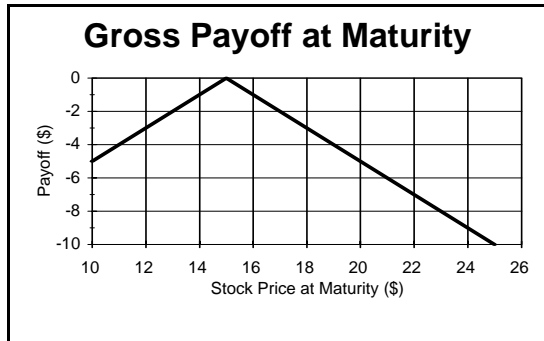

Instructions: 120 minutes. Closed book. You are permitted use of a financial calculator and two sheets (8½" x 11") of notes. Answer *all* questions. Each question carries the number of points indicated. Total points is 100. Show all work on the exam paper itself. If you do not have or can not calculate a number that you need, make an assumption and proceed with the rest of the question. *No credit will be given for illegible, unsupported or ambiguous answers.*

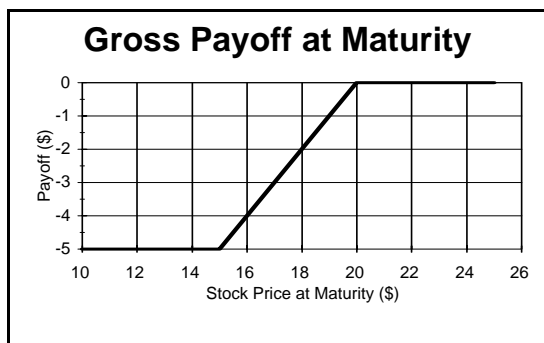
Practice Final Exam Questions

- I. [12 points] Suppose the ICAPM holds and investors care about $E[R_p(t)]$, $\sigma[R_p(t)]$ and $\sigma[R_p(t), MI(t)]$ where MI is a macroeconomic indicator and R_p is portfolio return.
- A. Explain why investors might care about $\sigma[R_p(t), MI(t)]$?
 - B. Does the market portfolio lie on the minimum variance frontier for the N risky assets in the economy? Explain why or why not?
 - C. Do all individuals hold the market portfolio in combination with the riskless asset? Explain why or why not? If not, characterize the portfolios they do hold.
 - D. Does the market portfolio lie on the minimum variance frontier for the N risky assets in the economy? Explain why or why not?
 - E. Two assets A and B have the same return covariance with the market portfolio. Will $E[R_A(t)]$ equal $E[R_B(t)]$? Explain why or why not?
- II. [20 points]
- A. If forward-spot parity holds, what is the one-year forward price of the French Franc, assuming that the current exchange rate is \$.20/Franc, the French one-year interest rate is 8% (expressed as an EAR) and the U.S. one-year rate is 6% (also expressed as an EAR)?
 - B. The one-year dividend yield on the S&P is 4%, the current one-year risk-free rate is 6% (expressed as an EAR) and the one-year forward price on the S&P index is 460. The dividends are paid at the end of 12 months.
 - 1. If forward-spot parity holds, what is the current level of the S&P index?
 - 2. If the current S&P index is 10 points higher than you calculated in the previous part, describe a risk-free arbitrage and compute the cash flows on it.
- III. [16 points] Describe a portfolio of puts, calls (with their exercise prices), bonds (with par value) and shares of stock that would have the indicated gross payoffs (i.e., terminal value, ignoring the amount paid/received to set up the portfolio). Each put and each call must have 1 share of stock as the underlying, be European and expire at the terminal date. The bonds must be discount bonds which mature at the terminal date.

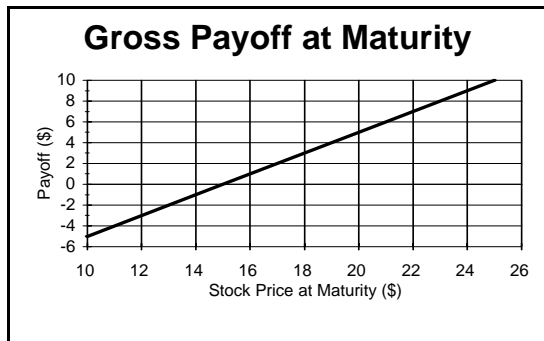
A.



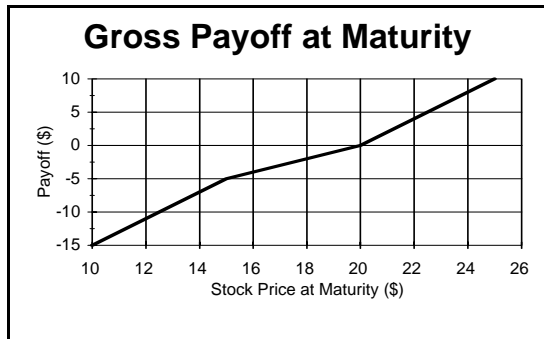
B.



C.



D.



IV. [16 points] The common stock of Sternco is currently trading at \$40 per share (up from

- \$25 at the beginning of the year). Sternco is currently “in play” as a takeover target and is not expected to pay any dividends for the next 6 months. You believe that if management is successful at repelling all offers, the stock will fall significantly, but if they are unsuccessful, the stock will rise. You want to profit from either outcome. The risk-free rate is 10% (continuously compounded annual rate) and a 6-month call option with an exercise price of \$40 is selling at \$4.
- A. A dealer offers you a 6-month European put option with an exercise price of \$40. What is a fair price for this option?
 - B. Propose a strategy to take advantage of your beliefs that uses one or more of the following instruments: the stock, 6-month call options with an exercise price of \$40, 6-month European put options with an exercise price of \$40 and discount bonds maturing in 6 months.
 - C. One month ago the firm trying to take over Sternco publicly announced its intentions. This announcement caused Sternco’s stock price to increase and the volatility of Sternco’s stock return to increase. What would have happened to the price of European calls on Sternco’s stock on the announcement date?
- V. [5 points] If forward-spot parity holds, what is the current spot price of the English Pound (expressed as U.S. dollars per English pound), assuming that the one year forward rate is \$1.80/£, the English one-year interest rate is 7% (expressed as an EAR) and the U.S. one-year rate is 9% (also expressed as an EAR)?
- VI. [8 points]
- A. Today’s price for a 12 month discount bond (face value=100) is 95 and today’s price for a 24 month discount bond (face value=100) is 90. The current spot price for 1 oz of gold is \$450. Suppose that the cost of carrying gold is zero. In the absence of arbitrage, what is today’s forward price for 1 oz of gold to be delivered in 12 months?
 - B. If forward-spot parity holds, what is the one year forward price of a yen (expressed as English pounds per yen), assuming that the current spot rate is £0.0256/yen, the English one-year interest rate is 8% (expressed as an EAR) and the Japanese one-year rate is 10% (also expressed as an EAR).
- VII. [18 points] The common stock of Rosh is currently trading at \$10 per share (down from \$15 at the beginning of the year). Rosh has commenced testing of a cure for the common cold. You believe that if the testing is successful, the stock’s price will rise significantly, but if it is unsuccessful, the stock’s price will fall. You want to profit from either outcome while putting a lower bound on your loss if you are wrong. The risk-free rate is 6% (continuously compounded annual rate) and a 6-month European call option with an exercise price of \$6 is selling at \$7. Rosh is not expected to make any dividend payments in the next 6 months.

