Online Appendix

Proofs of Propositions

Details of some steps in the Proofs of Lemma 1 and Lemma 2 are available in the Technical Appendix after the main proofs of all results.

Proof of Proposition 1

It cannot be optimal for the manufacturer to choose $W > V^\ell$. The maximal supply chain profit in the latter case is $\lambda V^h$. Since $\lambda < \frac{V^\ell}{V^h}$, the manufacturer will earn a higher profit from setting $W = V^\ell$, even if the franchise fee is zero.

Therefore, suppose the manufacturer chooses some $W \leq V^\ell$. Since consumer valuations are observed in the offline world, a price dispersion equilibrium along the lines exhibited by Varian (1980) holds for each consumer valuation, $V^\ell$, and $V^h$. It is straightforward to show, following Varian (1980), that there is no pure strategy equilibrium. We construct a symmetric, mixed strategy equilibrium, following the argument of Varian.

Each retailer observes the type of each consumer, and hence charges a price contingent on this type. If a retailer sold only to its captive uninformed segment, the optimal price to type $i$ $(i = \ell, h)$ is just $V^i$. Now, suppose, for each retailer, $P_i$ $(i = \ell, h)$ is randomly chosen over $[\hat{P}_i, V^i]$. Then, its profit from consumer type $i$ at any price in this interval must be the same, and must equal the profit at price $V^i$. Define $\gamma = \frac{\alpha_u + \alpha_p}{2}$ (so that $1 - \alpha_u - \alpha_p = 1 - 2\gamma$). At the price $V^i$, a retailer sells only to its captive segment, and its profit from consumer type $i$ is $\lambda^i \gamma (V^i - W)$.

Let $G^i(P) = \text{Prob}(P \geq P^i)$. Then, at some price $P$ in the support of its mixed strategy, a retailer sells to its captive segment, and also captures $G^i(P)$ of the competitive segment. Hence, its profit from consumer type $i$ is $\lambda^i \left[ \gamma + G^i(P)(1 - 2\gamma) \right] (P - W)$, where $\lambda^h = \lambda$ and $\lambda^\ell = 1 - \lambda$. Hence, $\left[ \gamma + G^i(P)(1 - 2\gamma) \right] (P - W) = \gamma (V^i - W)$. This implies $G^i(P) = \frac{\gamma}{1 - 2\gamma} \left( \frac{V^i - P}{p - W} \right)$ (where $\frac{\gamma}{1 - 2\gamma} = \frac{\alpha_u + \alpha_p}{2(1 - \alpha_u - \alpha_p)}$).

The lower bound on the support of the mixed strategy is found by setting $G^i(\hat{P}) = 1$, which yields $\hat{P}^i = \frac{1 - 2\gamma}{1 - \gamma} W + \frac{\gamma}{1 - \gamma} V^i$. Substituting for $\gamma$, we have $\hat{P}^i = \frac{2(1 - \alpha_u - \alpha_p)}{2 - \alpha_u - \alpha_p} W + \frac{\alpha_u + \alpha_p}{2 - \alpha_u - \alpha_p} V^i$.

Next, we show that this is an equilibrium. Note that $G^i(V^i) = 0$, so the mixed strategy has no mass points. Consider retailer 1. For all prices $P \in [\hat{P}^i, V^i]$, retailer 1 earns the same profit from consumer type $i$ (by construction). If it charges $P > V^i$, it loses all consumers of type $i$, leading to a lower profit. If it charges $P < \hat{P}^i$, it captures the same market share as at $\hat{P}^i$, $(1 - \frac{\alpha_u + \alpha_p}{2})$, at a lower price. Hence, it makes a lower profit than at $\hat{P}^i$. Therefore, retailer 1 has no profitable deviation.
By symmetry, neither does retailer 2. Hence, the strategies postulated constitute an equilibrium.

By construction, the expected gross profit (i.e., revenue less variable cost) of each retailer before deducting the discovery cost and the franchise fee is equal for all prices in $[\hat{P}_i, V_i]$. At the price $V_i$ for each $i$, this expected gross profit is just $\lambda V^h + (1 - \lambda) V^\ell - W = V^\ell + \lambda(V^h - V^\ell) - W$. Further, the proportion of consumers visiting each retailer is $(1 - \alpha_u - \alpha_p)$ (by symmetry). Hence, for each retailer, the consumer discovery cost is $(1 - \alpha_u - \alpha_p)\delta$.

Netting out discovery cost, the expected gross profit of each retailer is

$$\pi = \frac{\alpha_u + \alpha_p}{2} [V^\ell + \lambda(V^h - V^\ell) - W] - \left(1 - \frac{\alpha_u + \alpha_p}{2}\right) \delta.$$

At any fixed wholesale price $W$, it is optimal for the manufacturer to set the franchise fee $F$ to extract the entire gross profit of the retailers, net of discovery cost. Hence, $F = \frac{\alpha_u + \alpha_p}{2} [V^\ell + \lambda(V^h - V^\ell) - W] - \left(1 - \frac{\alpha_u + \alpha_p}{2}\right) \delta$. This also implies that the retailers earn a zero profit, as stated in part (iv) of the Proposition.

Now, the manufacturer’s profit is

$$\Pi^\circ = 2F + W = (\alpha_u + \alpha_p) [V^\ell + \lambda(V^h - V^\ell) - W] - (2 - \alpha_u - \alpha_p) \delta + W$$

$$= (\alpha_u + \alpha_p) [V^\ell + \lambda(V^h - V^\ell)] - (2 - \alpha_u - \alpha_p) \delta + (1 - \alpha_u - \alpha_p) W.$$

Since $W \leq V^\ell$, this is clearly maximized at $W^* = V^\ell$. Substituting $W = V^\ell$ into the equilibrium pricing and profit expressions derived above yields parts (i), (ii), (iii) of the Proposition. Part (v) follows immediately from observing that, since the retailers earn zero profit, total channel profit $S^\circ$ equals the manufacturer’s profit $\Pi^\circ$.

**Proof of Proposition 2**

Suppose both retailers sign up with the infomediary. Consider Table 2, which shows the prices observed by each consumer segment if the infomediary enrolls both retailers. Note that each consumer that observes a referral price $P^r_1$ from $D_1$ also observes a referral price $P^r_2$ from $D_2$, and vice versa. That is, neither retailer has a captive segment which observes a referral price from only that retailer. Hence, in setting $P^r_1$ and $P^r_2$, Bertrand competition prevails, and in equilibrium these prices are both set to marginal cost, $W$.

Now, consider $D_1$, and its choice of $(P_1(V^h), P_1(V^\ell))$. Neither of these prices will be less than $W$. Suppose both these prices strictly exceed $W$, all sales in the partially informed and fully informed segments will occur at the referral prices, $P^r_1$ or $P^r_2$. Since only the uninformed segment
buys at \( P_1(V^h) \) or \( P_1(V^f) \), it is optimal to set \( P_1(V^h) = V^h \) and \( P_1(V^f) = \max\{V^f, W\} \) (since there is no competition from \( D_2 \) in this segment). Note that it must be that \( W \leq V^h \), else no consumer buys and the manufacturer earns a zero profit. However, it may be that \( W > V^f \); to allow for this possibility, we set \( P_1(V^f) = \max\{V^f, W\} \).

By symmetry, the best response of \( D_2 \) is exactly similar. Hence, the profit of each retailer, before deducting discovery cost and the franchise fee, is \( \lambda V^h + (1 - \lambda)V^f - W \) if \( W \leq V^f \), and \( \lambda(V^h - W) \) if \( W > V^f \).

Now, suppose \( D_1 \) chooses to not enrol with the infomediary. It has the option of setting \( P_1(V^h) = V^h \) and \( P_1(V^f) = \max\{V^f, W\} \) and ignoring the partially and fully informed segments. Hence, the minimum profit it earns, before deducting discovery cost and franchise fee, is \( \lambda V^h + (1 - \lambda)V^f - W \) if \( W \leq V^f \), and \( \lambda(V^h - W) \) if \( W > V^f \). This is exactly the same profit it earns if it did enrol with the infomediary.

