Holdups and Overinvestment in Capital Markets*

André Kurmann

Federal Reserve Board

February 10, 2012

Abstract

This paper analyzes the problem of firms that need to make investment decisions in capital markets characterized by trading frictions and ex-post bargaining. Trading frictions make switching from one capital supplier to another costly, thus implying that the match between firms and suppliers generates surplus. Ex-post bargaining implies that suppliers can appropriate part of this surplus. Firms react strategically to the resulting holdup problem by overinvesting so as to reduce their marginal productivity and thus the negotiated price of capital. In a multifactor setting, the holdup problem in capital markets interacts with holdup problems in labor markets that typically lead to underinvestment and overemployment. This presents firms with a trade-off that has non-trivial equilibrium effects and that – depending on the substitutability of capital and labor and the firm’s bargaining power in each market – can neutralize or exacerbate the distortionary effects each of the holdup problems has on its own.

Keywords: Holdup problems, Trading frictions, Bargaining, Investment.

JEL Codes: D23, D24, E22,

*The views expressed in this paper do not necessarily represent the views of the Federal Reserve System or the Federal Open Market Committee. I thank Randy Wright for encouraging me to explore the topic and Stan Rabinovich for outstanding research assistance. I also thank three anonymous referees, David Arseneau, Gabriele Camera, Sanjay Chugh, Guido Menzio, Adriano Rampini, Michael Reiter, Eric Smith, Etienne Wasmer as well as seminar participants at CERGE-EI, the University of Maryland, Wharton, the SED, the Vienna Macro workshop, the CMSG and the Federal Reserve Banks of Atlanta, Cleveland, Philadelphia and St. Louis for helpful comments. Financial support from the SSHRC and the hospitality of the Wharton School at the University of Pennsylvania, where part of this paper was completed, is gratefully acknowledged.
1 Introduction

Factor specificity and contract incompleteness are pervasive phenomena in economic transactions and considered to be a major source of inefficiency. Whenever one party expends resources that increase the value of a productive relationship relative to outside options (i.e. specificity) and other parties can appropriate some of the rents arising from the investment (i.e. contract incompleteness), a holdup problem arises. Holdup problems typically reduce the incentive to invest (e.g. Simons, 1944; Grout, 1984) and, in general equilibrium, lead to underutilization of resources, missing technology adoption, and excessive destruction (e.g. Caballero and Hammour, 1998a).

The present paper examines the consequences of holdup problems in capital markets and shows that the frictions usually associated with underinvestment – factor specificity and contract incompleteness – can lead to exactly the opposite outcome: overinvestment. The result arises naturally in an environment where firms need to decide on investment before meeting capital suppliers in decentralized capital markets and negotiating over the price. The trading frictions associated with decentralized markets imply that the match between firms and suppliers generates surplus. Ex-post price negotiation implies that suppliers can appropriate part of this surplus. Hence, a holdup problem arises. As long as technology exhibits decreasing returns in capital, firms strategically react by overinvesting ex-ante so as to reduce marginal productivity and thus the price of capital.

The forces behind overinvestment are, from a mechanical point of view, the same as the ones that lead to overemployment in labor markets with worker replacement costs and ex-post bargaining, as analyzed originally by Stole and Zwiebel (1996a,b) and extended to a modern general equilibrium labor search context by Smith (1999); Cahuc and Wasmer (2001a,b); and Cahuc, Marque and Wasmer (2008).\textsuperscript{1} The novelty of the present paper is to argue that similar frictions are relevant for capital markets and, on their own, lead to overinvestment. The paper then extends the analysis to a multifactor setting with holdup problems in different markets and shows that the strategic reaction of firms has non-trivial equilibrium effects.

The assumption that capital markets are subject to trading frictions and ex-post bargaining contrasts with much of the investment literature, which focuses on different types of adjustment costs and credit constraints but maintains that the price of capital is determined competitively

\textsuperscript{1}Also see Wolinsky (2000) for a dynamic extension of Stole and Zwiebel’s (1996a) partial equilibrium analysis.
in a frictionless market. This assumption effectively eliminates strategic considerations the firm may have with respect to the price of capital. Yet, a wide variety of capital – ranging from real assets such as structures and equipment to financial assets such as bank loans, fixed-income debt and derivatives – trade in decentralized or so-called ‘over-the-counter’ markets. Since much of this capital is specific in terms of quality, task or location, the search for and vetting of suppliers is likely to involve both material and opportunity costs. These costs give rise to match surplus that needs to be split somehow, thus opening the door to bargaining.2

Recent empirical work suggests that for many decentralized capital markets, trading frictions and bargaining are not just a theoretical curiosity but quantitatively relevant.3 As a result, a burgeoning literature has emerged that attempts to explain the different phenomena with search-theoretic models.4 The present paper shares with this literature the premise that search and bargaining are important features of many decentralized capital markets but focuses on the normative question of holdups and inefficient investment. This focus is motivated by the observation that investment often requires lengthy planning during which market research is conducted, financing is obtained, and permits are issued. These time-to-plan constraints can be substantial and have been used widely in both the business cycle and the real options literature.5 But the same constraints also imply that the firm must make important investment decisions before it can meet with suppliers to bargain over the price.6

---

2See Duffie, Garleanu and Pedersen (2005, 2007) for examples of decentralized financial asset markets that are subject to trading frictions and bargaining. See Rauch (1999) and Nunn (2007) for a list of real asset markets without organized exchange nor reference prices in trade publications.

3See Pulvino (1998) and Gavazza (2011a) for the quantitative relevance of trading frictions and bargaining in commercial aircraft markets – presumably one of the most homogenous and frictionless real asset markets – as well as Ramey and Shapiro (1998, 2001); Maksimovich and Phillips (2001); Eisfeldt and Rampini (2006) or Kurmann and Petrosky-Nadeau (2007) for evidence on trading frictions and bargaining in real asset markets in general. For quantitative evidence of trading frictions and bargaining in over-the-counter financial markets, see Dell’Ariccia and Garibaldi (2005); Green, Li and Schuerhoff (2010); or Afonso and Lagos (2011a) among others.


5See Kydland and Prescott (1982); Christiano and Todd (1996); and Edge (2000) for applications of time-to-plan in business cycle models; and Dixit and Pyndick (1994) for a review of the importance of time-to-plan for the real options literature. Time-to-plan in these works is motivated by empirical evidence in Mayer (1960), Jorgenson and Stephenson (1967), Krainer (1968) among others who find that investment lags behind its determinants by 6 to 12 quarters and that the initial planning phase requires little resource costs relative to the actual investment outlay.

6The idea that firms need to decide on projects prior to the actual investment is similar in spirit to the analysis of innovation and ‘the market for ideas’ by Silveira and Wright (2010).
bargaining over the price – captures the essence of what is meant here by *ex-post* bargaining and what prevents firms and suppliers from committing ex-ante to alternative arrangements that would circumvent holdup problems. More generally, since investment is often a complex and incremental process, contract terms may also be renegotiated after the match between firms and suppliers has occurred so as to adjust to changing circumstances.\(^7\) The maintained assumption of the paper is therefore that contracts are non-binding or at least not binding for very long.

The analysis of the paper proceeds as follows. Section 2 derives the overinvestment result in a basic dynamic general equilibrium model with random search and ex-post bargaining. Apart from a modification to endogenize the supply of capital, the model mirrors the labor search environment that Smith (1999); Cahuc and Wasmer (2001a,b); and Cahuc, Marque and Wasmer (2008) use to analyze the implications of holdups in labor markets. The random search assumption is appealing because of its empirical relevance and its convenience for equilibrium analysis. At the same time, and as discussed in the main text, the overinvestment result emerges independently of the search friction as long as one maintains that specificity prevents firms from costlessly replacing a capital supplier with another. Furthermore, while the model resembles in many ways the neoclassical growth benchmark, capital should be understood broadly and include not only physical capital but also any type of intermediate input and financial asset that is traded in decentralized markets.

Ex-post bargaining is modeled as in Stole and Zwiebel (1996a) and captures the idea that due to the above discussed complexities of investment, firms and workers cannot enter into ex-ante binding contracts. In particular, at any time before production starts, a matched capital supplier may enter into pairwise renegotiation with the firm. Likewise, a firm may call in any of its supplier for renegotiation. Within each pairwise negotiation, the firm and the supplier play an alternating-offer game with exogenous probability of breakdown as in Binmore, Rubinstein and Wolinsky (1986). The resulting unique subgame-perfect equilibrium price is identical for all suppliers and solves the generalized Nash bargaining problem over the marginal match surplus. This solution is a function of the firm’s infra-marginal productivities because the involved parties understand that if negotiations

\(^7\)For example, Caballero (2007, page 64) argues: "[In ideal contracts], the plan for making such investments, the duration of the relationship, the rent-division mechanism, and the multiple dimensions that characterize each factor’s participation, must be pre-specified from the start and made fully contingent on the future profitability of the production unit, on factors that determine its evolving prospects, and on the various events, both aggregate and idiosyncratic, that govern each factor’s outside opportunity costs. A variety of problems of observability, verifiability, enforceability, and sheer complexity, make such ideal contracts rarely feasible. Thus, agents enter into arrangements...that leave plenty of room for ex post discretion."
with one supplier break down, the price with all other suppliers is renegotiated based on the new marginal match surplus. As long as technology exhibits decreasing returns in capital, the firm reacts to this situation by overinvesting ex-ante so as to reduce marginal productivity and thus the ex-post bargained price of capital.

