Problem Set 2 Solution

I. Buying Stock on Margin: BKM, Chapter 3, Question 17 (for part b. assume that interest on the broker’s loan does not accrue until the end of the year).

Part a.

A. One Answer.

1. You have $5000 today.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash 5000</td>
<td>Net Worth 5000</td>
</tr>
<tr>
<td>Total Asset 5000</td>
<td>Total Liab &amp; Net W. 5000</td>
</tr>
</tbody>
</table>

2. You borrow $5000 today.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash 10000</td>
<td>Loan 5000</td>
</tr>
<tr>
<td></td>
<td>Net Worth 5000</td>
</tr>
<tr>
<td>Total Asset 10000</td>
<td>Total Liab &amp; Net W. 10000</td>
</tr>
</tbody>
</table>

3. You invest the $10000 cash in 400 shares of Telecom. Percent Margin = 5000/10000 = 50%.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Telecom (400@$25) 10000</td>
<td>Loan 5000</td>
</tr>
<tr>
<td></td>
<td>Net Worth 5000</td>
</tr>
<tr>
<td>Total Asset 10000</td>
<td>Total Liab &amp; Net W. 10000</td>
</tr>
</tbody>
</table>

4. At the end of the period, the value of Telecom shares grows 10% to $27.50 while the amount outstanding on the loan grows 8%. Percent Margin = (11000-5400)/11000 = 50.909%.
5. At the end of the period, close out the position.

\[
\begin{array}{|c|c|}
\hline
\text{Assets} & \text{Liabilities} \\
\hline
\text{Cash} & 5600 \\
\hline
\text{Total Asset} & 5600 \\
\hline
\end{array}
\]

6. Your return is:
\[
\frac{\text{Net Worth at End} - \text{Net Worth at Start}}{\text{Net Worth at Start}} = \frac{5600 - 5000}{5000} = 12\%.
\]

B. Second Answer.

At start, $10000 is invested in Telecom by buying 400 shares.

Profit
\[
= \text{Value of 400 Telecom shares at end} - \text{Loan Outstanding at end} - \text{Net Worth at Start}
\]
\[
= 400 \times ($25 \times 1.1) - ($5000 \times 1.08) - $5000
\]
\[
= $11000 - $5400 - $5000 = $600.
\]

Return = Profit/\{Net Worth at Start\} = $600/$5000 = 12%.

Part b. Know
Margin = (Value of Stock Position - Loan Outstanding)/ Value of Stock Position.

So
\[
0.30 = \frac{(400 \times \text{Price of Telecom} - $5000)}{(400 \times \text{Price of Telecom})}
\]
\[
0.30 \times (400 \times \text{Price of Telecom}) = (400 \times \text{Price of Telecom} - $5000)
\]
\[
280 \times \text{Price of Telecom} = $5000
\]

Price of Telecom = $17.85.

II. Short-selling when the Stock Pays a Dividend: BKM, Chapter 3, Question 20 (assume the interest rate on any required margin for the short position is zero).

Assume that there is the margin requirement is 50%. For short sales,
Margin = Net Worth in Broker’s Account / Value of Stock.

So need to have a net balance in broker’s account of 0.5 x (100 x $14) =$700. So need to deposit $700 into the broker’s account on 1/1.

A. One Answer.

1. On 1/1, you borrow 100 shares of Zenith at $14.00 per share. The loan is denominated in shares of IBM not in dollars.
2. You sell the borrowed shares at $14 per share on 1/1.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 sh @ 14</td>
<td>1400</td>
</tr>
<tr>
<td>Cash</td>
<td>700</td>
</tr>
<tr>
<td>Total Asset</td>
<td>2100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>2100</td>
</tr>
<tr>
<td>less Commission</td>
<td>-50</td>
</tr>
<tr>
<td>(100 sh @ 0.5)</td>
<td></td>
</tr>
<tr>
<td>Total Asset</td>
<td>2050</td>
</tr>
</tbody>
</table>

3. On the 3/1, Zenith pays a $2 dividend per share which you have to pay. (Assume that Zenith’s price is still $14 per share.)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>2050</td>
</tr>
<tr>
<td>less Dividend</td>
<td>-200</td>
</tr>
<tr>
<td>(100 sh @ 2)</td>
<td></td>
</tr>
<tr>
<td>Total Asset</td>
<td>1850</td>
</tr>
</tbody>
</table>

4. On the 4/1, Zenith’s price is $9 per share.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>1850</td>
</tr>
<tr>
<td>Total Asset</td>
<td>1850</td>
</tr>
</tbody>
</table>
5. To close out the position on 4/1, take the following steps:

   (1) purchase 100 shares of Zenith stock.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>950</td>
</tr>
<tr>
<td>less Commission (100 sh @ 0.5)</td>
<td>-50</td>
</tr>
<tr>
<td>100 sh @ 9</td>
<td>900</td>
</tr>
<tr>
<td>Total Asset</td>
<td>1800</td>
</tr>
</tbody>
</table>

   (2) repay the stock loan.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>900</td>
</tr>
<tr>
<td>Total Asset</td>
<td>900</td>
</tr>
</tbody>
</table>

   Profit = Net Worth (4/1) - Net Worth (1/1) = $900 - $700 = $200.

   B. Second Answer.

   Profit
   = Proceeds Sale of 100 sh at $14 on 1/1 - Cost of Purchasing 100 sh at $9 on 4/1
   - Payment of Dividend of $2 per share on 100 sh - 2 x Commission on 100 sh at $0.50
   = $1400 - $900 - $200 -2x $50 = $200.

III. Expected Return, Return Standard Deviation, Covariance and Portfolios:

<table>
<thead>
<tr>
<th>State</th>
<th>Probability</th>
<th>Asset A</th>
<th>Asset B</th>
<th>Riskless Asset</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom</td>
<td>0.25</td>
<td>24%</td>
<td>14%</td>
<td>7%</td>
</tr>
<tr>
<td>Normal Growth</td>
<td>0.5</td>
<td>18%</td>
<td>9%</td>
<td>7%</td>
</tr>
<tr>
<td>Recession</td>
<td>0.25</td>
<td>2%</td>
<td>5%</td>
<td>7%</td>
</tr>
</tbody>
</table>

   A. What is the expected return on each asset?

   $E[R_A] = 0.25 \times 24\% + 0.5 \times 18\% + 0.25 \times 2\% = 15.5\%.$
   $E[R_B] = 0.25 \times 14\% + 0.5 \times 9\% + 0.25 \times 5\% = 9.25\%.$
   $E[R_f] = R_f = 0.25 \times 7\% + 0.5 \times 7\% + 0.25 \times 7\% = 7\%.$

   B. What is the standard deviation of return on each asset?
First, calculate variance

\[
\sigma_{R_A}^2 = 0.25 \times (24 \times 24) + 0.5 \times (18 \times 18) + 0.25 \times (2 \times 2) - (15.5 \times 15.5) = 307 - 240.25 = 66.75.
\]

\[
\sigma_{R_B}^2 = 0.25 \times (14 \times 14) + 0.5 \times (9 \times 9) + 0.25 \times (5 \times 5) - (9.25 \times 9.25) = 95.75 - 85.5625 = 10.1875.
\]

\[
\sigma_{R_f}^2 = 0.25 \times (7 \times 7) + 0.5 \times (7 \times 7) + 0.25 \times (7 \times 7) - (7 \times 7) = 49 - 49 = 0.
\]

Then calculate standard deviation

\[
\sigma_{R_A} = 8.1701\%.
\]

\[
\sigma_{R_B} = 3.1918\%.
\]

\[
\sigma_{R_f} = 0\%.
\]

C. What is the correlation and covariance between the returns on

1. assets A and B?

Covariance:

\[
\sigma_{R_A,R_B} = 0.25 \times (24 \times 14) + 0.5 \times (18 \times 9) + 0.25 \times (2 \times 5) - (15.5 \times 9.25) = 167.5 - 143.375 = 24.125.
\]

Correlation:

\[
\rho_{R_A,R_B} = \frac{\sigma_{R_A,R_B}}{\sigma_{R_A} \sigma_{R_B}} = \frac{24.125}{8.1701 \times 3.1918} = 0.9251
\]

2. asset A and the riskless asset?

Covariance:

\[
\sigma_{R_A,R_f} = 0.25 \times (24 \times 7) + 0.5 \times (18 \times 7) + 0.25 \times (2 \times 7) - (15.5 \times 7) = 108.5 - 108.5 = 0.
\]

Correlation:

\[
\rho_{R_A,R_f} = \frac{\sigma_{R_A,R_f}}{\sigma_{R_A} \sigma_{R_f}} = 0/0 \text{ which is not well defined.}
\]

3. asset B and the riskless asset?