That is, enrolling with the infomediary does not increase the profit of a retailer. Hence, the infomediary cannot charge a referral fee higher than zero. 

\[ \text{Lemma 1} \]

Consider the case of only infomediary referrals, with the infomediary enrolling only one retailer. There exists a wholesale price \( \hat{W} < V^f \) with the following property: Suppose the manufacturer chooses a wholesale price \( W \in [\hat{W}, V^f] \). Then, there is an equilibrium in which

(i) \( P_2(V^h) = V^h \), and the prices \( P_1(V^h) \) and \( P_2^r \) are randomly chosen from \( [\hat{P}^h, V^h] \) where \( \hat{P}^h = W + \frac{\alpha_u(V^h - W)}{2(1 - \alpha_u)(P - W)} \). Further, \( G_2^r(P) = \frac{\alpha_u(V^h - P)}{2(1 - \alpha_u)(P - W)} \) and \( G_1^h(P) = \frac{\alpha_u(V^h - W)}{2(1 - \alpha_u)(P - W)} \), with a mass point at \( V^h \) equal to \( \frac{2\alpha_u}{\alpha_u - \alpha_p} \).

(ii) the prices \( P_1(V^f) \) and \( P_2(V^f) \) are randomly chosen from \( [\hat{P}^f, V^f] \), where \( \hat{P}^f = W + \frac{(\alpha_u + 2\alpha_p)(V^f - W)}{2 - \alpha_u} \). The price distributions satisfy \( G_1^f(P) = \frac{\hat{P}^f - P}{2 - \alpha_u} - \frac{\alpha_u}{2(1 - \alpha_u - \alpha_p)} \frac{P - \hat{P}^f}{P - W} \), with a mass point at \( V^f \) equal to \( \frac{2\alpha_u}{\alpha_u - \alpha_p} \), and \( G_2^f(P) = \frac{\alpha_u + 2\alpha_p}{2(1 - \alpha_u - \alpha_p)} \frac{V^f - P}{P - W} \).

(iii) \( D_2 \) has a higher gross profit than \( D_1 \). Specifically, the expected gross profits are as follows:

\[
\begin{align*}
E(\pi_1) &= \frac{\alpha_u}{2}(\lambda V^h + (1 - \lambda)V^f - W) + \alpha_p(1 - \lambda)(V^f - W), \\
E(\pi_2) &= \frac{\alpha_u}{2}(\lambda V^h + (1 - \lambda)V^f - W) + \alpha_p \left( \frac{\alpha_u}{2 - \alpha_u} \right) \lambda(V^h - W) \\
&\quad + (1 - \alpha_u - \alpha_p) \left( \lambda \frac{\alpha_u}{2 - \alpha_u} (V^h - W) + (1 - \lambda) \frac{2\alpha_p}{2 - \alpha_u} (V^f - W) \right). 
\end{align*}
\]

\[ \text{Proof of Lemma 1} \]

If the infomediary has enrolled only one retailer, retailers are asymmetric with respect to their
market coverages. The arguments of Narasimhan (1988), who considers price dispersion with asymmetric firms, can be extended to our setting in a straightforward manner to show that there is no pure strategy equilibrium.

We first use the intuition of the equilibrium exhibited by Narasimhan (1988) to construct a candidate equilibrium, given a wholesale price $W$. In Step 2, we demonstrate that the candidate strategies do indeed constitute an equilibrium. At this step, requiring that $D_2$ not make a higher profit by deviating to some other strategy requires $W$ to be sufficiently close to $V_1$, which leads to an implicit definition of the threshold wholesale price $\hat{W}$.

**Step 1:** Construction of candidate equilibrium in mixed strategies.

Suppose the manufacturer charges some wholesale price $W$. Consider $P_1(V^h)$, the price charged by $D_1$ to the high consumer type. Suppose $P_1(V^h)$ is chosen from some distribution with support $[\hat{P}^h, V^h]$. In equilibrium, $D_1$ should make the same profit by charging any price $P$ in the support of the mixed strategy as from charging the monopoly price $V^h$. Let $G^*_1(P) = \text{Prob}(P_2^r \geq P)$.

Then, $\frac{\alpha_2}{2} (P - W) + (1 - \alpha_u - \alpha_p + \alpha_p)(P - W) G^*_1(P) - F = \frac{\alpha_2}{2} (V^h - W) - F$, which implies $G^*_1(P) = \frac{\alpha_u (V^h - P)}{2(1 - \alpha_u)(P - W)}$. Setting $G^*_1(P) = 1$ yields the lower bound of the support of the strategy, $\hat{P}^h = W + \frac{\alpha_u (V^h - W)}{2 - \alpha_u}$.

Now consider the price charged by $D_1$ to high valuation consumers, $P_1(V^h)$. Suppose $P_1(V^h)$ is randomly chosen from some region $[\hat{P}_1^h, V^h]$. In equilibrium, the lower bound $\hat{P}_1^h$ must equal $\hat{P}^h$. Suppose $\hat{P}_1^h < \hat{P}^h$. Then, by charging $\hat{P}_1^h + \varepsilon$ (for some $\varepsilon \in (0, \hat{P}^h - \hat{P}_1^h)$), $D_1$ earns a higher profit than from any price $P \in (\hat{P}_1^h, \hat{P}^h + \varepsilon)$. Hence, it cannot be an equilibrium to have $\hat{P}_1^h < \hat{P}^h$. By the same logic, it cannot be that $\hat{P}^h < \hat{P}_1^h$, so it must be that $\hat{P}_1^h = \hat{P}^h$.

Now, for $D_2$, the profit from any price $P$ in the support of its mixed strategy $P^*_2$ should be equal to that from charging the lower bound $\hat{P}^h$. First, note that the highest price that $P^*_2$ will be set to is $V^h$. Further, in equilibrium $P_1(V^h)$ is being randomized. Hence, consumers of type $V^h$ who observe $P^*_2$ will buy either at $P_1(V^h)$ or at $P^*_2$. Let $G^*_1(P) = \text{Prob}(P_1^h \geq P)$. Then, $\lambda(1 - \alpha_u)(P - W) G^*_1(P) = \lambda(1 - \alpha_u)(\hat{P}^h - W) G^*_1(P)$ which implies $G^*_1(P) = \frac{(\hat{P}^h - W)}{(P - W)} = \frac{\alpha_u (V^h - W)}{(2 - \alpha_u)(P - W)}$.

Consider $P_2(V^h)$. $D_2$ is using $P^*_2$ to compete with $P_1(V^h)$ for high value consumers. Hence, we conjecture that $P_2(V^h)$ is set to $V^h$ to capture maximum profit from the captive uninformed segment.

Finally, consider $P_1(V^\ell)$ and $P_2(V^\ell)$. Let $G^*_j(P) = \text{Prob}(P^\ell_j \geq P)$. Since $P^*_2$ is always greater than $V^\ell$, no consumer of type $V^\ell$ will buy at $P^*_2$. Hence, the equilibrium strategies for consumers of type $V^\ell$ exactly parallel those demonstrated by Narasimhan (1988). From this we can
determine that $G_1^\ell(P) = \frac{\hat{P}^\ell - W}{P - W} - \frac{\alpha_u}{2(1 - \alpha_u - \alpha_p)} \frac{P - \hat{P}^\ell}{P - W}$, with a mass point at $V^\ell$ equal to $\frac{2\alpha_p}{2 - \alpha_u}$, and $G_2^\ell(P) = \frac{\alpha_u + 2\alpha_p}{2(1 - \alpha_u - \alpha_p)} \frac{V^\ell - P}{P - W}$, where $\hat{P}^\ell = W + \frac{\alpha_u + 2\alpha_p(V^\ell - W)}{2 - \alpha_u}$.