Section 3 extends the analysis to a multifactor setting. First, the paper assesses the robustness of the overinvestment result to a second capital factor that is either a substitute or a complement in production. Second, the paper introduces labor alongside capital to analyze how the holdup problem in the capital market interacts with a standard holdup problem arising from search and ex-post bargaining in the labor market that, on its own, would lead to underinvestment and overemployment. In both cases, a double holdup problem arises that can be analyzed using the innovative application of spherical coordinate techniques by Cahuc, Marque and Wasmer (2008). The analysis yields a straightforward partial equilibrium prediction: if the two factors are complements (substitutes), the firm’s incentive to overinvest / overemploy is mitigated (aggravated) because this increases (decreases) marginal productivity of the other factor, thus worsening (attenuating) the holdup problem in the second factor market.

In general equilibrium, the firm’s reaction to the double holdup problem has non-trivial effects that depend on the form of the production function and the firm’s bargaining power in each market. For the special case where the firm’s bargaining power is identical across markets, the equilibrium ratio of the two factors of production is efficient because the incentive to overinvest / overemploy in one market is exactly offset by the incentive to overinvest / overemploy in the other market. If, in addition, production is constant-returns-to-scale, the equilibrium level of each factor is also efficient because marginal productivity in this case is independent of firm size. This knife-edge case illustrates that multiple holdup problems do not necessarily exacerbate each other as is often the case in the literature (e.g. Aruoba, Waller and Wright, 2011). If production is decreasing-returns-to-scale instead, the equilibrium level of each factor remains inefficiently high, independent of whether the two factors are complements or substitutes, because marginal productivity in this case decreases with firm size. This result illustrates that the overinvestment result is surprisingly robust to the introduction of a second factor of production.

Once the firm’s bargaining power differs across the two markets, the equilibrium implications of the double holdup problem become more involved. If the two factors are complements, then there
is relative overinvestment / overemployment in the factor for which the firm has lower bargaining power because this is the market where the holdup problem is relatively worse. Furthermore, if bargaining powers in the two markets are sufficiently far apart, the distortion from the double holdup problem becomes so large as to result in inefficiently low equilibrium levels of either or both factors. Implications can be quite different when the two factors are substitutes. In particular, for sufficiently low firm bargaining power in the two markets and sufficient substitutability between the two factors, there is equilibrium overinvestment / overemployment in the market where the firm has higher bargaining power because this reduces the surplus in the other market where the holdup problem is relatively worse.

The difference in results between partial equilibrium and general equilibrium in a multifactor setting is striking and highlights the importance of analyzing multiple holdup problems in general equilibrium. More generally, the results offer a cautionary tale that holdup problems in the labor market do not necessarily lead to underinvestment and overemployment as is typically emphasized in the literature. Instead, it all depends on the specifics of the production function and the relative importance of holdup problems in each market.

Section 4 concludes by discussing policy implications of overinvestment, and why the holdup problem identified in this paper may offer interesting explanations for a number of empirical observations.

2 Overinvestment in a basic model

The basic model is intentionally kept simple to convey intuition. Capital is the only factor of production and its allocation from suppliers to firms is governed by a random search matching function. Once new matches have occurred, firms and suppliers (whether newly matched or incumbent) bargain over the price of capital. In other words, capital is rented. This renting (or leasing) assumption is motivated both by empirical and theoretical considerations. Empirically, a substantial fraction of both financial and real assets are not purchased outright but borrowed.\textsuperscript{8} Theoretically, in the

\textsuperscript{8}For example, Graham, Lemmon and Schallheim (1998) report that operating leases and capital leases make up 42% and 6% of fixed claims in the 1981-1992 Compustat database (which contains mostly large firms for which leasing is less common). Gavazza (2011a), in turn, finds that 52% of all commercial aircraft are leased. Also see The Economist (2010) for a report on the large projected impact of accounting rule changes with respect to leased capital. The decision to lease instead of purchasing can be optimal for a firm because of tax considerations (Graham,
dynamic context of the present model, the assumption that capital is purchased (or, more generally, that rental rates with suppliers are not all negotiated at the same time) would introduce an additional strategic motive for the firm to overinvest. While interesting, this second strategic motive would considerably complicate the analysis without changing any of the results shown here. Further details are provided at the end of this section, together with a discussion of the robustness of the results to a variety of other alternative assumptions.

Apart from a modification to endogenize the supply of capital, the model mirrors the labor search environment that Smith (1999), Cahuc and Wasmer (2001a,b) and Cahuc, Marque and Wasmer (2008) use to study overemployment. Derivations are therefore relegated to the appendix.

2.1 Environment

The economy is populated by a continuum of infinitely-lived atomistic firms $i \in [0, 1]$ and capital suppliers $\omega \in [0, 1]$. Time is discrete and discounted at rate $\beta = (1 + r)^{-1}$. Both firms and capital suppliers derive linear utility from consuming a numeraire good that firms produce with capital stock $k$ using technology $f(k)$. This technology is twice differentiable, strictly increasing and concave in $k$ and satisfies the usual Inada conditions.

The allocation of capital from suppliers to firms occurs in two stages. In the first stage, firms post ‘ventures’ $v$ at flow cost $\gamma$ to search for new capital while capital suppliers transform numeraire into ‘liquid’ capital $l$ to search for firms with open ventures. There is no resource cost for this transformation. However, capital suppliers bear an opportunity cost because committing liquid capital to the market reduces current consumption independent of whether the liquid capital is matched with a firm or not. The allocation of liquid capital to ventures is governed by a random-search matching function $m(V, L) \leq \min(V, L)$, where $V \equiv \int_0^1 v \, di$ and $L \equiv \int_0^1 l \, d\omega$ denote the total mass of new ventures and liquid capital, respectively. For simplicity, this function is assumed to exhibit constant-returns-to-scale with $\lim_{V \to 0} m_V(\theta) = 1$ and $\lim_{V \to \infty} m_V(\theta) = 0$, where $m_V(\theta) \equiv \frac{\text{Lemmon and Schallheim, 1998}}{\text{or financial constraints (Eisfeldt and Rampini, 2009).}}$

---

9The assumption of no resource cost is made for simplicity. None of the results would change if it was instead assumed that the transformation of numeraire into liquid capital was subject to a convex adjustment cost.
\[ \partial m(V, L)/\partial V \text{ and } \theta \equiv V/L \] denotes capital market tightness.\(^{10}\) Accordingly, each venture matches with a unit of liquid capital with probability \( p(\theta) \equiv m(V, L)/V \) and each unit of liquid capital matches with a venture with probability \( q(\theta) \equiv m(V, L)/L \).

In the second stage, newly matched capital joins the firm’s existing capital stock net of depreciation to form the firm’s new capital stock. Firms then produce and open new ventures while capital suppliers combine unmatched liquid capital with numeraire to search for new ventures.

Given these assumptions, the evolution of a firm’s productive capital stock is described by\(^{11}\)

\[
k' = (1 - \delta)k + p(\theta)v,
\]

where \( \delta \) denotes the exogenously given rate of depreciation. Likewise, the evolution of a capital supplier’s matched capital rented out to firms is described by

\[
k' = (1 - \delta)\kappa + q(\theta)l,
\]

where \( \kappa \) is the matched capital stock of the capital supplier. Finally, unmatched capital returning to the capital supplier in the beginning of next period is defined as

\[
u' = (1 - q(\theta))l.
\]

The temporal distinction in these equations highlights the sequential nature of the investment process: first, firms open ventures and capital suppliers produce liquid capital; second, ventures match with liquid capital and unmatched liquid capital returns to the suppliers. Also notice that while the environment mostly resembles one of physical capital accumulation, capital can be easily

\(^{10}\)To provide an example of a matching function that satisfies the different properties, consider

\[
m(V, L) = \frac{VL}{(L^x + V^x)^{1/x}}
\]

with \( \chi > 1 \). This formulation has been used by Den Haan, Ramey and Watson (2000) and many others in the labor search literature. In the present context, the denominator \( (L^x + V^x)^{1/x} \) can be interpreted as the number of submarkets in which the capital market is segmented. A match occurs when a liquid capital unit and a venture are in the same submarket. The other ventures and liquid capital units remain unmatched. Hence, if liquid capital \( L \) and ventures \( V \) are assigned randomly to submarkets (due, for example, to information imperfections about potential suppliers and firms), then the total number of matches is \( VL/(L^x + V^x)^{1/x} \).

\(^{11}\)Since firms and capital suppliers are atomistic, the law of large numbers applies such that the expected number of matches equals the realized number of matches.
interpreted as an intermediate good (with the depreciation rate taking the value $\delta = 1$) or a financial asset. In the latter case, $\delta$ would be interpreted as a separation rate; and equation (3) would be modified to accommodate returns of separated capital. More generally, the model could be easily extended to allow for reallocation of used capital.

2.2 Efficient allocation

Before analyzing the holdup problem, it is useful to describe the (constrained) efficient allocation. The planner’s problem that achieves this allocation is

$$P(K, U) = \max_{L,V} \left[ f(K) - \gamma V - (L - U) + \beta P(K', U') \right]$$

s.t. $K' = (1 - \delta)K + m(V, L)$

$$U' = L - m(V, L),$$

where $K = \int_0^1 kd\zeta = \int_0^1 k\zeta d\omega$ and $U = \int_0^1 u\zeta$ denote the aggregate stocks of matched and unmatched capital, respectively. The term $f(K) - \gamma V - (L - U)$ is the current-period aggregate utility from numeraire consumption, consisting of aggregate output by firms minus resources expended for opening new ventures and committing net new liquid capital to the market.\(^{12}\)

The analysis focuses exclusively on steady state equilibria where $X = X'$ for all variables. Appendix A.1 provides an explicit derivation of the planner’s problem. Here, a simple characterization of the resulting equilibrium allocation is given.