- A. A dealer offers you a 6-month European put option with an exercise price of \$6. What is a fair price for this option?
- B. Propose a strategy to take advantage of your beliefs that uses one or more of the following instruments: the stock, 6-month European call options with an exercise price of \$6, 6-month European put options with an exercise price of \$6 and discount bonds maturing in 6 months.

One month ago Rosh announced that testing of the potential cure was about to commence but that its researchers were less optimistic than earlier about the likelihood of success. This announcement caused Rosh's stock price to decrease and the volatility of Rosh's stock return to increase.

- C. What would have happened to the price of European calls on Rosh's stock on the announcement date? Explain your reasoning.
 - D. What would have happened to the price of European puts on Rosh's stock on the announcement date? Explain your reasoning.
- VIII. [16 points] A large computer manufacturer Pear has \$1M of riskless debt. The equity of Pear has a Beta with respect to the market of 1.1. Assume that the CAPM holds for annual returns and that Pear pays annual dividends. The riskless rate is 8% per annum and the expected annual return on the market is 20%. Pear has just paid a dividend of \$2 per share and Pear's dividend per share is expected to grow at 15% per year forever. It has a plowback (retention) ratio of 0.75 and the expected return on book equity each year is a constant (ROE).
- A. What is the expected annual return on Pear's stock?
- Use the constant dividend growth model to answer the following questions.
- B. What is the current price of Pear stock?
 - C. What is Pear's current earnings per share after interest?
 - D. What is the expected return on book equity (ROE) for Pear?
- IX. [13 points] Today's price for a 12 month discount bond (face value=100) is 93 and today's price for a 6 month discount bond (face value=100) is 96. Today's forward price for delivery of IBM stock in 1 year is 132. IBM will pay a dividend of \$5 in six months. The current price of IBM stock is 125. Describe an arbitrage opportunity (if one exists) and show that it is an arbitrage opportunity.

X. [19 points] Today is the 2/15/95. The following information is available.

Government Notes. (principal=100)

Rate	Maturity	Price
6	Feb 96	102.875

U.S. Treasury Strips. (face value=100)

Maturity	Aug 95	Feb 96	Aug 96	Feb 97
Spot Price	?	97	93	90

- A. Six months later (on the 8/15/95), the price of the Feb 96 strip is 98.
 1. What is the price of the 6% Feb 96 note on the 8/15/95?
 2. What is the 6-month holding period return on the 6% Feb 96 note from 2/15/95 to 8/15/95?
- B. What is the forward price today (2/15/95) for delivery of the Feb 97 strip on 8/15/96?
- C. What is today's price of the Aug 95 strip?

Solutions

I.

- A. Individuals could have a multi-period horizon and $MI(\text{end } t)$ may be correlated with individuals' human capital values (at the end of period t).
- B. Because investors care about $\sigma[R_p(t), MI(\text{end } t)]$, investors do not all want to hold combinations of the riskless asset and the mean-variance tangency portfolio. Investors who like negative $\sigma[R_p(t), MI(\text{end } t)]$ may be prepared to hold a risky portfolio with a flatter-sloped Capital Allocation Line than that of the tangency portfolio because that portfolio offers more negative covariance with $MI(\text{end } t)$ than the tangency portfolio. Thus, the market portfolio is not the tangency portfolio and need not lie on the minimum variance frontier for the N risky assets.
- C. All individuals do not hold portfolios that are combinations of the market portfolio and the riskless asset. Because investors care about $\sigma[R_p(t), MI(\text{end } t)]$, investors do not want to hold combinations of the riskless asset and the mean-variance tangency portfolio. Instead, all individuals hold portfolios that are combinations of:
 - a. the riskless asset.
 - b. the market portfolio.
 - c. a hedging portfolio which is highly correlated with $MI(\text{end } t)$.

An investor's portfolio weights for the three assets depend, in part, on how much the investor cares about $\sigma[R_p(t), MI(\text{end } t)]$. Thus, all investors are not holding

- combinations of the market portfolio and the riskless asset.
- D. A and B could have very different expected returns due to very different covariances with $MI(\text{end } t)$.

II.

- A. Covered interest rate parity says:

$$S^{S/f}(0) = F_T^{S/f}(0) [1 + y^{*f}_1(0)] / [1 + y^{*s}_1(0)]$$

which implies

$$F_T^{S/f}(0) = S^{S/f}(0) [1 + y^{*s}_1(0)] / [1 + y^{*f}_1(0)] = 0.2 \{1.06/1.08\} = 0.1963.$$

B.

1. Using spot forward parity with negative carrying costs:

$$F_1(0) / [1 + y^{*}_1(0)] - C(1) / [1 + y^{*}_1(0)] = S(0)$$

$$F_1(0) - \{-D(1)\} = S(0) [1 + y^{*}_1(0)]$$

$$F_1(0) = S(0) [1 + y^{*}_1(0)] - D(1)$$

$$F_1(0) = S(0) [1 + y^{*}_1(0)] - S(0) 0.04$$

$$460 = S(0) [1 + 0.06 - 0.04]$$

$$\Rightarrow S = 450.98$$

2. If $S(0) = 460.98$, then:

Action	Today	at T=1 year
sell index at 0 and close out at T=1	+460.98	-S(T)-460.98(.04)
invest in 1 year discount bonds with face value {460 + 460.98(.04)}	-{460 + 460.98(.04)}/1.06 = 451.36	{460 + 460.98(.04)}
buy 1 stock index future today with delivery at T=1	0	S(T)-460
Total	9.62	0

III. These are not the only correct answers.

- A. Sell 1 C(X=15) and sell 1 P(X=15).
 B. Buy 1 P(X=15) and sell 1 P(X=20).
 C. Buy 1 C(X=15) and sell 1 P(X=15).
 D. Sell 1 P(X=20), sell 1 P(X=15) and buy 2 C(X=20).

IV.

- A. Use put call parity to value the put.

$$S(0) = C_{40,0.5}(0) - P_{40,0.5}(0) + 40 e^{-\frac{1}{2} \cdot 0.1}$$

$$P_{40,0.5}(0) = C_{40,0.5}(0) + 40 e^{-0.1 \times \frac{1}{2}} - S(0) = 4 + 38.05 - 40 = 2.05.$$

- B. Buy 1 P(X=40) and buy 1 C(X=40).
 C. Using Black Scholes pricing model, know $C(0)$ is increasing in $S(0)$ and is increasing in σ . So both the stock price increase and stock return volatility

increase cause the prices of calls on the stock to increase.

