Personalized Pricing and Quality Customization

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We embed the principal–agent model in a model of spatial differentiation with correlated consumer preferences to investigate the competitive implications of personalized pricing and quality allocation (PPQ), whereby duopoly firms charge different prices and offer different qualities to different consumers, based on their willingness to pay. Our model sheds light on the equilibrium product-line pricing and quality schedules offered by firms, given that none, one, or both firms implement PPQ. The adoption of PPQ has three effects in our model: it enables firms to extract higher rents from loyal customers, intensifies price competition for nonloyal customers, and eliminates cannibalization from customer self-selection. Contrary to prior literature on one-to-one marketing and price discrimination, we show that even symmetric firms can avoid the well-known Prisoner's Dilemma problem when they engage in personalized pricing and quality customization. When both firms have PPQ, consumer surplus is nonmonotonic in valuations such that some low-valuation consumers get higher surplus than high-valuation consumers. The adoption of PPQ can reduce information asymmetry, and therefore sellers offer higher-quality products after the adoption of PPQ. Overall, we find that while the simultaneous adoption of PPQ generally improves total social welfare and firm profits, it decreases total consumer surplus.

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1. Introduction

Personalized pricing has often been defined as the ability to charge different prices to different consumers. The price offered to a consumer whose valuation for a product or service is known may be higher or lower than the posted uniform price charged by firms that lack the sophistication to target individual consumers. Various technologies exist today that allow firms to identify and track individual customers. For example, the online data provider Lexis-Nexis sells to virtually every user at a different price (Shapiro and Varian, 1999). Amazon once experimented offering personalized discounts based on each individual's purchase history (Choudhary et al., 2005). Similarly, there are examples of firms not only charging personalized prices but also offering personalized quality or services too. In the enterprise software applications market, conventional pricing is based on one-to-one negotiations, which is another form of personalized pricing. At the same time, there is also a trend toward customizing the product to suit clients' needs as well as offering a personalized level of service quality through the use of one-to-one repair schedules and uptime guarantees.

In this paper, we use the term personalized pricing and quality, or PPQ, to refer to the case in which a firm can implement a pricing policy and offer a quality schedule based on complete knowledge of the willingness to pay of each consumer. We examine the following questions: (i) How does the presence of technologies that facilitate PPQ affect equilibrium price and quality schedules? (ii) How is consumer surplus and social welfare affected by the adoption of PPQ technologies? (iii) When do firms benefit from the adoption of PPQ technologies in the equilibrium? Because the amount of information required for implementing PPQ is high, in practice firms may not know valuations precisely. Hence, our results should be interpreted as the solution to an important limiting case that provides a useful benchmark—the case of perfect information.

To investigate these research questions, we need to solve a competitive second-degree price discrimination model and its extension to first-degree price discrimination. Some prior work exists in this domain. Recent work on customer recognition and behavior-based price discrimination includes Villas-Boas (1999). Much of the recent work on perfect price discrimination has been done either in the context of horizontal or vertical product differentiation with a single product (Thisse and Vives, 1988; Shaffer and Zhang, 1995; Chen and Iyer, 2002; Bhaskar and To, 2004; Liu and Serfes, 2004; Taylor, 2004; Choudhary et al., 2005). To the best of our knowledge, there exist two approaches for solving the competitive second-degree price discrimination case. In the first approach, prior work has used a single dimensional model of consumer preferences in which consumer types represent both quality and brand preferences (Spulber, 1989; Stole, 1995). In the second approach, prior work has used a two-dimensional model in which buyers' quality preferences are independent of their brand preferences (Armstrong and Vickers, 2001; Rochet and Stole, 2002). We adopt the first approach in this paper, and the reasons for doing so are deferred to Section 2. Intuitively, the equilibrium solutions of our baseline model are qualitatively similar to those in the standard model developed by Mussa and Rosen (1978). In this model, when the seller adopts PPQ, we assume that it can execute perfect price discrimination in both price and product quality, and simultaneously restrict buyers' self-selection opportunities. As a result, there exist three effects from the adoption of PPQ in our model: (i) firms can better extract rents from loyal customers who have stronger brand preferences; (ii) the adoption of PPQ may aggravate the competition for nonloyal customers who have weaker brand preferences; (iii) the adoption of PPQ can eliminate self-selection by buyers, and hence, the well-known quality degradation and welfare loss due to product line cannibalization is eliminated.

Our main findings are as follows. First, in a duopoly setting, we show that the adoption of PPQ technologies can reduce information asymmetry, and therefore sellers offer higher-quality products after the adoption of PPQ. Although the adoption of PPQ may aggravate price competition, we show that the equilibrium prices are generally higher than in the absence of PPQ. Specifically, even the firm without PPQ may increase product prices when its competitor adopts PPQ.

Second, we show that the adoption of PPQ by both firms has a differential impact on consumer surplus. In particular, consumers located closer to the middle of the market—who are the least loyal to either firm, are those who are the most better-off (in terms of their surplus) when both firms adopt PPQ technologies. Intuitively, in the absence of PPQ, it is important for firms to leave some information rents for their most loyal (higher valuation) consumers so that it can prevent cannibalization induced by intrafirm product competition. This leads to positive surplus for the higher valuation consumers. However, with PPQ there is no potential for such cannibalization and as a result, firms do not need to leave any information rents for consumers. Consequently, the most loyal consumers are left with zero surplus.

Third, with regard to the welfare analysis, we show that compared with the scenario without PPQ, when one firm adopts PPQ, it always increases its quality level while the other firm keeps its quality schedule unchanged. In this paper, the social welfare is determined by the level of product quality. In the absence of PPQ, only the most loyal customers receive the socially optimal quality level whereas all other customers receive degraded products. As a result, the adoption of PPQ always improves social welfare in this model. This effect is robust and is true even without the distributional assumptions on customer types.

Lastly, in contrast to prior studies, we show that the adoption of PPQ technologies by competing firms can make even symmetric (or identical) firms better-off. A number of recent papers (Shaffer and Zhang, 1995; Bester and Petrakis, 1996; Choudhary et al., 2005), have shown that when firms offer one-to-one promotions or other forms of customized pricing, it generally leads to a Prisoner's Dilemma that leaves all firms worse-off compared to the scenario when they do not offer customized pricing. These papers are based on *ex ante* symmetric firms. In a model of vertical differentiation, Choudhary et al. (2005) show that a higher-quality firm can actually be worse off when it implements personalized pricing. Corts (1998) and Shaffer and Zhang (2000) find that targeted promotions need not necessarily lead to a Prisoner's Dilemma. However, they allow for at most one promotional price by symmetric firms, and their result accrues due to an alleviation of price competition. A closely related paper by Shaffer and Zhang (2002) shows that in a model that includes both horizontal and vertical differentiation and a positive cost of targeting customers, Prisoner's Dilemma can be avoided with perfect price discrimination but only with asymmetric firms (when firms are dissimilar in market size, *ex ante*).

Our paper shows that even *symmetric* firms are better-off when they engage in PPQ, and thus avoid the Prisoner's Dilemma. In our model, this result arises because of the "quality enhancement" effect. With PPQ, firms can provide higher qualities to each consumer without the fear of intrafirm product cannibalization that occurs in situations with customer self-selection. This occurs also because targeting in our paper enables a firm to allocate a better product at higher prices to each individual consumer, which leads to a higher rent extraction ability for each firm. This effect offsets the price competition effect and makes it profitable for symmetric firms to engage in PPQ. It is useful to note that we ignore the possibility of mistargeting, which results, for example, when a firm mistakenly perceives some price-sensitive customers as price-insensitive and charges them high prices.[1]

Our work is also related to the emerging stream of research on product customization that shows that firms should not make symmetric investments in product customization technology (Dewan et al., 2003; Bernhardt et al., 2007). Syam and Kumar (2005) show that

1. Liu and Serfes (2004) also consider imperfect information in a spatial price discrimination model and find that when the quality of information is low, firms unilaterally commit not to price discriminate.

customization helps firms increase the prices of the standard products as well thereby leading to higher profits. They also find conditions under which *ex ante* symmetric firms will adopt asymmetric strategies. Alptekinoglu and Corbett (2004) compare mass production with mass customization, and show that under competition the mass producer offers lower product variety. Notice that most of the existing models discuss horizontally differentiated markets and price may be the same for all consumers. Our work is different from all of these papers because firms in our model do not make decisions between offering standardized versus customized products in a horizontal differentiation setting. They always produce the same number of verticality differentiated products, that is, the length of the product line is fixed. Basically, without PPQ, firms can only post a price menu and let customers decide which to buy whereas with PPQ, firms can target each consumer with a specific (price, quality) offer. Moreover, our paper combines both personalized pricing and one-to-one quality allocation in the same theoretical framework. For a recent literature survey on papers related to personalization and customization, see Arora et al. (2008).

The rest of the paper is organized as follows. Section 2 describes the model setup in detail. Section 3 presents the second-stage subgame's equilibria wherein neither firm, one firm or both firms can have PPQ. We then proceed to Section 4 to compare prices, consumer surplus, social welfare, and firm profits in the equilibrium. Findings of the first-stage PPQ adoption game are presented in Section 5. Section 6 concludes this paper. All proofs have been relegated to the Appendix.

2. Model

We consider personalized pricing and quality allocation in a duopoly model. Two multi-product firms compete in both the quality and price of the products they offer. Each firm's product line consists of a continuum of qualities, as in prior literature (Mussa and Rosen, 1978). In this framework, a firm's focus is on the choice of price as a function of quality rather than the choice of quality levels itself. This is because the implicit assumption in such models is that a firm's product line length is fixed: all possible quality levels are produced by firms.

These two firms locate at the two ends of a straight line from 0 to 1, offering a continuum of products differentiated in quality. The firm located at the left is denoted as firm L while that located on the right is denoted as firm R. Consumer types are denoted by the parameter θ where $\theta \in [0, 1]$ with a uniform distribution. The type parameter θ represents two concepts: a consumer's marginal valuation for quality and a consumer's brand preference. A consumer has positive utility for one unit only. If either of the two products offers a positive net utility, the consumer buys the one that maximizes their surplus. Otherwise, they choose not to buy any product. We assume quasi-linear utility functions. Formally, the gross utility to a consumer with type θ buying from the firm located at 0, firm L, is

$$u^L(q, \theta) = q \times (1 - \theta), \tag{1}$$

while his utility in buying from the firm located at 1, firm R, is

$$u^R(q, \theta) = q \times \theta. \tag{2}$$

This is the setup used in Spulber (1989) and Stole (1995) to model competitive nonlinear pricing. From either firm's perspective, θ or $1 - \theta$ is the marginal willingness to pay for quality of buyers. q is the product quality offered by firms. Hence, from one firm's perspective, the setup is consistent with the standard nonlinear pricing models in the applied economics literature that models vertical differentiation (e.g., Mussa and Rosen, 1978). The uniqueness of this setup is that the consumer type θ simultaneously denotes consumer's quality preference as in vertical differentiation and brand preference as in horizontal differentiation. A given consumer has a different marginal valuation for each of the two brands: θ versus $1 - \theta$. Implicitly, this assumes that larger the brand preference of a given consumer for one firm, the larger is the quality preference of that consumer for that firm's products. A consumer's marginal valuation of quality is determined by his preferences over horizontal product characteristics: a consumer who prefers brand L over brand R derives more utility from an increase in the quality of good L rather than increase in the quality of good R. Basically, a simple interpretation of our model is a market in which there are customers who have very high marginal WTP for quality for the products of one firm but not for products of the other firm. This setup maps a scenario wherein two firms sell branded products and have groups of brand loyal customers. It also captures the fact that customers who do not have loyalty toward any particular brand, have an average marginal willingness to pay (WTP) for quality for either brand. Hence, consumers who purchase goods of higher quality also have stronger brand preferences. Note that this setup does not change the ordinary interpretation of product qualities (q): any buyer still strictly prefers higher-quality than lower-quality products of either firm.