**Proposition 1.** There exists a unique efficient allocation $\{K^P, \theta^P\}$ that solves

$$\gamma = m_V(\theta) \left[ \frac{f_K(K)}{r + \delta} - \beta \right]$$

$$1 - \beta = m_L(\theta) \left[ \frac{f_K(K)}{r + \delta} - \beta \right]$$

**PROOF:** Appendix A.1.

\(^{12}\)As mentioned above, this definition of consumption makes clear that while transforming numeraire into liquid capital can be done at no resource cost, there is an opportunity cost for liquid capital because it reduces current consumption. This is analogous to investment in a frictionless neoclassical model only that now, aggregate net investment by capital suppliers is defined as $L - U$. 9
Equation (4) says that the cost of an additional venture equals the marginal increase in matches with respect to ventures times the discounted net surplus of an additional match. Equation (5) says that the opportunity cost of an additional unit of liquid capital equals the marginal increase in matches with respect to liquid capital times the discounted net surplus of an additional match. Combining the two equations to eliminate the marginal net surplus yields the following implicit solution for the efficient level of capital market tightness

\[ \theta^P = \frac{1 - \beta}{\gamma} \left( \frac{1 - \epsilon(\theta^P)}{\epsilon(\theta^P)} \right), \]  

where \( \epsilon(\theta) \equiv m_L(\theta)/q(\theta) \) and \( 1 - \epsilon(\theta) \equiv m_V(\theta)/p(\theta) \) denote the matching elasticities with respect to liquid capital and ventures, respectively. Given \( \theta^P \), either (4) or (5) then pins down the efficient matched capital stock \( K^P \); and the efficient levels of liquid capital \( L^P \), ventures \( V^P \) and unmatched capital \( U^P \) are computed from the two constraints and the definition of capital market tightness.

### 2.3 Decentralized allocation

In the decentralized economy, the surplus from matching is split via rental rate \( \rho \). Following Stole and Zwiebel (1996a), it is assumed that at any time before production starts, a matched supplier may enter into pairwise negotiation with the firm. Likewise, a firm may call in any matched supplier for negotiation. As discussed in the introduction, this assumption of continuous pairwise negotiation captures the idea that due to various complexities of the investment process, contracts are non-binding or at least not binding for very long. As Stole and Zwiebel (1996a) show formally, the negotiation protocol implies that each supplier (whether newly matched or incumbent) is the marginal supplier and negotiates an identical rental rate over the marginal match surplus. Since this marginal match surplus is a function of the productivity of capital, firms consider the marginal match surplus and thus the rental rate as endogenous in their venture posting decision; i.e. \( \rho = \rho(k) \). By contrast, suppliers ex-ante consider the rental rate as exogenous because they match with a random set of firms and in each match provide only a small portion of any given firm’s capital stock.

Given these considerations about the rental rate, the problem of a firm entering the period with
matched capital stock $k$ is

$$J(k) = \max_v \left[ f(k) - \rho(k)k - \gamma v + \beta J(k') \right]$$

subject to (1); and the problem of a supplier entering the period with matched capital $\kappa$ and unmatched liquid capital $u$ is

$$V(\kappa, u) = \max_l \left[ \rho \kappa - l + u + \beta V(\kappa', u') \right]$$

subject to (2) and (3). In steady state, the firm’s problem leads to the following demand for capital

$$\rho(k) = f_k(k) - \rho_k(k)k - (r + \delta) \frac{\gamma}{\rho(\theta)}.$$

(7)

The term $\rho_k(k)k$ takes into account the firm’s strategic investment motive with respect to the rental rate and is described further below. Without this term, (7) would reduce to a standard capital demand condition, stating that the firm’s marginal product of capital equals the rental rate plus investment adjustment cost (here, the average venture posting cost per unit of capital). In turn, the supplier’s problem implies the following steady state condition for new liquid capital

$$1 = q(\theta) - \frac{\rho}{r + \delta} + (1 - q(\theta))\beta.$$  

(8)

Committing one unit of liquid capital costs one unit of numeraire. With probability $q(\theta)$, the liquid capital gets matched with a firm yielding a discounted present-value of $\rho/(r + \delta)$. With probability $1 - q(\theta)$, the liquid capital remains unmatched and returns to the supplier in the beginning of next period, implying a discounted value of $\beta$.

As described above, Stole and Zwiebel’s (1996a) assumption of pairwise continuous bargaining implies that the rental rate is a function of the marginal match surplus, defined as $J_k(k) + V_\kappa(\kappa, u) - V_u(\kappa, u)$ where $J_k(k)$, $V_\kappa(\kappa, u)$ and $V_u(\kappa, u)$ denote the different marginal values of the firm and the supplier (see appendix A.2 for formal definitions). Following Stole and Zwiebel (1996a) one more step, if within each pairwise negotiation, the firm and the supplier play an alternating-offer game with exogenous probability of breakdown as in Binmore, Rubinstein and Wolinsky (1986), then
the unique subgame-perfect equilibrium solves the generalized Nash bargaining problem over the marginal match surplus; i.e. the renegotiation-proof rental rate with each of the suppliers is such that 
\[ \phi J_k(k) = (1-\phi) [V_u(\kappa, u) - V_u(\kappa, u)] \]
where \( \phi \) denotes the bargaining power of capital suppliers. In steady state, this rental rate can be expressed as
\[
\rho(k) = \phi [f_k(k) - \rho_v(k)k] + (1 - \phi) \frac{r + \delta}{1 + r}. \tag{9}
\]

The rental rate is a weighted average of the firm’s and the supplier’s respective outside option (which together define marginal surplus), with the weights depending on the relative bargaining power of the two parties. The supplier’s outside option is to have an additional unmatched unit of capital in the beginning of next period, which has a discounted annuity value of \((r + \delta)/(1 + r)\). The firm’s outside option is to produce with one marginal unit of capital less, which reduces production by \( f_k(k) \) and changes total rental cost by \(-\rho_v(k)k\). This latter effect is a direct consequence of the continuous bargaining assumption: if negotiations with a supplier break down, the resulting change in surplus triggers negotiation of a new rental rate with all remaining suppliers.

To derive an explicit expression for \( \rho_v(k) \), notice that (9) is a non-homogenous linear differential equation of \( \rho \) in \( k \) with solution
\[
\rho(k) = k^{-\frac{1}{\delta}} \int_0^k z^{1-\phi} f_z(z) dz + (1 - \phi) \frac{r + \delta}{1 + r}. \tag{10}
\]

The rental rate is a function of the contribution of each unit of capital to the firm’s production. Intuitively, this dependence on infra-marginal productivities arises because the change in rental rate if negotiations with one supplier break down depends itself on what would happen to the rental rate if there was a hypothetical breakdown with one of the remaining capital suppliers – a recursive argument that can be carried through all the way to the first capital supplier (see Stole and Zwiebel, 1996a for a demonstration). Given (10), the firm’s capital demand in (7) can be expressed as

\[
\rho(k) = OI(k) \times f_k(k) - (r + \delta) \frac{\gamma}{p(\theta)}. \tag{11}
\]

where
\[
OI(k) \equiv 1 - \frac{k^{-\frac{1}{\delta}} \int_0^k z^{\frac{1}{\delta}} f_{zz}(z) dz}{f_k(k)}. \tag{12}
\]
is defined as the ‘overinvestment factor’, analogous to the definition of the overemployment factor in Cahuc, Marque and Wasmer (2008). This overinvestment factor captures the essence of the firm’s strategic response to the holdup problem and has the following properties.

**Proposition 2.** For \( \phi > 0 \) and \( f_z(z) \leq 0 \) with strict inequality over some segment of \( z \in [0, k] \), \( OI(k) > 1 \) and \( \partial OI(k)/\partial \phi > 0 \).

PROOF: Appendix A.2.

\( OI(k) > 1 \) together with the fact that \( OI(k) \times f_k(k) \) is decreasing in \( k \) (see Appendix A.2) implies that the firm’s investment exceeds the level warranted by its marginal productivity. Intuitively, by increasing its capital stock, the firm drives down productivity and with it the rental rate, thus alleviating the holdup problem. \( \partial OI(k)/\partial \phi > 0 \), in turn, implies that the larger the share of the surplus the supplier can appropriate, the larger the firm’s incentive to overinvest.

To examine the effects of the holdup problem in general equilibrium, combine the rental rate in (9) with the firm’s capital demand in (11) to obtain

\[
\frac{\gamma}{p(\theta)} = (1 - \phi) \left[ \frac{OI(k) \times f_k(k)}{r + \delta} - \beta \right].
\]  

(13)

This equation implies that the firm’s average cost per match equals its share of the match surplus. Likewise, supply condition (8) can be combined with the rental rate in (9) to obtain

\[
\frac{1 - \beta}{q(\theta)} = \phi \left[ \frac{OI(k) \times f_k(k)}{r + \delta} - \beta \right].
\]  

(14)

This equation implies that in equilibrium, the capital supplier’s average opportunity cost per match equals its share of the match surplus. After aggregation over all firms and suppliers, the two equations lead to the following characterization of the decentralized allocation.

**Proposition 3.** There exists a unique decentralized allocation \( \{K^D, \theta^D\} \) that solves (13) and (14). This allocation is always inefficient. In particular, for \( \phi = \epsilon(\theta^P) \), \( \theta^D = \theta^P \) and \( K^D > K^P \).

PROOF: Appendix A.2.

To provide intuition for this proposition, combine (13) with (14) to eliminate the bracketed
expression. This yields a unique solution for capital market tightness

\[ \theta_B = \frac{1 - \beta}{\gamma} \frac{1 - \phi}{\phi}, \tag{15} \]

which closely resembles the socially efficient solution in (6), with the difference that the bargaining power \( \phi \) replaces the match elasticity \( \epsilon(\theta) \). If \( \phi = \epsilon(\theta) \), the bargaining power is just strong enough that the firm and the supplier’s matching probabilities reflect the externality from adding another venture, respectively another unit of liquid capital to the market. Hence \( \theta_B = \theta^P \). This is the equivalent of Hosios’ (1990) famous efficiency condition in labor search models with bargaining.

