is decreasing in $r_H$.

d. In a $\hat{\theta}_2$ equilibrium, $\alpha$ is increasing in $r_H$.

The effects of the general increase in the cost of financing on housing prices are similar to those of an increase in $r_F$ alone. The following proposition summarizes the results.

**Proposition 9**

a. In both the no-intermediation and fully-intermediated equilibria, equiproportionate increases in $r_H$ and $r_F$ lower housing prices.

b. In a partially-intermediated equilibrium,

i. equiproportionate increases in $r_H$ and $r_F$, holding $\theta$ fixed, lower housing prices;

ii. across steady-state equilibria and holding $S_\Delta = 0$, equiproportionate changes in $r_H$ and $r_F$, whether positive or otherwise, cause $p_{FB}$ to increase (decrease) as long as to follow the interest rate shocks are increases (decreases) in $\theta$ and $\alpha$ and for $R \in [0, 1 + z]$; the same effect is felt on $p_{FS} = p_H$ for $R$ in neighborhoods of $R = 0, 1,$ and $1 + z$.

Notice that by (b.i), if to follow the equiproportionate increases in $r_H$ and $r_F$ is a decline in flippers’ presence, housing prices must unambiguously fall, just as when an increase in $r_F$ alone causes $\theta$ and $\alpha$ to fall will lower housing prices for sure. More generally, (b.ii) is concerned with how prices may change when the interest rate shocks may be followed by either an increase or a decline in $\theta$, as in the situations covered in Proposition 6. Also as in Proposition 6, here prices will increase if $\theta$ and $\alpha$ happen to rise to follow the interest rate shocks, positive or otherwise, if the values of $R$ are appropriately chosen. The last restrictions are sufficient, but not necessary, conditions, and that the conclusions should hold under weaker conditions.
3.7 Welfare

In a steady-state equilibrium where $S_\Delta = 0$, asset values for matched and mismatched homeowners, renters, and flippers are given by respectively,\(^{21}\)

\[ V_M = \frac{(\eta + 2r_H)\upsilon}{r_H(2\delta + \eta + 2r_H)}, \quad (29) \]

\[ V_U = \frac{\eta\upsilon}{r_H(2\delta + 2r_H + \eta)}, \quad (30) \]

\[ V_R = \frac{(r_F - r_H)\eta\upsilon}{r_Fr_H(2\delta + \eta + 2r_H)}, \quad (31) \]

\[ V_F = \frac{\eta\upsilon}{r_F(2\delta + \eta + 2r_H)}. \quad (32) \]

It is straightforward to verify that $V_M$, $V_U$, and $V_F$ are all increasing in $\theta$. Any homeowners—matched or mismatched, end-users or flippers—benefit from the higher housing prices in a tighter market. The asset value for households in rental housing $V_R$, however, is decreasing in $\theta$ if $r_F < r_H$, which is a necessary condition for the multiplicity of equilibrium (Lemma 8a). In this case, would-be buyers are worse off with the higher housing prices in the tighter market. Now, suppose both $\hat{\theta}_1$ and $\hat{\theta}_2$ are steady-state equilibria with the two $\theta$ lying within the interval $[\hat{\theta}_L, \hat{\theta}_U]$. In this case, homeowners are better off in the $\hat{\theta}_2$ equilibrium than in the $\hat{\theta}_1$ equilibrium, whereas renters are better off in the second than in the first equilibria. The two steady-state equilibria then cannot be Pareto-ranked. The same holds for comparisons between the $\hat{\theta}_U$ and $\hat{\theta}_2$ equilibria (Figure 7) and between the $\hat{\theta}_2$ and the $\hat{\theta}_L$ equilibria (Figure 8).

Even though the equilibria cannot be Pareto-ranked, perhaps they can be ranked by aggregate welfare as measured by the sum of the asset values for all agents,

\[ W = n_MV_M + n_UV_U + n_RV_R + n_FV_F. \]

At where $S_\Delta = 0$, the asset values are given by (29)-(32). Again, consider a comparison between the $\hat{\theta}_1$ and $\hat{\theta}_2$ steady-state equilibria. Substituting from (53) to (56) in the Appendix for the various steady-state measures of agents and simplifying,

\[ W = \frac{\eta\upsilon}{2\delta + \eta + 2r_H} \left( \frac{2H}{\eta + \delta} + \frac{H - 1}{r_F^2} + \frac{1}{r_H^2} \right). \]

This expression is guaranteed to be increasing in $\theta$ for large $H$. In this case, there is a larger aggregate asset value in the tighter and higher-priced $\hat{\theta}_2$ equilibrium, where

\(^{21}\)The first two equations are from (36) and (37), respectively. The last two are from (38) and (35), respectively, both evaluated at $S_\Delta = 0$.\]
more transactions are intermediated. For smaller $H$, however, $W$ above is decreasing in $\theta$, so that there is only a smaller $W$ in the $\theta_2$ equilibrium than in the $\theta_1$ equilibrium. Thus, it seems that the equilibria cannot in general be ranked by even aggregate asset value. While all agents, except for households in rental housing, benefit from the higher housing prices in the more active $\theta_2$ equilibrium and that more households are matched in the steady state amid a shorter average Time-Between-Matches, $W$ needs not be higher. Would-be buyers in rental housing are more numerous and they suffer a lower asset value with the higher prices. Such a negative impact on $W$ can more than offset the positive effects of a more active market, especially when there is a small housing stock. Intuitively, given a small $H$, there are few house owners to benefit from the higher prices and faster turnover, while there are many would-be buyers to suffer from the same higher prices and the longer wait for owner-occupied housing. When owner-occupied houses are scarce to begin with, leaving more houses vacant in the hands of flippers can be disproportionately costly. Conversely, in a market endowed with a large $H$, it is hardest to sell and flippers’ role in speeding up turnover is most valued.