There is also empirical motivation for modeling consumer preferences in this way. It comes from the observation that in some markets, consumers who purchase higher qualities are more brand-loyal than those who purchase lower qualities. For example, in the auto industry, some consumers have a stronger preference toward highly fuel-efficient cars that may rate lower on another dimension like vehicle safety while others have a strong preference toward SUVs (sports utility vehicles) that rate higher on safety but lower on fuel efficiency. This is well documented within the car market segments when the brand, as shown in Goldberg (1995), Berry et al. (1995), and Feenstra and Levinsohn (1995). For another example, in the enterprise software industry, competing products are usually differentiated along the dimension of "software with more features" versus "software that is easy to learn and use." Specifically, the database enterprise software industry is a good example. It approximates a duopolistic industry with Oracle and Microsoft commanding most of the market. Oracle provides a powerful, stable database package but it is also notoriously difficult to learn and use. In contrast, Microsoft SQL server is much more user-friendly but has fewer features and is considered less stable by users. In practice, larger companies are likely to care more about features and stability in enterprise software. For example, large banks and brokerage firms prefer to have zero fault tolerance (e.g., $\theta \to 1$): they have very high marginal valuation for stability but a relatively lower valuation for reducing training costs. On the other hand, smaller and medium-sized firms are likely to care more about ease of learning in order to save on staff training expenses (e.g., $\theta \to 0$), and relatively less about advanced features because they typically need to perform basic operations. Similar examples exist in the consumer packaged software industry. For example, MBA students and industry professionals typically prefer using SPSS for data analysis because of its user-friendly design whereas academic researchers typically prefer more sophisticated packages such as SAS or STATA so that they can execute its advanced features. Thus academic researchers are likely to have a higher marginal valuation for improvements in the quality of STATA but a lower valuation for any improvements in SPSS. Other examples exist in the industries in which most buyers have been locked-in to one brand and the switching cost is higher for higher-quality products. In commercial airplanes (Boeing vs. Airbus) industry, there are significant brand-specific investments from buyers' perspectives especially due to lock-in effects. Because of switching costs, consumers may have higher marginal valuation for the quality of one firm's products but lower marginal valuation for quality of the other firm's product. Note that products like automobiles, enterprise software, and aircrafts can be highly customized according to buyers' specifications and their prices are often personalized based on negotiations.[2]

2. Another example is the OTC (over-the-counter) pharmaceutical products market. The loyalty of a consumer toward a specific brand is usually built upon his/her prior

As mentioned in the introduction, the other approach is to model quality and brand preferences independently. In a related work, Ghose and Huang (2007) look at the potential value of personalization in a two-dimensional spatial model with independent consumer preferences along the horizontal and vertical dimensions. That model is built upon the two-dimensional competitive second-degree price discrimination model developed by Rochet and Stole (2002). It is important to note that these two models are parallel cases and that our current model is not a special case of the two-dimensional model. Moreover, the key research question is different in Ghose and Huang (2007) in that they analyze whether personalization along a single or dual dimension is more profitable compared to no personalization.

The sequence of the game in our paper is as follows: two firms simultaneously decide whether to adopt PPQ or not. When they adopt PPQ, they incur a nonzero fixed cost and can perfectly identify each consumer's type. In the second stage, the adoption of PPQ is common knowledge and firms announce their product line pricing in the second stage accordingly. The solution concept is subgame perfect Nash equilibrium (SPNE). When neither firm has access to PPQ, prices are chosen simultaneously by both firms. When only one firm has access to PPQ, the firm without PPQ chooses its price first. After observing this firm, the firm with access to PPQ sets a menu of prices. This setting is widely adopted in the literature (see, e.g., Thisse and Vives, 1988; Choudhary et al., 2005). When both firms have PPQ, the order of moves at stage 2 does not affect the equilibrium outcome; we again posit that prices are chosen simultaneously. Once prices are chosen, at the last stage of the game (stage 3), consumers decide which, if any, product to buy.[3]

experience with that OTC medicine; he/she will stick to the brand that is more effective on him/her. For example, some customers may have no preference between Claritin and Benadryl (allergy-relief medicines) because both have similar effects on them (these customers have $\theta = 0.5$ in this model). Some other customers are likely to have stronger preferences for either brand. These customers are very likely to have a higher marginal WTP for higher-quality products from their favorite brand but a lower marginal WTP for higher-quality products of the other brand because the other brand is less effective on them ($\theta = 0$ or 1 in this model). Similarly, consider the food and restaurant industry. Although some consumers prefer to eat spicy or high-calorie food (say, burgers from McDonalds), others may be more health conscious and prefer relatively bland but low-calorie food (say, sandwiches from Subways).

3. Note that with PPQ each consumer receives a single (price, quality) offering from the firm in accordance with their types. Hence, it is not critical for consumers to observe the menu before purchase in scenarios with PPQ. In contrast, when a firm does not have PPQ, a consumer can choose any pair from a menu of prices and qualities. In this case, consumers do need to observe the menu of prices and qualities. This is feasible and common in practice too. For example, using sources on the Internet, or direct mail order

Consistent with the prior literature, we assume that firms have a marginal cost of production which is invariant with the quantity, but depends on the quality of the product (Moorthy, 1988). That is, both firms have the same cost function, but depending on the quality schedules they choose, their marginal costs may differ in equilibrium. Each firm has a constant marginal cost for producing the good, denoted by c. For analytical tractability and to highlight the impact of the cost function on different decision variables, we use the following function: $c(q) = q^{\alpha}/\alpha, \alpha > 1$.

3. Second-Stage Equilibrium with or without PPQ

3.1 Neither Firm Has PPQ

First, we consider the benchmark case when neither firm has access to PPQ (we call this the No-PPQ case). This baseline model is exactly the game solved in Spulber (1989). In this subgame, each firm offers a menu of prices, $p(q)$, for all consumer types θ. The decision variable $p(q)$ of each firm can be equivalently written as $q(\theta)$ and $p(\theta)$ because each consumer will self-select the contract designed for his type in equilibrium.[4] As shown by Spulber (1989), in equilibrium, each firm occupies an interval of consumers. Specifically, there exists a marginal customer, denoted by $\theta = B$, who feels indifferent between each firm's lowest-quality product. As a result, the objective function of firm R is given by

$$\max_{p^R(\theta), q^R(\theta)} \pi_N^R, \text{ where } \pi_N^R = \int_B^1 \left[p^R(\theta) - \frac{(q^R)^{\alpha}(\theta)}{\alpha} \right] d\theta, \tag{3}$$

subject to the following constraints:

- **(IC):** Each consumer of type θ chooses the $q^R(\theta)$ and $p^R(\theta)$ that the firm designed for him. $\theta = \arg\max_t \ \theta \times q^R(t) - p^R(t), \forall \theta \in [B, 1]$.
- **(IR1):** Each consumer of type θ receives a utility level that is higher than 0. $s^R(\theta) \geq 0, \forall \theta \in [B, 1]$.
- **(IR2):** The marginal consumer B gets the same surplus from each firm and hence, is indifferent between buying from firm R and firm L. That is, $s^R(B) = s^L(B)$.

catalogs consumers can observe the different prices firms charge for different possible configurations of the product.

4. Rather than considering all possible pricing functions, the revelation principle ensures that the firm can restrict its attention to direct mechanisms—that is, contracts in which one specific quality–price pair is designed for each consumer, and in which it is rational and optimal for the consumer to choose the price and quality pair that was designed for him or her. This type of transformation is standard in models of price screening (see, for instance Armstrong, 1996).

Here, we use superscripts to denote the variables of firm L or R. Let π_N^L and π_N^R denote the profit of firm L and firm R, respectively, in the No-PPQ case. Let $s(\theta)$ denote the consumer's surplus function. For ease of comparison with the model developed by Mussa and Rosen (1978), the (IC) and (IR1) are written in a way that seem to apply only for $\theta \in [B, 1]$. It is important to note that in this duopoly model, customers are free to choose any product from either firm. However, because of the specification of (1) and (2), it is straightforward to show that in equilibrium, firm R serves $[B, 1]$ whereas firm L serves $[0, B]$. This is because compared with the marginal customer, customers between $[0, B)$ receive a strictly higher surplus from firm L's products but a lower surplus from firm R's products. Because the marginal customers do not switch to buy firm R's products, customers between $[0, B)$ have even a lesser of an incentive to do so. As a result, when solving for the maximization problem of firm R, we can ignore the (IC) of customers in [0, B) because customers of firm L will not choose contracts designed for firm R's customers. Hence, the profit maximization problem can be stated as above, which is also consistent with the results shown in Spulber (1989).

As in prior work, intuitively, the (IC) constraint ensures that a consumer prefers the contract that was designed for him/her, and the (IR1) constraint guarantees that each consumer accepts his designated contract. In the absence of competition from the other firm, (IC) and (IR1) are essentially the same as those in standard models such as Mussa and Rosen (1978). From Mussa and Rosen (1978), we know that the marginal customer receives zero surplus and the closer to $\theta = 1$ a customer locates, the higher the surplus he/she receives. However, in the presence of competition, because the marginal customer can switch to the other firm, he/she may receive a positive surplus. Therefore, the profit-maximization problem deviates from the standard one due to the additional constraint (IR2).

The next step is to apply the standard transformation on the decision variable $p^R(\theta)$ to consumer surplus function $s^R(\theta) \equiv u^R(q^R(\theta), \theta) - p^R(\theta)$. We substitute the pricing schedule, $p^R(\theta)$, by the consumer surplus function, $s^R(\theta)$ given that $p^R(\theta) = u^R(q^R(\theta), \theta) - s^R(\theta)$. Based on equation (3), the simplified objective function for firm R can be rewritten as

$$\max_{q^R(\theta), s^R(\theta)} \pi_N^R, \text{ where } \pi_N^R = \int_B^1 \left[\theta q^R(\theta) - s^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}\right] d\theta, \tag{4}$$

Similarly, the optimization problem for firm L can be derived as follows:

$$\max_{q^L(\theta),s^L(\theta)} \pi_N^L, \text{ where } \pi_N^L = \int_0^B \left[(1-\theta)q^L(\theta) - s^L(\theta) - \frac{(q^L)^\alpha(\theta)}{\alpha}\right] d\theta. \quad (5)$$

Both (4) and (5) are subject to associated (IC), (IR1), and (IR2). The detailed derivations are provided in the Appendix. This leads to our first result.

Proposition 1: *The equilibrium prices, quality schedules, and surplus functions of the No-PPQ case are as follows:*

$$q^L(\theta) = \begin{cases} (1-2\theta)^{1/(\alpha-1)} & \text{if } \theta \in [0, 1/2]; \\ 0 & \text{if } \theta \in (1/2, 1]; \end{cases}$$

$$q^R(\theta) = \begin{cases} 0 & \text{if } \theta \in [0, 1/2); \\ (2\theta-1)^{1/(\alpha-1)} & \text{if } \theta \in [1/2, 1]; \end{cases}$$

$$s^L(\theta) = \frac{\alpha-1}{2\alpha}(1-2\theta)^{\alpha/(\alpha-1)}, \theta \in [0, 1/2];$$

$$s^R(\theta) = \frac{\alpha-1}{2\alpha}(2\theta-1)^{\alpha/(\alpha-1)}, \theta \in [1/2, 1];$$

$$p^L(\theta) = \begin{cases} (1-2\theta)^{1/(\alpha-1)} \left(\frac{-2\theta+\alpha+1}{2\alpha}\right) & \text{if } \theta \in [0, 1/2]; \\ 0 & \text{if } \theta \in (1/2, 1]; \end{cases}$$

$$p^R(\theta) = \begin{cases} 0 & \text{if } \theta \in [0, 1/2); \\ (2\theta-1)^{1/(\alpha-1)} \left(\frac{2\theta+\alpha-1}{2\alpha}\right) & \text{if } \theta \in [1/2, 1]. \end{cases}$$

Intuitively, in the equilibrium, duopoly firms compete by lowering their price menu by a constant while keeping the quality schedule at the same level. The amount of constant discount is exactly equal to the consumer surplus offered to the marginal customer, $s^R(B) = s^L(B)$, which is zero in the Mussa and Rosen (1978) model. It is easy to verify that in this proposition, $s^R(B) = s^L(B) = 0$. However, this is not a general result but a special case when customers are uniformly distributed from 0 to 1. In Section 5, when customers are uniformly distributed in a narrower interval (e.g., 0.2 to 0.8), $s^R(B) = s^L(B) > 0$ in equilibrium. In other words, in this "corner solution," duopoly firms behave as if they were local monopolists and hence, the solutions are essentially similar to those in standard monopoly solutions.