Comparing (13) and (14) with the social planner counterparts in (4) and (5), it is then straightforward to see that the decentralized allocation must be inefficient because \( OI(K) > 1 \) for any \( K \). Since \( OI(K) \times f_K(K) \) is decreasing in \( K \) by Proposition 2, this implies that \( K^D > K^P \) at \( \phi = \epsilon(\theta^P) \); i.e. there is overinvestment in general equilibrium.

For \( \phi \neq \epsilon(\theta^P) \), the inefficiency due to the holdup problem interacts with the externality from the search friction. Specifically, a lower \( \phi \) reduces the firm’s incentive to overinvest (i.e. by Proposition 2, \( OI(K) \) is increasing in \( \phi \)). At the same time, a lower \( \phi \) decreases capital suppliers’ incentive to provide liquid capital and increases the firm’s incentive to post ventures. As detailed in Appendix A.2, it is possible to show that taken together, the net effect of lowering \( \phi \) below \( \epsilon(\theta^P) \) is a decrease in \( K^D \) and that for sufficiently low values of \( \phi \), the equilibrium capital stock falls below the efficient level. Vice versa, as \( \phi \) increases above \( \epsilon(\theta^P) \), \( K^D \) locally increases even further above \( K^P \) but ultimately decreases below the efficient level.\(^{13}\)

### 2.4 Behind overinvestment

The basic model embodies a number of assumptions made for tractability reasons. To obtain a better sense of the fundamental forces behind overinvestment, the section closes with a discussion

\(^{13}\) \( K^D \) therefore follows a general inverted U-shape in \( \phi \), which is similar to the relationship between equilibrium employment and worker bargaining power in a standard labor search model with holdup problems (e.g. Cahuc and Wasmer, 2001b). At the same time, the equilibrium implications of \( \phi \neq \epsilon(\theta^P) \) are quite different from the labor search case. Absent holdup problems, a fall in workers’ bargaining power below the elasticity of the matching function implies an inefficiently high equilibrium employment. With holdup problems, the effect of changing bargaining power is therefore always ambiguous and implies that there exist worker bargaining powers for which the decentralized equilibrium is efficient. The reason for this difference is that in the standard search model, labor force participation (respectively search intensity by workers) is fixed.
of the different assumptions and how they relate to the holdup literature.

As emphasized in the introduction, two of the main ingredients for overinvestment are specificity and absence of ex-ante binding contracts. While both of these frictions are crucial, their exact form is not. Consider specificity first. In the basic model, specificity arises because matching between firms and suppliers is subject to search frictions that cost firms \( \frac{\gamma}{p(\theta)} \) per supplier. However, the overinvestment result would obtain even in the extreme case of no venture posting cost (i.e. \( \gamma = 0 \)) as long as one maintains that firms cannot replace a supplier with another in the same period in case negotiations break down.\(^{14}\) In other words, the overinvestment result does not hinge on the details of the search friction. By the same argument, overinvestment would arise if firms were able to obtain new capital from already matched suppliers as long as this new capital cannot be delivered instantaneously (otherwise, there would be costless replacement).\(^{15}\)

Second, consider absence of ex-ante binding contracts. In the model, this friction is implemented through the assumption of continuous pairwise bargaining after the match has occurred. However, the overinvestment result would obtain for any alternative bargaining protocol as long as suppliers can appropriate some of the match surplus ex-post. Furthermore, as Stole and Zwiebel (1996a) demonstrate, the solution of the continuous pairwise bargaining game is equivalent to the Shapley values of a corresponding cooperative game. Hence, the absence of cooperation among suppliers is by itself not important. What is important, however, is that suppliers cannot form a perfect coalition ex-ante and consider the firm’s capital stock as an endogenous variable when making liquid capital decisions. In the model, such ex-ante coalition-building is ruled out because prior to matching when the firm makes its investment decision, new suppliers do not know either incumbent suppliers or other new potential suppliers. As discussed in the introduction, the same sequentiality constraint is what prevents firms and suppliers from entering ex-ante binding agreements that would circumvent the holdup problem; e.g. long-term contracts (e.g. Williamson, 1975); reallocation of property rights (e.g. Klein, Crawford and Alchian, 1978; Grossman and Hart, 1985; Hart and Moore, 1990); punishment schemes (e.g. MacLeod and Malcomson; 1993); or price posting with

\(^{14}\)If \( \gamma = 0 \), the firm would post an infinity of ventures, which would drive the price of capital down to the supplier’s reservation value and allow the firm to capture the entire match surplus (see Cahuc and Wasmer, 2001b for a demonstration of this result in the labor search context).

\(^{15}\)In the model, this restriction is imposed by assuming that suppliers pre-produce before entering the matching market and that search frictions prevent instantaneous reallocation of unmatched capital to matched firms. Alternatively, one could impose a convex cost for delivery of additional capital within the same period.
commitment (Acemoglu and Shimer, 1999).

This last point raises the interesting question of how results would change if, upon matching, firms purchased rather than rented capital from suppliers; i.e. if there was an ex-post reallocation of property rights or, equivalently, an ex-post long-term contract. As shown in an earlier version of the paper, the overinvestment result would still emerge under this alternative purchasing assumption (or, more generally, the long-term leasing assumption) except that the firm would only bargain with newly matched suppliers and thus, the resulting price next period would be a function of infra-marginal productivities between \( k' \) (the capital stock if negotiations with all new suppliers are successful) and \((1 - \delta)k\) (the capital stock in case negotiations with all new suppliers break down). In addition, the purchasing assumption would introduce a second strategic motive for the firm because \((1 - \delta)k\) (the lowest possible capital stock if all negotiations break down) and therefore the price depends on capital accumulation in previous periods. As shown formally by Kurmann and Rabinovich (2012), this intertemporal consideration provides an additional source of overinvestment, therefore reinforcing the results derived here.

Next, consider production. In the basic model, capital is a perfectly divisible and homogenous input to production. In reality, capital projects are often indivisible and heterogenous. Stole and Zwiebel (1996a) derive their overhiring result in a discretized environment and thus, factor indivisibilities per se are not a problem. The only thing that would change is that \( k \) and therefore \( \rho(k) \) would no longer be continuous, thus unnecessarily complicating the equilibrium analysis.\(^{16}\) Heterogeneity in capital inputs is more difficult to analyze and is therefore treated in detail in the next section for an environment with two capital inputs.

Finally, consider the assumption of decreasing returns to capital. If \( f_k(k) \) was constant, the firm would have no incentive to overinvest because the marginal surplus of a match and therefore the rental rate would not be a function of capital. By the same argument, if firms produced with constant-returns-to-scale technology in capital and another factor (e.g. labor) that is hired ex-post in a competitive spot market, the overinvestment result would disappear. This is because with constant-returns-to-scale technology, the marginal productivity of capital is proportional to the price.

\(^{16}\)It is also possible to show that the assumption of atomistic suppliers represents a limiting case that occurs when the search cost per supplier is zero. If the search cost per supplier was positive instead, the firm would optimally obtain equal amounts of capital from a finite number of suppliers. This would make the overinvestment motive worse.
of the other factor, which, in a competitive spot market, is taken as exogenous by the firm.\textsuperscript{17} The incentive to overinvest reappears, however, under the assumption that the firm’s optimal decision for the other factor occurs before the price of capital is negotiated. If, in addition, this other factor is also subject to holdup problems, then the firm’s strategic behavior in each market has non-trivial general equilibrium effects. These effects are analyzed in detail in the next section.

The assumption of decreasing returns to capital in the basic model is one of the key differences to an earlier analysis of specificity and contract incompleteness in capital markets by Caballero and Hammour (1998a) that implies underinvestment rather than overinvestment. More precisely, Caballero and Hammour impose that the productivity in joint relationships is exogenous and thus, the concept of the firm as an independent optimizing entity is absent. Furthermore, Caballero and Hammour assume that suppliers have a convex cost structure and "...that each type of factor in the production unit forms a coalition that bargains as a single agent" (fn. 13, p. 733). Their analysis therefore places all the power to react strategically against holdup problems in the hands of the suppliers. This comparison highlights the crucial role of microeconomic structure for the analysis of holdup problems, and one of the main contribution of the paper is to show that two relatively innocuous assumptions – diminishing returns to capital and absence of ex-ante binding contracts due the decentralized, sequential nature of investment – overturn Caballero and Hammour’s underinvestment result.

3 Overinvestment with multiple factors of production

This section extends the analysis to an economy with multiple factors of production in which each factor market is subject to search frictions and ex-post bargaining. The partial equilibrium analysis of the resulting multiple holdup problem borrows from Cahuc, Marque and Wasmer (2008) who, in the context of a labor search model, use spherical coordinate techniques to derive a closed-form expression for wages. As they show, existence and uniqueness of an equilibrium is impossible to establish for general functional forms in such a multi-factor setting. This section therefore focuses on two examples that are of particular interest for the present analysis and for which existence of a unique general equilibrium allocation can be established.

\textsuperscript{17}See Cahuc and Wasmer (2001a) who show this result in a labor search context.
The first example is a decreasing-returns-to-scale production function in two capital inputs. The main objective of this example is to assess the robustness of the overinvestment result to the introduction of a second capital input that is either a substitute or a complement in the production function. The second example is a constant-returns-to-scale production function in capital and labor. The focus of this example is to analyze how the holdup problem in the capital market interacts with a standard holdup problem in the labor market that on its own leads to underinvestment and overemployment. Notice that all the results derived in the two-capital example carry through to a variant of the capital-labor example with decreasing-returns-to-scale. This variant is therefore not analyzed separately.