4 Time-series relations among housing price, transaction volume, and vacancy

By Propositions 5, 7, and 9, the model predicts housing price, transaction volume, and vacancy should move together over time from one to another steady-state equilibria. The positive time-series relation between housing price and transaction volume is well-known and numerous models have been constructed to account for it. Unique to our analysis is that vacancy should also move in the same direction with the two variables.

The prediction is not obviously inconsistent with the pictures depicted in Figures 1-3 in the Introduction. In a more systematic analysis, we first verify that in the 1991-I to 2010-IV sample period, the three variables are all $I(1)$ at conventional significance levels. Next, we find that the variables are indeed cointegrated by the Johansen Cointegration Test under all the usual trend assumptions. In particular, assuming a quadratic deterministic trend, the Trace test indicates 2 cointegrating equations:

$$\text{Price} - 13034 \times \text{Transaction} = 0,$$
$$\text{Transaction} - 3.87 \times \text{Vacancy} = 0,$$

which together imply that there exist long-run positive relations among the three variables. With other trend assumptions, the Trace tests only indicate 1 cointegrating equation. In a single cointegrating equation with non-zero coefficients for all three variables, the Johansen Cointegration Test indicates 2 cointegrating equations:

$$\text{Price} - 13034 \times \text{Transaction} = 0,$$
$$\text{Transaction} - 3.87 \times \text{Vacancy} = 0,$$

which together imply that there exist long-run positive relations among the three variables. With other trend assumptions, the Trace tests only indicate 1 cointegrating equation. In a single cointegrating equation with non-zero coefficients for all three variables, the Johansen Cointegration Test indicates 2 cointegrating equations:

22 Masters (2007) is also a model in which intermediation in a search and matching environment can be wasteful.
variables, the three cannot move together in the same direction. But if one imposes a priori two cointegrating equations in the estimation, the same qualitative results survive.

5 Quantitative predictions on volatility

Given the possible multiplicity of equilibrium and that an interest rate shock may have important effects on the extent of intermediation, the model can be consistent with a volatile housing market. The question remains as to how important quantitatively such channels can be. In this section, we calibrate the model to several observable turnovers characteristics of the U.S. housing market and study by how much housing prices can fluctuate in response to small interest rate shocks and across steady-state equilibria.

We begin the analysis by assuming a C-D matching function, whereby \( \eta(\theta) = a\theta^b \). By lemmas 2-7 in Section 2, given \( \alpha \), the model’s implications on various aspects of turnovers only depend on \( \delta \), \( H \), and the parameters of the matching function but not on interest rates and preference parameters. Hence, to match values of the three observable turnover variables, namely, transaction volume, vacancy, and TOM, it suffices to calibrate \( \delta \), \( H \), \( a \), and \( b \).\(^{23}\) The parameter values chosen in this exercise are reported in the left panel of Table 1. In the right panel, we then report the lower and upper bounds and the medians of the three turnover variables, which correspond to values of \( \alpha \) equal to 0, 1, and 0.5, respectively, that follow.\(^{24}\) While the values of transaction volume and vacancy appear to be well within their respective ranges as depicted in Figures 1-3, only the lower bound of the calibrated values of TOM is within the range of 3-6 months typically reported in studies of the U.S. housing market.\(^{25}\) Moreover, to the extent that we are choosing the values of four parameters to calibrate three observable variables, there appears to be one degree of freedom too many. With one apparently free parameter and in the absence of any a priori information on what the appropriate value for \( b \) may be, a more innocuous parameterization is perhaps to just set \( b = 0.5 \) at the outset, so as to give equal weights for the masses of buyer and seller in the matching function. However, it turns out that when setting \( b = 0.5 \) a priori and then calibrating the other parameters to match smaller values for TOM, the model inevitably predicts values for vacancy that are far too large in comparison to its empirical counterpart if the model is to match the observed values of transaction volume in the mean time. Given that TOM is notoriously difficult to measure accurately, it is probably prudent to set higher priority to match the

\(^{23}\)Recall that \( H \) is the stock of owner-occupied houses relative to the population of households choosing owner-occupied housing. As such it should not be calibrated to the observed value of the homeownership rate.

\(^{24}\)Transaction volume is measured as a percentage of the housing stock \( H \) as in Figures 1 and 3.

\(^{25}\)For example, Haurin et al. (2010) and Knight (2002).
observed values of vacancy than to match those for TOM.26

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Calibrations</th>
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<tbody>
<tr>
<td></td>
<td>Parameters</td>
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<tr>
<td></td>
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<tr>
<td>(H)</td>
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<td>(a)</td>
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<td>(b)</td>
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<tr>
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<tr>
<td>TOM</td>
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Table 1: Calibrating Turnovers

The four remaining parameters \(\{r_H, r_F, \nu, q\}\) serve to pin down the equilibrium values of the three turnover variables within the ranges reported in Table 1. More importantly for our purpose, the chosen parameterization would help answer how much volatility the model housing market can generate. Now, insofar as only the ratio \(z = q/\nu\), but not the absolute values of \(q\) and \(\nu\), matters for where equilibrium lies and for the percentage price differences across equilibria, we first normalize \(\nu = 1\). If \(r_H\) should correspond to the mortgage interest rate for end-user households, a not unreasonable value is maybe 8%. Our choices for \(z = q\) (given \(\nu = 1\)) and \(r_F\) are dictated by the objective of calibrating the model to possess a multiplicity of equilibrium. The results of interest are reported in Table 2. In all cases, we set \(z = 1.6\). The results in columns 3-5 and 6-8, respectively, correspond to values for \(r_F\) equal to 2.967% and 2.964%.27 In both cases, the model has three equilibria, at \(\theta = \theta_1, \theta_2,\) and \(\theta_U\), as in Figure 7. We also calculate equilibrium for a few other values of \(r_F\), under which equilibrium is unique. The results are reported in the other columns of the table. Prices and aggregate asset values in the table are normalized by their respective equilibrium values for a \(r_F\) of 3.03%.