V. Use the covered interest parity theorem:

$$S^{\$/\pounds}(0) = F_T^{\$/\pounds}(0) [1 + y^{\pounds}_T(0)]^T / [1 + y^{\$}_T(0)]^T = 1.8 \times [1.07] / [1.09] = 1.77.$$

VI.

- A. In general, $S(0) = F_T(0) d_T(0)$. Here, $450 = F_1(0) 0.95$. So $F_1(0) = 473.68$.
 B. In general, $S^{\pounds/y}(0) = F_T^{\pounds/y}(0) [1 + y^y_T(0)]^T / [1 + y^{\pounds}_T(0)]^T$. Here $\pounds 0.0256/y = F_1^{\pounds/y}(0) 1.1/1.08$. So $F_1^{\pounds/y}(0) = \pounds 0.02513/y$.

VII.

- A. Using put-call parity, $S(0) = C_{X,T}(0) - P_{X,T}(0) + X e^{-r^*T}$.
 So $10 = 7 - P_{6,1/2}(0) + 6 e^{-0.06 \times 1/2}$ giving $P_{6,1/2}(0) = 2.82$.
 B. Buy 1 put and buy 1 call.
 C. Impact on $C(0)$ is indeterminant since the drop in $S(0)$ causes $C(0)$ to drop while the increase in stock return volatility causes $C(0)$ to increase.
 D. Impact on $P(0)$ is positive since the drop in $S(0)$ causes $P(0)$ to increase while the increase in stock return volatility also causes $P(0)$ to increase.

VIII.

- A. Using the SML, $E[R_{IBX}] = 8\% + 1.1 (20\% - 8\%) = 21.2\%$.
 B. The constant growth DDM says:
 $P^{IBX}(0) = D^{IBX}(0) [1 + g^{IBX}] / \{E[R_{IBX}] - g^{IBX}\} = 2 [1 + 0.15] / \{0.212 - 0.15\} = 37.10$.
 C. $E^{IBX}(0) = D^{IBX}(0) / (1 - b^{IBX}) = 2 / 0.25 = 8$.
 D. $ROE^{IBX} = g^{IBX} / b^{IBX} = 0.15 / 0.75 = 0.2$.

IX. Using forward spot parity, the 12 month forward price implies a spot price of

$$\begin{aligned} S(0) &= F_T(0) d_T(0) - C(t_c) d_{tc}(0) \\ &= 132 \times 0.93 + 5 \times 0.96 = 127.56 > 125. \end{aligned}$$

which implies that the 1 year forward price is too high relative to stock price. So the arbitrage involves selling the 1 year forward contract and buying the stock. An arbitrage position can be constructed as follows:

Strategy	Today	6 months	12 months
Buy 1 IBM today and sell in 12 months	-125	5	S(1)
Sell a forward contract today to deliver IBM in 12 months	0	0	132-S(1)
Sell 12 month discount bonds today with face value of 132 and buy at maturity	$132 \times 0.93 = 122.76$		-132
Sell 6 month discount bonds today with face value of 5 and buy at maturity	$5 \times 0.96 = 4.8$	-5	
Net Cash Flow	2.56 > 0	0	0

X.

A.

- $P^{6\% \text{ Feb } 96}(8/15/95) = 103$ $d_{1/2}(8/15/95) = 103 \times 0.98 = 100.94$.
- $h^{6\% \text{ Feb } 96}(8/15/95) = \{100.94 + 3 - 102.875\} / 102.875 = 1.0352\%$ expressed as an effective 6 month return.

B. $d_{1\frac{1}{2},2}(2/15/95) = d_2(2/15/95) / d_{1\frac{1}{2}}(2/15/95) = 0.90 / 0.93 = 0.9677$. So the forward price today for delivery of the Feb 97 strip on 8/15/96 is 96.77.

C. Know $P^{6\% \text{ Feb } 96}(2/15/95) = 102.875 = 3 d_{1/2}(2/15/95) + 103 d_1(2/15/95)$. and know $d_1(2/15/95) = 0.97$. So $102.875 = 3 d_{1/2}(2/15/95) + 103 \times 0.97$ which gives $d_{1/2}(2/15/95) = 0.9883$. Thus, today's price of the $\frac{1}{2}$ -year strip is 98.83.

Practice Final Exam

- I. [15 points] VLP has just paid a dividend of \$0.95 per share. It has a plowback (retention) ratio of .4, and a current market price of \$12. Dividends are paid annually and the CAPM holds for annual returns. The expected annual return on the market is 11% and the riskless rate is 6%. VLP's stock has a Beta with respect to the market of 1.8. The expected return on book equity each year is a constant (ROE).

A. What is the expected annual return on VLP's stock?

Use the constant dividend growth model to answer the following questions.

- What is the expected growth rate of dividends for VLP stock?
- What is the expected return on book equity (ROE) for VLP?
- What is the book value of equity (per share) today for VLP?
- If you buy VLP stock today and hold it for one year, what is your expected holding period return?

- II. [7 points] WT Co has assets worth \$15M and has equity with a market value of \$10M. The Beta of WT's assets (with respect to the market portfolio) is 1.5 while the Beta of

WT's equity (with respect to the market portfolio) is 2.

- A. What is the market value of WT's debt?
- B. What is the Beta of WT's debt (with respect to the market portfolio)?
- C. Is WT's debt riskless?

III. [12 points] XYZ Co pays monthly dividends and its current dividend per share is \$0.25. XYZ's dividend per share is expected to grow at the same rate per month indefinitely and its current share price is \$20. The riskless rate has always been 0.8% per month and will remain at 0.8%. A market model regression of XYZ's monthly stock return on the market's monthly return has a slope coefficient of 1.2. The expected monthly return on the market portfolio is 1.3% and its standard deviation is 0.9%. Suppose the CAPM holds.

- A. What is the intercept of the regression of XYZ's monthly stock return on the market's monthly return?
- B. What is the expected monthly return on XYZ's equity?
- C. What is expected dividend growth rate for XYZ?

IV. [12 points] Let $DEF(\text{Jan})$ be the difference in the yield on a long term low-grade corporate bond and the yield on a long term government bond at the end of January and let $R_M(\text{Jan})$ be the January return on the market portfolio. Suppose each individual cares about $\{E[R_p(\text{Jan})], \sigma[R_p(\text{Jan})], \sigma[R_p(\text{Jan}), DEF(\text{Jan})]\}$ when forming his/her portfolio p for January. The following additional information is available:

i	$E[R_i(\text{Jan})]$	$\beta^*_{i,M}$	$\beta^*_{i,DEF}$
Pink	1.73%	1.3	0.25
Grey	1.34%	0.9	0.1

where $\beta^*_{i,M}$ and $\beta^*_{i,DEF}$ are regression coefficients from a multiple regression (time-series) of $R_i(t)$ on $R_M(t)$ and $DEF(t)$:

$$R_i(t) = \phi_{i,0} + \beta^*_{i,M} R_M(t) + \beta^*_{i,DEF} DEF(t) + e_i(t).$$

Also know that riskless rate for January, $R_f(\text{Jan})$, 0.7%.