3.1 Two capital factors

The environment is identical to the basic model except that firms now produce with technology $f(k_1, k_2)$, where $k_1$ and $k_2$ denote two different capital factors that are rented from two separate $[0, 1]$ continua of capital suppliers. This technology is strictly increasing and concave in each of its arguments, satisfies the usual Inada conditions, and exhibits decreasing-returns-to-scale. Each capital market is subject to allocation frictions characterized by matching functions $m(V_1, L_1)$, $m(V_2, L_2)$ and market tightness $\theta_1 \equiv V_1/L_1$, $\theta_2 \equiv V_2/L_2$, respectively.\textsuperscript{18}

3.1.1 Efficient allocation

As in the basic model, it is helpful to start with the efficient allocation. The planner’s problem is

$$
P(K_1, U_1, K_2, U_2) = \max_{L_1, V_1, K_2, V_2} \left[ f(K_1, K_2) - \gamma_1 V_1 - \gamma_2 V_2 - (L_1 - U_1) - (L_2 - U_2) + \beta P(K'_1, U'_1, K'_2, U'_2) \right]
$$

s.t. \quad K'_i = (1 - \delta_i) K_i + m(V_i, L_i)

$$
U'_i = L_i - m(V_i, L_i),
$$

where $K_i = \int_0^1 k_idt = \int_0^1 \kappa_i d\omega_i$, $i = 1, 2$ and so forth for the other aggregate quantities. As shown in Appendix B.1, for general production functions, the solution to this problem does not necessarily

\textsuperscript{18}To save on notation, the two matching frictions and the corresponding match probabilities are described by the same functional forms. Technically, this means that the matching frictions in the two markets are identical. Nothing about the results would change if different functional forms were used instead so as to allow for different matching frictions in each market.
exist and, if it exists, is not necessarily unique. Given the decreasing-return-to-scale assumption, however, the following characterization of the equilibrium obtains.\textsuperscript{19}

**Proposition 4.** For homogenous production functions of degree $d < 1$ with

$$f_{K_2}(K_1, K_2) f_{K_1K_1}(K_1, K_2) - f_{K_1}(K_1, K_2) f_{K_1K_2}(K_1, K_2) < 0 \forall (K_1, K_2),$$

(16)

there exists a unique efficient allocation $\{K^P_i, \theta^P_i\}_{i=1}^2$ that solves

$$\theta_i = \frac{1 - \beta}{\gamma_i} \frac{1 - \epsilon(\theta_i)}{\epsilon(\theta_i)}$$

(17)

$$A^P_i(\theta^P_i) \equiv (r + \delta_i) \left( \frac{\gamma_i}{\epsilon(\theta_i)(1 - \epsilon(\theta_i))} + \beta \right) = K^{d-1}_i f_i(R, 1)$$

(18)

where $R \equiv K_1/K_2$ and $f_i(R, 1)$ denotes the partial derivative of $f(R, 1)$ to its $i$-th argument.

**PROOF:** Appendix B.1.

Condition (17) is equivalent to (6) in the basic model and uniquely pins down tightness in each of the capital markets. Given $\theta^P_1$ and $\theta^P_2$, condition (18) is equivalent to (4) and uniquely defines $R^P_i, K^P_1$ and $K^P_2$ as long as the two capital inputs are not too substitutable with each other (i.e. $f_{K_1K_2}(K_1, K_2)$ is either positive or not too negative so as to satisfy condition (16)). As Appendix B.1 shows, a straightforward functional form for which this condition is always satisfied is the CES case $f(k_1, k_2) = [k_1^\sigma + k_2^\sigma]^{d/\sigma}$ where $0 < d < 1$ and $\sigma < 1$ is the elasticity of substitution. The efficient equilibrium values of the other aggregates are then computed from the different constraints and the definition of capital market tightness.

### 3.1.2 Decentralized allocation

Analogous to the basic model, the rental rate between firms and capital suppliers in each market is determined ex-post through continuous pairwise bargaining over the marginal match surplus using the alternating-offer protocol of Binmore, Rubinstein and Wolinsky (1986). As long as $k_1$ and $k_2$ are not perfect substitutes, the resulting rental rates are a function of both capital stocks of the firm; i.e. $\rho_1 = \rho_1(k_1, k_2)$ and $\rho_2 = \rho_2(k_1, k_2)$. Firms internalize this effect in their venture decision.

\textsuperscript{19}The assumption of decreasing-returns-to-scale is in itself not necessary except that the present environment is too linear in other dimensions for there to be a determinate solution if production was constant-returns-to-scale.
but capital suppliers do not because they match with a random set of firms and in each match provide only an atomistic portion of the firm’s capital stock. Consequently, the problem of a firm entering the period with capital stocks $k_1$ and $k_2$ is

$$J (k_1, k_2) = \max_{v_1, v_2} [f (k_1, k_2) - \rho_1 (k_1, k_2) k_1 - \rho_2 (k_1, k_2) k_2 - \gamma_1 v_1 - \gamma_1 v_2 + \beta J (k'_1, k'_2)]$$

$$s.t. \quad k'_i = (1 - \delta_i) k_i + p (\theta_i) v_i,$$

for $i = 1, 2$. In turn, the problem of a supplier in market $i$ entering the period with matched capital $\kappa_i$ and unmatched liquid capital $u_i$ is

$$V (\kappa_i, u_i) = \max_{l_i} [\rho_i \kappa_i - (l_i - u_i) + \beta V (\kappa'_i, u'_i)]$$

$$s.t. \quad \kappa'_i = (1 - \delta_i) \kappa_i + q (\theta_i) l_i$$

$$u'_i = (1 - q (\theta_i)) l_i,$$

for $i = 1, 2$. The resulting steady state demand for capital of type $i$ is

$$\rho_i (k_1, k_2) = f_{ki} (k_1, k_2) - \rho_{1,ki} (k_1, k_2) k_1 - \rho_{2,ki} (k_1, k_2) k_2 - (r + \delta_i) \frac{\gamma_i}{p (\theta_i)}.$$ \hspace{1cm} (19)

The terms $\rho_{1,ki} (k_1, k_2) k_1$ and $\rho_{2,ki} (k_1, k_2) k_2$ capture the firm’s strategic reaction to the double holdup problem, as explained below. The suppliers’ problem, in turn, is identical to the one in the basic model. The steady state condition for optimal supply of liquid capital in market $i$ is therefore the same as in (8); i.e.

$$(1 - \beta) = q (\theta_i) \left[ \frac{\rho_i}{r + \delta_i} - \beta \right].$$ \hspace{1cm} (20)

The rental rate for capital of type $i$ that obtains from the continuous pairwise bargaining assumption solves $\phi_i J_{ki} (k_1, k_2) = (1 - \phi_i)[V_{ki} (\kappa_i, u_i) - V_{ui} (\kappa_i, u_i)]$, where $\phi_i$ denotes the bargaining power of the capital supplier in market $i$. In steady state, this rental rate can be expressed as

$$\rho_i (k_1, k_2) = \phi_i \left[ f_{ki} (k_1, k_2) - \rho_{1,ki} (k_1, k_2) k_1 - \rho_{2,ki} (k_1, k_2) k_2 \right] + (1 - \phi_i) \frac{r + \delta_i}{1 + r}.$$ \hspace{1cm} (21)

for $i = 1, 2$. As in the basic model, the rental rate is a weighted average of the firm’s and the
supplier’s outside option. The only difference now is that the firm’s outside option of producing with one less unit of \( k_i \) affects rental costs in both markets. This is because for general production functions, a decrease in \( k_i \) affects the marginal productivity of both capital types. As a result, the remaining matched suppliers from both markets come in to negotiate a new rental rate, the effect of which is captured by \( \rho_{1,k_i} (k_1, k_2) k_1 \) and \( \rho_{2,k_i} (k_1, k_2) k_2 \).

Deriving an explicit expression for these two terms involves solving the system of two non-homogenous linear differential equations formed by (21). Using spherical coordinate techniques as in Cahuc, Marque and Wasmer (2008) yields (see Appendix B.2 for details)

\[
\rho_i (k_1, k_2) = \int_0^1 z^{-\phi_i} f_i (k_1 z^{\chi_{1i}}, k_2 z^{\chi_{2i}}) dz + (1 - \phi_i) \frac{r + \delta_i}{1 + r},
\]

where \( f_i (k_1 z^{\chi_{1i}}, k_2 z^{\chi_{2i}}) \) is defined as the partial derivative of \( f(k_1 z^{\chi_{1i}}, k_2 z^{\chi_{2i}}) \) with respect to its \( i \)-th argument; and \( \chi_{ij} \equiv \frac{1 - \phi_i}{\phi_i} \frac{\phi_j}{1 - \phi_j} \) measures the bargaining power of a capital supplier in market \( i \) relative to a capital supplier in market \( j \). Similar to the basic model, the rental rate for capital \( k_i \) is a function of both its infra-marginal productivities. If these inframarginal productivities are a function of both capital stocks, the rental rate depends on both capital stocks.