Step 2: Checking no-deviation conditions given the above strategies.

Next, we prove that the conjectured strategies constitute an equilibrium. Since the details are somewhat lengthy, this proof is provided in the Technical Appendix after the main proofs of all results.

The threshold value of $W$ emerges from the no-deviation conditions for $D_2$. Suppose $D_2$ deviates and chooses $P_2^e(V^\ell) = V^\ell, P_2^e \leq V^\ell$ and $P_2(V^h) = V^h$. Then, comparing its profit after deviation to its equilibrium profit, we find that the deviation is unprofitable if and only if

$$(1 - \alpha_u)(\hat{P}^\ell - W) \leq \lambda(1 - \alpha_u)(\hat{P}^h - W) + (1 - \alpha_u - \alpha_p)(1 - \lambda)(V^\ell - W) \frac{2\alpha_p}{2 - \alpha_u}.$$ 

Clearly, if $W = V^\ell$, the inequality above is strictly satisfied, since in this case $\hat{P}^\ell = V^\ell = W$, and the LHS is zero with the RHS strictly positive. Hence, for $W$ sufficiently close to $V^\ell$, it must be satisfied as well. Solving the equality for $W$ yields the threshold value $\hat{W}$.

This Step proves parts (i) and (ii) of the Lemma.

Step 3: Determining expected profit for retailers.

We now proceed to derive the expected profit of each retailer when $W \geq \hat{W}$. Retailer $D_1$ sells to all of its captive segment, of size $\frac{\alpha_u}{2}$. In the other two segments, of size $(1 - \alpha_u)$, it sells to the high consumer type, and only if $P_2^e > P_1(V^h)$, which happens with probability $G_2^\ell(P_1(V^\ell))$ at a price $P_1(V^\ell)$. In the partially informed segment of size $\alpha_u$ it sells to all the low type consumers. In the fully informed segment of size $1 - \alpha_u - \alpha_p$, it sells to the low types only if $P_1(V^\ell) < P_2(V^\ell)$ which happens with the probability $G_2^\ell(P_1(V^\ell))$. Therefore, the gross profit of retailer $D_1$ (i.e., ignoring discovery costs and the franchise fee) may be written as

$$E(\pi_1^\ell) = \frac{\alpha_u}{2}(\lambda P_1(V^h) + (1 - \lambda)P_1(V^\ell) - W) +$$
$$\alpha_p \left( \lambda G_2^\ell(P_1(V^h)) P_1(V^h) - W + (1 - \lambda)(P_1(V^\ell) - W) \right) +$$
$$(1 - \alpha_u - \alpha_p) \left( \lambda G_2^\ell(P_1(V^h)) P_1(V^h) - W + (1 - \lambda)G_2^\ell(P_1(V^\ell)) (P_1(V^\ell) - W) \right)$$

Substituting $P_1(V^h) = V^h$ and $P_1(V^\ell) = V^\ell$, the expected gross profit of $D_1$ is

$$E(\pi_1^\ell) = \frac{\alpha_u}{2}(\lambda V^h + (1 - \lambda)V^\ell - W) + \alpha_p(1 - \lambda)(V^\ell - W).$$ \hspace{1cm} (2)
Suppose \( D_2 \) chooses some prices \( P'^r_2, P'_2(V^h), P(V^\ell) \). For now, ignore discovery costs and franchise and referral fees—none of these terms change as the prices \( P'^r_2, P'_2(V^h), P'_2(V^\ell) \) change. Then, the gross profits for \( D_2 \) can be written as follows.

\[
\pi_2(P'^r_2, P'_2(V^h), P'_2(V^\ell)) = \frac{\alpha_u}{2} \left( \lambda P'_2(V^h) + (1 - \lambda) P'_2(V^\ell) - W \right) + \\
\frac{\alpha_p}{\lambda} \left( \lambda G_h(P'^r_2(P'_2 - W) + (1 - \lambda) G^\ell(P'^r_2(P'_2 - W) - \lambda \min\{P'^r_2, P'_2(V^h)\}) - W \right) + \\
\left\{ (1 - \alpha_u - \alpha_p) \left( \lambda G_h(\min\{P'^r_2, P'_2(V^h)\}) - W \right) + \\
(1 - \lambda) G^\ell(\min\{P'^r_2, P'_2(V^\ell)\}) - W \right\}
\]

The expected gross profit in equilibrium of \( D_2 \) can be determined by substituting \( P'^r_2 = V^h, P'_2(V^h) = V^h \), and \( P'_2(V^\ell) = V^\ell \) (since any choice of \( P'^r_2, P'_2(V^\ell) \) in the stated range leads to the same profit). This leads to an equilibrium expected gross profit for \( D_2 \) given by

\[
E(\pi'_2) = \frac{\alpha_u}{2} (\lambda V^h + (1 - \lambda) V^\ell - W) + \frac{\alpha_p}{\lambda} \left( \lambda (V^h - W) \right) + \\
(1 - \alpha_u - \alpha_p) \left( \lambda \frac{\alpha_u}{2 - \alpha_u} (V^h - W) + (1 - \lambda) \frac{2\alpha_p}{2 - \alpha_u} (V^\ell - W) \right).
\]

Hence, \( E(\pi'_2) > E(\pi'_1) \), as stated in part (iii) of the Lemma.

**Proof of Proposition 3**

Since both dealers must earn a non-negative expected profit, the optimal franchise fee for the manufacturer is the fee that reduces the minimum of the two retailers’ profits to zero. Lemma 1 shows that \( D_1 \) has a lower gross profit than \( D_2 \). In addition, \( D_1 \) has a greater number of consumers visiting its physical facility. Hence, the discovery cost for \( D_1 \), \( 1 - \frac{\alpha_u}{2} \), exceeds the discovery cost for \( D_2 \), \( 1 - \frac{\alpha_u + \alpha_p}{2} \). Therefore, \( D_1 \) has the lower gross profit net of discovery cost. Hence, the optimal franchise fee is to set \( F \) equal to the gross profit of \( D_1 \) net of discovery cost, or

\[
F^* = \frac{\alpha_u}{2} [V^\ell + \lambda(V^h - V^\ell) - W] + \alpha_p(1 - \lambda)(V^\ell - W) - \left( 1 - \frac{\alpha_u}{2} \right) \delta.
\]

Given \( F^* \), the infomediary sets the maximum referral fee at which \( D_2 \) earns a non-negative profit. This is defined by \( K^* \) such that \( E(\pi'_2) - \left( 1 - \frac{\alpha_u + \alpha_p}{2} \right) \delta - F^* - K^* = 0 \), or

\[
K^* = E(\pi'_2) - \left( 1 - \frac{\alpha_u + \alpha_p}{2} \right) \delta - F^* = \frac{\lambda \alpha_u (1 - \alpha_u)}{2 - \alpha_u} (V^h - V^\ell) + \alpha_p \delta.
\]

Hence, both retailers earn zero profit, as stated in part (iv) of the Proposition.

The total profits of the manufacturer are now \( \Pi^r = 2F^* + W \), so can be written as

\[
\Pi^r = \alpha_u [V^\ell + \lambda(V^h - V^\ell)] + 2\alpha_p (1 - \lambda) V^\ell + [1 - \alpha_u - 2\alpha_p (1 - \lambda)] W - (2 - \alpha_u) \delta.
\]
This expression is increasing in $W$ if $(1 - \alpha_u - 2(1 - \lambda)\alpha_p) > 0$. Hence, if this condition is satisfied, the manufacturer should set $W$ as high as possible in the range $\hat{W}$; i.e., to $V^\ell$. Setting $W = V^\ell$ in equation (7) yields that the manufacturer profit is

$$\Pi^\ell = \alpha_u \lambda (V^h - V^\ell) + V^\ell - (2 - \alpha_u)\delta. \quad (8)$$

This expression is non-negative if $\delta \leq \frac{V^\ell + \alpha_u \lambda (V^h - V^\ell)}{2 - \alpha_u}$.