- A. What is the risk premium for bearing $\beta^*_{i,M}$ risk?
- B. What is the expected January return on the market portfolio $E[R_M(\text{Jan})]$?
- C. What is the risk premium for bearing $\beta^*_{i,DEF}$ risk?
- D. Is the market portfolio on the minimum variance frontier of the risky assets in the economy? Why or why not?

V. [15 points] Today's price for a 6 month discount bond is 97 and today's price for a 12 month discount bond is 94. Today's forward price for 1 oz of gold to be delivered in 6 months is

\$465. There are no carrying costs associated with holding gold.

- A. In the absence of arbitrage, what must be the spot price of 1 oz of gold today?
- B. Suppose that today's forward price for 1 oz of gold to be delivered in 12 months is \$475 and that you do not know the spot price of gold today (so the spot price of gold need not equal the price you calculated in part A above). However, you can buy and sell gold today. Is there an arbitrage opportunity? If so, describe it and demonstrate that it is in fact an arbitrage opportunity.

VI. [15 points] Consider the following two strategies involving options on BG stock which does not pay dividends.

- (1) Buy a European put expiring in 6 months on BG stock with a strike price of \$80 and buy a European call expiring in 6 months on BG stock with a strike price of \$80. (A straddle.)
- (2) Buy a European put expiring in 6 months on BG stock with a strike price of \$90 and buy a European call expiring in 6 months on BG stock with a strike price of \$70.
- A. Draw the payoff (ignoring any purchase price paid or received to enter the position) in 6 months for each strategy as a function of the price of BG stock at that time. Be sure to clearly indicate the angle of any lines you draw.
- B. Which strategy would cost more? Why?
- C. Replicate strategy (1) using only: BG stock; discount bonds maturing in 6 months (state the face value of bonds bought or sold); and, European puts on BG stock that expire in 6 months and have a strike price of \$80.

VII. [14 points] Today is the 2/15/95. The following information is available.

Government Bonds and Notes.

Rate	Maturity	Price
4	Feb 96	98.84
6	Aug 97	102

U.S. Treasury Strips.

Maturity	Spot Price	Forward Price for delivery on 8/15/95	Forward Price for delivery on 2/15/96	Forward Price for delivery on 8/15/96
Aug 95	97			
Feb 96	?	?		
Aug 96	93	95.876	?	
Feb 97	?	93.814	95.789	97.849

- A. What is the price of the Feb 96 strip?
 B. What is the price of the Feb 97 strip?

VIII. [10 points] The current stock price of FG Co is \$15 and the yield on a 1 year discount bond expressed as an EAR is 8%. Suppose the current price of a European put on FG Co that expires in one year and has a strike price of \$18 is \$1.50. Describe an arbitrage opportunity and show that it is in fact an arbitrage opportunity.

Practice Final Exam Solution

I.

- A. Using the SML:

$$E[R] = 6\% + 1.8 (11\% - 6\%) = 15\%.$$

- B. The constant growth DDM says:

$$P(0) = D(0) [1 + g] / \{E[R] - g\}$$

which implies

$$12 = 0.95 [1+g] / \{0.15 - g\}$$

$$12 \{0.15 - g\} = 0.95 + 0.95g$$

$$g = 0.85/12.95 = 0.06564.$$

- C. $ROE = g/b = 0.06564/0.4 = 0.16409.$

- D. First need to determine expected earnings in one year's time:

$$E[D(1)] = D(0) [1+g] = 0.95 [1+0.06564] = 1.012358.$$

$$E[E(1)] = E[D(1)]/(1 - b) = 1.012358/0.6 = 1.68726.$$

Then can calculate book value of equity today:

$$K(0) = E[E(1)]/ROE = 1.68726 / 0.16409 = 10.2825.$$

- E. Know that the expected holding period return on VLP's equity is 15%. So the answer is 15%.

II.

A. $B = V - S = 15M - 10M = 5M.$

B. Know

$$\beta_V = (B/V) \beta_B + (S/V) \beta_S$$

and so

$$\beta_B = (V/B) [\beta_V - (S/V) \beta_S] = (15/5) [1.5 - (10/15) 2] = 0.5$$

- C. Since $\beta_B \neq 0$, the debt is not riskless.

III.

A. XYZ lies on the SML. So $\alpha_{XYZ,M} = (1 - \beta_{XYZ,M}) R_f = (1 - 1.2) \times 0.8\% = -0.16\%.$

B. XYZ lies on the SML:

$$E[R_{XYZ}(t)] = R_f + \beta_{XYZ,M} \{E[R_M(t)] - R_f\} = 0.8\% + 1.2 \{1.3\% - 0.8\%\} = 1.4\%.$$

C. Use the constant growth DDM:

$$P_{XYZ}(0) = D_{XYZ}(0) [1+g] / \{E[R_{XYZ}(t)] - g\}$$

$$20 = 0.25 [1+g] / \{0.014 - g\}$$

$$20 \{0.014 - g\} = 0.25 [1+g]$$

$$0.28 - 20g = 0.25 + 0.25g$$

$$g = 0.03/20.25 = 0.00148 = 0.148\% \text{ per month.}$$

IV. Know ICAPM holds. So

$$E[R_i(\text{Jan})] = R_f(\text{Jan}) + \beta_{i,M}^* \lambda_M^* + \beta_{i,DEF}^* \lambda_{DEF}^*$$

$$\text{where } \lambda_M^* = E[R_i(\text{Jan})] - R_f(\text{Jan}).$$

A. Using the above formula for Pink and Grey:

Pink: $1.73 = 0.7 + 1.3 \lambda_M^* + 0.25 \lambda_{DEF}^*$

Grey: $1.34 = 0.7 + 0.9 \lambda_M^* + 0.10 \lambda_{DEF}^*$

$$\text{Now Pink} \Rightarrow \lambda_{DEF}^* = 4 (1.03\% - 1.3 \lambda_M^*)$$

which can be substituted into Grey to obtain

$$1.34 = 0.7 + 0.9 \lambda_M^* + 0.10 \times 4 (1.03\% - 1.3 \lambda_M^*).$$

It follows that $\lambda_M^* = 0.6\%$ and $\lambda_{DEF}^* = 1\%$.

So the risk premium for bearing $\beta_{i,M}^*$ risk λ_M^* is 0.6%.

- B. $E[R_i(\text{Jan})] = \lambda^*_M + R_f(\text{Jan}) = 0.6\% + 0.7\% = 1.3\%$.
 C. From above, the risk premium for bearing $\beta^*_{i,DEF}$ risk λ^*_{DEF} is 1%.
 D. The market portfolio is not on the minimum variance frontier for the risky assets since

$$E[R_i(\text{Jan})] = R_f(\text{Jan}) + \beta^*_{i,M} \lambda^*_M + \beta^*_{i,DEF} \lambda^*_{DEF}$$

implies

$$E[R_i(\text{Jan})] \neq E[R_{0,M}(\text{Jan})] + \beta_{i,M} \{E[R_M(\text{Jan})] - E[R_{0,M}(\text{Jan})]\}$$

where $E[R_{0,M}(\text{Jan})]$ is the expected return on a risky asset uncorrelated with the market portfolio and $\beta_{i,M} = \text{cov}[R_i(\text{Jan}), R_M(\text{Jan})]/\sigma^2[R_M(\text{Jan})]$.