Given this solution, the firm’s demand for capital \( k_i \) in (19) can be expressed similarly to equation (11) for the basic model; i.e.

\[
\rho_i (k_1, k_2) = OI_i (k_1, k_2) \times f_{k_i} (k_1, k_2) - (r + \delta_i) \frac{\gamma_i}{p (\theta_i)},
\]

only that now, the ‘overinvestment factor’ takes on a more complicated form

\[
OI_i (k_1, k_2) = 1 - \frac{k_1 \int_0^1 z^{-\phi_1 + \chi_{1i}} f_{i1} (k_1 z^{\chi_{1i}}, k_2 z^{\chi_{2i}}) dz + k_2 \int_0^1 z^{-\phi_1 + \chi_{2i}} f_{i2} (k_1 z^{\chi_{1i}}, k_2 z^{\chi_{2i}}) dz}{f_{k_i} (k_1, k_2)}.
\]

Consider demand for \( k_1 \) (i.e. \( i = 1 \)). As in the basic model, diminishing marginal productivity \( (f_{11} < 0) \) increases \( OI_1 (k_1, k_2) \), pushing the firm to overinvest in order to reduce the holdup problem in the market for \( k_1 \). At the same time, \( k_1 \) affects the marginal productivity of \( k_2 \) and thus the holdup problem in the market for \( k_2 \). If the two capital inputs are substitutes (i.e. \( f_{12} < 0 \)), a larger \( k_1 \) lowers the marginal productivity of \( k_2 \), thus reinforcing the tendency to overinvest. If, by contrast, the two capital inputs are complements (i.e. \( f_{12} > 0 \)), then the firm faces a trade-off
because a larger $k_1$ worsens the holdup problem in the market for $k_2$.

The implications for complements can be nicely illustrated for the case of Cobb-Douglas technology $f(k_1, k_2) = k_1^{\alpha_1} k_2^{\alpha_2}$ with $\alpha_1 + \alpha_2 = d < 1$. For this specification, the overinvestment factor in (24) simplifies to

$$OI_i = \frac{1}{1 - \phi_i (1 - \chi_{i1} \alpha_1 - \chi_{i2} \alpha_2)}.$$  

For equal bargaining powers, $\chi_{i1} = \chi_{i2} = 1$ and $OI_1 = OI_2 > 1$: the firm has an incentive to overinvest equally in both factors so as to take advantage of decreasing-returns-to-scale. If $\phi_1 > \phi_2$, then the overinvestment motive is stronger for the first type of capital than for the second one (i.e. $OI_1 > OI_2$). Furthermore, if $\phi_1$ is sufficiently large compared to $\phi_2$ such that $1 - \chi_{21} \alpha_1 - \alpha_2 < 0$, the overinvestment motive for the second type of capital is reverted; i.e. $OI_2 < 1$. These partial-equilibrium implications are equivalent to the ones discussed in the labor search context by Cahuc, Marque and Wasmer (2008).

To analyze the general equilibrium effects of the firm’s overinvestment motives, combine (19) and (20), respectively, with (22) to eliminate rental rates. After aggregation over firms and capital suppliers, the following solution obtains.

**Proposition 5.** For homogenous production functions of degree $d < 1$ with

$$\int_0^1 z^{\frac{1}{\gamma_1}} f_{11} (Rz, z^{\chi_{12}}) dz \times A^D_i (\theta_2) - \int_0^1 z^{\frac{1-d}{\gamma_2}} f_{12} (R z^{\chi_{21}}, z) dz \times A^D_i (\theta^D_i) < 0 \forall (K_1, K_2),$$

there exists a unique decentralized allocation $\{K^D_i, \theta^D_i\}_{i=1}^2$ that solves

$$\theta_i = \frac{1 - \beta}{\gamma_i} \frac{1 - \phi_i}{\phi_i}$$  

$$A^D_i (\theta^P_i) \equiv (r + \delta_i) \left( \frac{\gamma_i}{p(\theta_i)(1 - \phi_i)} + \beta \right) = K^{d-1}_i OI_i(R, 1) f_i(R, 1).$$

This allocation is always inefficient. In particular:

1. For $\phi_1 = \epsilon(\theta^P_1) = \epsilon(\theta^P_2) = \phi_2$, $\theta^D_1 = \theta^P_1 = \theta^P_2 = \theta^D_2$ and $R^D = R^P$ with $K^D_1 > K^P_1$, $K^D_2 > K^P_2$.

2. For $\phi_1 = \epsilon(\theta^P_1) > \epsilon(\theta^P_2) = \phi_2$, $\theta^D_1 = \theta^P_1 < \theta^P_2 = \theta^D_2$ and

   (a) $R^D > R^P$ if $f_{12} > 0$ with $K^D_1 > K^P_1$, $K^D_2 > K^P_2$ for $\phi_1 - \phi_2$ sufficiently small;
(b) $R^D < R^P$ and $K^D_2 > K^P_2$ around $\phi_1 = \phi_2 = \phi$ if $f_{12} < 0$ for $f_{12}$ sufficiently negative and $\phi$ sufficiently large.

**PROOF:** Appendix B.2.

The first part of the Proposition is analogous to Proposition 4. Condition (26) pins down market tightness $\theta^D_1$ and $\theta^D_2$. Given $\theta^D_1$ and $\theta^D_2$, condition (27) then define a unique solution for $R^D$, $K^D_1$ and $K^D_2$ as long as the two capital inputs are not too substitutable such that condition (25) holds.

The second part of the Proposition analyzes the consequences of the double holdup problem. For all the cases considered, market tightness is assumed to be efficient (i.e. $\phi_1 = \epsilon(\theta^P_1)$ and $\phi_2 = \epsilon(\theta^P_2)$) so as to isolate the effects of the double holdup problem from the effects of search externalities. First, when match elasticities and therefore market tightness are the same in the two markets, the equilibrium ratio of the two capital stocks is efficient ($R^D = R^P$) but there is overinvestment in each market ($K^D_1 > K^P_1$ and $K^D_2 > K^P_2$). As Appendix B.2 shows, this overinvestment result is entirely driven by the assumption of decreasing-returns-to-scale and does not depend on whether the two capital inputs are substitutes or complements. This result contrasts with the partial equilibrium results discussed above, which showed that everything else constant, complementarity between the two capital inputs generally reduces the incentive to overinvest in each market. In general equilibrium, however, firms need to balance the effect of overinvesting in one market with the indirect effect on the other market. When the bargaining power of the firm in each market is the same, the two effects cancel each other out and the only incentive for overinvestment left is the one from decreasing-returns-to-scale. This difference in results emphasizes the importance of studying the effects of holdups in general equilibrium.

Once the firm’s bargaining power in the two markets is different (but still such that there are no search externalities), the situation becomes more complicated. As Appendix B.2 shows, if the capital factors are complements (i.e. $f_{12} > 0$), $\phi_1 > \phi_2$ implies relative overinvestment in the first capital factor; i.e. $R^D > R^P$. However, only for $\phi_1$ sufficiently close to $\phi_2$ is it also the case that the levels of each factor are inefficiently high (i.e. $K^D_1 > K^P_1$, $K^D_2 > K^P_2$). Intuitively, if the difference between $\phi_1$ and $\phi_2$ is too large, then the distortions from the double holdup become so large so as to result in inefficiently low absolute levels of capital stocks. This can be nicely illustrated with the above Cobb-Douglas example. In this case, equilibrium implies $(K^D_i/K^P_i)^{1-d} = OI^+_i OI^{-\alpha_i}_j$.
with \( \lim_{\phi_i \to 1} OI_i = 1/\alpha_i \) and \( \lim_{\phi_i \to 1} OI_j = 0 \). Hence, equilibrium capital stocks \( K^D_1 \) and \( K^P_2 \) both converge to 0 as \( \phi_i \to 1 \); i.e. everything else constant, firms have an incentive to overinvest maximally in the \( \phi \)-th capital type and nothing in other capital type. But since the two capital stocks are complements in production, the equilibrium outcome is that investment drops to 0.

Finally, if the factors are substitutes (i.e. \( f_{12} < 0 \)), then it can be shown that in the neighborhood of \( \phi_1 = \phi_2 = \phi \), \( \phi_1 > \phi_2 \) implies relative underinvestment in the first factor (i.e. \( R^D < R^P \)) as long as the two factors are sufficiently substitutable and the firm’s bargaining power in each market is sufficiently small (i.e. \( \phi \) sufficiently large). For example, in the CES technology case presented above, this condition can be expressed in closed-form around \( \phi_1 = \phi_2 = \phi \) as (see Appendix B.2)

\[
\sigma - d > \frac{1 - \phi}{\phi}.
\]

Intuitively, when the two factors are sufficiently substitutable (i.e. \( \sigma - d \) large) and the holdup problem in each market is important (i.e. \( \phi \) large), then \( \phi_1 > \phi_2 \) implies that the firm substitutes towards the capital factor with the lower bargaining power so as to depress the price of the other factor. Furthermore, by the substitutability of the two factors, \( R^D < R^P \) implies that \( K^D_2 > K^P_2 \); i.e. there is overinvestment in the market where the firm has higher bargaining power. As before, these implications are quite different from the partial equilibrium results, thus emphasizing the importance of analyzing holdup problems in general equilibrium.

### 3.2 Capital and labor

The environment consists of firms producing with technology \( f(k, n) \), where \( k \) and \( n \) denote capital and labor that are rented from a \([0, 1]\) continuum of atomistic capital suppliers and a \([0, 1]\) continuum of atomistic workers, respectively. This technology is strictly increasing and concave in each of its arguments, satisfies the usual Inada conditions, and exhibits constant-returns-to-scale.

The capital market is subject to exactly the same search friction as in the basic model, characterized by matching function \( m(V_k, L) \) and market tightness \( \theta_k \equiv V_k/L \). The evolution of a firm’s productive capital stock, respectively of a capital supplier’s rented capital stock and unmatched capital is therefore described by equations (1)-(3).

The labor market is modeled as in Smith (1999), Cahuc and Wasmer (2001a,b) and Cahuc,
Marque and Wasmer (2008). Workers are infinitely-lived; discount the future at the same rate \( \beta = (1 + r)^{-1} \) as firms and capital suppliers; and have linear preferences for the numeraire good.\(^{20}\) When employed, workers are paid wage \( w \) that is continuously rebargained with the firm, as will be described below. When unemployed, workers obtain flow value \( b < w \) from non-market activity.