Now, the manufacturer can also choose a price $W > V^\ell$. Since retail prices must be at least as high as wholesale prices, the low valuation consumers are shut out of the market. Thus, the maximum gross profit attainable in the supply chain is $\lambda V^h$. Of course, retailers incur the same discovery cost as before, so the maximum profit in the supply chain net of discovery cost is $\lambda V^h - (2 - \alpha_u)\delta$. Since $\lambda < \frac{V^\ell}{V^h}$, it follows that this profit is less than $V^\ell - (2 - \alpha_u)\delta$. Hence, this profit is less than $\Pi^\ell$. Therefore, the optimal wholesale price is $W = V^\ell$, proving part (i) of the Proposition.

Setting $W = V^\ell$ in equations (5), (6), and (7) yields the expressions for $F^\ell, K^\ell,$ and $\Pi^\ell$ shown in Appendix A1 for the “Infomediary referral only case.” From these expressions, it follows that $F^\ell < F^o$ (part (ii) of the proposition), and $K^\ell$ is increasing in $\lambda, \alpha_p,$ and $\delta$ (part (iii)). Finally, the total channel profit is defined as $S^\ell = \Pi^\ell + K^\ell$; inspecting the expression for $S^\ell$ in Appendix A1, the total channel profit exceeds $S^o$ when $\alpha_p$ is sufficiently small.

**Lemma 2** Consider the case with both manufacturer and infomediary referral services. There exists a wholesale price $\hat{W}_m < V^\ell$ with the following property: Suppose the manufacturer chooses a wholesale price $W \in [\hat{W}_m, V^\ell]$. Then, there is an equilibrium in which:

(i) $P_1 (V^\ell), P_1 (V^h), P_2 (V^\ell), P_2 (V^h)$ and $P_1$ are set exactly as in Lemma 1,

(ii) $P^m_2 = V^h$, and $P^m_1$ is randomly chosen over $[\hat{P}^h, V^h]$, where $\hat{P}^h = W + \frac{\alpha_u (V^h - W)}{(2 - \alpha_u)\beta}$. Further, $G^o_1 (P) = \frac{\alpha_u (V^h - W)}{(2 - \alpha_u)\beta}$, with a mass point at $V^h$ equal to $\frac{\alpha_u}{2 - \alpha_u}$.

(iii) $D_2$ has a higher gross profit than $D_1$. Specifically, the expected gross profits are as follows:

$$E(\pi_1^m) = \beta E(\pi_1^\ell) + (1 - \beta) \frac{\alpha_u}{2} \lambda (V^h - W)$$

$$E(\pi_2^m) = \beta E(\pi_2^\ell) + (1 - \beta) \frac{\alpha_u}{2} (\lambda V^h + (1 - \lambda)V^\ell - W) + \alpha_p \left( \frac{\alpha_u}{2 - \alpha_u} \right) \lambda (V^h - W)$$

$$+ (1 - \alpha_u - \alpha_p) \left( \frac{\alpha_u}{2 - \alpha_u} (V^h - W) + (1 - \lambda) \frac{2\alpha_p}{2 - \alpha_u} (V^\ell - W) \right).$$

**Proof of Lemma 2**

We proceed with a series of steps. In steps 1, 2, and 3, we build upon Narasimhan (1988), who shows a price dispersion equilibrium with asymmetric firms, to determine candidate strategies for
an equilibrium, and show that a deviation on a single price does not help a retailer. In step 4, we show that these strategies indeed constitute an equilibrium by ruling out all possible deviations by a retailer.

**Step 1**

First, suppose \( \beta = 0 \), so that there are no consumers at the physical stores. Suppose also that \( P_2^m = V^h \), and that \( P_1^m \) and \( P_r^* \) are chosen randomly. From the profit invariance condition of a mixed strategy equilibrium, \( D_1 \) should make the same gross profit from any price \( P \) in the support of its mixed strategy as it would at a monopoly price. Since \( D_1 \) cannot differentiate across consumer types when \( \beta = 0 \), it must be the case that its monopoly price is \( P_1^m = V^h \) (when \( W \) is sufficiently close to \( V^f \), a price of \( V^h \) yields a higher gross profit from the captive segment than a price of \( V^f \)). Hence, \( \alpha_u (P - W) + (1 - \alpha_u)(P - W) G_2^f(P) = \frac{\alpha_u}{2} (V^h - W) \) and \( G_2^f(P) = -\frac{\alpha_u (v^h - P)}{2(1 - \alpha_u)(P - W)} \). Therefore, the distribution of \( P_r^* \) is identical to that in Lemma 1. This further yields that \( \hat{P}^h = \frac{\alpha_u (v^h - W)}{2 - \alpha_u} + W \), as before. Similarly for \( D_2 \), profit from pricing at any \( P \in [\hat{P}^h, V^h] \) should be the same as the profit from pricing at \( \hat{P}^h \). Hence, \( \frac{\alpha_u}{2} (P^m_2 - W) + (1 - \alpha_u) G_1^m(P) (P - W) = \frac{\alpha_u}{2} (P^m_2 - W) + (1 - \alpha_u) G_1^m(\hat{P}^h) (\hat{P}^h - W) \) which implies \( G_1^m(P) = \frac{(\hat{P}^h - W)}{(P^m_2 - W)} = \frac{\alpha_u (v^h - W)}{2 - \alpha_u (P - W)} \).

Next, for the \( \beta = 0 \) case, we show that neither retailer can gain from deviating on just one price from the strategies exhibited in the Proposition. In Step 4 below, we rule out deviations by a retailer in two or more of the prices it quotes.

Note that \( P_1^m = V^h \) is the monopoly price that for \( D_1 \) in its captive segment, as long as \( W \) is sufficiently close to \( V^f \). If \( P_r^* \) is set to any price above this, \( D_2 \) will make no sales at \( P_r^* \), so it must price at or below \( V^h \). Further, by construction, \( G_1^m(P) \) and \( G_2^f(P) \) are best responses by the dealers, so a deviation to prices below \( \hat{P}^h \) is not profitable either.

Finally, consider \( P_2^m \). First, observe that any price above \( V^h \) is sub-optimal, compared to \( V^h \), since it loses all consumers in this segment. Suppose \( D_2 \) sets \( P_2^m = P < V^h \). There are three effects on profit, as compared to charging \( P_2^m = V^h \).

a) In its captive segment, of size \( \frac{\alpha_u}{2} \), it loses \( ((1 - \lambda) + \lambda) \frac{\alpha_u}{2} (V^h - P) = \frac{\alpha_u}{2} (V^h - P) \),

b) in the segment of mass \((1 - \alpha_u - \alpha_p)\), if \( P < P_r^* < P_1^m \), it cannibalizes its own sales, and loses an amount \((1 - \lambda) + \lambda (1 - \alpha_u - \alpha_p) \) \( G_1^m(P) G_2^f(P) \) \text{Prob}(P_r^* < P_1^m \mid P_r^* > P) \{ E(P_r^* \mid P < P_r^* < P_1^m) - P \}, \) where \( E(P_r^* \mid P < P_r^* < P_1^m) \) is the expected price at which the cannibalized sales were being made (the conditioning event is that \( P < P_r^* < P_1^m \)),

(c) finally, in the segment of mass \((1 - \alpha_u - \alpha_p)\), if \( P < P_1^m < P_r^* \), it wins some sales over from \( D_1 \), leading to a gain \( ((1 - \lambda) + \lambda (1 - \alpha_u - \alpha_p) \) \( G_1^m(P) G_2^f(P) \) \text{Prob}(P_1^m < P_r^* \mid P < P_1^m) \) \( (P - W) \).
Replacing the relevant expressions for \( G^m_1(P) \) and \( G^m_2(P) \), and evaluating the conditional probabilities and expectations, we find that, in overall terms, the firm loses some profit. Hence, it will not deviate to \( P^m_2 < V^h \).