A value of \(r_F\) equal to 3.03% is a useful benchmark since at this and for any larger \(r_F\), the unique equilibrium is a no-intermediation equilibrium whereby \(\theta = \bar{\theta}_L\). As \(r_F\) falls to 2.968%, column 2 reports that the unique equilibrium becomes one of a partially-intermediated equilibrium at \(\theta = \bar{\theta}_1\). In the mean time, transactions

26Interestingly, Cole and Rogerson (1999) find that the Mortensen-Pissarides (1994) model also has to over-predict unemployment duration—the labor market counterpart of TOM for the housing market—for it to match the usual business-cycle facts.

27A \(r_F\) five-percentage-point below \(r_H\) can make sense if the flippers, but not end-user households, tend to choose mortgages with zero initial or negative amortization, short interest rate reset periods, or low introductory teaser interest rates. Such mortgages obviously are ideal for flippers who plan to sell quickly for short-term gains. Amromin et al. (2012) find that borrowers who take out such “complex” mortages are usually high income individuals with good credit scores. Foote et al. (2012) find that periods of interest rate resets do not tend to trigger significant increases in defaults, consistent with the finding of Amromin et al. that the borrowers of such mortgages are sophisticated investors. Barlevy and Fisher (2010) find that interest-only mortages are used much more heavily in cities with the most rapid increase in housing prices. And then Haughwout et al. (2011) find that states that have undergone the most rapid price increase are states where the share of transaction involving flippers is highest.
<table>
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<th>4</th>
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<td>$r_f$</td>
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<td>2.967%</td>
<td>2.964%</td>
<td>2.963%</td>
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<td>$\hat{\theta}_2$</td>
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<td>$\hat{\theta}_2$</td>
<td>$\hat{\theta}_U$</td>
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<tr>
<td>Trans Volume</td>
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<td>9.57%</td>
<td>6.69%</td>
<td>8%</td>
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<td>4.32%</td>
<td>2.65%</td>
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<td>1.05</td>
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<td>1.08</td>
<td>1.06</td>
<td>1.07</td>
<td>1.08</td>
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</table>

Table 2: Volatility; $r_h = 0.08$, $z = 1.6$

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<tr>
<td>$r_f$</td>
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<td>2.07%</td>
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<td></td>
</tr>
<tr>
<td>Trans Volume</td>
<td>4.44%</td>
<td>8.87%</td>
<td>9.57%</td>
</tr>
<tr>
<td>Vacancy</td>
<td>0%</td>
<td>3.99%</td>
<td>4.32%</td>
</tr>
<tr>
<td>Search mkt price</td>
<td>1</td>
<td>1.24</td>
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<tr>
<td>Inv mkt price</td>
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<tr>
<td>Asset value</td>
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<td>1.08</td>
<td>1.09</td>
</tr>
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</table>

Table 3: Volatility; $r_h = 0.065$, $z = 2$;
are more numerous, more houses become vacant, and most importantly, prices are 12-17% higher than the price in the no-intermediation benchmark. As $r_F$ falls further to 2.967%, $\theta = \hat{\theta}_2$ and $\hat{\theta}_U$ also become equilibrium. Housing prices now can be 30% higher than in the no-intermediation benchmark. In all, seemingly insignificant interest rate shocks can indeed generate sizeable housing price volatility when the given interest rate shock dislocates the model housing market from a no- or low-intermediation equilibrium and sends it to one where many transactions are intermediated. In contrast, with no changes in equilibrium $\theta$, prices in columns 5, 8, and 9 are practically identical amid the given changes in $r_F$.

Fixing $r_F$ at some given value, volatility can also arise when the model housing market coordinates from one to another equilibrium. Columns 3-5 and 6-8 of Table 2 show that prices can differ up to about 10% across steady-state equilibria absent any interest rate shocks. While a 10% housing price volatility appears modest, the model, however, has the potential to imply somewhat larger price volatility across steady-state equilibria. In Table 3, we report equilibrium values for $z = 2$, a $r_H$ of 6.5% and a $r_F$ of 2.07%. In this case, the three equilibria the model possesses are at $\theta = \hat{\theta}_L$, $\hat{\theta}_2$, and $\hat{\theta}_U$, as in Figure 8. Search market price in the $\hat{\theta}_U$ equilibrium is now 26% higher than that in the $\hat{\theta}_L$ equilibrium.

While prices in the steady-state equilibria in Tables 2 and 3 can differ up to 25-30%, the last rows of the two tables show that welfare, as measured by aggregate asset values, differs much less, by up to only about 8-9%. This is not surprising in light of the analysis in Section 3.7. Whereas homeowners benefit from the higher prices and faster turnover, would-be buyers in rental housing are worse off by the same higher prices and longer wait for owner-occupied housing. The welfare gains from intermediation are bounded by the losses buyers suffer amid a tighter market.

6 Concluding remarks

By allowing for the presence of flippers, without any assumed or acquired heterogeneity and endogenous search efforts, our model predicts a positive relation between housing price and TOM in the cross section, a relation found in numerous empirical studies. Our model can also generate the well-known relation between price and transaction volume in the time series. Previous models rely on preference and construction shocks and increasing returns in the matching technology to generate such relations. In our model, such relations are the relations among the different steady-state equilibria, as well as from interest rate shocks. Unique to our analysis is that vacancy should move together with price and transaction volume. This relation appears to be borne out in the data.