As in Mussa and Rosen (1978), given the continuous product lines (where there is quality level available for every possible consumer type θ), there is a fear of cannibalization because some high-valuation consumers might end up buying the lower-quality product. Consequently, firms need to leave some information rents for the high-valuation consumers (consumers located closer to 0 or 1) in order to prevent them from buying lower-quality products. Basically without PPQ, firms have to "reward" their loyal customers to prevent them from buying lower-quality products.

Formally, note that the total surplus generated by firm R is $\theta q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}$. This implies that the socially optimal quality level (first-best solution) is given by $q^R(\theta) = \theta^{1/(\alpha-1)}$. By comparing this quality level with the optimal quality schedule actually offered by the firm, we find that the quality received by each consumer is lower than the socially optimal level (except for the highest type whose $\theta = 1$). This degradation of quality happens because of the potential for cannibalization from self-selection. This proposition shows that competition in this model setup does not improve the quality degradation phenomenon because while competition affects the price schedules by a constant, it does not affect quality schedules at all. This result also suggests that the range of qualities offered by both the firms is [0, 1] because no firm will produce products with quality higher than the socially optimal level in this model.

3.2 Only One Firm Has PPQ

Next, we analyze a situation in which only one firm has access to technologies that facilitate PPQ. Without loss of generality, we assume that among these two firms, only firm R has PPQ. To be consistent with prior research (Thisse and Vives, 1988; Liu and Serfes, 2004; Choudhary et al., 2005, Liu and Zhang, 2006), we analyze the setting in which the PPQ firm makes its pricing decision after the No-PPQ firm.[5] At stage 1 of this scenario, firm L (the firm without PPQ) announces its menu and allows consumers to self-select a particular quality and price from its product line. At stage 2, firm R (the firm with PPQ) targets every consumer with a specific quality and price in accordance with their type. In the final stage, consumers choose which firm to buy from and

5. This setting has been widely adopted in the literature due to the following reason. A simultaneous choice of pricing in this asymmetric game does not lead to a pure strategy Nash equilibrium. Similar to prior work, this is true in our model as well. Further, the mixed strategy equilibrium is also analytically intractable.

demand is realized. The solution concept of this section is subgame perfect Nash equilibrium.

Given any strategy of firm L, in equilibrium, firm R will offer the socially optimal level of quality to maximize its profit because it can perfectly target consumers to avoid cannibalization. Generally, whenever one firm acquires PPQ, it does not need to consider the cannibalization problem because consumers can now be allocated the price and quality pair exactly in accordance with their valuation. Let π_R^L and π_R^R denote the profit of firm L and firm R, respectively, in this case. Formally, the maximization problem of firm R can be written as

$$\max_{q^R(\theta), s^R(\theta)} \pi_R^R(\theta), \text{ where } \pi_R^R(\theta) = \theta q^R(\theta) - s^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}, \forall\theta \in [0, 1]. \tag{6}$$

Firm R sets the price, or equivalently, sets the surplus function $s^R(\theta)$, such that each consumer's surplus exactly matches his/her surplus from the outside opportunity, which is either equal to zero or equal to the surplus from buying from firm L. Given R's strategy described above, L's optimization problem is the same as that in the No-PPQ case given by equation (5) except that (IR2), is replaced by the socially optimal surplus curve of firm R given as follows:

$$s^L(B) = \max_{q^R(\theta)} \left[\theta q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}\right]\Bigg|_{\theta=B}. \tag{7}$$

If firm L were to offer less than the socially optimal surplus of firm R, then firm R could potentially poach L's consumers by offering lower prices and by adjusting quality. The potential for poaching exists because firm R can perfectly identify each consumer, and in particular, it can lower its price to marginal cost for the consumer at the boundary. Thus, firm L can retain the marginal consumer at B (i.e., maintain its market share) only if its surplus $s^L(B)$ equals the socially optimal surplus offered by firm R. To derive the equilibrium prices of firm R, we also need to know the outside opportunities of firm R's customers ($\theta \in [B, 1]$). We assume that firm L does not offer any additional contracts (price–quality pairs) to firm R's consumers. In other words, the outside opportunity of firm R's customers is equal to $(1-\theta)q^L(B) - p^L(B)$. This leads to the following proposition.

Proposition 2: *In the case when only one firm has PPQ, the optimal prices, quality schedules, and surplus functions are as follows:*

$$q^L(\theta) = \begin{cases} (1-2\theta)^{1/(\alpha-1)} & \text{if } \theta \in [0, B]; \\ q^L(B) & \text{if } \theta \in (B, 1]; \end{cases}$$
$$q^R(\theta) = \theta^{1/(\alpha-1)}, \theta \in [0, 1];$$
$$s^L(B) = \left(1 - \frac{1}{\alpha}\right)(1-2B)^{\alpha/(\alpha-1)} - B^{\alpha/(\alpha-1)};$$
$$s^L(\theta) = s^L(B) - \frac{\alpha-1}{2\alpha}(1-2B)^{\alpha/(\alpha-1)} + \frac{\alpha-1}{2\alpha}(1-2\theta)^{\alpha/(\alpha-1)}, \theta \in [0, B];$$
$$s^R(\theta) = \max[0, (1-\theta)q^L(B) - p^L(B)], \theta \in [B, 1];$$
$$p^L(\theta) = \begin{cases} (1-\theta)(1-2\theta)^{1/(\alpha-1)} - s^L(\theta) & \text{if } \theta \in [0, B]; \\ p^L(B) & \text{if } \theta \in (B, 1]; \end{cases}$$
$$p^R(\theta) = \begin{cases} 0 & \text{if } \theta \in [0, B); \\ \theta^{\alpha/(\alpha-1)} - s^R(\theta) & \text{if } \theta \in [B, 1]. \end{cases}$$

The marginal consumer's type is given by $B = [(\frac{2\alpha-1}{\alpha-1})^{\frac{\alpha-1}{\alpha}} + 2]^{-1}$. When $c(q) = q^2/2$, $B = 0.27$. Although general expressions for $s(B)$ and prices are analytically tractable, the math is not easily parsable and so we do not present it in the main body of the paper.

3.3 Both Firms Have PPQ

In this case, both firms have complete knowledge of each consumer's type and are able to implement PPQ. We term this the Both-PPQ case and derive the Nash equilibrium of this game. Because both firms have full information about consumer preferences for price and quality, they engage in a Bertrand-type price–quality competition at the individual consumer level. The following analysis shows that in equilibrium the firm located closer to a given consumer will set a price schedule such that it can exactly match the consumer surplus offered by its rival. This firm can appropriate the remaining surplus from this transaction and thus will offer a socially optimal level of quality. The firms' profit functions are given by

$$\max_{q^L(\theta), s^L(\theta)} \pi^L_{Both}(\theta), \text{ where } \pi^L_{Both}(\theta) = (1-\theta)q^L(\theta) - s^L(\theta) - \frac{(q^L)^\alpha(\theta)}{\alpha}, \tag{8}$$

$$\max_{q^R(\theta),s^R(\theta)} \pi^R_{Both}(\theta),\ \text{where}\ \pi^R_{Both}(\theta) = \theta q^R(\theta) - s^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}. \tag{9}$$

where $s^L(\theta)$ and $s^R(\theta)$ are equal to the socially optimal surplus offered by the rival firm. Formally,

$$\begin{aligned} s^R(\theta) &= \max_{q^L(\theta)}\left[(1-\theta)q^L(\theta) - \frac{(q^L)^\alpha(\theta)}{\alpha}\right] \\ &= \left(1-\frac{1}{\alpha}\right)(1-\theta)^{\alpha/(\alpha-1)},\ \theta \in [1/2, 1], \end{aligned}$$

$$s^L(\theta) = \max_{q^R(\theta)}\left[\theta q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}\right] = \left(1-\frac{1}{\alpha}\right)\theta^{\alpha/(\alpha-1)},\quad \theta \in [0, 1/2].$$

It is important to notice that in equilibrium, firm R offers a surplus which is equal to the socially optimal surplus of firm L. If R's surplus is less than the socially optimal surplus offered by L, L would be able to poach on R's consumers by increasing quality or decreasing price. If R's surplus is more than that of L, it is not maximizing its profit. Hence, it is optimal for firm R to increase its price to the profit maximizing level.

Given the kind of competition that will ensue between the two firms, we can determine the surplus functions $s^L(\theta)$, $s^R(\theta)$, and hence point out the optimal price schedules. All consumers whose $\theta \in [1/2, 1]$, buy from firm R in equilibrium. Similarly, all consumers whose $\theta \in [0, 1/2]$, buy from firm L in equilibrium. Basically, the equilibrium price from the firm located closer to θ is set so that consumers feel indifferent between buying from two firms. The equilibrium price offered by the farther firm is set to marginal cost due to Bertrand price competition. This leads to the following result.

Proposition 3: *The optimal prices, quality schedules, and surplus functions when both firms have PPQ are as follows:*

$$q^L(\theta) = (1-\theta)^{1/(\alpha-1)}, \theta \in [0, 1];$$

$$q^R(\theta) = \theta^{1/(\alpha-1)}, \theta \in [0, 1];$$

$$s^L(\theta) = \left(1-\frac{1}{\alpha}\right)\theta^{\alpha/(\alpha-1)},\ \theta \in [0, 1/2];$$

$$s^R(\theta) = \left(1-\frac{1}{\alpha}\right)(1-\theta)^{\alpha/(\alpha-1)},\ \theta \in [1/2, 1];$$

$$p^L(\theta) = \begin{cases} (1-\theta)^{\alpha/(\alpha-1)} - \left(1 - \dfrac{1}{\alpha}\right)\theta^{\alpha/(\alpha-1)} & \textit{if } \theta \in [0, 1/2]; \\ 0 & \textit{if } \theta \in (1/2, 1]; \end{cases}$$

$$p^R(\theta) = \begin{cases} 0 & \textit{if } \theta \in [0, 1/2); \\ \theta^{\alpha/(\alpha-1)} - \left(1 - \dfrac{1}{\alpha}\right)(1-\theta)^{\alpha/(\alpha-1)} & \textit{if } \theta \in [1/2, 1]. \end{cases}$$

Note that compared to the No-PPQ case the adoption of PPQ actually decreases the product line quality range of a firm, from [0, 1] to [1/2, 1]. That is, because qualities and prices are now targeted (with PPQ), firms do not need to degrade qualities. Intuitively this occurs because from a firm's perspective, there is no fear of cannibalization in this case. Recall that because firms can allocate qualities by targeting consumers directly with PPQ, there are no consumer self-selection problems. Consequently, firms do not have any incentive to degrade qualities offered to their customers. Thus, they provide their loyal customers products with better quality which results in higher prices as well. This leads to higher profits than the No-PPQ case. On the other hand, despite offering their competitor's loyal customers with lower qualities and lower prices (both firms' prices fall to marginal cost in their respective rival's turfs) they are unable to poach on their competitor's territory.[6]

Note that these findings do not depend on the distributional distribution of θ at all. That is, θ can be drawn from any probability distribution but the results in Proposition 3 will remain the same. A comparison of quality schedules offered in three subgames reveals that when a firm adopts PPQ, it increases the quality offered to each consumer. However, the firm without PPQ keeps its quality schedule unchanged. When both firms adopt PPQ, their qualities are always higher than the No-PPQ qualities. This enables them to offer a higher quality than in the No-PPQ case and charge higher prices. Comparisons of other results are discussed in detail in the next section.

4. Discussions of the Second-Stage Equilibria

4.1 Prices

We plot the price curves for quadratic and cubic cost functions in Figures 1 and 2 for each of the three cases: (i) neither firm has PPQ, (ii) one firm (firm R, without loss of generality) has PPQ, and (iii) both firms have PPQ. Interestingly, note that when $\alpha = 2$, the price functions

6. The implicit notion here is that consumers buy from the firm offering a higher-quality product even if the surplus offered by both firms is exactly the same.