**Step 2** Suppose \( \beta = 1 \). Then, the strategies exhibited constitute an equilibrium. This step follows immediately from Lemma 1; for \( \beta = 1 \), the game reduces to the game in Table 3.

**Step 3:** Suppose the strategies in Step 1 describe an equilibrium for \( \beta = 1 \). Then, for all values of \( \beta \in (0, 1) \), the strategies exhibited constitute an equilibrium.

Notice that \( G^f_2(P) \), the distribution of \( P^f_2 \), is exactly identical in the two cases \( \beta = 0 \) and \( \beta = 1 \). Further, there is no consumer who observes both an offline price and a manufacturer referral price. That is, \( P_1(V^h), P_1(V^f), P_2(V^h), P_2(V^f) \) are set as best responses only to each other and \( P^f_2 \), and are not affected by \( P^m_1, P^m_2 \). Similarly, \( P^m_1, P^m_2 \) are set as best responses only to each other and \( P^f_2 \). Hence, it is immediate that, given that \( G^f_2(P) \) is the same in both cases, when \( \beta > 0 \), \( P_1(V^h), G^f_1(P), P_2(V^h), P_2(V^f), \) and \( G^m_1(P), P^m_2 \), are mutual best responses. Finally, since \( G^f_2(P) \) is a best response for both the \( \beta = 0 \) and \( \beta = 1 \) cases, it must continue to be so when \( \beta \in (0, 1) \).

**Step 4:** The strategies in the statement of the Lemma constitute an equilibrium: Neither dealer has an incentive to deviate.

The details of this step are provided in the Technical Appendix.

In the candidate equilibrium, the expected gross profit of \( D_1 \) in the web segment can be found by setting \( P^m_1 = V^h \) (since all strategies in the support of the distribution yield equal profit). Hence, the expected profit of \( D_1 \) in the web segment is \( \frac{\alpha_u}{2} \lambda (V^h - W) \). Therefore, the overall expected profit of \( D_1 \) is

\[
E(\pi^m_1) = \beta E(\pi^f_1) + (1 - \beta) \frac{\alpha_u}{2} \lambda (V^h - W). \tag{9}
\]

Since the strategy of \( D_1 \) in the web segment, \( P^m_1 \), has a mass point at \( V^h \), the expected profit of \( D_2 \) in the web segment is found by setting \( P^m_2 \) to just less than \( V^h \). In the limit, as \( plw \) approaches \( V^h \), the expected gross profit of \( D_2 \) from the web segment is \( \lambda \left[ \frac{\alpha_u}{2} + \frac{\alpha_u(1 - \alpha_u)}{2 - \alpha_u} \right] (V^h - W) = \lambda \left( \frac{\alpha_u(4 - 3\alpha_u)}{2(2 - \alpha_u)} \right) (V^h - W) \). By inspection, this exceeds the gross profit of \( D_1 \) from the web segment.

The overall gross profit of \( D_2 \) is

\[
E(\pi^m_2) = \beta E(\pi^f_2) + (1 - \beta) \lambda \left( \frac{\alpha_u}{2} \right) \left( \frac{4 - 3\alpha_u}{2 - \alpha_u} \right) (V^h - W). \tag{10}
\]
Since $D_2$ earns more than $D_1$ in both offline and online segments, its gross profit is higher than that of $D_1$. This proves part (iii) of the Lemma.

As we show in the Technical Appendix, if $D_2$ instead deviates to a strategy with $P_2^e(V^e) = V^e, P_2^m = V^e, P_2(V^h) = V^h$, and $P_r^e$ randomized over $[\hat{P}, V^e]$, the profit after deviation is

$$
\beta \frac{\alpha_u}{2} \left( \lambda V^h + (1 - \lambda)V^e - W \right) + \beta \alpha_r (V^e - W) +
\beta (1 - \alpha_u - \alpha_p)(V^e - W) + (1 - \beta) \left( 1 - \frac{\alpha_u}{2} \right) (V^e - W).
$$

(11)

If $W = V^e$, the expression in equation (11) reduces to $\beta \lambda \alpha_u (V^h - V^e)$, and exactly equals the first term in equation (10). Since the second term in (10) are also positive, the equilibrium profit is strictly higher. Hence, for values of $W$ close enough to $V^e$, the deviated profit must continue to be less than the equilibrium profit. Setting the expressions in equations (10) and (11) equal yields the threshold value $\hat{W}_m$.

This proves parts (i) and (ii) of the Lemma.

Proof of Proposition 4

At any wholesale price $W$, the optimal franchise fee is the expected gross profit of $D_1$ net of discover cost. For $W \in [\hat{W}_m, V^e]$, we have

$$
F^m = \beta \left\{ \frac{\alpha_u}{2} \left[ (1 - \lambda)(V^e - W) + \lambda(V^h - W) \right] + \alpha_p (1 - \lambda)(V^e - W) - \left( 1 - \frac{\alpha_u}{2} \right) \hat{W}_m \right\} + (1 - \beta) \lambda \alpha_u \left[ V^h - W \right].
$$

(12)

The total sales of the manufacturer are given by $[1 - (1 - \lambda)(1 - \beta)]$, since low valuation consumers in the online segment $(1 - \beta)$ do not buy the good. Hence, the total profit of the manufacturer is given by

$$
\Pi^m = [1 - (1 - \lambda)(1 - \beta)] W + 2F^m
= \alpha_u \left[ \lambda V^h + \beta(1 - \lambda)V^e \right] + 2\beta \alpha_p (1 - \lambda)V^e - \beta \left( 2 - \alpha_u \right) \hat{W}_m
+ [\lambda(1 - \alpha_u) + \beta(1 - \lambda)(1 - \alpha_u - 2\alpha_p)] W,
$$

(13)

which is maximized at $W = V^e$ if $[\lambda(1 - \alpha_u) + \beta(1 - \lambda)(1 - \alpha_u - 2\alpha_p)] > 0$.

As in the infomediary-only case, choosing a wholesale price $W$ less than $\hat{W}_m$ leads to more aggressive price competition at the retail level, and a resultant fall in retailer gross profit and hence the manufacturer’s franchise fee. Suppose for now that it is optimal to choose a wholesale price less than or equal to $V^e$, rather than one greater than $V^e$. Then, the optimal wholesale price in the region $[\hat{W}_m, V^e]$ is $W = V^e$. 

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It remains to show that choosing \( W > V^\ell \) is not optimal. Among all \( W \) in this range, the optimal is \( W = V^h \). This will imply that retailers set \( W = V^h \) as well, leading to total revenue in the channel \([1 - (1 - \lambda)(1 - \beta)]V^h\). The discovery cost in the channel remains \( \beta \left( 1 - \frac{\alpha_u}{2} \right) \delta \). Thus, the total channel profit that can be extracted by the manufacturer is \([1 - (1 - \lambda)(1 - \beta)]V^h - \beta \left( 1 - \frac{\alpha_u}{2} \right) \delta \). Comparing this to the profit when \( W = V^\ell \), it is immediate that, if \( \beta \) is sufficiently high, the manufacturer earns a higher profit from setting \( W = V^\ell \), as stated in part (i) of the proposition.

Substituting \( W = V^\ell \) into equations (12) and (13) yields the expressions for \( F^m \) and \( \Pi^m \) shown in Appendix A1 for the case in which both manufacturer and infomediary referral services are present. Given these expressions, \( F^m > F^I \), but \( \Pi^m > \Pi^I \) if and only if \( \delta > \frac{(1 - \lambda)V^h}{2 - \alpha_u} \), proving part (ii) of the proposition. \( \Pi^m \) can also be written as:

\[
\Pi^m = [1 - (1 - \beta)(1 - \lambda)]V^\ell + \alpha_u \lambda (V^h - V^\ell) - \beta (2 - \alpha_u) \delta \]  (14)

from which it follows that \( \Pi^m \geq 0 \) whenever \( \delta \leq \frac{V^\ell + \alpha_u \lambda (V^h - V^\ell)}{2 - \alpha_u} \), as assumed earlier.