- V. Know that the price of 6 month discount bond is related to the yield on a six month bond expressed as an EAR as follows:

$$97 = 100/[1+y^*_{0.5}(0)]^{0.5} \text{ which implies } 0.97 = 1/[1+y^*_{0.5}(0)]^{0.5}.$$

Know that the price of 12 month discount bond is related to the yield on a 12 month bond expressed as an EAR as follows:

$$94 = 100/[1+y^*_1(0)] \text{ which implies } 0.94 = 1/[1+y^*_1(0)].$$

- A. Since the carrying costs associated with gold are zero, forward spot parity says:
 $S(0) = F_{0.5}(0)/[1+y^*_{0.5}(0)]^{0.5}$

which implies a spot price of

$$S(0) = 465 \times 0.97 = 451.05.$$

- B. Using forward spot parity, the 12 month forward price implies a spot price of
 $S(0) = 475 \times 0.94 = 446.5$

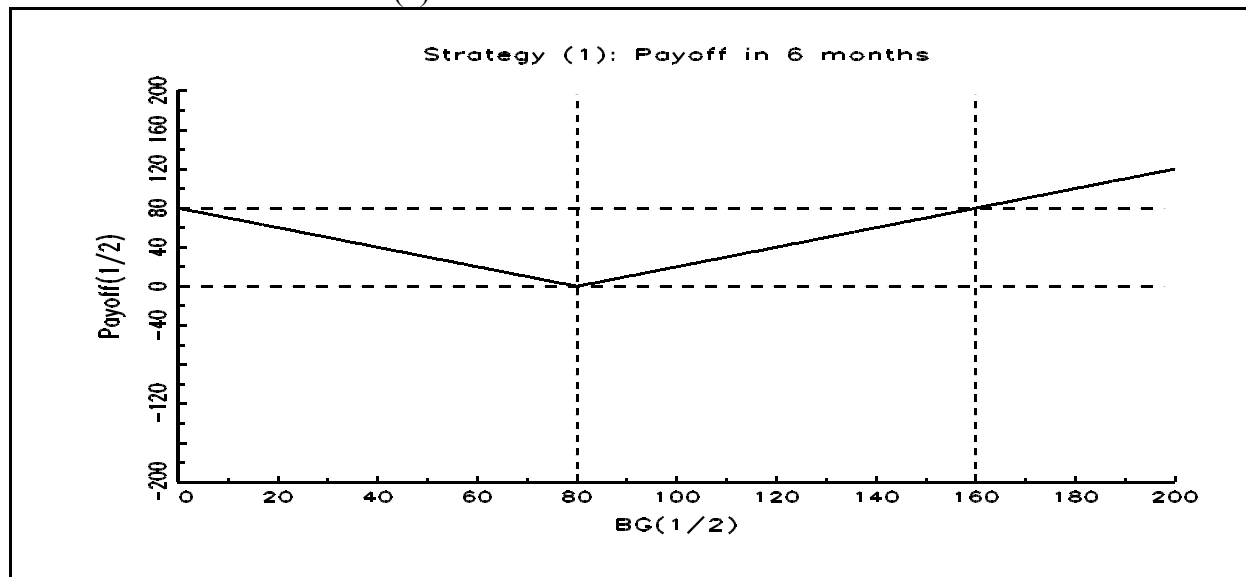
which implies that the 6 month forward price is too high relative to the 12 month forward price. So the arbitrage involves selling the 6 month forward contract and buying the 12 month forward contract. An arbitrage position can be constructed as follows:

Strategy	Today	6 months	12 months
Buy 1 oz gold today and sell in 6 months	$-S(0)$	$S(\frac{1}{2})$	
Sell a 6 month forward contract today	0	$465 - S(\frac{1}{2})$	
Sell 6 month discount bonds today with face value of 465 today and hold to maturity	$465 \times 0.97 = 451.05$	-465	
Sell 1 oz gold today and buy in 12 months	$S(0)$		$-S(1)$
Buy a 12 month forward contract today	0		$S(1) - 475$
Buy 12 month discount bonds today with face value of 475 today and hold to maturity	$-475 \times 0.94 = -446.5$		475
Net Cash Flow	4.55	0	0

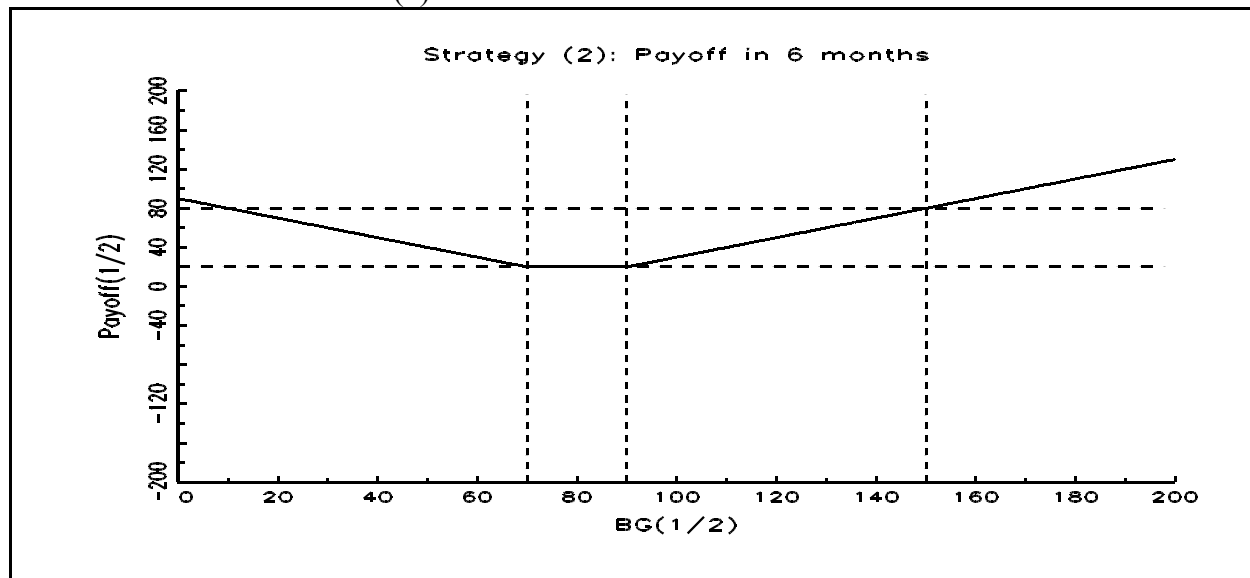
VI.

A. The payoff diagrams are:

(1)



(2)



B. Since strategy (2) pays off more than strategy (1) irrespective of the price of the underlying in 6 months, $BG(1/2)$, no arbitrage implies that strategy (2) must cost more.

C. Replicating strategy:

Buy 2 European puts with strike price of 80 which expire in 6 months.