If flippers can survive in both slack and tight markets, the multiplicity of equilibrium can be a natural outcome in a frictional housing market. Such a model housing market can then be consistent with a substantial amount of volatility. Undoubtedly, our analysis cannot be the complete analysis of “speculative bubbles” in the housing
market. Credit market conditions, market psychology, and the dynamics of price movements must also feature prominently. Nevertheless, we show that even in the absence of such factors, the interaction of the strength of the incentives to sell quickly to flippers and the influence of these agents’ activities on market tightness suffices to imply an intrinsically volatile housing market.
7 Appendix

7.1 Lemma

Lemma 9 For $\Delta \leq 0$, so that $\max \{V_R + p_{FB}, V_U\} = V_U,$

$$p_H = \frac{((r + r_H)(r + 2r_F) - ((1 - \alpha)(r + r_F) + r_F)\mu r + (2\delta + \eta + 2r_H)(\eta + 2r_H)q}{(2\delta + \eta + 2r_H)(\alpha\mu F + \eta r_H + 2r_F r_H)},$$

$$p_{FS} = \frac{((\eta + r_F)((\eta + 2r_H) - (1 - \alpha)\mu r + (2\delta + \eta + 2r_H)q)}{(2\delta + \eta + 2r_H)(\alpha\mu F + \eta r_H + 2r_F r_H)},$$

$$p_{FB} = V_F = \frac{\eta((\eta + 2r_H) - (1 - \alpha)\mu r + (2\delta + \eta + 2r_H)q)}{(2\delta + \eta + 2r_H)(\alpha\mu F + \eta r_H + 2r_F r_H)},$$

$$V_M = \frac{(r_H(2\delta + \eta + 2r_H)\mu r - r_H(\eta + 2r_F)(2\delta + \eta + 2r_H)q}{r_H(2\delta + \eta + 2r_H)(\eta + 2r_F) + \alpha\mu F}.$$  \hspace{1cm} (33)

For $\Delta > 0$, so that $\max \{V_R + p_{FB}, V_U\} = V_R + p_{FB}$ and $\alpha = 1,$

$$p_H = \frac{(\eta r_H - \mu r_F + 2\eta r_F + 2r_H r_F + q^2) r + (2\eta r_H + 2\delta r_F + 4r_H r_F + \eta^2)q}{(\eta + 2r_H)(\eta r_F + 2r_H + 2r_H r_F)},$$

$$p_{FS} = \frac{(r_F + \eta)(v + q)}{r_H(\eta + 2r_F) + r_F(\mu + 2\delta)},$$

$$p_{FB} = V_F = \frac{\eta(v + q)}{r_H(\eta + 2r_F) + r_F(\mu + 2\delta)},$$

$$V_M = \frac{(r_H(\eta + 2r_F) + \mu r_F)v - 2\delta r_F q}{r_H(\eta + 2r_F) + r_F(\mu + 2\delta)},$$

$$V_U = \frac{\eta}{\eta + 2r_H r_H(\eta + 2r_F) + r_F(\mu + 2\delta)},$$

$$V_R = \frac{\mu r_F v - (r_H(\eta + 2r_F) + 2\delta r_F) q}{r_H(\eta + 2r_F) + r_F(\mu + 2\delta)}.$$  \hspace{1cm} (40)
Lemma 10

a. For $r_F < \hat{r}_F$, $S_\Delta$ is U-shaped, with a well-defined minimum. Write $S_\Delta^* = \min_\theta S_\Delta$.

i. For small $r_F$, $S_\Delta^* > 0$.

ii. $\partial S_\Delta^*/\partial r_F < 0$. As $r_F$ increases, before $r_F$ reaches $\hat{r}_F$, $S_\Delta^* = 0$ at some $\theta = \hat{\theta}^*$.

iii. Thereafter, as $r_F$ continues to increase, $S_\Delta^*$ falls below 0, and that the two roots of $S_\Delta = 0$, $\hat{\theta}_1$ and $\hat{\theta}_2$ diverge as $\partial \hat{\theta}_1/\partial r_F < 0$ but $\partial \hat{\theta}_2/\partial r_F > 0$.

iv. As $r_F \to \hat{r}_F$, $\hat{\theta}_2 \to \infty$ and $\hat{\theta}_1 \to \hat{\theta}_U$ for some limiting value $\hat{\theta}_U$.

b. For $r_F \geq \hat{r}_F$, $S_\Delta$ becomes downward-sloping throughout.

i. At $r_F = \hat{r}_F$, the unique root of $S_\Delta = 0$, $\hat{\theta} = \hat{\theta}_U$, the limiting value of $\hat{\theta}_2$.

ii. Thereafter, $\partial \hat{\theta}/\partial r_F < 0$; while as $r_F$ becomes arbitrarily large, $\hat{\theta} \to \hat{\theta}_L$ for some finite $\hat{\theta}_L < 1$.

7.2 Proofs

Proof of Lemma 1

Combine (5), (6), and

$$\mu = \frac{\eta}{\theta} = \eta \frac{S}{B} = \eta \frac{n_U + n_F}{n_R}$$

yields the result of the lemma.