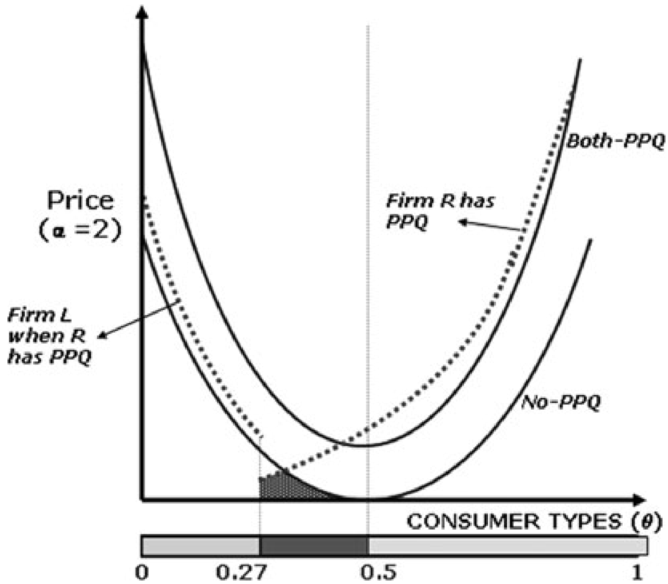

FIGURE 1. PRICE WITH OR WITHOUT PPQ ($\alpha = 2$)

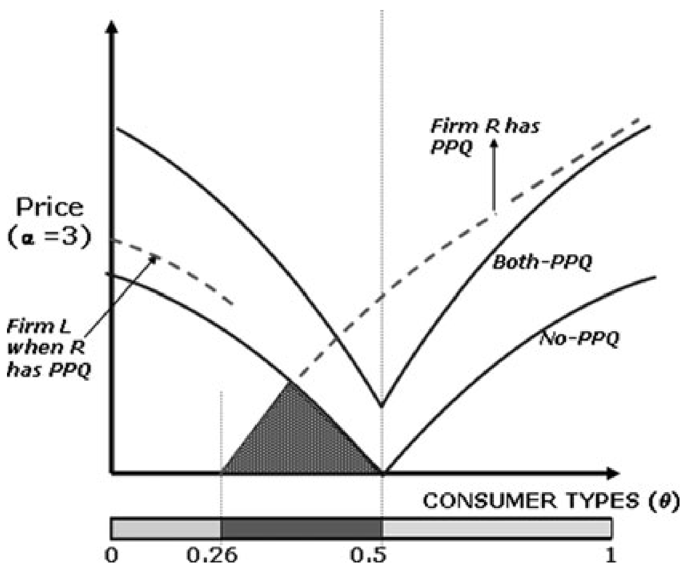

FIGURE 2. PRICE WITH OR WITHOUT PPQ ($\alpha = 3$)

are convex, while when $\alpha = 3$, the price functions are concave.[7] The thick continuous U-shaped curves indicate the price function when both firms have PPQ or when neither firm has PPQ. It is immediate to see that firm prices are always higher in the Both-PPQ case. The dotted discontinuous curve represents the price function for the case when only one firm (firm R) has PPQ. Note that when only firm R has PPQ, firm L offers a higher price compared to the No-PPQ case but lower than the Both-PPQ case. On the other hand, firm R's price is higher than its price in the No-PPQ and the Both-PPQ cases. This leads to the following corollary.

Corollary 1: *Suppose the cost function is quadratic or cubic ($\alpha = 2$ or 3). (i) Then, the adoption of PPQ by both firms leads to higher prices for all consumers compared to the No-PPQ case. (ii) When only one firm adopts PPQ, the firm without PPQ increases its price to all its consumers, compared to the No-PPQ case. However, some potential consumers of the firm without PPQ, buy from the PPQ firm at lower prices than in the No-PPQ case.*

Consider the case when $\alpha = 2$. When R is the only firm that offers PPQ, its market coverage extends across the region where $\theta \in [0.27, 1]$ while firm L covers the market where $\theta \in [0, 0.27]$. Notice that when firm R has PPQ, firm L's price is always higher than its No-PPQ price. However, firm R's price is lower than firm L's No-PPQ price in the region of $\theta \in [0.27, 0.38]$. Thus, consumers in this region get a lower price. At first sight, this is a counter-intuitive result because prices from competing duopoly firms are strategic complements. It is intuitive to see that Firm R will offer a very low price in the region of $\theta \in [0.27, 0.38]$ where it is at a disadvantage in order to exploit its information advantage from PPQ. However, it is surprising that Firm L should react by raising its product prices in the equilibrium. Essentially the intuition of this finding is as follows: because firm R (the firm with PPQ) knows the preferences of each consumer, it has the flexibility to target some of its rival's consumers. Firm L (the firm without PPQ) knows that firm R can offer a lower quality and lower price at the margin, and thus lure away some of its own consumers, especially those with relatively weaker preferences for its products (customers whose type $\theta \in [0.27, 0.5]$). Although firm L can respond

7. The intuition behind this comes from the fact that a price charged to a consumer is determined by two effects: (i) that of the offered quality (quality effect) and (ii) that of the information rent left for the consumers. These two forces have countervailing effects and thus the net shape of the pricing function depends on which of the two forces dominate. Moreover, as α increases it becomes relatively more costly to offer higher-quality products. Hence, quality schedules become more concave, and the pricing function also becomes more concave.

strategically by lowering its price to prevent this poaching, it is less profitable for firm L to do so because firm L needs to worry about its own cannibalization effect. On the contrary, by increasing its price it is able to extract a higher surplus from its loyal customers (customers whose type $\theta \in [0, 0.27]$) who have a stronger preference for its products. This results in higher overall profits than those accruing from undercutting firm R and engaging in a head–head competition for some less profitable customers. Consequently, firm L offers a higher price compared to the No-PPQ case.

We point out to the readers that this result is based on the stylized assumptions of the utility function in our model. In other words, it is applicable to cases in which the quality and brand preference of buyers are close to the specification of the present paper. It may not be universally true, particularly when customers have uncorrelated preferences along two different dimensions of quality and brand.

4.2 Consumer Surplus

Proposition 4: *When both firms have PPQ, consumer surplus is non-monotonic in valuations in that low-valuation consumers get higher surplus compared to high-valuation consumers. Specifically, for all $\theta \in [0, \widehat{\theta}]$, and for all $\theta \in [1 - \widehat{\theta}, 1]$, consumers get lower surplus when both firms have PPQ in contrast to the No-PPQ scenario. Thus, when both firms have PPQ consumers located in the middle of the market have the highest surplus in contrast to the No-PPQ scenario wherein these consumers (in the middle) have the lowest surplus.*

In the No-PPQ scenario, the fact that consumers in the middle (or those that have the lowest inclination to buy from either firm) have the lowest surplus. The intuition is exactly the same as those in the literature Mussa and Rosen (1978) where the lowest consumer type gets a *zero* surplus. Here as well, the local monopolist captures the entire surplus of the consumer at the boundary ($\theta = 0.5$) as can be seen in Figure 3. On the other hand, in the Both-PPQ scenario, consumer surplus provided by one firm is determined by its rival's socially optimal welfare curve. We can indeed verify that the surplus provided by each firm to a given consumer increases as the consumer's location gets closer to the rival firm as stated in the beginning of this section. As a result, consumers located in the middle receive a higher consumer surplus, with the highest surplus accruing to the consumer located at $\theta = 0.5$. Interestingly, this result suggests that consumers who are the least loyal to either firm, are those who are the most well-off when both firms adopt PPQ. Thus, we show in Figure 3 that consumer surplus is monotonic

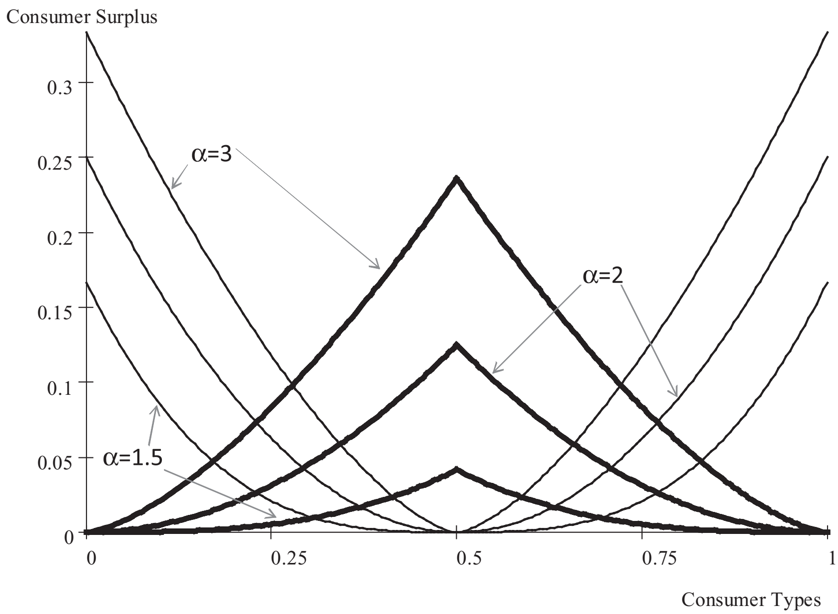

FIGURE 3. CONSUMER SURPLUS FOR DIFFERENT α. (THICK LINES: BOTH-PPQ CASES; THIN LINES: NO-PPQ CASES.)

(nonmonotonic) in valuations depending on whether firms do not have (have) access to such PPQ technologies.[8]

In addition to this proposition, we also note that as the cost of quality decreases (α increases), the optimal quality offered to any consumer also increases. Hence, the surplus accruing to any consumer also increases with α. This is true when both firms have PPQ as well as when neither firm has PPQ (except for the consumer located at $\theta = 0.5$). Lastly, we find that the total consumer surplus is highest when neither firm has access to PPQ. Thus, the adoption of PPQ enables the firms to extract the maximum rent from consumers. Once again, the additional rents from quality enhancement outweigh the price competition effect from personalized pricing leading to a lower consumer surplus.[9]

8. Prior literature in Hotelling models (e.g., Ulph and Vulcan, 2000) have shown that if transportation costs do not increase fast with distance then all consumers get lower prices (and higher surplus) when firms practice personalized pricing. This is in contrast to our results where we show that the most loyal consumers get zero surplus while the least loyal consumers get positive surplus, and that the size of these "loyal" segments is driven by the convexity of the cost function (α) parameter.

9. Note that this is in contrast to Choudhary et al. (2005) who show that total consumer surplus is highest when both firms engage in personalized pricing. In their model this result occurs because firms could only personalize prices—the products offered to all

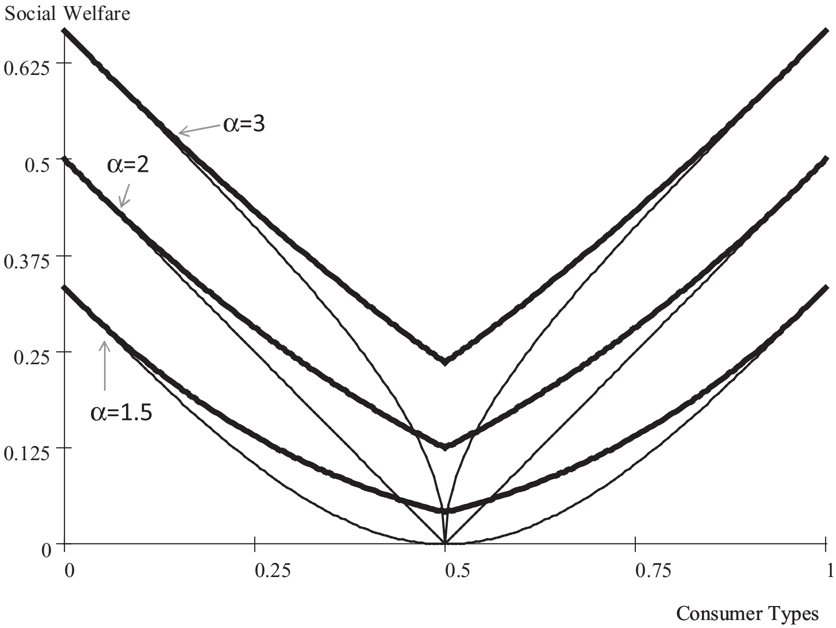

FIGURE 4. SOCIAL WELFARE FOR DIFFERENT α. (THICK LINES: BOTH-PPQ CASES; THIN LINES: NO-PPQ CASES.)

4.3 Welfare

We plot the welfare curves in Figure 4 for each of the two cases as before: neither firm has PPQ and both firms have PPQ. We define welfare of a consumer as the sum of the firm's profit from that consumer and the surplus accruing to that consumer. Note from Figure 4 that the total welfare is highest when both firms adopt PPQ. Next, we show that the adoption of PPQ by one firm (e.g., firm R) has interesting welfare implications.