Buy 1 BG stock.

Sell 6 month discount bond with a face value of 80.

VII.

A. Can recover the yield on a six month discount bond using the Aug 95 strip:
 $y_{1/2}(\text{Feb } 95) = \{100/97 - 1\} \times 2 = 6.18557\%$.

Can then recover the price of the Feb 96 strip using the Feb 96 note by splitting it into its two payments:

(1) The first coupon payment is paid in Aug 95. Thus, the law of one price says it can be discounted using $y_{1/2}(\text{Feb } 95)$ to get its value on 2/15/95:

$$(4/2)/(1+0.0618557/2) = 2 \times 0.97 = 1.94.$$

(2) The value of the second and final cash flow of $\{100 + (4/2)\} = 102$ can be calculated by subtracting the value of the Aug 95 coupon from the price of the note:

$$98.84 - 1.94 = 96.9.$$

(3) The yield on a one year discount bond can then be obtained since 96.9 is the price on 2/15/95 of a cash flow of 102 in Feb

96:

$$y_1(\text{Feb } 95) = \{[102/96.9]^{0.5} - 1\} \times 2 = 5.1957\%.$$

(4) The price of the Feb 96 strip can then be determined:

$$100/(1+y_1(\text{Feb } 95)/2)^2 = 100/(1+0.051957/2)^2 = 95.$$

B. Need to use spot forward parity with the Feb 97 strip as the underlying:

1. One way:

$$S(0) = F_{0.5}(0)/[1+y_{0.5}^*(0)]^{0.5} = 93.814 \times 0.97 = 91.$$

2. A second way:

$$S(0) = F_1(0)/[1+y_1^*(0)]^1 = 95.789 \times 0.95 = 91.$$

3. A third way:

$$S(0) = F_{1.5}(0)/[1+y_{1.5}^*(0)]^{1.5} = 97.849 \times 0.93 = 91.$$

VIII. Know that the put must satisfy the following:

$$P_{18,1}(0) \geq \max\{18/[1+y_1^*(0)]^1 - S(0), 0\}.$$

But

$$P_{18,1}(0) = 1.50 < \max\{18/[1+y_1^*(0)]^1 - S(0), 0\} = 18/[1.08]^1 - 15 = 1.6667.$$

So an arbitrage opportunity exists.

Strategy	Now	1 Year	
		$S(1) < 18$	$S(1) \geq 18$
Buy a European put option at 0 with exercise price of 18 and hold until expiration in 1 year	-1.5	$[18 - S(1)]$	0
Buy FG stock at 0 and sell in 1 year	-15	$S(1)$	$S(1)$
Sell a 1 year discount bond with face value of 18 and close out at maturity	$18/[1+0.08]$ =16.6667	-18	-18
Net Cash Flow	0.1667	0	$S(T) - 18 \geq 0$

Foundations of Finance
Final Exam Spring 1997
Prof. Anthony Lynch

Instructions: 120 minutes. Closed book. You are permitted use of a financial calculator and two sheets (8½" x 11") of notes. Answer *all* questions. Each question carries the number of points indicated. Total points is 100. Show all work on the exam paper itself. If you do not have or can not calculate a number that you need, make an assumption and proceed with the rest of the question. *No credit will be given for illegible, unsupported or ambiguous answers.*

GOOD LUCK!

Name: _____

Student #: _____

Signature: _____

- I. [18 points] Firm ABC is required to make a \$8M payment in 2 years and a \$10M payment in 5 years. The yield curve is flat at 6% APR with semi-annual compounding.
- A. What is the value today of the liability stream?
 - B. What is the duration of the liability stream?
 - C. What is the duration of a 10-year strip?
 - D. What is the value today of a 10-year strip with a face value of 100?
 - E. Firm ABC wants to form a portfolio using 1-year and 10-year U.S. strips to fund the payments. How much (\$ value today) of each strip must the portfolio contain for it to still be able to fund the payments after a shift in the yield curve that leaves the curve flat?
- II. [7 points] A large furniture manufacturer Akia has \$1M of risky debt. The equity of Akia has a Beta with respect to the market of 1.3. Assume that the CAPM holds for annual returns and that Akia pays annual dividends. The riskless rate is 10% per annum and the expected annual return on the market is 18%. Akia has just paid a dividend of \$5 per share and Akia's dividend per share is expected to grow at 15% per year forever. It has a plowback (retention) ratio of 0.75.
- A. What is the expected annual return on Akia's stock?

Use the constant dividend growth model to answer the following question.

- B. What is the current price of Akia stock?

- III. [11 points] Suppose the I-CAPM holds and investors care about $\{E[R_p(t)], \sigma[R_p(t)], \sigma[R_p(t), \text{TERM}(t)]\}$ where $\text{TERM}(t)$ is the difference between the yield on a long term bond and the yield on a short term bond at the end of month t . The riskless rate is 0.8% and the following information is available:

i	$E[R_i(t)]$	$\beta_{i,M}^*$	$\beta_{i,\text{TERM}}^*$
M (market)	1.3%	?	?
XDM	?	1.1	0.6
QL	1.5%	0.8	0.4

where $\beta_{i,M}^*$ and $\beta_{i,\text{TERM}}^*$ are the slope coefficients from the multivariate regression of asset i 's monthly return on the market portfolio's monthly return and TERM :

$$R_i(t) = \phi_{i,0} + \beta_{i,M}^* R_M(t) + \beta_{i,\text{TERM}}^* \text{TERM}(t) + e_i(t).$$

What is the expected return on XDM?

- IV. [11 points] Today's price for a 12 month discount bond (face value=100) is 92 and today's price for a 24 month discount bond (face value=100) is 85. The current spot price for 1 oz of gold is \$400. Suppose that the cost of carrying 1 oz of gold for 12 months is zero.
- A. In the absence of arbitrage, what is today's forward price for 1 oz of gold to be delivered in 12 months?
- B. If the forward price is \$2 higher than you calculated in part A, describe the arbitrage opportunity that is available and show that it is an arbitrage.
- V. [21 points] Consider the following two strategies involving options on SLT stock which does not pay dividends.
- (1) Buy a European put expiring in 1 year on SLT stock with a strike price of \$100 and buy a European call expiring in 1 year on SLT stock with a strike price of \$100. (A straddle.)
 - (2) Buy a European put expiring in 1 year on SLT stock with a strike price of \$120 and buy a European call expiring in 1 year on SLT stock with a strike price of \$80.
- A. Draw the payoff (ignoring any purchase price paid or received to enter the strategy) in 1 year for each strategy as a function of the price of SLT stock at that time. Be sure to clearly indicate the angles of any lines you draw.
- B. Which strategy would cost more? Why?
- C. Replicate strategy (1) using only: SLT stock; discount bonds maturing in 1 year (state the face value of bonds bought or sold); and, European puts on SLT stock that expire in 1 year and have a strike price of \$100.
- D. Suppose SLT announces that it has increased its research and development budget three fold for the next year. While the announcement does not move the price of SLT

stock today, there is an expectation that SLT will either lose the R&D money (causing a big drop in price during the year) or discover a new pharmaceutical that will cause a big increase in SLT's price during the year. Consequently, the announcement increases the volatility of SLT's stock return. How does this announcement affect the cost of entering strategy (2) today? Explain why.