Proof of Lemma 2

By Lemma 1,

$$n_U = \frac{1 - \alpha}{\alpha} n_F.$$  \hfill (45)

Use (1) and (2) to write

$$n_R = 1 - H + n_F.$$  \hfill (46)

Then, by (45) and (46),

$$\theta = \frac{B}{S} = \frac{n_R}{n_U + n_F} = \frac{1 - H + n_F}{\frac{1 - \alpha}{\alpha} n_F + n_F} = \frac{1 - H + n_F}{n_F}.$$  \hfill (47)

Solve the equation for $n_F$,

$$n_F = \frac{\alpha}{\theta - \alpha} (1 - H).$$  \hfill (48)

Next, by (5) and (1),

$$\mu n_R = \delta (1 - n_U - n_R).$$
Rearrange and then substitute from (45) and (46),
\[(\mu + \delta) (1 - H + n_F) = \delta \left(1 - \frac{1 - \alpha}{\alpha} n_F\right).\]
Solve the equation for \(n_F\),
\[n_F = \alpha \frac{\delta H - \mu (1 - H)}{\delta + \mu \alpha}. \quad (49)\]
Setting the LHSs of (48) and (49) equal yields (9). The LHS is equal to \((1 - (1 - \alpha) H) \delta > 0\) at \(\theta = 0\) but is negative for arbitrarily large \(\theta\) given the concavity of \(\eta\). A solution is guaranteed to exist. Differentiating with respect to \(\theta\) yields
\[
\frac{\partial \eta}{\partial \theta} (1 - H) - H \delta,
\]
which is positive for small \(\theta\) but negative otherwise. The solution to (9) must then be unique, and that the LHS is decreasing in \(\theta\) at where it vanishes. Given that the LHS of the equation is increasing in \(\alpha\), \(\partial \theta / \partial \alpha > 0\). The lower and upper bounds \(\tilde{\theta}_L\) and \(\tilde{\theta}_U\) are given by the respective solutions to
\[
\delta + \eta (1 - H) - (1 + \theta) H \delta = 0, \quad (50)
\]
\[
\delta + \eta (1 - H) - \theta H \delta = 0. \quad (51)
\]
By straightforward manipulation of the second equation, \(\tilde{\theta}_U > 1/H > 1\).

**Proof of Lemma 3**  
*Comparative statics*— Solve (9) for
\[\alpha = \frac{\theta \delta H - (1 - H) (\eta + \delta)}{\delta H}. \quad (52)\]
Substituting (52) into (48) yields
\[n_F = \frac{\theta \delta H - (1 - H) (\eta + \delta)}{\eta + \delta}. \quad (53)\]
Substituting (53) into (45) and (46), respectively, yields
\[n_U = \frac{\eta (1 - H) + \delta (1 - \theta H)}{\eta + \delta}, \quad (54)\]
\[n_R = \frac{\theta \delta H}{\eta + \delta}. \quad (55)\]
Finally, by (1), (54), and (55),
\[n_M = \frac{\eta H}{\eta + \delta}. \quad (56)\]
The comparative statics in the Lemma can be obtained by differentiating (53)-(56), respectively, with respect to \( \theta \), and then noting that \( \partial \tilde{\theta} / \partial \alpha > 0 \).

**Boundary values**—At \( \alpha = 0 \), by (8), \( n_F = 0 \). Then, by (46), \( n_R = 1 - H \) and by (2), \( n_M = H - n_U \). To obtain the equation for \( n_U \), substitute (2) for \( n_M \) into (6), set \( n_F = \alpha = 0 \) and solve the equation for

\[
\tilde{\theta}_L = \eta^{-1} \left[ \frac{\delta H - n_U}{n_U} \right].
\]

Substituting the expression into (50) yields

\[
\frac{1 - H}{n_U} - \eta^{-1} \left[ \frac{\delta H - n_U}{n_U} \right] = 0. \quad (57)
\]

At \( \alpha = 1 \), by (6), \( n_U = 0 \). And then by (46), \( n_R = 1 - H + n_F \). Thus, with (1), \( n_M = H - n_F \). The equation for \( n_F \) is obtained by first substituting (2) for \( n_M \) into (8), setting \( n_U = 1 \) and \( \alpha = 0 \) and solving the equation for

\[
\tilde{\theta}_U = \eta^{-1} \left[ \frac{\delta H - n_F}{n_F} \right].
\]

Substituting the expression into (51) yields

\[
\frac{1 - H}{n_F} + 1 - \eta^{-1} \left[ \frac{\delta H - n_F}{n_F} \right] = 0. \quad (58)
\]

**Proof of Lemma 5**—By (50),

\[
\frac{1 - H}{\delta H} = \frac{\theta_L}{\delta + \eta(\theta_L)} \leq \frac{\theta}{\delta + \eta(\theta)}, \quad (59)
\]

since \( \theta \geq \tilde{\theta}_L \). Substitute (2) into (10) and then from (52) and (56),

\[
TV = \alpha \delta n_M + (H - n_M) \eta = \frac{\eta \delta H}{\delta + \eta} (1 + \theta) - \eta (1 - H). \quad (60)
\]

Differentiating and simplifying,

\[
\frac{\partial TV}{\partial \theta} = \delta H \left( \frac{\partial \eta \delta (1 + \theta)}{\delta + \eta} - \frac{\partial \eta 1 - H}{\delta H} + \frac{\eta}{\delta + \eta} \right) \geq \delta H \left( \frac{\eta' \delta (1 + \theta)}{(\delta + \eta)^2} - \frac{\theta \partial \eta / \partial \theta}{\delta + \eta} + \frac{\eta}{\delta + \eta} \right) > 0,
\]

where the first inequality is by (59) and the second inequality by the concavity of \( \eta \). But then \( \partial \tilde{\theta} / \partial \alpha > 0 \); hence \( \partial TV / \partial \alpha > 0 \).
Proof of Lemma 6  Substituting from (2), (60), and (56) and simplifying, (11) becomes
\[ TOM = \left( (1 + \theta) \eta - \frac{1 - H}{H} \eta (\delta + \eta) \right)^{-1}. \]
Differentiating with respect to \( \theta \),
\[
-TO M^{-2} \times \left( \eta + \theta \frac{\partial \eta}{\partial \theta} - \frac{1 - H}{H} \frac{\partial \eta}{\partial \theta} (\delta + 2\eta) \right) < \]
\[
-TO M^{-2} \times \left( \eta + \theta \frac{\partial \eta}{\partial \theta} - \frac{\theta}{\delta + \eta} \frac{\partial \eta}{\partial \theta} (\delta + 2\eta) \right) = \]
\[
-TO M^{-2} \times \left( \frac{\eta}{\delta + \eta} (\delta + \eta - \theta \frac{\partial \eta}{\partial \theta}) \right) < 0,
\]
where the first inequality is by (59) and the second by the concavity of \( \eta \). The lemma follows given that \( \partial \bar{\theta}/\partial \alpha \).