Corollary 2: *Suppose the cost function is quadratic* ($\alpha = 2$ or $\alpha = 3$). *(i) When both firms adopt PPQ, social welfare is highest for the transactions with all consumers. (ii) When only one firm adopts PPQ, the social welfare from the transaction with some consumers is lower than that in the No-PPQ and the Both-PPQ cases.*

The intuition of the first result is as follows. In the No-PPQ scenario only the highest consumer type (that located at $\theta = 1$ or $\theta = 0$) gets

consumers were the same. Hence, the competitive effect of aggravated price competition led to lower prices than in the scenario when firms did not practice personalized pricing.

the socially optimal quality. In the Both-PPQ case all consumers get the socially optimal quality. Because both firms can identify each consumer, they do not need to offer degraded qualities in order to prevent possible cannibalization in which the higher consumer types choose lower qualities. That is, firms can maintain the incentive compatibility constraints without having to lower the quality offered to a given consumer. Because the total welfare depends only on qualities, the welfare is highest in the Both-PPQ case for all types. Note that because this result is true for all consumer types individually, this finding does not depend on the distributional distribution of θ at all. That is, θ can be drawn from any probability distribution and the adoption of PPQ always leads to the highest total welfare.

The intuition for the second result is similar to that for Corollary 1. When one firm has PPQ (say firm R, for example), while all the immediate consumers of the PPQ firm (those located between 0.5 and 1) get a socially optimal quality, only the highest type of the firm without PPQ (firm L in this case) gets the socially optimal quality. The remaining consumers of firm L (located between 0 and 0.27) as well those consumers of firm L (located between 0.27 and 0.5) who have been poached by firm R get less than socially optimal quality. That is, the misallocation effect arises because in a socially optimal situation consumers whose $\theta \in [0.27, 0.5]$ should have ideally bought from firm L. However, when firm R has PPQ, it induces some of L's consumers (those with $\theta \in [0.27, 0.5]$) to buy from it by offering them lowering qualities at lower prices.

4.4 Firm Profits

Proposition 5: *The adoption of PPQ by both firms does not lead to a Prisoner's Dilemma. Compared to the No-PPQ case, both firms are always better-off adopting PPQ for any α.*

From Figure 5A, we can observe that the profits in the Both-PPQ case are always higher than that in the No-PPQ case for any value of α. Although we have not discussed what are the equilibria in the first-stage PPQ adoption game, this proposition alone is sufficient to eliminate the existence of Prisoner's dilemma. The intuition driving this finding is that given the setup of quality choices in this model, there exists the well-known quality degradation phenomenon for firms without PPQ. As a result, the adoption of PPQ has three effects in a competitive setting: first, it enables firms to better extract rents from loyal customers; second, it intensifies competition (particularly for the nonloyal customers) when both firms adopt PPQ; and lastly,

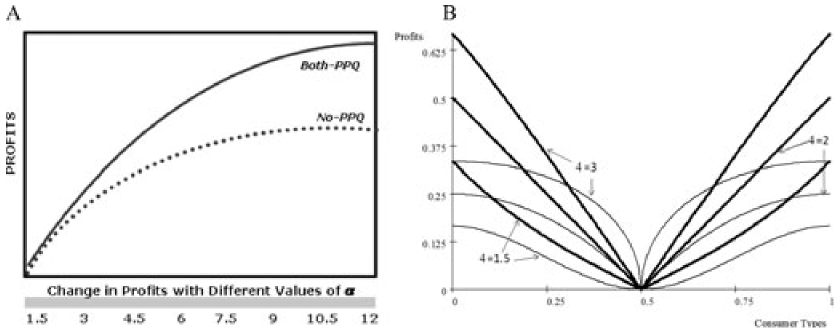

FIGURE 5. (A) PROFITS WITH OR WITHOUT PPQ FOR DIFFERENT α. (B) FIRM PROFITS FOR DIFFERENT α. (THICK LINES: BOTH-PPQ CASES; THIN LINES: NO-PPQ CASES.)

it eliminates intrafirm cannibalization and enables firms to save the information rents typically left for customers. The first two effects have been discussed in the literature but third effect in which moving to PPQ eliminates the own product (intrafirm) competition and replaces it with interfirm competition on quality and price, is unique to our paper. Note that the rent-extraction in the present study is higher because firms offer better products in the Both-PPQ case. Furthermore, we can show that because of the third effect, the simultaneous adoption of PPQ may actually enhance firm profits in equilibrium. In the extreme case, under this parameter setting a Prisoner's dilemma never occurs. This conclusion can be inferred from Figure 5B as well. In that figure, we can find that with PPQ, firms can extract larger gains from their loyal customers whereas the gains (when $\alpha = 3$) from the nonloyal customers ($\theta \to 0.5$) is minimal. The reason is that as $\theta \to 0$ or 1, those customers are located further away from the other firm and hence, are essentially captive to the firm located closer to them. Hence, the rent-extraction effect is higher, price competition effect is lower, and the forfeited information rent is also higher.[10]

To sum up, together with the results in the previous two sections, we can conclude that the simultaneous adoption of PPQ generally improves total social welfare and total firm profits. However, it decreases total consumer surplus because of the (quality-enhanced) rent-extraction effect and because the gains accruing to firms from the elimination of self-selection dominate the aggravated price competition

10. Recall that in the second-degree price discrimination model, the consumers with higher marginal valuation enjoy higher information rents. The information rents are transferred to sellers after the sellers adopt PPQ because PPQ takes away buyers' self-selection options.

effect. However, from Figure 5B, we can observe that the profit gain brought forth by PPQ accrues mainly from the loyal customers located close to 0 or 1. Hence, intuitively, when there are fewer loyal customers, the benefit from adopting PPQ will be lower. In the next section, when we discuss the first-stage PPQ adoption game, we will generalize our distributional assumption of θ from [0, 1] to $[r, 1-r]$ to further examine the benefit to firms from PPQ adoption.

5. First-Stage PPQ Adoption Game

Next, we investigate when and which firm will adopt PPQ in the equilibrium, when adopting PPQ entails a cost. Suppose in the very first stage, each firm decides whether or not to adopt the PPQ technology at a fixed cost of F. In the second stage, firms play a simultaneous pricing game when both firms have PPQ or when both firms do not have PPQ. They play a sequential pricing game when only one firm has PPQ. The payoffs of the subgames have been reported in Section 3. We are interested in determining the range of fixed costs over which the adoption of PPQ leads to a positive outcome for both firms or a negative outcome such as a Prisoner's Dilemma where both firms are worse-off in comparison to the scenario when neither of them have PPQ. In order to determine the impact of the market size of loyal customers on each firm's optimal strategies, we generalize the range over which customers are uniformly distributed. In particular, we consider a stylized symmetric example in which customer type θ is distributed between $[1-r, r]$. From this setup, we are able to analyze the situation when each firm's loyal segment changes equally.

The payoff matrix of the first-stage game is reported in Table I. The subscripts refer to the four possible cases: neither firm has PPQ, only firm L has PPQ, only firm R has PPQ, and both firms have PPQ. The superscripts, L and R, refer to firms L and R, respectively. All the revenue figures are straightforward generalizations of the baseline analytical solutions presented in the previous sections. The numerical

Table I.
General Payoff Matrix of the First-Stage PPQ Adoption Game

Payoffs	R, No-PPQ	R, PPQ
L, No-PPQ	(π_N^L, π_N^R)	$(\pi_R^L, \pi_R^R - F)$
L, PPQ	$(\pi_L^L - F, \pi_L^R)$	$(\pi_{Both}^L - F, \pi_{Both}^R - F)$

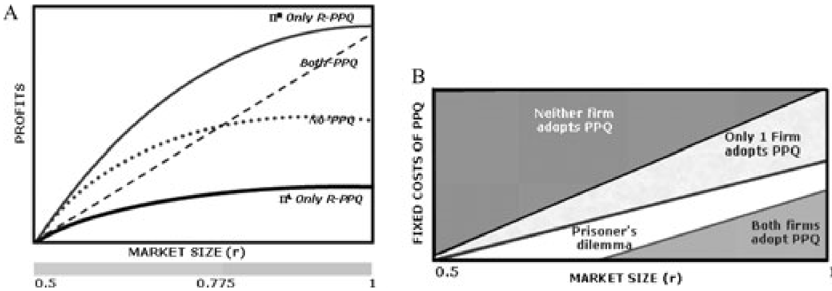

FIGURE 6. (A) FIRM PAYOFFS WITH PPQ (B) PPQ ADOPTION

solutions of the firms' revenues (payoffs exclude *F*) with a change in the value of *r* are shown in Figure 6A and Figure 6B illustrates the cases of Nash Equilibria.

Given these figures, we have the following result.

Observation 1: *Suppose* $\alpha = 2$. *(i) When the customer types are uniformly distributed in* $[1 - r, r]$ *and* $r > 0.775$, *the profit of each firm is higher after both firms adopt PPQ. When* $r \leq 0.775$, *the profit of each firm is smaller after both firms adopt PPQ. (ii) Moreover, it is not always profitable for a firm to adopt PPQ when its competitors were to have PPQ. That is, an asymmetric equilibrium is possible.*

First, we can observe that when *r* is larger than 0.775, it is possible to have situations in which both firms are better-off after the adoption of PPQ. From this, we conclude that when both firms have a larger loyal segment, it is less likely that the adoption of PPQ will lead to a Prisoner's Dilemma. On the other hand, if both firms have fewer loyal customers, the adoption of PPQ will lead to a Prisoner's Dilemma when *F* is small enough (Figure 6B). This finding is consistent with the findings in Section 4.4: after the adoption of PPQ, loyal customers suffer from stronger rent-extraction and lost information rents from self-selection, but do not enjoy the benefit of intensified competition because they are located far from the other firm. As a result, lower the proportion of loyal customers, the lower the profits of the firm from PPQ adoption.

The second observation is that when *F* is higher than the difference in the profits of firm L between the Both-PPQ and Only-R PPQ cases, if R adopts PPQ, L will not adopt it to facilitate a level-playing field. That can be observed also from Figure 6B. When we fixed the value of *r*, there exists a range of *F* that will lead to asymmetric equilibrium in which only one firm adopts PPQ. In other words, in that region even with symmetric firms we have two asymmetric Nash equilibria in which only one firm adopts PPQ. Thus, we find that depending on the costs of adopting PPQ, it is not a dominant strategy for a firm to adopt PPQ even if its competitor adopts PPQ.

6. Discussion and Conclusion

Firms' are increasingly realizing that the ability to establish attractive value propositions and turn them into personalized and compelling offers across the right channel for the right customer at the most opportune moment—drives customer relationships, and profits. This has led to a widespread adoption of CRM and personalization technologies by firms in different industries such as long distance telecommunications, industrial products, mobile telephone service, hotels, IT hardware, financial services, online retailing, credit cards, etc., in order to influence their customer acquisition and retention strategies. Moreover, increasing availability of flexible manufacturing technologies is facilitating quality enhancement through customization.

Our novelty consists in combining both personalized pricing and targeted quality allocation in the same theoretical framework. Our model highlights how firms should allocate product or service qualities, and prices, and how in turn, such targeting decisions impact the surplus of consumers, and overall social welfare. In contrast to prior work, we show that quality enhancement through targeted quality allocation leads to less aggravated price competition by strengthening the opportunities for rent extraction for firms, when firms are able to personalize prices as well. Thus, the adoption of PPQ technologies such as customer relationship management systems (CRM) and flexible manufacturing systems (FMS) by competing firms can make even *symmetric* firms better-off. That is, when firms can better target the allocation of qualities and prices, and offer a broader product line, competition becomes less intense because a greater proportion of the potential consumers now has a higher willingness to pay for the firms' products. Prior work (Shaffer and Zhang, 2002) has identified situations where asymmetric firms can avoid the Prisoner's Dilemma through the market share effect. Our goal is to identify circumstances when even *symmetric* firms can avoid a Prisoner's Dilemma because of the quality enhancement effect. We account for the cost of PPQ technologies that can include, for instance, the cost of FMS in the case that the product quality is enhanced, or the cost incurred in providing personalized services for each consumer. Even after explicitly accounting for such costs, we find regions where symmetric firms are better-off after engaging in PPQ.