VI. [22 points] Today is the 2/15/95. The following information is available.

Government Bonds and Notes.

Rate	Maturity	Price
4	Feb 97	96
6	Aug 98	97

U.S. Treasury Strips.

Type	Maturity	Price
ci	Aug 95	98
ci	Feb 96	94
ci	Aug 96	90

- A. Based on the above information, what is the implied price today of a 2 year discount bond with a face value of 100?
- B. Suppose a Feb 97 strip is priced at 88 today? Is there an arbitrage opportunity? If so, describe an arbitrage and show that it is in fact an arbitrage.
- C. What is the forward rate available today for the 1 year period starting in 6 months time? Express the rate as an APR with semiannual compounding.

VII. [10 points] KL is a non-dividend paying stock. A European call on KL stock expiring in 1 year with an exercise price of \$20 has a price of \$5 today. KL's stock is expected to be worth \$19 in 6 months time and the price today of a discount bond (with a \$100 face value) maturing in 6 months is 96. Today's forward price of KL stock for delivery in 1 year is \$20.

- A. Do you have enough information to determine KL's stock price today? If so, calculate KL's stock price today and explain your calculations?
- B. Do you have enough information to determine the price today of a European put on KL stock expiring in a year with an exercise price of \$20? If so, calculate the put price today and explain your calculations?

Foundations of Finance
Final Exam Spring 1997 Answer Key
Prof. Anthony Lynch

I.

- A. $L(0) = 8M/(1.03^4) + 10M/(1.03^{10}) = 7.108M + 7.441M = 14.549M$.
 B. $D^L(0) = 2 \times (7.108/14.549) + 5 \times (7.441/14.549) = 0.977 + 2.557 = 3.534$.
 C. $D^{10}(0) = 10$.
 D. $P^{10}(0) = 100/(1.03^{20}) = 55.367$.
 E. To immunize the liability, need to match asset value to the liability value. So let ω_1 be the fraction of the \$14.549M invested in the 1-year strip. Also need to match the duration of the assets to the duration of the liabilities:

$$D^L(0) = 2.8820 = D^A(0) = \omega_1 D^1(0) + (1 - \omega_1) D^{10}(0) = \omega_1 1 + (1 - \omega_1) 10.$$

And so $\omega_1 = (10 - 3.534)/9 = 0.7184$ and the \$-value today invested in 1-year strips $A^1(0) = 0.7184 \times 14.549M = 10.452M$. The \$-value today invested in 10-year strips $A^{10}(0) = (1 - 0.7184) \times 14.549M = 4.097M$.

II.

- A. In a CAPM world, Akia stock lies on the SML:
 $E[R_{Akia}] = R_f + \beta_{Akia,M} \{E[R_M] - R_f\} = 10\% + 1.3 \{18\% - 10\%\} = 20.4\%$.
 B. $P_{Akia}(0) = D_{Akia}(0) \{1 + g_{Akia}\} / \{E[R_{Akia}] - g_{Akia}\} = 5(1 + 0.15) / (0.204 - 0.15) = 106.48$.

III. Since each individual cares about $\{E[R_p(t)], \sigma[R_p(t)], \sigma[R_p(t), TERM(t)]\}$ when forming his/her portfolio p for period t, it follows that any asset satisfies:

$$E[R_i(t)] = R_f + \beta_{i,M}^* \lambda_M^* + \beta_{i,TERM}^* \lambda_{TERM}^*$$

Also know that $\lambda_M^* = E[R_M(t)] - R_f$. So

$$E[R_{QL}(t)] = R_f + \beta_{QL,M}^* \lambda_M^* + \beta_{QL,TERM}^* \lambda_{TERM}^*$$

$$1.8\% = 0.8\% + 0.8 \times \{1.3\% - 0.8\%\} + 0.4 \times \lambda_{TERM}^*$$

which implies

$$\lambda_{TERM}^* = 0.3\% / 0.4 = 0.75\%.$$

So then the expected return on Yellow can be calculated

$$E[R_{XDM}(t)] = R_f + \beta_{XDM,M}^* \lambda_M^* + \beta_{XDM,TERM}^* \lambda_{TERM}^*$$

$$= 0.8\% + 1.1 \times 0.5\% + 0.6 \times 0.75\%$$

$$= 1.8\%.$$

IV.

- A. $S(0) = F_T(0) d_T(0)$. So $400 = F_1(0) 0.92$ and $F_1(0) = 400/0.92 = 434.78$.
 B. The forward price is too high which implies that you want to sell forward contracts and buy the underlying:

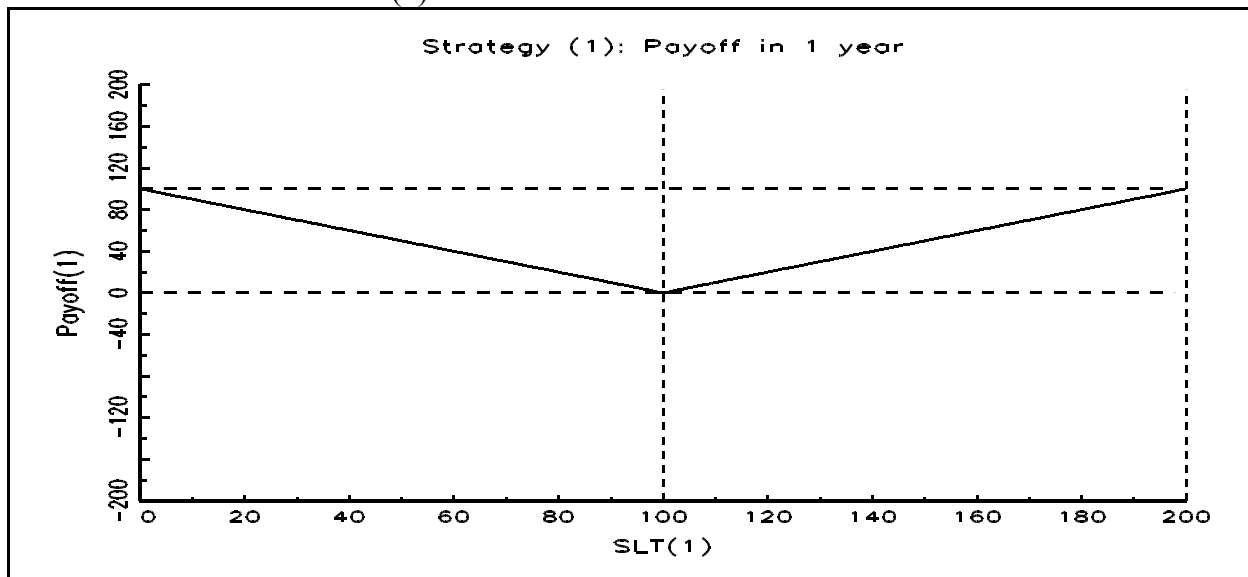
Strategy	0	1
Sell a forward contract on 4/97 which delivers 1 oz of gold on 4/98	0	$436.78 - S(1)$
Buy 1 oz of gold on 4/97 and sell on 4/98	-400	$S(1)$
Sell 1-yr U.S. T-bills on 4/97 with face value of 520	436.78×0.92	-436.78
Net Cash Flow	1.84	0

which is an arbitrage opportunity.