Proof of Lemma 7  Substituting from (5), (6), and then (1),
\[ TBM = \frac{1}{\mu} + \frac{1 - \alpha}{\eta} = \frac{1 - n_M}{n_M}, \]
a decreasing function of \( n_M \). But where \( \partial n_M/\partial \alpha > 0 \), there must be a smaller average TBM.

Proof of Lemma 8  With \( \lim_{\theta \to 0} \eta = 0 \),
\[ \lim_{\theta \to 0} S_{\Delta} = \mu - 2 (\delta + r_H) z = \infty, \]
since \( \lim_{\theta \to 0} \mu = \infty \). With \( \lim_{\theta \to \infty} \mu = 0 \),
\[ \lim_{\theta \to \infty} S_{\Delta} = \left( \frac{r_H}{r_F} - 1 - z \right) \eta - 2 (\delta + r_H) z, \]
which is equal to positive infinity/a finite negative number/negative infinity for \( r_F \geq \bar{r}_F \). Differentiating,
\[
\frac{\partial S_{\Delta}}{\partial \theta} = \left( \frac{r_H}{r_F} - 1 - z \right) \frac{\partial \eta}{\partial \theta} + \frac{\partial \mu}{\partial \theta}, \tag{61}
\]
which is guaranteed negative if \( r_F \geq \bar{r}_F \). In this case, \( S_{\Delta} \) starts out equal to positive infinity and falls continuously towards either a finite negative number or negative infinity. A unique \( \theta \) then solves \( S_{\Delta} = 0 \). On the other hand, if \( r_F < \bar{r}_F \), \( S_{\Delta} \) starts out and ends up equal to positive infinity. It must therefore be initially decreasing but eventually increasing. If the condition in the Lemma holds, (61) changes sign just once. This can be shown by differentiating (61) and evaluating at where it is equal to zero, which leads to an expression which is positive if the condition holds. Then, at where (61) vanishes, \( S_{\Delta} \) is convex.
Proof of Lemma 10  By (20), lim_{r_F \to 0} S_\Delta = \infty. Thus, for arbitrarily small r_F, S_\Delta^* > 0. This proves (a.i). Differentiating (20) and by the Envelope Theorem,

$$\frac{\partial S_\Delta^*}{\partial r_F} = -\frac{r_H}{r_F^2} \eta < 0.$$  

This proves the first part of (a.ii). As to the second part, notice that

$$\lim_{r_F \to 0} S_\Delta = \mu - 2(\delta + r_H) z,$$

which is minimized at \( \theta \to \infty \), yielding a negative \( S_\Delta^* \) in the limit. Given that \( S_\Delta^* \) is continuous in \( r_F \), \( S_\Delta^* = 0 \) must hold before \( r_F \) has reached \( \hat{r}_F \). For (a.iii), differentiating \( S_\Delta = 0 \) and for \( i = 1, 2 \),

$$\frac{\partial \hat{\theta}_i}{\partial r_F} = \frac{r_H \eta}{\frac{\partial S_\Delta}{\partial \theta}}.$$

Where \( S_\Delta \) is decreasing at \( \hat{\theta}_1 \) and increasing at \( \hat{\theta}_2 \), \( \frac{\partial \hat{\theta}_1}{\partial r_F} < 0 \) and \( \frac{\partial \hat{\theta}_2}{\partial r_F} > 0 \). For (a.iv), notice that \( \partial S_\Delta / \partial \theta \), as given by \( (61) \), can only remain positive as \( r_F \to \hat{r}_F \) if \( \theta \to \infty \) in the interim. This proves \( \hat{\theta}_2 \to \infty \) as \( r_F \to \hat{r}_F \). The limiting value for \( \hat{\theta}_1 \) is given by the solution to

$$\mu(\hat{\theta}_1) = 2(\delta + r_H) z.$$

Denote this as \( \hat{\theta}_U \). The negativity of \( \frac{\partial \hat{\theta}_1}{\partial r_F} \) is due to the same reason for the negativity of \( \frac{\partial \hat{\theta}_1}{\partial r_F} \). The limiting value of \( \hat{\theta} \) as \( r_F \) becomes arbitrarily large is given by the solution to

$$\eta(\hat{\theta}_L) \left( \frac{1}{\hat{\theta}_L} - (1 + z) \right) = 2(\delta + r_H) z.$$

Given that the RHS is positive and finite, \( \hat{\theta}_L \) is finite and satisfies \( \hat{\theta}_L < \frac{1}{1 + z} < 1 \). This completes the proof of (b).

Proof of Proposition 1  To establish existence, we apply Kakutani’s fixed point theorem to show that \( F \) has a fixed point. First, the unit interval is clearly a compact, convex, and nonempty subset of the one-dimensional Euclidean space. Second, since \( \hat{\theta}(\alpha) \) is defined for all \( \alpha \in [0, 1] \) and is positive-valued, and that \( \hat{\alpha} \) is nonempty for all \( \theta > 0 \), \( F \) must be positive-valued and nonempty for all \( \alpha \in [0, 1] \). Whenever \( F \) is multi-valued, \( F \) is the entire unit interval. Then it must be convex. Finally, with \( \hat{\theta} \) continuous and \( \hat{\alpha} \) possessing a closed graph by virtue of the continuity of \( S_\Delta \), \( F \) must have a closed graph as well. Then by Kakutani’s fixed point theorem, \( F \) has a fixed point.
Proof of Proposition 2  First define the graph of the \( \hat{\alpha} (\theta) \) correspondence as being non-increasing over a given interval of \( \theta \) if for any two \( \theta' \) and \( \theta'' \) in the interval, where \( \theta' < \theta'' \), no element of \( \hat{\alpha} (\theta') \) is strictly greater than any element of \( \hat{\alpha} (\theta'') \).