Part (iii) of the Proposition is determined by substituting \( W = V^\ell \) into retailers’ expected gross profit in Lemma 2, subtracting out discovery cost for each retailer, and setting \( K^m \) equal to the difference in expected gross profit net of discover cost between \( D^2 \) and \( D^1 \). This yields the expression for \( K^m \) shown in Appendix A1, from which it is immediate that \( K^m < K^I \). Part (iv) follows immediately; if retailers do not earn a zero net profit, either \( F \) or \( K \) should be raised. Finally, part (v) follows by adding manufacturer and infomediary profit.

Proof of Proposition 5

(i) Consider the condition in Proposition 4, \([\lambda (1 - \alpha_u) + \beta (1 - \lambda)(1 - \alpha_u - 2\alpha_p)] > 0 \). There are two cases to consider:

Case 1: Suppose \((1 - \alpha_u - 2\alpha_p) > 0 \). Then, the LHS of the condition consists of the sum of two positive terms, so the expression is positive.

Case 2: Suppose \((1 - \alpha_u - 2\alpha_p) < 0 \). Then, the LHS is declining in \( \beta \), so is minimized at \( \beta = 1 \).

However, at \( \beta = 1 \), the condition reduces to \([1 - \alpha_u - 2\alpha_p (1 - \lambda)] > 0 \), exactly the same as the condition in Proposition 3. Hence, for all \( \beta < 1 \), the condition in Proposition 4 is satisfied over a wider range of parameters than the corresponding condition in Proposition 3.

Finally, note that if \( \beta \) is sufficiently low, from the proof of Proposition 4, the manufacturer
should set \( W = V^h \). Regardless, the wholesale price is set to greater than or equal to \( V^f \) over a wider range of parameters than in the infomediary-only case.

(ii) From the expressions in Appendix A1,

\[
\Pi^m - \Pi^f = (1 - \beta)[(2 - \alpha_u)\delta - (1 - \lambda)V^f]
\]

\[
S^m - S^f = (1 - \beta)[(2 - \alpha_u - \alpha_p)\delta - (1 - \lambda)V^f].
\]

Hence, if \( \delta \in \left(\frac{(1 - \lambda)V^f}{2 - \alpha_u}, \frac{(1 - \lambda)V^f}{2 - \alpha_u - \alpha_p}\right) \), it follows that \( \Pi^m > \Pi^f \) and \( S^m < S^f \).

**Proof of Proposition 6**

(i) Consider the sales of \( D_1 \) in the \( (1 - \beta) \) segments. Its expected sales amount to

\[
(1 - \beta)\lambda \left( \frac{\alpha_u}{2} + \int_{\beta \rho_u}^{V^h} (1 - \alpha_u) G_2'(P) d(1 - G_1'(P))dP \right) = \lambda(1 - \beta) \left( \frac{1 - \alpha_u}{2 - \alpha_u} - \frac{\alpha_u}{2} \right). 
\]

The total sales in the web segment are \( \lambda(1 - \beta) \). Thus, \( D_1 \) has lower sales than \( D_2 \) in this segment if it has less than half of the total sales in the segment; that is, if \( \frac{1 - \alpha_u}{2 - \alpha_u} - \frac{\alpha_u}{2} < \frac{1}{2} \), which is true for all values of \( \alpha_u < 1 \).

(ii) As shown in Lemma 2 (ii), the manufacturer referral price of \( D_2 \) is \( P^m_2 = V^h \). The expected infomediary price of \( D_2 \), \( E(P^r_2) \) does not change from that shown in Lemma 1. Substituting \( W = V^f \) into the expression in Lemma 1 (i), \( P^r_2 \) is randomly chosen over the range \( [V^f + \frac{\alpha_u(V^h - V^f)}{2 - \alpha_u}, V^h] \).

Thus, the average infomediary referral price of \( D_2 \) is less than \( V^h \).

**Proof of Proposition 7**

In order to eliminate the infomediary the manufacturer needs to set \( W = V^h \) and absorb all the discovery costs of the retailers. Any \( W < V^h \) will lead to positive profits for the retailer enrolled with the infomediary which the infomediary can extract using its referral fee. At \( W = V^h \), all the low valuation customers are shut out of the market and so the total profits of the manufacturer are

\[
\lambda V^h - \beta(2 - \alpha_u)\delta. 
\]

(15)

The manufacturer’s profit in the manufacturer referral services case is given by equation (14). Comparing this with the expression (15) yields the critical value of \( \beta \) as stated in the Proposition. The statement of the Proposition then follows.
Technical Appendix

Proof of Lemma 1

Details of Step 2:

Having constructed the equilibrium, we prove that the conjectured strategies constitute an equilibrium. Consider $D_1$ first. Since all its sales are offline, it knows the consumer type before it chooses its price for each consumer. Hence, a deviation in $P_1^h(V^h)$ or $P_1^l(V^l)$ does not affect its profit from consumers of type $V^l$ or $V^h$, respectively. That is, it is sufficient to rule out deviations in each of $P_1^h(V^h)$ and $P_1^l(V^l)$ in isolation. By construction, $G_1^h$ and $G_1^l$ are best responses, ruling out such deviations.

Next, consider firm 2. This dealer is choosing three prices, $P_2^h(V^h), P_2^l(V^l), P_2^r$. Since all three are chosen jointly to maximize its profits, to show that its choices are optimal, we must consider joint deviations in these prices.

For clarity, we first show that each price is optimal given the other two prices $D_2$ charges, and then consider joint deviations in two or more prices.

First, consider $P_2^h(V^h)$. At any price $P \in [V^l, V^h]$, $D_2$ sells only to its own captive segment, $\alpha_2$, of the high type consumer. Since sales are unchanged at all these prices, $V^h$ is optimal in this set.

Finally we show that it is not optimal for $D_2$ to set $P_2^h(V^h) < V^h$. Suppose it does charge $P_2^h(V^h) < V^h$. There are three effects on profit, compared to charging $P_2^h(V^h) = V^h$:

(a) In its captive segment, it loses $\alpha \frac{\alpha_2}{2} (V^h - P_2^h(V^h))$.

(b) In the fully informed segment (of mass $(1 - \alpha_u - \alpha_p)$), it was not making sales at all; it was fighting for the high types using $P_2^r$ by randomizing it between $(\tilde{P}^h, V^h)$. By reducing its price below $V^h$, in the segment of mass $(1 - \alpha_u - \alpha_p)$, if $P < P_2^r < V^h$, it cannibalizes its own sales, and loses an amount $\lambda (1 - \alpha_u - \alpha_p) G_1^h(P) G_2^l(P) \text{Prob}(P_2^r < P_1^h(V^h) \mid P_2^r > P) \{E(P_2^r - P_2^h(V^h))\}$, where $E(P_2^r \mid P < P_2^r < P_2^h(V^h))$ is the expected price at which the cannibalized sales were being made (the conditioning event is that $P < P_2^r < P_1^h(V^h)$).

(c) finally, in the segment of mass $(1 - \alpha_u - \alpha_p)$, if $P < P_1^h(V^h) < P_2^r$, it wins some sales over from $D_1$, leading to a gain $\lambda (1 - \alpha_u - \alpha_p) G_1^h(P) G_2^l(P) \text{Prob}(P_1^h(V^h) < P_2^r \mid P < P_1^h(V^h)) (P - W)$.