Another implication of our analysis is that the adoption of CRM technologies leads to an increase in the quality level of the entire product line of a firm. This is relevant for a firm's pricing and product line decision because the adoption of PPQ negates the threat from intra-firm competition that was prevalent in the absence of PPQ. Basically, firms that adopt PPQ only need to consider interfirm competition, and hence it is optimal for them to offer a significant quality/service improvement.

We show that the adoption of PPQ by both firms has a differential impact on consumer surplus. Consumers located closer to the middle of the market, who are the least loyal to either firm or have the lowest willingness to pay for either firm's products, are those who are the most better-off when both firms adopt PPQ technologies. This is in contrast to a scenario when neither firm has PPQ, when the very same consumers who are least likely to buy either firms' products, are the most worse-off. From a public policy perspective, our analysis of social welfare highlights that social welfare is highest when both firms adopt PPQ. Indeed even if one firm adopts PPQ, social welfare is higher than the situation where neither firm has PPQ. However, in such a case, the total welfare for some consumers can be lower because of the misallocation of products. In particular, because some customers of the firm without PPQ end up buying from the firm with PPQ at lower prices and lower qualities, we see a decrease in social welfare for those regions.

Our paper has several limitations, some of which can be fruitful areas of research. For example, we have only considered symmetric cost functions for both firms. Some firms may have operational efficiencies that can give rise to less convex production costs when customizing quality. It would be interesting to see how firms' strategies change under such scenarios. Another interesting extension would be to study competition in markets with discrete segments such as loyals and switchers, when firms adopt nonlinear pricing schedules. A third area of related research would be to allow competing firms to invest in loyalty building measures, such as switching costs, before they invest in PPQ. Finally, we do not consider consumers making strategic choices in revealing information about their preferences. One could consider a scenario where higher valuation consumers might want to mimic lower types and vice versa, in anticipation that some consumers are left with positive surplus while others are not when firms engage in PPQ. Incorporating such a situation is beyond the scope of this paper but it might be an interesting extension to pursue in a related framework. We hope our research paves the way for more future work in this domain.

Appendix

A.1 Neither Firm Has PPQ

Recall that each firm offers a continuous menu of prices and qualities. Because consumers choose any contract $(p(\theta), q(\theta))$ from the menu, the incentive compatibility (IC) condition for consumers is given by

$$s^R(\theta) = \max_t \ \theta \times q^R(t) - p^R(t). \tag{A1}$$

From the first-order condition of (A1) and using the envelope theorem, we have the following lemma.

Lemma A1: $\frac{ds^R(\theta)}{d\theta} = q^R(\theta)$ *and* $\frac{ds^L(\theta)}{d\theta} = -q^L(\theta)$.

Proof of Lemma A1. First, recall that each firm maintains a menu of prices and qualities. Because consumers choose any contract from the menu, the incentive compatibility condition for consumers is given by

$$s^R(\theta) = \max_t \theta \times q^R(t) - p^R(t). \tag{A2}$$

The first-order condition is

$$\theta \times \frac{\partial q^R(t)}{\partial t} - \frac{\partial p^R(t)}{\partial t} = 0. \tag{A3}$$

This equation holds at $t = \theta$ because consumers self-select the price and quality pair designed for them. By differentiating equation (A2), we have

$$\begin{aligned} \frac{ds^R(\theta)}{d\theta} &= q^R(\theta) + \theta \times \frac{\partial q^R(\theta)}{\partial \theta} - \frac{\partial p^R(\theta)}{\partial \theta}, \\ &\Rightarrow \frac{ds^R(\theta)}{d\theta} = q^R(\theta). \end{aligned} \tag{A4}$$

In the second equation, the last two terms are zero because of the first-order condition as shown above in equation (A3). Using the same procedure, it can be shown that

$$\frac{ds^L(\theta)}{d\theta} = -q^L(\theta). \qquad \square$$

Proof of Proposition 1. Lemma A1 implies that

$$s^R(\theta) = s^R(B) + \int_B^\theta q^R(t)\,dt;$$

$$s^L(\theta) = s^L(B) + \int_\theta^B q^L(t)\,dt.$$

Note that the IC constraint ensures that a consumer prefers the contract that was designed for him, and the IR constraint guarantees that each consumer type accepts his designated contract. Hence, in this case (IC) implies that the slope of $s^R(\theta)$ is equal to $q^R(\theta)$ as shown in Lemma A1. In this model, competition between two firms affects only the surplus to the consumer at the boundary (which, for example, is equal to $s^R(B)$ for firm R), which is a constant. This implies that two firms compete by lowering the pricing schedule by a constant, $s^R(B)$. Higher consumer types will receive higher surplus; this is termed as information rent in the nonlinear pricing literature. This implies that whenever the firm increases the quality offered to any consumer, it has to leave higher information rents to higher consumer types in order to avoid cannibalization during self-selection.

As a result, our decision variables can be further simplified as $q(\theta)$ and $s(B)$. Substituting for $s^R(\theta)$, the simplified objective function for firm R can be rewritten as

$$\max_{q^R(\theta),s^R(\theta)} \pi_N^R, \text{ where } \pi_N^R = \int_B^1 \left[\theta q^R(\theta) - s^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}\right] d\theta,$$
$$\text{s.t. } s^R(\theta) \geq 0, s^L(B) = s^R(B).$$

After substituting for the value of $s^R(\theta)$, the optimization problem becomes equal to

$$\max_{q^R(\theta),s^R(B)} \pi_N^R, \text{ where } \pi_N^R$$
$$= \int_B^1 \left[\theta q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha} - s^R(B) - \int_B^\theta q^R(t)\,dt\right] d\theta.$$

Changing the order of integration of the last term in the bracket,[11] we can simplify the objective function as

$$\pi_N^R = \int_B^1 \left[\theta q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha} - s^R(B) - q^R(\theta)(1-\theta)\right] d\theta, \tag{A5}$$

$$= \int_B^1 \left[(2\theta - 1) q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha} - s^R(B)\right] d\theta. \tag{A6}$$

11. $\int_B^1 [\int_B^\theta q^R(t)\,dt] d\theta = \int_B^1 [\int_t^1 q^R(t) d\theta]\,dt = \int_B^1 q^R(t)(1-t)\,dt = \int_B^1 q^R(\theta)(1-\theta) d\theta$

Similarly, the optimization problem for firm L is given as follows:

$$\max_{q^L(\theta),s^L(\theta)} \pi_N^L, \text{ where } \pi_N^L = \int_0^B \left[(1-\theta)q^L(\theta) - s^L(\theta) - \frac{(q^L)^\alpha(\theta)}{\alpha}\right] d\theta,$$
$$\text{s.t. } s^L(\theta) \geq 0, s^L(B) = s^R(B). \tag{A7}$$

After substituting for the value of $s^L(\theta)$, the optimization problem becomes equal to

$$\max_{q^L(\theta),s^L(B)} \pi_N^L, \text{ where } \pi_N^L$$
$$= \int_0^B \left[(1-2\theta)q^L(\theta) - \frac{(q^L)^\alpha(\theta)}{\alpha} - s^L(B)\right] d\theta. \tag{A8}$$

The optimal quality schedule can be determined by maximizing the integrand point wise (the terms in the bracket). This leads to the following lemma.

Lemma A2: *The equilibrium quality schedules are $q^R(\theta) = (2\theta - 1)^{1/(\alpha-1)}$ and $q^L(\theta) = (1-2\theta)^{1/(\alpha-1)}$.*

Proof. Differentiating terms in the bracket of (A6) with respect to $q^R(\theta)$, we have

$$\theta - (q^R)^{\alpha-1}(\theta) - (1-\theta) = 0.$$
$$\Longrightarrow q^R(\theta) = (2\theta-1)^{1/(\alpha-1)}$$

The solution of firm L can be derived in a similar manner. □

To find the solution of $s^R(B)$, we differentiate the objective functions with respect to $s^R(B)$ and derive the following lemma by Leibniz theorem.

Lemma A3: *The equilibrium consumer surplus at the boundary is given by $s^R(B) = s^L(B) = 0$.*

Proof. Define the terms in the bracket of (A6) as X. Using Leibniz theorem, we have

$$\frac{d\pi_N^R}{ds^R(B)} = \int_B^1 \frac{\partial X}{\partial s^R(B)} d\theta - X|_{\theta=B} \times \frac{dB}{ds^R(B)}.$$

As a result, by differentiating (A6) with respect to $s^R(B)$, we have

$$\underbrace{\int_B^1 -1 \cdot d\theta}_{1^{\text{st}} \text{ term}} - \underbrace{\left[(2B-1)q^R(B) - \frac{(q^R)^\alpha(B)}{\alpha} - s^R(B)\right]}_{2^{\text{nd}} \text{ term}} \underbrace{\frac{dB}{ds^R(B)}}_{3^{\text{rd}} \text{ term}} = 0. \tag{A9}$$

These terms represent the costs and benefits that accrue to firm R if it changes its price offered to consumer at B by one unit. Intuitively, when that price is lowered by 1 unit, all products' prices will be lowered by 1 to sustain the IC condition. The first term represents the aggregate loss in revenue from all existing consumers of firm R. The second and third terms together represent the gain in revenue from attracting some potential consumers in firm L's territory. Specifically, the second term represents the profit from the marginal consumer and the third term represents the gain in market share from infra-marginal consumers that occurs by lowering price by one unit. From Lemma A1, we know that

$$\begin{aligned}\left.\frac{d\theta}{ds^R(\theta)}\right|_{\theta=B} &= \left.\frac{d\theta}{ds^L(\theta)}\right|_{\theta=B}, \text{ (given that} s^R(B) = s^L(B))\\ &= \frac{1}{-q^L(B)}.\end{aligned}$$

Substituting this back to (A9), we have

$$(B-1) - \left[(2B-1)q^R(B) - \frac{(q^R)^\alpha(B)}{\alpha} - s^R(B)\right]\frac{1}{-q^L(B)} = 0.$$

After rearranging terms the above equation can be written as

$$s^R(B) = q^L(B)(B-1) + (2B-1)q^R(B) - \frac{(q^R)^\alpha(B)}{\alpha}. \tag{A10}$$

In the symmetric equilibrium, $B = 1/2$. Moreover, from Lemma A2 we know that $q^L(B) = q^R(B) = 0$. Substituting these in equation (A10) we have $s^R(B) = 0$.

Given the quality schedules derived in Lemma A2. Next, we can derive the consumer surplus functions by definitions, $s^L(\theta) = 0 + \int_\theta^{1/2} q^L(t)\,dt$ and $s^R(\theta) = 0 + \int_{1/2}^\theta q^R(t)\,dt$. The optimal price schedules are derived by substituting $p^L(\theta) = (1-\theta)q^L(\theta) - s^L(\theta)$ and $p^R(\theta) = \theta\, q^R(\theta) - s^R(\theta)$. Hence, the optimal prices are given by

$$\begin{aligned}p^L(\theta) &= (1-\theta)(1-2\theta)^{1/(\alpha-1)} - \frac{\alpha-1}{2\alpha}(1-2\theta)^{\alpha/(\alpha-1)},\\ &= (1-2\theta)^{1/(\alpha-1)}\left(\frac{-2\theta+\alpha+1}{2\alpha}\right).\\ p^R(\theta) &= \theta(2\theta-1)^{1/(\alpha-1)} - \frac{\alpha-1}{2\alpha}(2\theta-1)^{\alpha/(\alpha-1)},\\ &= (2\theta-1)^{1/(\alpha-1)}\left(\frac{2\theta+\alpha-1}{2\alpha}\right).\end{aligned} \tag{A11}$$

□

Proof of Total Welfare, Surplus, and Profits. Because firms are symmetric, it is sufficient to derive the results for any one firm. Without loss of generality, consider firm L. Then the total surplus is given by

$$s_N^L = \int_0^{1/2} \frac{\alpha - 1}{2\alpha}(1 - 2\theta)^{\alpha/(\alpha-1)} d\theta = \frac{(\alpha - 1)^2}{4\alpha(2\alpha - 1)} = s_N^R.$$

The total welfare function is given by

$$\begin{aligned} w_N^L &= \int_0^{1/2} [u^L(q(\theta), \theta) - c(q(\theta))]\, d\theta, \\ &= \int_0^{1/2} \left[(1 - \theta)(1 - 2\theta)^{1/(\alpha-1)} - \frac{1}{\alpha}(1 - 2\theta)^{\alpha/(\alpha-1)}\right] d\theta, \\ &= \frac{3\,(\alpha - 1)^2}{4\alpha(2\alpha - 1)}. \end{aligned}$$