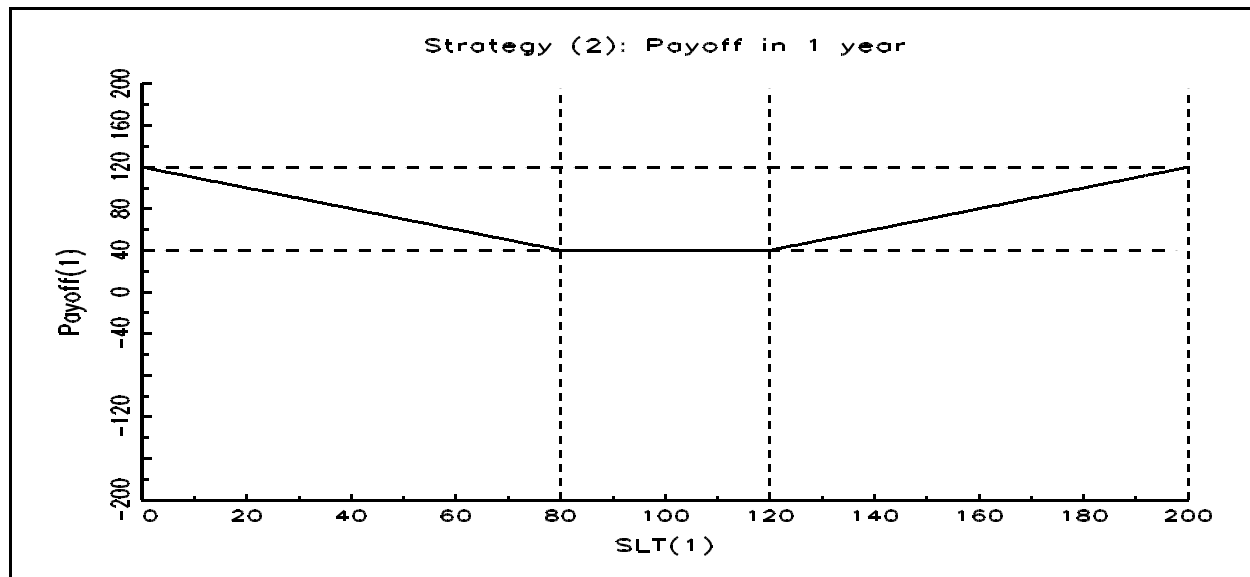
V.

A. The payoff diagrams are:

(1)



(2)



B. Since strategy (2) pays off more than strategy (1) irrespective of the price of the underlying in 1 year, $SLT(1)$, no arbitrage implies that strategy (2) must cost more.

C. Replicating strategy:

Buy 2 European puts with strike price of 100 which expire in 1 year.

Buy 1 SLT stock.

Sell 1-year discount bonds with a face value of 100.

D. The cost of entering strategy (2) increases since the increase in stock return volatility causes $C(0)$ and $P(0)$ to increase.

VI.

A. Using the strip prices:

$$d_{1/2}(\text{Feb } 95) = 0.98$$

$$d_1(\text{Feb } 95) = 0.94$$

$$d_{1\frac{1}{2}}(\text{Feb } 95) = 0.90$$

Applying the law of one price to the 4% Feb 97 note:

$$96 = (4/2) d_{1/2}(\text{Feb } 95) + (4/2) d_1(\text{Feb } 95) + (4/2) d_{1\frac{1}{2}}(\text{Feb } 95) + (100 + [4/2]) d_2(\text{Feb } 95) \\ = 2 \times 0.98 + 2 \times 0.94 + 2 \times 0.9 + 102 \times d_2(\text{Feb } 95)$$

which implies $d_2(\text{Feb } 95) = 0.88588$ and so a Feb 97 strip with a face value of 100 has a price of 88.588 on 2/15/95.

B. Since the price of the Feb 97 strip implied by the shorter maturity strips and the 4% Feb 97 coupon bond is greater than the strip's actual price, there is an arbitrage opportunity which involves buying the Feb 97 strip and selling a synthetic Feb 97 strip.

Let a be the number of 4% Feb 97 notes that you buy, b be the number of Aug 96 strips that you buy, c be the number of Feb 96 strips that you buy, and d be the number of Aug 95 strips that you buy. Want to choose a , b , c , and d so that the net cash flows at 2/15/97, 8/15/96, 2/15/96, and 8/15/95 are all zero:

Position	2/15/95	8/15/95	2/15/96	8/15/96	2/15/97
Buy 1 x Feb 97 strip	-88				100
Buy a x 4% Feb 97 note	$-a \times 96$	$a \times 2$	$a \times 2$	$a \times 2$	$a \times 102$
Buy b x Aug 96 strip	$-b \times 90$			$b \times 100$	
Buy c x Feb 96 strip	$-c \times 94$		$c \times 100$		
Buy d x Aug 95 strip	$-d \times 98$	$d \times 100$			
Net	Cash(2/95)	0	0	0	0

Now $a = -100/102 = -0.98039$. So sell 0.98039 of the 4% Feb 97 note.

So $b \times 100 + a \times 2 = 0$ implies $b = 0.01961$. Notice that $b = c = d = 0.01961$. So buy 0.01961 of the Aug 96, Feb 96 and Aug 95 strips.

The position is an arbitrage opportunity since:

$$\text{Cash}(2/95) = -88 + 0.98039 \times 96 - 0.01961 \times 90 - 0.01961 \times 94 - 0.01961 \times 98 = 0.588 > 0.$$

- C. First calculate $d_{\frac{1}{2}, \frac{1}{2}}(\text{Feb } 95) = d_{1\frac{1}{2}}(\text{Feb } 95)/d_{\frac{1}{2}}(\text{Feb } 95) = 0.90/0.98 = 0.918367$. Then calculate the forward rate:

$$f_{\frac{1}{2}, 1\frac{1}{2}}(\text{Feb } 95) = 2 \{ [1/d_{\frac{1}{2}, 1\frac{1}{2}}(\text{Feb } 95)]^{\frac{1}{2}} - 1 \} = 2 \{ [1/0.918367]^{\frac{1}{2}} - 1 \} = 8.6997\%.$$

VII.

- A. No.
- B. Yes. The put price, $P_{20,1}(0)$, must equal the call price, $C_{20,1}(0)$, of \$5. Since the strike price equals the forward price, the payoff in 1 year from long the call and short the put exactly equals the payoff from long the forward contract with delivery in 1 year. But by definition, no money changes hands today with the forward contract, so the cost of the position involving the put and the call must also be zero: i.e., $C_{20,1}(0) - P_{20,1}(0) = 0$.