Replacing the relevant expressions for $G_1^h(P)$ and $G_2^l(P)$, and evaluating the conditional probabilities and expectations, we find that, in overall terms, the firm incurs a net loss from deviation, given by

$$\alpha_u (V^h - P_2^h) (1 + \frac{(\alpha_2 (1 - \alpha_u - \alpha_p) (V^h - W) (2(V^h - P)(V^h + P - 2W))}{2(2 - \alpha_u)(1 - \alpha_u)(V^l - W)(V^h + P - 2W)(P - W)}$$
\[
\frac{(3P + V^h - 4W)(V^h - W)(\log^{(P-W)})}{2(2 - \alpha_u)(1 - \alpha_u)(V^h - W)(V^h + P - 2W)(P - W)}.
\]

Hence, it will not deviate to \(P_2(V^h) < V^h\).

By construction, \(P_2(V^\ell)\) and \(P^r_2\) are best responses. Hence, deviating in one of these alone cannot improve the profit of \(D_2\).

Next, we look at possible joint deviations for \(D_2\). Suppose \(D_2\) chooses some prices \(P^r_2, P_2(V^h), P_2(V^\ell)\). For now, ignore discovery costs and franchise and referral fees—none of these terms change as the prices \(P^r_2, P_2(V^h), P_2(V^\ell)\) change. Then, the gross profits for \(D_2\) can be written as follows.

\[
\pi_2(P^r_2, P_2(V^h), P_2(V^\ell)) = \alpha_u\left(\frac{\lambda_p P_2(V^h) + (1 - \lambda)P_2(V^\ell) - W}{2}\right)
+ \alpha_p\left(\lambda G^h_1(P^r_2)(P^r_2 - W) + (1 - \lambda)G^e_1(P^r_2)(P^r_2 - W) 1_{P^r_2 \leq V^\ell}\right) +
\left\{(1 - \alpha_u - \alpha_p)\left(\lambda G^h(P^r_2, P_2(V^h))(\min\{P^r_2, P_2(V^h)\}) - W\right) + \right.
\left.(1 - \lambda)G^e_1(\min\{P^r_2, P_2(V^h)\})(\min\{P^r_2, P_2(V^h)\}) - W)\right\}
\]

The first term is the profits of the dealer from the high and low valuation customers in the uninformed segment of mass \(\alpha_u\). The second term indicates its profit from the low valuation and high valuation customers in the partially informed segment of mass \(\alpha_p\). Recall that in this segment, the consumers are seeing two prices, \(P^r_2\) and \(P_2(V^h)\) or \(P^r_2\) and \(P_2(V^\ell)\) (depending on whether they are high types or low types). Hence for \(D_2\) to make any sales, there should exist a positive probability that \(P^r_2 > P_2(V^h)\) and \(P^r_2 > P_2(V^\ell)\). The third term indicates its profits from the high valuation and low valuation customers in the fully informed segment. Recall that in this segment, the consumers are seeing two prices of \(D_2\), \(P^r_2, P_2(V^h)\) or \(P^r_2, P_2(V^\ell)\) (depending on whether they are high types or low types). Hence any sales that \(D_2\) makes in these segments will occur only at the minimum of \((P^r_2, P_2(V^h))\) for the high types and minimum of \((P^r_2, P_2(V^h))\) for the low types.

The gross profit in equilibrium of \(D_2\) can be determined by substituting \(P^r_2 = V^h, P_2(V^h) = V^h,\) and \(P_2(V^\ell) = V^\ell\) (since any choice of \(P^r_2, P_2(V^\ell)\) in the stated range leads to the same profit). This leads to an equilibrium gross profit for \(D_2\) given by

\[
\pi^*_2 = \alpha_u\left(\frac{\lambda V^h + (1 - \lambda)V^\ell - W}{2}\right) + \alpha_p\left(\frac{\alpha_u}{2 - \alpha_u}\right)\lambda(V^h - W) +
\left\{(1 - \alpha_u - \alpha_p)\left(\frac{\alpha_u}{2 - \alpha_u}(V^h - W) + (1 - \lambda)\frac{2\alpha_p}{2 - \alpha_u}(V^\ell - W)\right)\right\}
\]

Now, consider a deviation by firm 2. Note that it will never choose \(P_2(V^\ell)\) outside the range \([\hat{P}^\ell, V^\ell]\). Any price higher than \(V^\ell\) leads to no sales at the price \(P_2(V^\ell)\), so such prices are dominated
by $V^\ell$. Similarly, any price lower than $\hat{P}^h$, given the strategy of $D_1$, is dominated by $\hat{P}^\ell$. Hence, we consider deviations by firm 2 in $P'_{2}$ and $P_{2}(V^h)$. The following deviations are feasible:

1. Suppose $V^\ell < P'_{2} < \hat{P}^h$ and $P_{2}(V^h) < V^h$. There could be two possibilities here:

   (i) $P'_{2} < P_{2}(V^h)$
   (ii) $P'_{2} > P_{2}(V^h)$.

   Consider equation (3) and case (i) first. From the first scenario, it turns out that $\min\{P'_{2}, P_{2}(V^h)\}$ is less than $\hat{P}^h$. So $G^h_1(\min\{P'_{2}, P_{2}(V^h)\}) = 1$. In the same vein, $G^\ell_1(\min\{P'_{2}, P_{2}(V^\ell)\}) = 0$ and $(\min P - W) < (\hat{P}^h - W)$. If the deviation occurs, the deviated profits are given by

\[
\pi_{2} = \frac{\alpha_{u}}{2}(\lambda P + (1 - \lambda)V^\ell - W) + \alpha_{p}\lambda(P - W) + (1 - \alpha_{u} - \alpha_{p})\lambda(P - W). \quad (18)
\]

Since $P < \hat{P}^h$, $\frac{V^h - W}{P - W} \geq \frac{2 - \alpha_{u}}{\alpha_{u}}$, from the definition of $\hat{P}^h$. Thus it is shown that each of the terms in equation (18) turn out to be lower than or equal to the corresponding terms in equation (17). Hence, $D_2$ does not have a profitable deviation.

From the second scenario, in the claimed equilibrium, we have $P'_{2}$ being randomized between $(\hat{P}^h, V^h)$. Hence, it again turns out that $\min\{P'_{2}, P_{2}(V^h)\}$ is less than $\hat{P}^h$. Thus, $D_2$ does not have a profitable deviation.

2. Suppose $P'_{2} \leq V^\ell$. Then, it cannot be profitable to set $P_{2}(V^h)$ such that $V^\ell < P_{2}(V^h) < V^h$. This is because, in the fully informed segment, no consumer buys at the price $P_{2}(V^h)$ (since all these consumers also see the lower price $P'_{2}$). Further, in the uninformed segment, this leads to lower profit from the high valuation consumers.

   Further, if $P'_{2} < V^\ell$, it cannot be profitable to set $P_{2}(V^h) < V^\ell$. Again, $D_2$ earns less in the uninformed segment; in the remaining fully informed segment, at best, it cannibalizes sales it would otherwise have made at the price $P'_{2}$ (if $P_{2}(V^h) < P'_{2}$).

3. Next, suppose that $P'_{2} \leq V^\ell$, and $P_{2}(V^\ell) < P'_{2}$, with $P_{2}(V^h) = V^h$. In a similar manner to (1) above, it can be shown that $D_2$ cannot deviate to any profitable equilibrium since it only loses profits in the uninformed segments.

4. Finally we check for the case when $P_{2}(V^\ell) = V^\ell$, $P'_{2} \leq V^\ell$ and $P_{2}(V^h) = V^h$. From equations (3) and (17) we find that the deviation is unprofitable iff $(1 - \alpha_{u})(\hat{P}^\ell - W) \leq \lambda(1 - \alpha_{u})(\hat{P}^h -
Proof of Lemma 2

Details of Step 4

Consider $D_1$ first. In the physical ($\beta$) segment, since all its sales are offline, it knows the consumer type before it chooses its price for each consumer. Hence, a deviation in $P_1(V^h)$ or $P_1(V^f)$ does not affect its profit from consumers of type $V^f$ or $V^h$, respectively. That is, it is sufficient to rule out deviations in each of $P_1(V^h)$ and $P_1(V^f)$ in isolation. Similarly, any deviation in $P_1^m$ does not affect its profit in the physical segment. In the web $(1-\beta)$ segment, by construction, $G^m_1$ is a best responses, ruling out such deviations. Finally, since any change in $P_1(V^h)$ or $P_1(V^f)$ does not impact sales made by $D_1$ at $P_1^m$ in the web segment, any joint deviation in all 3 prices does not fetch higher profits.