Covariance:

\[
\sigma_{R_B,R_f} = 0.
\]

Correlation:

\[
\rho_{R_B,R_f} = \frac{\sigma_{R_B,R_f}}{\sigma_{R_B} \sigma_{R_f}} = 0/0 \text{ which is not well defined.}
\]
Using Dividend Yield Information: Suppose the following data is to be used by Ms Q (a risk-averse investor) to form a portfolio that consists of the small firm fund and T-bills.

\[
E[R_{\text{Small}}(t)] = 1.369\% \\
\sigma[R_{\text{Small}}(t)] = 8.779\%
\]

\[
E[\text{DP}(\text{start } t)] = 4.446\% \\
\sigma[\text{DP}(\text{start } t)] = 1.513\%
\]

\[
\sigma[\text{DP}(\text{start } t), R_{\text{Small}}(t)] = 1.967
\]

where \( \text{DP}(\text{start } t) \) is the dividend yield on the S&P 500 known at the start of month \( t \). \( R_{\text{Small}}(t) \) is the return on the small firm fund in month \( t \).

A. What is the intercept and slope coefficients from a regression of \( R_{\text{Small}}(t) \) (dependent variable) on \( \text{DP}(\text{start } t) \)?

Slope: \( \varphi_{\text{Small}, \text{DP}} = \frac{\sigma[R_{\text{Small}}(t), \text{DP}(\text{start } t)]}{\sigma[\text{DP}(\text{start } t)]^2} = 1.967/((1.513 \times 1.513) = 0.859
\)

Intercept: \( \mu_{\text{Small}, \text{DP}} = E[R_{\text{Small}}(t)] - \varphi_{\text{Small}, \text{DP}} E[\text{DP}(\text{start } t)] = 1.369 - 0.859 \times 4.446 = -2.451.
\)

B. What is the standard deviation of the residual from the regression of \( R_{\text{Small}}(t) \) on \( \text{DP}(\text{start } t) \)?

Defining \( e_{\text{Small,DP}}(t) \) to be the residual from the regression of \( R_{\text{Small}}(t) \) on \( \text{DP}(\text{start } t) \), then

\[
\sigma[R_{\text{Small}}(t)]^2 = \varphi_{\text{Small}, \text{DP}}^2 \sigma[\text{DP}(\text{start } t)]^2 + \sigma[e_{\text{Small,DP}}(t)]^2.
\]

So

\[
\sigma[e_{\text{Small,DP}}(t)]^2 = \sigma[R_{\text{Small}}(t)]^2 - \varphi_{\text{Small}, \text{DP}}^2 \sigma[\text{DP}(\text{start } t)]^2
\]

\[
= 8.779^2 - 0.859^2 \times 1.513^2 = 75.381
\]

and \( \sigma[e_{\text{Small,DP}}(t)] = 8.682. \)

C. Suppose it is the end of March 1997, Ms Q does not know \( \text{DP} \) and the return on T-bills for April is 0.3%.

1. What is the April return on the small firm fund that Ms Q expects?

\[E[R_{\text{Small}}(t)] = 1.369\%\]

2. What is the volatility of the April return on the small firm fund, given Ms Q’s information?

\[\sigma[R_{\text{Small}}(t)] = 8.779\%\]

D. Suppose it is the end of March 1997, Ms Q knows that \( \text{DP} \) is 2% and the return on T-bills for April is 0.3%.
1. What is the April return on the small firm fund that Ms Q expects?
\[ \mu_{Small,DP} + \varphi_{Small,DP} \cdot DP(start \ Apr) = -2.451 + 0.859 \times 2 = -0.733\% . \]

2. What is the volatility of the April return on the small firm fund, given Ms Q’s information?
Since Ms Q observes DP, the volatility of the April return on the small firm fund is the volatility of the regression residual:
\[ \sigma[e_{Small,DP}(t)] = 8.682\% . \]

E. Suppose it is the end of October 1997, Ms Q does not know DP and the return on T-bills for November is 0.4%.
1. What is the November return on the small firm fund that Ms Q expects?
2. What is the volatility of the April return on the small firm fund, given Ms Q’s information?

The answer to this question is the same as for part C.

F. Suppose it is the end of October 1997, Ms Q knows that DP is 5% and the return on T-bills for November is 0.4%.
1. What is the November return on the small firm fund that Ms Q expects?
\[ \mu_{Small,DP} + \varphi_{Small,DP} \cdot DP(start \ Nov) = -2.451 + 0.859 \times 5 = 1.844\% . \]
2. What is the volatility of the April return on the small firm fund, given Ms Q’s information?
Since Ms Q observes DP, the volatility of the April return on the small firm fund is the volatility of the regression residual:
\[ \sigma[e_{Small,DP}(t)] = 8.682\% . \]