Finally, profits are given by

$$\pi_N^L = w_N^L - s_N^L = \frac{(\alpha - 1)^2}{2\alpha(2\alpha - 1)}. \tag{A12}$$

□

A.2 Only Firm R Has PPQ

Proof of Proposition 2. In this case, recall that we solve a sequential pricing game because the simultaneous pricing game does not have a pure strategy Nash Equilibrium. Without loss of generality, let R be the firm with PPQ. In stage 2, given firm L's quality and pricing schedules, firm R will set the price so that the consumers feel indifferent between buying from L or R. Because firm R can appropriate the additional rents from the transaction with each consumer type θ, firm R will set its quality schedule equal to the socially optimal quality schedule. Formally, firm R chooses $q^R(\theta), s^R(\theta)$ to maximize

$$\pi_R^R(\theta) = \theta q^R(\theta) - s^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}, \forall\theta \in [0, 1].$$

Note that it is a simple polynomial function of $q^R(\theta)$, the optimal solution is given by

$$\begin{aligned} \frac{\partial \pi_R^R(\theta)}{\partial q^R(\theta)} &= \theta - (q^R)^{\alpha-1}(\theta) = 0. \\ &\implies q^R(\theta) = \theta^{1/(\alpha-1)} \end{aligned}$$

Back to the first stage, L's optimization problem is the same as that in the No-PPQ case except that the individual rationality constraint (IR2) is now different (please see below). Because this does not affect the optimal quality schedule for firm L, it is the same as that in the No-PPQ case and is equal to the following:

$$q^L(\theta) = (1 - 2\theta)^{1/(\alpha-1)}.$$

Next we determine the surplus function of firm L. Note that the surplus offered by firm L will depend on firm R's socially optimal surplus curve. If firm L were to offer less than the socially optimal surplus of firm R, then firm R could potentially poach L's consumers by offering lower prices. The potential for poaching exists because R can perfectly identify each consumer. L's optimization problem is the same as that in the No-PPQ case except that the individual rationality constraint (IR2), instead of being given by $s^L(B) = s^R(B)$, is replaced by the socially optimal surplus curve of firm R. Specifically, it is given by

$$s^L(B) = \max_{q^R(\theta)} \left[\theta q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}\right]\Bigg|_{\theta=B}.$$

Similar to the Proof in Proposition 1, differentiating (A8) with respect to $s^L(B)$, we have

$$\frac{\partial \pi_R^L(\theta)}{\partial s^L(B)} = -B + \left[(1-2B)q^L(B) - s^L(B) - \frac{(q^L)^\alpha(B)}{\alpha}\right]\frac{dB}{ds^L(B)} = 0. \tag{A13}$$

The difference is that $s^L(B)$ is given by

$$s^L(B) = \max_{q^R(B)} Bq^R(B) - \frac{(q^R)^\alpha(B)}{\alpha} = \left(1-\frac{1}{\alpha}\right)B^{\alpha/(\alpha-1)}.$$

It follows that

$$\frac{dB}{ds^L(B)} = B^{-1/(\alpha-1)}.$$

Substituting this back in (A13), it follows that

$$\begin{aligned} s^L(B) &= -B \cdot B^{1/(\alpha-1)} + \left[(1-2B)q^L(B) - \frac{(q^L)^\alpha(B)}{\alpha}\right], \\ &= \left(1-\frac{1}{\alpha}\right)(1-2B)^{\alpha/(\alpha-1)} - B^{\alpha/(\alpha-1)}. \end{aligned}$$

Given that the marginal consumer feels indifferent between buying from firm L and firm R, we have

$$
\begin{aligned}
s^L(B) &= w^R(B) \\
&\Leftrightarrow \left(1-\frac{1}{\alpha}\right)(1-2B)^{\alpha/(\alpha-1)} - B^{\alpha/(\alpha-1)} = \left(1-\frac{1}{\alpha}\right)B^{\alpha/(\alpha-1)}, \\
&\Leftrightarrow B = \left[\left(\frac{2\alpha-1}{\alpha-1}\right)^{(\alpha-1)/\alpha} + 2\right]^{-1}.
\end{aligned}
$$

As a consequence, the consumer surplus function of firm L is given by

$$
\begin{aligned}
s^L(\theta) &= s^L(B) + \int_\theta^B (1-2t)^{1/(\alpha-1)}\,dt, \\
&= s^L(B) - \left(\frac{\alpha-1}{2\alpha}\right)(1-2B)^{\alpha/(\alpha-1)} + \left(\frac{\alpha-1}{2\alpha}\right)(1-2\theta)^{\alpha/(\alpha-1)}.
\end{aligned}
$$

Next we derive the consumer surplus function for firm R. Firm R sets the price, equivalently $s^R(\theta)$, such that each consumer's surplus exactly matches his/her surplus from the outside opportunity. Recall that the outside opportunity of R's consumers is either 0 or equal to the surplus offered by firm L which is determined by the contract offered to the marginal consumer $(q^L(B), s^L(B))$. As a result, the consumer surplus function of firm R is given by

$$
s^R(\theta) = \max(0, (1-\theta)q^L(B) - p^L(B)).
$$

Note that we already have derived the expressions for $(q^L(\theta), s^L(\theta))$, and $(q^R(\theta), s^R(\theta))$. Hence, by substituting the relevant expressions in $p(\theta) = u(q(\theta), \theta) - s(\theta)$,we have

$$
\begin{aligned}
p^L(\theta) &= (1-\theta)(1-2\theta)^{1/(\alpha-1)} - s^L(\theta), \quad \theta \in [0, B], \\
p^R(\theta) &= \theta^{\alpha/(\alpha-1)} - s^R(\theta), \quad \theta \in [B, 1]. \qquad \square
\end{aligned}
$$

Proof of Total Welfare, Surplus, and Profits. Due to the fact that B, $s^L(B)$, and $s^L(\theta)$ do not have simple closed-form solutions, we cannot present the prices and profits in closed-form solutions. However, we can derive the relevant expressions for a given value of α. For example, when $\alpha = 2$, we find that

$$
B = 2 - \sqrt{3} = 0.27
$$

Because firm L moves first, we derive the relevant expressions for surplus, price, and welfare functions, respectively, as follows:

$$s^L(B) = \frac{7}{2} - 2\sqrt{3}.$$

$$s^L(\theta) = \frac{1}{2}(-2\theta + 2\theta^2 + 2\sqrt{3} - 3).$$

$$p^L(\theta) = -2\theta + \theta^2 - \sqrt{3} + \frac{5}{2}.$$

The total surplus, welfare, and profit functions for firm L are given as follows:

$$s_R^L = \int_0^B s^L(\theta)d\theta = \frac{1}{2}\sqrt{3} - \frac{5}{6}.$$

$$w_R^L = \int_0^B w^L(\theta)d\theta = \frac{3}{2}\sqrt{3} - \frac{5}{2}.$$

$$\pi_R^L = \int_0^B \pi^L(\theta)d\theta = 5 - (3)^{\frac{3}{2}}.$$

Given all these solutions in the first stage, we can derive the optimal consumer surplus schedule of firm R. Firm R offers zero surplus to some of its consumers and then offers positive surplus to those consumers who are located closer to firm L. Hence, we need to derive the location of the marginal consumer of firm R that obtains a positive surplus. This is given by the equating the surplus from outside opportunity (in this case the surplus offered by firm L) to zero.

$$\begin{aligned} 0 &= (1-\theta^M)q^L(B) - p^L(B) \\ &= (1-\theta^M)(1-2B) - p^L(B) \\ &= (1-\theta^M)[1-2(2-\sqrt{3})] - \left[-2(2-\sqrt{3}) + (2-\sqrt{3})^2 - \sqrt{3} + \frac{5}{2}\right] \\ &\Rightarrow \theta^M = \left\{\frac{1}{2\sqrt{3}-3}\left[\sqrt{3} - (2-\sqrt{3})^2 - \frac{3}{2}\right]\right\} = 0.345. \end{aligned}$$

Consequently, the total consumer surplus, welfare, and profit of firm R are

$$s_R^R = \int_{(2-\sqrt{3})}^{0.34530} \left[(1-\theta)\cdot(1-2B) - \left(-2B + B^2 - \sqrt{3} + \frac{5}{2}\right)\right]d\theta = 0.0138.$$

$$w_R^R = \int_B^1 w^R(\theta)d\theta = \frac{5}{6}(\sqrt{3}-1)(2-\sqrt{3}).$$

$$\pi_R^R = \frac{1}{72}(738\sqrt{3} - 1263). \qquad \square$$

A.3 Both Firms Have PPQ

Proof of Proposition 3. In this case, both firms know exactly each consumer's type. These two firms engage in a competition similar to Bertrand competition. In equilibrium, both firms offer a socially optimal level of quality.

The firm located closer to a consumer will set the price such that the consumer surplus exactly matches the highest possible consumer surplus offered by the other firm. The rival firm sets price at marginal cost. Neither firm will deviate by offering a lower price to its rivals' customers because no such action can bring in additional profit. Hence, the profit functions of the firms are given as follows:

$$\pi_{Both}^L(\theta) = (1-\theta)q^L(\theta) - s^L(\theta) - \frac{(q^L)^\alpha(\theta)}{\alpha};$$

$$\pi_{Both}^R(\theta) = \theta q^R(\theta) - s^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}.$$

Note that as before, firms still optimize with respect to both quality and surplus. Moreover, due to the perfect targeting of consumers there are no self-selection problems, and thus there is no potential for cannibalization. Hence, firms do not have to consider any IC constraints from the consumers' point of view. Therefore, the optimal quality schedules are determined by

$$\frac{\partial \pi_{Both}^L(\theta)}{\partial q^L(\theta)} = (1-\theta) - (q^L)^{\alpha-1}(\theta) = 0,$$

$$\Leftrightarrow q^L(\theta) = (1-\theta)^{1/(\alpha-1)}.$$

$$\frac{\partial \pi_{Both}^R(\theta)}{\partial q^R(\theta)} = \theta - (q^R)^{\alpha-1}(\theta) = 0,$$

$$\Leftrightarrow q^R(\theta) = \theta^{1/(\alpha-1)}.$$

Both of these are the socially optimal quality schedules (first-best solutions).

Given the nature of the price competition between the two firms, we can determine $s^L(\theta)$, $s^R(\theta)$, and hence demonstrate the optimal price schedules. When $\theta \in [1/2, 1]$, consumers buy from firm R in equilibrium. At the same time, the equilibrium price from firm L is equal to its marginal cost, $\frac{(q^L)^\alpha(\theta)}{\alpha}$, because of Bertrand price competition. The equilibrium price from firm R is set at a level so that consumers feel indifferent between buying from firm R and firm L.

$$\begin{aligned} s^R(\theta) &= (1-\theta)q^L(\theta) - p^L(\theta) \\ &= \left(1-\frac{1}{\alpha}\right)(1-\theta)^{\alpha/(\alpha-1)}. \end{aligned}$$

Similarly, we can derive the consumer surplus function of firm L. This is given by

$$\begin{aligned} s^L(\theta) &= \theta q^R(\theta) - p^R(\theta) \\ &= \left(1-\frac{1}{\alpha}\right)\theta^{\alpha/(\alpha-1)}. \end{aligned}$$

The social welfare functions are given by

$$\begin{aligned} w^L(\theta) &= u^L(q(\theta),\theta) - c(q(\theta)) \\ &= \left(1-\frac{1}{\alpha}\right)(1-\theta)^{\alpha/(\alpha-1)}, \theta \in [0,1/2]. \\ w^R(\theta) &= u^R(q(\theta),\theta) - c(q(\theta)) \\ &= \left(1-\frac{1}{\alpha}\right)\theta^{\alpha/(\alpha-1)}, \theta \in [1/2,1]. \end{aligned}$$

Because $p(\theta) = u(q(\theta),\theta) - s(\theta)$, the price charged by each firm is given by

$$\begin{aligned} p^L(\theta) &= (1-\theta)^{\alpha/(\alpha-1)} - \left(1-\frac{1}{\alpha}\right)\theta^{\alpha/(\alpha-1)}, \theta \in [0,1/2], \\ p^R(\theta) &= \theta^{\alpha/(\alpha-1)} - \left(1-\frac{1}{\alpha}\right)(1-\theta)^{\alpha/(\alpha-1)}, \theta \in [1/2,1]. \end{aligned} \tag{A14}$$