Next we rule out possible joint deviations for $D_2$. Suppose $D_2$ chooses some prices $P^{r}_2, P_2(V^h), P_2(V^f), P_2^m$. For now, ignore discovery costs and franchise and referral fees—none of these terms change as the prices $P^{r}_2, P_2(V^h), P_2(V^f), P_2^m$ change. Then, the gross profits for $D_2$ can be written as follows.

$$
\pi_2 = \beta \frac{\alpha_u}{2} (\lambda P_2(V^h) + (1-\lambda)P_2(V^f) - W) + \beta \alpha_p \left( \frac{\lambda G^h_1(P^r_2)(P^r_2 - W) + (1-\lambda)G^f_1(P^r_2)(P^r_2 - W)\ 1_{P^r_2 \leq V^f}}{2} \right) + \\
\left\{ \beta(1-\alpha_u - \alpha_p) \left( \frac{\lambda G^h_1(\min\{P^r_2, P_2(V^h)\})(\min\{P^r_2, P_2(V^h)\}) - W}{1-\lambda} \right) \right\} + \\
(1-\beta) \frac{\alpha_u}{2} (\lambda P^m_2 + (1-\lambda)P^m_2 - W) + \beta \left( \frac{\lambda G^m_1(P^r_2)(P^r_2 - W) + (1-\lambda)G^m_1(P^r_2)(P^r_2 - W)\ 1_{P^r_2 \leq V^f}}{2} \right) + \\
\left\{ (1-\beta)(1-\alpha_u - \alpha_p) \left( \frac{\lambda G^m_1(\min\{P^r_2, P^m_2\})(\min\{P^r_2, P^m_2\}) - W}{1-\lambda} \right) \right\} \right\}
$$

The gross profit in equilibrium of $D_2$ can be determined by substituting $P^r_2 = V^h$, $P^m_2 = V^h$, $P_2(V^h) = V^h$, and $P_2(V^f) = V^f$ (since any choice of $P^r_2, P_2(V^f)$ in the stated range leads to the same profit). This leads to an equilibrium gross profit for $D_2$ given by

$$
\pi_2^* = \beta \frac{\alpha_u}{2} (\lambda V^h + (1-\lambda)V^f - W) + \beta \alpha_p \left( \frac{\alpha_u}{2-\alpha_u} \right) \lambda(V^h - W) +
$$
\[
\beta(1 - \alpha_u - \alpha_p) \left( \frac{\alpha_u}{2 - \alpha_u} (V^h - W) + (1 - \lambda) \frac{2\alpha_p}{2 - \alpha_u} (V^\ell - W) \right) + \\
(1 - \beta) \frac{\alpha_u}{2} (\lambda V^h + (1 - \lambda)V^\ell - W) + (1 - \beta) \alpha_p \left( \frac{\alpha_u}{2 - \alpha_u} \lambda (V^h - W) + \frac{\alpha_p}{2 - \alpha_u} (V^h - W) \right) 
\]

(20)

1. In the \( \beta \) segment, deviations in \( P_2(V^h), P_2(V^\ell), P_2^r \) can be ruled out as done earlier in Proposition 1.

2. Consider the \( (1 - \beta) \) segment. \( D_2 \) has two strategies \( P_2^r \) and \( P_2^m \). Suppose \( V^\ell < P_2^r < \hat{P}^h \) and \( P_2^m < V^h \). There could be two possibilities here:

(i) \( P_2^r < P_2^m \)

(ii) \( P_2^r > P_2^m \).

Consider equation (19) and case (i) first. From the first scenario, it turns out that \( \min P_2^r, P_2^m \) is less than \( \hat{P}^h \). So \( G_1^m(\min P_2^r, P_2^m) = 1 \) and \( \min (P - W) < (\hat{P}^h - W) \). Then it can be shown, similar to the Proof of Proposition 1 that each of the terms in equation (19) turn out to be lower than or equal to the corresponding terms in equation (20). Hence, \( D_2 \) does not have a profitable deviation.

From the second scenario, in the claimed equilibrium, we have \( P_2^r \) being randomized between \( (\hat{P}^h, V^h) \). Hence, it again turns out that \( \min \{P_2^r, P_2^m\} \) is less than \( \hat{P}^h \). Thus, \( D_2 \) does not have a profitable deviation.

3. Suppose \( P_2^r \leq V^\ell \). Then, it cannot be profitable to set \( P_2^m \) such that \( V^\ell < P_2^m < V^h \). This is because, in the fully informed segment, no consumer buys at the price \( P_2^m \) (since all these consumers also see the lower price \( P_2^r \)). Further, in the uninformed segment, this leads to lower profit from the high valuation consumers. Further, if \( P_2^r < V^\ell \), it cannot be profitable to set \( P_2^m < V^\ell \). Again, \( D_2 \) earns less in the uninformed segment; in the remaining fully informed segment, at best, it cannibalizes sales it would otherwise have made at the price \( P_2^r \) (if \( P_2^m < P_2^r \)).

4. Finally we check for the case when \( P_2(V^\ell) = V^\ell, P_2^r \leq V^\ell, P_2^m = V^\ell \) and \( P_2(V^h) = V^h \). The deviated profits are given by

\[
\beta \frac{\alpha_u}{2} (\lambda V^h + (1 - \lambda)V^\ell - W) + \beta \alpha_p(V^\ell - W) + \\
+ \beta (1 - \alpha_u - \alpha_p)(V^\ell - W) + (1 - \beta)(1 - \frac{\alpha_u}{2})(V^\ell - W). 
\]
Comparing this with equation (20), leads us to the critical value of the wholesale price $\hat{W}_m$. For wholesale prices $W \geq \hat{W}_m$, the deviation is not profitable. Thus, the specified strategies constitute an equilibrium for all $W \geq \hat{W}_m$. ■
Table 5: Prices observed by each consumer segment if infomediary enrols both retailers and search behavior is correlated with valuations

<table>
<thead>
<tr>
<th>Types</th>
<th>$\alpha_u$</th>
<th>$\alpha_p$</th>
<th>$\frac{\alpha_p}{\alpha_u}$</th>
<th>$1 - \alpha_u - \alpha_p$</th>
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<tbody>
<tr>
<td>HV Consumers</td>
<td>$P_1(V^H)$</td>
<td>$P_2(V^H)$</td>
<td>$P_{1r}^r, P_{2r}^r$</td>
<td>$P_{1r}^r, P_{2r}^r$</td>
</tr>
<tr>
<td>LV Consumers</td>
<td>$P_1(V^L)$</td>
<td>$P_2(V^L)$</td>
<td>$P_1(V^L)$</td>
<td>$P_2(V^L)$</td>
</tr>
</tbody>
</table>

Table 6: Prices observed by each consumer segment when infomediary enrolls only one retailer

<table>
<thead>
<tr>
<th>Types</th>
<th>$\alpha_u$</th>
<th>$\alpha_p$</th>
<th>$\frac{\alpha_p}{\alpha_u}$</th>
<th>$1 - \alpha_u - \alpha_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HV Consumers</td>
<td>$P_1(V^H)$</td>
<td>$P_2(V^H)$</td>
<td>$P_1(V^H), P_2(V^H), P_{2r}$</td>
<td>$P_1(V^H), P_2(V^H), P_{2r}$</td>
</tr>
<tr>
<td>LV Consumers</td>
<td>$P_1(V^L)$</td>
<td>$P_2(V^L)$</td>
<td>$P_1(V^L)$</td>
<td>$P_2(V^L)$</td>
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</tbody>
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