Then, if \( r_F \geq \tilde{r}_F \), the \( \hat{\alpha} (\theta) \) graph, as depicted in either Panel E or F of Figure 6, is non-increasing throughout. Then there can be one and only one \( \{ \alpha, \theta \} \) pair at which the upward-sloping \( \hat{\alpha} (\theta) \) in Figure 4 can meet a non-increasing \( \hat{\alpha} (\theta) \). Next, by (20), \( \lim_{r_F \to 0} S_\Delta = \infty \). Hence for arbitrarily small \( r_F \), \( S_\Delta \) stays positive for all \( \theta \) and the \( \hat{\alpha} (\theta) \) graph is as depicted in Panel A of Figure 6. Clearly, the unique equilibrium is \( \theta = \theta_U \) and \( \alpha = 1 \).

Otherwise, the \( \hat{\alpha} (\theta) \) graph must be like the two depicted in Panels C and D in Figure 6. In this case, for all \( \theta \geq \bar{\theta}_2 \), \( S_\Delta \geq 0 \), so that \( 1 \subset \hat{\alpha} (\theta) \). Thus if \( \hat{\theta}_2 \leq \theta_U \), \( \alpha = 1 \) and \( \theta = \hat{\theta}_U \) is equilibrium. Given that \( \hat{\theta}_2 \in [\bar{\theta}_L, \bar{\theta}_U] \), either that \( \hat{\theta}_1 \in [\bar{\theta}_L, \hat{\theta}_U] \) or that \( \hat{\theta}_1 < \hat{\theta}_L \). In the first case, \( \hat{\theta}_1 \) and \( \alpha = \hat{\alpha} (\hat{\theta}_1) \) is obviously equilibrium. In the second case, with \( \bar{\theta}_L \in (\hat{\theta}_1, \hat{\theta}_2) \) and since for \( \bar{\theta} \in (\hat{\theta}_1, \hat{\theta}_2) \), \( \hat{\alpha} (\bar{\theta}) = 0 \), \( \theta = \hat{\theta}_L \) and \( \alpha = 0 \) is equilibrium. This proves that \( \hat{\theta}_2 \in [\bar{\theta}_L, \bar{\theta}_U] \) is sufficient for multiplicity given an \( \hat{\alpha} (\theta) \) graph as depicted in either Panel C or D in Figure 6. To show that it is also necessary, notice that if \( \hat{\theta}_2 \notin [\bar{\theta}_L, \bar{\theta}_U] \), \( \hat{\alpha} (\theta) \) is non-increasing throughout \( [\bar{\theta}_L, \bar{\theta}_U] \), and as in where \( r_F \geq \tilde{r}_F \), there can be just one point at which \( \hat{\alpha} (\theta) \) and \( \hat{\alpha} (\theta) \) intersect.

Proof of Proposition 3  Given that for small \( r_F \), the \( \hat{\alpha} (\theta) \) correspondence is given by that in Panel A of Figure 6, (a) follows immediately. For large \( r_F \), the \( \hat{\alpha} (\theta) \) correspondence tends to that in Panel F. In this case, \( \theta \) can remain equal to \( \theta_U \) only if the entire \( \hat{\alpha} (\theta) \) schedule lies to the left of \( \hat{\theta}_L \); i.e., \( \bar{\theta}_U \leq \hat{\theta}_L \). Given that \( \hat{\theta}_U > 1 \) (Lemma 2) but \( \hat{\theta}_L < 1 \) (Lemma 9), the condition cannot hold. Hence, for large \( r_F \), in equilibrium, \( \alpha < 1 \) and \( \theta < \hat{\theta}_U \). Next, if \( \hat{\theta}_L \leq \hat{\theta}_U \), in the limit when \( r_F \) becomes arbitrarily large, the upward-sloping \( \hat{\alpha} \) must meet the horizontal axis at where \( \hat{\alpha} (\theta) = 0 \), in which case equilibrium is \( \alpha = 0 \) and \( \theta = \hat{\theta}_L \). Otherwise, the upward-sloping \( \hat{\alpha} \) would meet the vertical segment of \( \hat{\alpha} \), yielding a \( \hat{\theta} \) equilibrium, where \( \alpha > 0 \). This proves (b). For (c), note that as \( r_F \) increases, the set of \( \theta \) over which \( 1 \subset \hat{\alpha} (\theta) \) shrinks and the set of \( \theta \) over which \( 0 \subset \hat{\alpha} (\theta) \) expands. Parts (d) and (e) are direct corollaries of (a.iii) and (b.ii) of Lemma 10, given that \( \hat{\alpha} (\theta) \) is downward-sloping.