The allocation of workers to firms is subject to a matching friction and occurs in two stages. In the first stage, firms open job vacancies \( v_n \) at flow cost \( \gamma_n \) and search for available workers. Available workers consist of the unemployed from the previous period plus workers who have been newly separated from their jobs.\(^{21}\) Let \( 1 - N \) denote the number of unemployed workers in the beginning of the period and \( sN \) the number of newly separated workers, with \( s \) denoting the exogenous rate at which workers separate from firms. Then \( 1 - N + sN \) defines the number of available workers. Matching between job vacancies and available workers occurs according to matching function \( m(V_n, 1 - N + sN) \), with \( V_n \equiv \int_0^1 v_n dt \) denoting the total mass of job vacancies.\(^{22}\) As in the capital market, it is assumed that this matching function exhibits constant-returns-to-scale and satisfies \( \lim_{V_n \to 0} m(\theta_n) = 1 \), \( \lim_{V_n \to \infty} m(\theta_n) = 0 \) with \( \theta_n = V_n/(1 - N + sN) \) denoting labor market tightness. Accordingly, \( p(\theta_n) \equiv m(V_n, 1 - N + sN)/V_n \) and \( q(\theta_n) = m(V_n, 1 - N + sN)/(1 - N + sN) \) denote the probabilities of filling and finding a vacant job, respectively.

In the second stage, newly matched workers join the firm’s existing hires net of separations and produce. After production has taken place, \( sn \) workers separate from the firm and join the unemployed to search for new jobs. Given these assumptions, the evolution of a firm’s employment \( n \) is described by

\[
n' = (1 - s)n + p(\theta_n)v_n. \tag{28}
\]

\(^{20}\)It is straightforward to show that none of the results are affected if workers are risk averse.

\(^{21}\)The assumption that separated workers can reenter the labor market in the same period contrasts with the standard Mortensen-Pissarides setup, where separated workers reenter only the following period and thus spend at least one period in unemployment. The assumption has the advantage that zero matching frictions implies zero unemployment (also see Den Haan, Ramey and Watson, 2000).

\(^{22}\)Similar to the example with two capital inputs, the same functional form for the two matching functions is used so as to save on notation. None of the below results would be affected if different functional forms were used instead.
3.2.1 Efficient allocation

As before, it is useful to first describe the efficient allocation. The planner’s problem is

\[
P(K, U, N) = \max_{L, V_k, V_n} \left[ f(K, N) + b(1 - N + sN) - \gamma_k V_k - \gamma_n V_n - (L - U) + \beta P(K', U', N') \right]
\]

s.t. \( K' = (1 - \delta) K + m(V_k, L) \)

\[
U' = L - m(V_k, L)
\]

\[
N' = (1 - s) N + m(V_n, 1 - N + sN),
\]

where \( K = \int_0^1 kd\tau = \int_0^1 kd\omega; U = \int_0^1 u\omega \) as in the basic model and \( N = \int_0^1 n\omega \). Similar to the model with two capital inputs, the solution to this problem does not necessarily exist for general production functions and, if it exists, is not necessarily unique (see Appendix C.1 for details). Under the assumption of constant-returns-scale technology, however, the following characterization of the equilibrium obtains.

**Proposition 6.** For constant-returns-to-scale technology \( f(k, n) \), there exists a unique efficient allocation \( \{K^P, N^P, \theta_K^P, \theta_N^P\} \) that solves

\[
\theta_K = \frac{1 - \beta}{\gamma_k} \frac{1 - \varepsilon(\theta_K)}{\varepsilon(\theta_K)} \quad (29)
\]

\[
A^P(\theta_K) \equiv (r + \delta) \left( \frac{\gamma_k}{p(\theta_K)(1 - \varepsilon(\theta_K))} + \beta \right) = f_1(R, 1) \quad (30)
\]

\[
N = \frac{q(\theta_N)}{s + (1 - s)q(\theta_N)} \quad (31)
\]

\[
B^P(\theta_N) \equiv b + \frac{\gamma_n}{1 - \varepsilon(\theta_N)} \left[ \frac{r + s}{p(\theta_N)} + (1 - s) \varepsilon(\theta_N) \theta_N \right] = f_2(R, 1) \quad (32)
\]

with \( R \equiv K/N \).

**Proof:** Appendix C.1.

As in the basic model, condition (29) pins down efficient capital market tightness \( \theta_K^P \). Conditions (30)-(31) then jointly determine the efficient levels of \( K^P, N^P \) and \( \theta_N^P \). The remaining efficient allocations \( N^P, K^P, L^P, V_K^P \) and \( V_N^P \) are computed from the different optimality conditions and constraints evaluated in steady state.\(^{23}\)

\(^{23}\)As emphasized in the beginning of the section, the definition of equilibrium would hold for more general ho-
3.2.2 Decentralized allocation

Analogous to the basic model, the rental rate between firms and matched capital suppliers is determined ex-post through continuous pairwise bargaining. Similarly, and following the assumptions in Stole and Zwiebel (1996a,b), Smith (1999), Cahuc and Wasmer (2001a,b) and Cahuc, Marque and Wasmer (2008), the wage of all employed workers (incumbents and newly matched) are negotiated after matching via pairwise bargaining. For the same reasons as in the previous example with two capital inputs, as long as capital and labor are not perfect substitutes, the resulting rental rate and wage rate are functions of both $k$ and $n$; i.e. $\rho = \rho(k, n)$ and $w = w(k, n)$. Firms internalize this effect in their optimal investment and hiring decisions. By contrast, suppliers and workers take $\rho$, respectively $w$ as exogenous because they contribute only an atomistic portion to the firm’s capital stock and employment. The problem of a firm entering the period with capital stock $k$ and employment $n$ is therefore

$$J(k, n) = \max_{v_k, v_n} \left[ f(k, n) - \rho(k, n) k - w(k, n) n - \gamma_k v_k - \gamma_n v_n + \beta J(k', n') \right]$$

subject to:

$$k' = (1 - \delta) k + p(\theta_k) v_k$$

$$n' = (1 - s) n + p(\theta_n) v_n.$$ 

The problem of a capital supplier, in turn, is exactly identical to the one in the basic model and not repeated here (see appendix C.2 for details). Workers, finally, do not take any optimal decisions (if not matched, they automatically search at no cost). The value of a matched (employed) worker is

$$E = w + (1 - s(1 - q(\theta_n))) \beta E' + s(1 - q(\theta_n)) S',$$

and the value of an unmatched (i.e. searching) worker is

$$S = b + q(\theta_n) \beta E' + (1 - q(\theta_n)) \beta S'.$$

On the capital side, the firm’s steady state demand for capital and the supplier’s optimal com-

mogenous production functions as long as $k$ and $n$ are not too substitutable (for constant-returns-to-scale they are necessarily complements). The derivations for this more general case are essentially equivalent to the ones for the two-capital case analyzed above.
dition for liquid capital are, respectively,

\[ p(k, n) = f_k(k, n) - \rho_k(k, n) k - w_k(k, n) n - (r + \delta) \frac{\gamma_k}{p(\theta_k)} \]  

(33)

\[ (1 - \beta) = q(\theta_k) \left[ \frac{\rho}{r + \delta} - \beta \right]. \]  

(34)

On the labor side, the firm’s steady state demand for new workers is

\[ w(k, n) = f_n(k, n) - \rho_n(k, n) k - w_n(k, n) n - (r + s) \frac{\gamma_n}{p(\theta_n)}. \]  

(35)

The terms \( \rho_k(k, n) k \) and \( w_k(k, n) n \) in (33) embody the firm’s strategic motive with respect to capital accumulation; i.e. the firm internalizes that an additional unit of capital affects rental rate negotiations with suppliers but also influences wage negotiations with workers. The terms \( \rho_n(k, n) k \) and \( w_n(k, n) n \) in (35) capture the same strategic considerations with respect to hiring.

The rental rate and the wage rate that come out of the continuous pairwise bargaining with alternating-offer protocol solve \( \phi_k J_k(k, n) = (1 - \phi_k) [V_k(\kappa, u) - V_u(\kappa, u)] \) and \( \phi_n J_n(k, n) = (1 - \phi_n)(E - S) \), respectively, with \( \phi_k \) and \( \phi_n \) denoting the bargaining power of capital suppliers and workers. After some rearrangement, this yields the following steady state expressions for the rental rate and the wage rate

\[ p(k, n) = \phi_k \left[ f_k(k, n) - \rho_k(k, n) k - w_k(k, n) n \right] + (1 - \phi_k) \frac{r + \delta}{1 + r} \]  

(36)

\[ w(k, n) = \phi_n \left[ f_n(k, n) - \rho_n(k, n) k - w_n(k, n) n \right] + (1 - \phi_n) w_R, \]  

(37)

where \( w_R \equiv b + \frac{\phi_n}{1 - \phi_n} \gamma_n (1 - s) \theta_n \) is the worker’s reservation wage. Together, (36) and (37) form a system of non-homogenous linear differential equations of \( p \) and \( w \) in \( k \) and \( n \). Application of the spherical coordinate solution techniques of Cahuc, Marque and Wasmer (2008) yields

\[ p(k, n) = \int_0^1 z^{1-\phi_k} f_1(kz, nz^{\chi_{kn}}) dz + (1 - \phi_k) \frac{r + \delta}{1 + r} \]  

(38)

\[ w(k, n) = \int_0^1 z^{1-\phi_n} f_2(kz^{\chi_{nk}}, nz) dz + (1 - \phi_n) w_R \]  