□

Proof of Total Welfare, Surplus, and Profits. Next, we present the closed-form solutions for the total welfare, surplus, and profits. Because firms are symmetric, it is sufficient to present the results from firm L. The total welfare in this case is given by

$$\begin{aligned} w^L_{Both} &= \int_0^{1/2} \left[\left(1-\frac{1}{\alpha}\right)(1-\theta)^{\alpha/(\alpha-1)}\right] d\theta \\ &= \frac{(\alpha-1)^2}{\alpha(2\alpha-1)}(1-2^{-(2\alpha-1)/(\alpha-1)}). \end{aligned}$$

The total consumer surplus is given by

$$
\begin{aligned}
s^L_{Both} &= \int_0^{1/2} \left[\left(1 - \frac{1}{\alpha}\right)\theta^{\alpha/(\alpha-1)}\right] d\theta \\
&= \frac{(\alpha-1)^2}{\alpha(2\alpha-1)} \cdot 2^{-(2\alpha-1)/(\alpha-1)}.
\end{aligned}
$$

The total profit is given by

$$
\begin{aligned}
\pi^L_{Both} &= w^L_{Both} - s^L_{Both} \\
&= \frac{(\alpha-1)^2}{2\alpha(2\alpha-1)}(2 - 2^{1/(1-\alpha)}). \qquad \text{(A15)}
\end{aligned}
$$

□

A.4 Comparisons

Proof of Corollary 1. First, by comparing the prices of firm L in the No-PPQ and Both-PPQ cases from equations (A11) and (A14), we can show the difference when $\alpha = 2$ is given by the following equation:

$$
(1-2\theta)\left(\frac{-2\theta+3}{4}\right) - \left[(1-\theta)^2 - \frac{1}{2}\theta^2\right] = \frac{1}{4}\left(2\theta^2 - 1\right) < 0, \ \forall\theta \in \left[0, \frac{1}{2}\right].
$$

Similarly, we can show that the price of firm R in the Both-PPQ case is higher than that in the No-PPQ case.

For the case when $\alpha = 2$, and firm R has PPQ, the price function of firm L is given by

$$
p^L_R(\theta) = \theta^2 - 2\theta - \sqrt{3} + \frac{5}{2}, \quad \theta \in [0, 2-\sqrt{3}].
$$

The price of firm L in the No-PPQ case is given by

$$
(1-2\theta)\left(\frac{-2\theta+3}{4}\right).
$$

Comparing these two expressions, we have

$$
\left(\theta^2 - 2\theta - \sqrt{3} + \frac{5}{2}\right) - (1-2\theta)\left(\frac{-2\theta+3}{4}\right) = \frac{7}{4} - \sqrt{3} = 0.0179.
$$

The last part of this corollary states that in the case when only firm R has PPQ, some consumers in L's market segment may receive lower prices from R. We can verify this by looking at the price of the marginal consumer located very close to $\theta = 2 - \sqrt{3}$. This is given by

$$\begin{aligned} p_R^R(\theta) &= \theta^2 - \max\left(0, 3\theta + 5\sqrt{3} - 2\theta\sqrt{3} - \frac{17}{2}\right), \\ &\implies p_R^R(2-\sqrt{3}) = 0.0359 < p_N^L(2-\sqrt{3}) = 0.286. \end{aligned} \qquad \square$$

Proof of Proposition 4. We first show that the surplus is lowest at $\theta = 1/2$ in the No-PPQ case.

$$\frac{ds^L(\theta)}{d\theta} = \frac{d}{d\theta}\left[\frac{\alpha-1}{2\alpha}(1-2\theta)^{\alpha/(\alpha-1)}\right] = -(1-2\theta)^{1/(\alpha-1)} < 0, \forall\theta \in \left[0, \frac{1}{2}\right).$$

$$\frac{ds^R(\theta)}{d\theta} = \frac{d}{d\theta}\left[\frac{\alpha-1}{2\alpha}(2\theta-1)^{\alpha/(\alpha-1)}\right] = (2\theta-1)^{1/(\alpha-1)} > 0, \forall\theta \in \left(\frac{1}{2}, 1\right].$$

Next, we show that the surplus is highest at $\theta = 1/2$ in the Both-PPQ case.

$$\frac{ds^L(\theta)}{d\theta} = \frac{d}{d\theta}\left[\left(1-\frac{1}{\alpha}\right)\theta^{\alpha/(\alpha-1)}\right] = \theta^{1/(\alpha-1)} > 0, \forall\theta \in \left[0, \frac{1}{2}\right).$$

$$\frac{ds^R(\theta)}{d\theta} = \frac{d}{d\theta}\left[\left(1-\frac{1}{\alpha}\right)(1-\theta)^{\alpha/(\alpha-1)}\right] = -(1-\theta)^{1/(\alpha-1)} < 0. \qquad \square$$

Proof of Corollary 2. The proof of (i) is immediate because the quality is the first-best solution and each consumer buys from the firm situated closer to him.

As to the proof of (ii), first note that when only one firm has PPQ, there are three regions in the market which we need to consider in order to derive the stated result. In the first region where $\theta \in [0, B]$, the welfare generated from firm L is the same as that in the No-PPQ case. This is because the quality schedule of the firm L (the No-PPQ firm) remains the same in each case. The second region under consideration extends from $\theta \in [B, \frac{1}{2}]$. We analyze the welfare in this region at the end. In the third region where $\theta \in [\frac{1}{2}, 1]$, the welfare generated by firm R is higher in this case compared to the No-PPQ case. This is because these consumers are located closer to R and get the socially optimal quality from firm R. Given these results, it is sufficient for us to compare the welfare in the second region. When $\alpha = 2$, the corresponding expressions for firm L and for firm R, respectively, are given by

$$w_N^L(\theta) = (1-\theta)(1-2\theta) - \frac{1}{2}(1-2\theta)^2, \theta \in \left[B, \frac{1}{2}\right],$$

$$w_R^R(\theta) = \theta \cdot \theta - \frac{1}{2}\theta^2, \theta \in \left[B, \frac{1}{2}\right].$$

Recall that $B = 2 - \sqrt{3}$. If we compare the welfare of the marginal consumer in the case when only R has PPQ, we can find that the welfare of this consumer is lower than what (s)he gets in the No-PPQ case as given by the following equation:

$$\left[w_N^L(\theta) - w_R^R(\theta)\right]\big|_{\theta=B} = \left(-\frac{1}{2}\right)\left(2\theta + \theta^2 - 1\right)\big|_{\theta=2-\sqrt{3}} = 0.196.$$

This establishes this corollary. □

Proof of Proposition 5. It is sufficient to compare the profits in the case in which both firms adopt PPQ with that when neither firm adopts PPQ. The expressions are given in (A12) and (A15), respectively.

$$\frac{(\alpha-1)^2}{2\alpha(2\alpha-1)} \ : \ \text{No-PPQ}$$

$$(2 - 2^{\frac{1}{1-\alpha}})\frac{(\alpha-1)^2}{2\alpha(2\alpha-1)} \ : \ \text{Both-PPQ}$$

Because $2 - 2^{\frac{1}{1-\alpha}}$ is greater than 1 for all $\alpha > 1$, we find that (A15) is always greater than (A12). Thus, the profit of the Both-PPQ case is always higher. □

References

Alptekinoglu, A. and C. Corbett, 2004, "Mass Customization versus Mass Production: Variety and Price Competition," *Manufacturing and Service Operations Management*, 6(1), 98–103.

Armstrong, M., 1996, "Multiproduct Nonlinear Pricing," *Econometrica*, 64(1), 51–75.

—— and J. Vickers, 2001, "Competitive Price Discrimination," *RAND Journal of Economics*, 32(4), 579–605.

Arora, N., X. Dreze, A. Ghose, J. Hess, R. Iyengar, B. Jing, Y. Joshi, V. Kumar, N. Lurie, S. Neslin, S. Sajeesh, M. Su, N. Syam, J. Thomas, and Z.J. Zhang, 2008, "Putting One-to-One Marketing to Work: Personalization, Customization and Choice," forthcoming in *Marketing Letters*.

Bernhardt D., Q. Liu, and K. Serfes, 2007, "Product Customization," *European Economic Review*, 51(6), 1396–1422.

Berry, S., J. Levinsohn, and A. Pakes, 1995, "Automobile Prices in Market Equilibrium," *Econometrica*, 63(4), 841–890.

Bester, H. and E. Petrakis, 1996, "Coupons and Oligopolistic Price Discrimination," *International Journal of Industrial Organization*, 14(2), 227–242.

Bhaskar, V. and T. To, 2004, "Is Perfect Price Discrimination Really Efficient? An Analysis of Free Entry," *RAND Journal of Economics*, 35(4), 762–776.

Chen, Y. and G. Iyer, 2002, "Consumer Addressability and Customized Pricing," *Marketing Science*, 21(2), 197–208.

Choudhary, V., A. Ghose, T. Mukhopadhyay, and U. Rajan, 2005, "Personalized Pricing and Quality Differentiation," *Management Science*, 51(7), 1120–1130.

Corts, K., 1998, "Third-Degree Price Discrimination in Oligopoly: All Out Competition and Strategic Commitment," *RAND Journal of Economics*, 29(2), 306–323.

Dewan, R., B. Jing, and A. Seidmann, 2003, "Product Customization and Price Competition on the Internet," *Management Science*, 49(8), 1055–1070.

Feenstra, R. C. and J. Levinsohn, 1995, "Estimating Markups and Market Conduct with Multidimensional Product Attributes," *Review of Economic Studies*, 62(1), 19–52.

Ghose, A. and K. Huang, 2007, "Personalization in a Two Dimensional Model," Working Paper, New York University.

Goldberg P. K., 1995, "Product Differentiation and Oligopoly in International Markets: The Case of the U.S. Automobile Industry," *Econometrica*, 63(4), 891–951.

Liu, Q. and K. Serfes, 2004, "Quality of Information and Oligopolistic Price Discrimination," *Journal of Economics & Management Strategy*, 13(4), 671–702.

Liu, Y. and Z. J. Zhang, 2006, "The Benefits of Personalized Pricing in a Channel," *Marketing Science*, 25(1), 97.

Moorthy, S., 1988, "Product and Price Competition in a Duopoly," *Marketing Science*, 7(2), 141–168.

Mussa, M. and S. Rosen, 1978, "Monopoly and Product Quality," *Journal of Economic Theory*, 18(2), 301–317.

Rochet, J.C. and L. Stole, 2002, "Nonlinear Pricing with Random Participation," *Review of Economic Studies*, 69(1), 277–311.

Shaffer, G. and Z.J. Zhang, 1995, "Competitive Coupon Targeting," *Marketing Science*, 14(4), 395–416.

—— and ——, 2000, "Pay to Switch or Pay to Stay: Preference-Based Price Discrimination in Markets with Switching Costs," *Journal of Economics & Management Strategy*, 9(3), 397–424.

—— and ——, 2002, "Competitive One-One Promotions," *Management Science*, 48(9), 1143–1160.

Shapiro, C. and H. Varian, 1999, *Information Rules: A Strategic Guide to the Network Economy*, Cambridge, MA: Harvard Business School Press.

Spulber, D., 1989, "Product Variety and Competitive Discounts," *Journal of Economic Theory*, 48(2), 510–525.

Stole, L., 1995, "Nonlinear Pricing and Oligopoly," *Journal of Economics & Management Strategy*, 4(4), 529–562.

Syam, N. and N. Kumar, 2005, "On Customized Goods, Standard Goods, and Competition," *Marketing Science*, 25(5), 525–537.

Taylor, T., 2004, "Consumer Privacy and the Market for Customer Information," *RAND Journal of Economics*, 35(4), 631–650.

Thisse, J. and X. Vives, 1988, "On the Strategic Choice of Spatial Price Policy," *American Economic Review*, 78(1), 122–137.

Ulph, D. and N. Vulcan, 2000, "Electronic Commerce and Competitive First-Degree Price Discrimination," Working Paper, Department of Economics, University College London.

Villas-Boas, M., 1999, "Dynamic Competition with Customer Recognition," *RAND Journal of Economics*, 30(4), 604–631.