Proof of Proposition 4  We begin with showing that price in the search market in the \( \theta_U \) equilibrium \( (p_{FS}) \) is higher than in the \( \hat{\theta}_2 \) equilibrium \( (p_H = p_{FS}) \). First, if \( \theta_U \) is equilibrium, \( p_{FS} \) as given by (40) must exceed \( p_H \) as given by (39), both evaluated at \( \theta = \hat{\theta}_U \). Next, it is straightforward but tedious to verify that with \( S_\Delta \geq 0 \) and
\(r_F < r_H, p_H\) as given by (39) is increasing in \(\theta\). Then the value of \(p_H\) with \(S_\Delta \geq 0\) and \(\theta = \theta_U\) exceeds the value with \(S_\Delta \geq 0\) and \(\theta = \hat{\theta}_2\) in case \(\theta_U > \hat{\theta}_2\). The latter, however, is identical to \(p_H\) with \(S_\Delta \leq 0\) as given by (33), when both expressions are evaluated at \(S_\Delta = 0\). Next, with \(\partial p_H / \partial \theta > 0\) as given by (24), search market housing price in the \(\hat{\theta}_2\) equilibrium must exceed that in the \(\hat{\theta}_1\) equilibrium. The final comparison is between search market prices in the \(\hat{\theta}_2\) and \(\hat{\theta}_L\) equilibria. At where \(\theta = \hat{\theta}_L\), search market housing price \(p_H\) is given by (23). Differentiating with respect to \(\theta\) yields an expression whose sign is the same as
\[
(2\delta + r_H) \theta^2 \frac{\partial \eta}{\partial \theta} + 2(\delta + r_H) \left( \eta - \theta \frac{\partial \eta}{\partial \theta} \right) + \eta^2 > 0.
\]
Hence, \(p_H\) given by (33), evaluated at \(\alpha = 0\), is increasing in \(\theta\). Then \(p_H\) at \(\theta = \hat{\theta}_L\) must fall below the value of (33) evaluated at \(\alpha = 0\) and \(\theta = \hat{\theta}_2\). The latter when also evaluated at \(S_\Delta = 0\) is just the search market housing price at \(\theta = \hat{\theta}_2\). This completes the proof that search market housing prices across steady-state equilibria can be ranked by the value of \(\theta\). Given that \(p_{FB} = \frac{\eta}{\eta + r_F} p_{FS}\), investment market housing prices are ranked in the same order as in search market housing prices.

**Proof of Proposition 8**  
Hold constant \(R\) and allow \(r_H\) to increase; by (28),
\[
\frac{\partial S_\Delta}{\partial r_H} = 2z < 0.
\]
For large \(r_H\) then, the \((\hat{\alpha} (\theta)\) graph cannot be like the ones in Panels A and B of Figure 6. For \(r_F \geq \hat{r}_F\) (i.e., \(R \leq 1 + z\)), so that there exists a unique root \(\hat{\theta}\) to \(S_\Delta = 0, \partial \hat{\theta} / \partial r_H < 0\), given that \(\partial S_\Delta / \partial r_H < 0\). By (28) and with \(R - 1 - z < 0\), \(\lim_{r_H \to \infty} \hat{\theta} = 0\). For \(r_F < \hat{r}_F\) (i.e., \(R > 1 + z\)), there are two roots \(\hat{\theta}_1\) and \(\hat{\theta}_2\) to \(S_\Delta = 0\). Again, given that \(\partial S_\Delta / \partial r_H < 0, \partial \hat{\theta}_1 / \partial r_H < 0\) and \(\partial \hat{\theta}_2 / \partial r_H > 0\). By (28) and with \(R - 1 - z > 0\), as \(r_H \to \infty\), there can just be two \(\hat{\theta}\) that solves \(S_\Delta = 0\), equal to zero and infinity; thus \(\lim_{r_H \to \infty} \hat{\theta}_1 = 0\) and \(\lim_{r_H \to \infty} \hat{\theta}_2 = \infty\). Parts (a)-(d) of the Propositions then follow immediately.

**Proof of Proposition 9**  
In a no-intermediation equilibrium, \(p_H\) is given by (33), evaluated at \(\alpha = 0\), which is independent of \(r_F\) but decreasing in \(r_H\). This proves the first part of (a). For the second part, substitute \(r_F = r_H R^{-1}\) into (40) and (41) and differentiate. For (b), substituting \(r_F = r_H R^{-1}\) into (24) and (25), respectively, yields,
\[
p_{FS} = p_H = \frac{(\eta + r_H R^{-1}) v}{(2\delta + \eta + 2r_H) r_H R^{-1}}, \quad (63)
\]
\[
p_{FB} = \frac{\eta v}{(2\delta + \eta + 2r_H) r_H R^{-1}}, \quad (64)
\]
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both of which are decreasing in $r_H$. Solve $S_\Delta = 0$ from (28) for $r_H$,

$$r_H = \eta \frac{R - 1 - z + 1/\theta}{2z} - \delta,$$

(65)

and substituting the result into (63) and (64), respectively, gives

$$p_{FS} = p_H = \frac{(2\eta zR + \eta R - \eta - \eta z + \eta/\theta - 2\delta z) q}{\eta (R - 1 + 1/\theta)(\eta R - \eta - \eta z + \eta/\theta - 2\delta z)}$$

(66)

$$p_{FB} = \frac{2\nu z^2 R}{(R - 1 + 1/\theta)(\eta R - \eta - \eta z + \eta/\theta - 2\delta z)}.$$

(67)

Differentiating (67) with respect to $\theta$ yields an expression whose sign is given by that of

$$\eta (R - 1 + 1/\theta - z) - 2\delta z - (R - 1 + 1/\theta) \left( \theta^2 \frac{\partial \eta}{\partial \theta} (R - 1 + 1/\theta - z) - \eta \right).$$

The expression is strictly positive at $R = 0$ and $R = 1 + z$ if the RHS of (65) is positive. And then differentiating twice with respect to $R$ yields

$$-2\theta^2 \frac{\partial \eta}{\partial \theta} < 0.$$

Thus, $p_{FB}$ in (67) must be increasing in $\theta$ for $R \in [0, 1 + z]$. For $p_{FS} = p_H$, differentiating (66) with respect to $\theta$ and evaluating at $R = 0$, 1, and $1 + z$, respectively, all yield a strictly positive expression as long as the RHS of (65) is positive. Then, $p_{FS} = p_H$ in (66) must be decreasing in $\theta$ for $R$ in neighborhoods of 0, 1, and $1 + z$. 
References