(39)

with \( \chi_{kn} \equiv \frac{1 - \phi_k}{\phi_k} \frac{\phi_n}{1 - \phi_n} \) and \( \chi_{nk} \equiv 1/\chi_{kn} \). As in the basic model, the price of capital is a weighted av-
verage of infra-marginal productivities with respect to capital. Similarly, the wage rate is a weighted average of infra-marginal productivities with respect to labor. Using these solutions, the capital and labor demands in (33) and (35) can be expressed as

\[ \rho(k, n) = \frac{O I(k, n) \times f_k(k, n) - (r + \delta) \frac{\gamma_k}{p(\theta_K)}} {f_k(k, n)} \]

\[ w(k, n) = \frac{O E(k, n) \times f_n(k, n) - (r + s) \frac{\gamma_n}{p(\theta_N)}} {f_n(k, n)} \]

with the overinvestment factor \( O I(k, n) \) and the overemployment factor \( O E(k, n) \) being defined as

\[ O I(k, n) = 1 - k \int_0^1 z^{\frac{1}{\sigma_k}} f_{11}(kz, nz^{\lambda_k}) dz + n \int_0^1 z^{\frac{1}{\sigma_n} - \frac{1}{\sigma_k}} f_{21}(kz^{\lambda_k}, nz) dz \]

\[ O E(k, n) = 1 - k \int_0^1 z^{\frac{1}{\sigma_k} - \frac{1}{\sigma_n} - \frac{1}{\sigma_k}} f_{12}(kz, nz^{\lambda_k}) dz + n \int_0^1 z^{\frac{1}{\sigma_n}} f_{22}(kz^{\lambda_k}, nz) dz \]

As before, these two definitions closely resemble the definition of the overemployment factor in Cahuc, Marque and Wasmer (2008) and so does their interpretation. Consider first the overinvestment factor \( O I(k, n) \). As in the basic model, decreasing marginal productivity of capital (i.e. \( f_{11} < 0 \)) pushes firms to overinvest in order to reduce the holdup problem in the capital market. At the same time, the firm needs to take into account that capital affects the marginal productivity of labor and thus the holdup problem in the labor market. Since capital and labor are complements (i.e. \( f_{12} > 0 \)) when production is constant-returns-to-scale, a higher capital stock worsens the holdup problem in the labor market, thus reducing the overinvestment motive of the firm. Exactly the same considerations apply to the firm’s hiring decision. One the one hand, larger employment reduces the marginal productivity of labor (i.e. \( f_{22} < 0 \)) and therefore the wage rate. On the other hand, larger employment increases the marginal productivity of capital (i.e. \( f_{21} > 0 \)) and therefore the rental rate. In a constant-returns-to-scale environment, the firm therefore faces a trade-off that, depending on the details of technology and the relative bargaining powers of capital suppliers and workers, may result in either overinvestment (i.e. \( O I(k, n) > 1 \)) or overemployment (i.e. \( O E(k, n) > 1 \)) or both.\(^{24}\)

The results offer a cautionary tale that holdup problems in the labor market do not necessarily

\(^{24}\)As mentioned before, if capital and labor were substitutes instead (in a setting with decreasing-returns-to-scale production), similar strategic motives as for the two-capital case would apply.
lead to underinvestment, as is typically emphasized in the literature (e.g. Grout, 1984; Caballero and Hammour, 1998a). Likewise, holdup problems in a multi-worker firm setting do not necessarily lead to overemployment as argued by Stole and Zwiebel (1996a), Smith (1999), Cahuc and Wasmer (2001a) and Cahuc, Marque and Wasmer (2008). Instead, it all depends on the importance of holdup problems in other factor markets.

To compute the equilibrium allocation, combine capital supply in (34) and capital demand in (40), respectively, with the rental rate in (38); and the wage rate in (39) with labor demand in (41). After aggregation over firms and capital suppliers, the two equations lead to the following characterization of the decentralized allocation.

**Proposition 7.** For constant-returns-to scale production function \( f(k,n) \), there exists a unique decentralized allocation \( \{K^D, N^D, \theta^D_K, \theta^D_N\} \) that solves

\[
\begin{align*}
\theta_K &= \frac{1 - \beta}{\gamma_k} \frac{1 - \phi_k}{\phi_k} \\
A^D(\theta_K) &\equiv (r + \delta) \left( \frac{\gamma}{p(\theta_K)(1 - \phi)} + \beta \right) = OI(R,1) \times f_1(R,1) \\
N &= \frac{q(\theta_N)}{s + (1 - s)q(\theta_N)} \\
B^D(\theta_N) &\equiv b + \frac{\gamma_n}{1 - \phi_n} \left[ \frac{r + s}{p(\theta_N)} + (1 - s)\phi_n\theta_N \right] = OE(R,1) \times f_2(R,1)
\end{align*}
\]

This allocation is generally inefficient except for one special case:

1. For \( \phi_K = \epsilon(\theta^P_K) = \epsilon(\theta^P_N) = \phi_n, \theta^D_K = \theta^P_K, \theta^D_N = \theta^P_N \) and \( R^D = R^P, K^D = K^P \) and \( N^D = N^P \); i.e. the decentralized allocation is efficient.

2. For \( \phi_K = \epsilon(\theta^P_K) > \epsilon(\theta^P_N) = \phi_n, R^D > R^P \) and \( K^D > K^P \) and \( N^D > N^P \) as long as \( \phi_k \) and \( \phi_n \) are sufficiently close together.

**PROOF:** Appendix C.2.

The above partial equilibrium analysis could be easily extended to a multi-worker setting as is done in Cahuc, Marque and Wasmer (2008) in the context of a perfectly competitive capital market. Their partial equilibrium analysis shows that if capital is substitutable to a type of labor with strong bargaining power (e.g. relatively low-skilled but highly unionized labor), then this provides incentive for the firm to overinvest. This incentive is reinforced if the capital market itself is subject to a holdup problem, as is the case in the present context. The present paper refrains from pursuing this multi-worker case further since even for very simple production functions, it is impossible to establish existence and uniqueness in general equilibrium.
The capital market part of the equilibrium is analogous to the basic model: equation (44) pins down capital market tightness $\theta^D_K$; and equation (45) pins down the equilibrium capital-labor ratio $R^D$. Given this ratio, equations (46) and (47) then jointly determine labor market tightness $\theta^D_N$ and employment $N^D$; and the equilibrium values of the other variables can be inferred using the different constraints and definitions.

As in the two-capital example above, the second part of the proposition considers only situations for which the bargaining powers $\phi_k$ and $\phi_n$ are such that $\theta^D_K = \theta^P_K$ and $\theta^D_N = \theta^P_N$ so as to isolate the general equilibrium implications of the double holdup problem from search externalities. Then, for the special case of equal bargaining power by suppliers and workers, the decentralized allocation is efficient. Intuitively, efficiency occurs because under constant-return-to-scale, the firm’s incentive to overinvest in capital is exactly offset by the firm’s incentive to overemploy.\(^{26}\) This knife-edge case illustrates that multiple holdup problems do not necessarily exacerbate each other as is often the case in the literature (e.g. Aruoba, Waller and Wright, 2011). Rather, if the factors subject to holdup problems are complementary in the firms’ objective, then depending on the relative bargaining powers of the different parties involved, the decentralized economy may be relatively close to efficiency even if each holdup problem on its own severely distort allocations.

Once bargaining powers of suppliers and workers are different (but still such that there are no search externalities), the capital-labor ratio departs from the efficient value. In particular, if $\phi_k > \phi_n$, the firm accumulates relatively more capital than labor because the holdup problem in the capital market is worse than in the labor market. If the difference between $\phi_k$ and $\phi_n$ is too large, then the distortions from the double holdup can become so large so as to result in inefficiently low levels of either capital or employment or both. As for the two-capital case, this result can be illustrated nicely for Cobb-Douglas production (see Appendix C.2).

4 Conclusion

This paper shows that two characteristics of many decentralized capital markets – specificity and absence of ex-ante binding contracts – lead to a potentially important holdup problem that provides

\(^{26}\)It is straightforward to show that under decreasing-returns-to-scale, the result from the two-capital example would arise: given equal bargaining powers, the firm would both overinvest and overemploy so as to lower productivity and with it the rental rate and the wage rate.
the firm with an incentive to overinvest. This result contrasts with much of the holdup literature, which typically assumes that the price of capital is determined competitively.\textsuperscript{27}

In general equilibrium, the holdup problem in capital markets interacts with externalities from trading frictions and holdup problems in other factor markets. Exploration in a multifactor setting with either two types of capital or capital and labor shows that the resulting equilibrium allocations are generally inefficient and depend crucially on the form of the production function and the firm’s bargaining power in the two markets. Whether these inefficiencies are important is therefore a quantitative question. Yet, the policy implications of the overinvestment channel are clearly important. For example, overinvestment provides a rationale for capital income taxation or, at the least, counteracts other forces that, on their own, imply underinvestment and capital income subsidies.

Overinvestment due to holdup problems may also help explain a number of empirical phenomena. On a microeconomic level, holdup problems in capital markets provide a new explanation for why we observe very large firms in capital-intensive industries. Likewise, holdups in capital markets may help explain lumpy investment behavior by firms and why financial performance varies across industries. On a macroeconomic level, Caballero and Hammour (1998b) argue that worsening holdups of labor on capital and factor substitutability provide an explanation for the sustained increase of the capital-labor ratio in different European countries from the 1970s through the 1990s. The analysis in Section 3 suggests that overinvestment due to holdup problems in capital markets amplifies these effects. Furthermore, the misallocation of resources implied by holdup problems in capital markets may provide another source for persistent productivity differences across countries.

\textsuperscript{27}De Meza and Lockwood (2004, 2007) explore alternative mechanisms due to coordination failure and heterogeneity across agents that imply overinvestment as a result of holdup problems. Outside the holdup literature, Chien and Lee (2008) or Lagos and Rocheteau (2008) present other mechanisms that can lead to overinvestment. Furthermore, in the industrial organizations literature, strategic behavior by incumbents in imperfectly competitive markets can generate overinvestment. See Tirole (1988) for a review.
References


