Problem Set 4 Solution: CAPM

I. **SML and the CAPM:**

A. In a CAPM world, all assets lie on the SML. So

\[ E[R_p] = R_f + \beta_{p,m} \{E[R_m] - R_f \} \]
\[ 20\% = 5\% + \beta_{p,m} \{15\% - 5\% \} \]
\[ \beta_{p,m} = 15\%/10\% = 1.5. \]

B.

1. The market has a Beta with respect to the market of 1. All assets plot on the SML including the market. So the market has the same expected return as the portfolio with a \( \beta_{p,m} \) of 1.

2. All assets lie on the SML. So an asset with a \( \beta_{p,m} = 0 \) has an expected return of:

\[ E[R_p] = R_f + \beta_{p,m} \{E[R_m] - R_f \} = 4\% + 0 \{12\% - 4\% \} = 4\%. \]

3. The expected return on the stock is given by:

\[ E[R] = R_f + \beta \{E[R_m] - R_f \} = 4\% + -0.5 \{12\% - 4\% \} = 0\%. \]

The intrinsic value of the stock is given by:

\[ V_0 = E[P_1 + D_1]/(1 + E[R]) = (41+ 3)/(1.00) = 44 \]

which is greater than its current price. So it is underpriced today.

II. **SML vs CML in the CAPM:** Assume that the CAPM holds in the economy. The following data is available about the market portfolio, the riskless rate and two assets, A and B. Remember \( \beta_{i,m} = \sigma[R_i, R_m]/(\sigma[R_m]^2) \).

<table>
<thead>
<tr>
<th>Asset i</th>
<th>( E[R_i] )</th>
<th>( \sigma[R_i] )</th>
<th>( \beta_{i,m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>m (market)</td>
<td>0.15</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0.096</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.07</td>
<td>0.6</td>
<td></td>
</tr>
</tbody>
</table>

\( R_f = 0.10. \)

A. What is \( \beta_{i,m} \) for \( i \) equal to the market portfolio (i.e., \( \beta_{m,m} \))?

\[ \beta_{m,m} = \sigma[R_m, R_m]/(\sigma[R_m]^2) = 1. \]
B. What is the expected return on asset A (i.e., $E[R_A]$)?

All assets plot on the SML:

$$E[R_i] = R_f + \beta_{i,M} \{E[R_M] - R_f \}$$

So

$$E[R_A] = R_f + \beta_{A,M} \{E[R_M] - R_f \} = 0.10 + 1.2 \{0.15-0.10\} = 0.16.$$  

C. What is the expected return on asset B (i.e., $E[R_B]$)?

Similarly,

$$E[R_B] = R_f + \beta_{B,M} \{E[R_M] - R_f \} = 0.10 + 0.6 \{0.15-0.10\} = 0.13.$$  

D. Does asset A plot:
   1. on the SML (security market line)?
   Yes.
   2. on the CML (capital market line)?

Formula for the CML:

$$E[R_i] = R_f + \sigma[R_i] \{E[R_M] - R_f \}/\sigma[R_M].$$

For A,

$$R_f + \sigma[R_A] \{E[R_M] - R_f \}/\sigma[R_M] = 0.10 + 0.096 \{0.15-0.10\}/0.08 = 0.16 = E[R_A]$$

as required for A to lie on the CML.

E. Does asset B plot:
   1. on the SML?
   Yes.
   2. on the CML?

For B,

$$R_f + \sigma[R_B] \{E[R_M] - R_f \}/\sigma[R_M] = 0.10 + 0.07 \{0.15-0.10\}/0.08 = 0.14375 > 0.13 = E[R_B]$$

and so B does not lie on CML.

F. Could any investor be holding asset A as her entire portfolio?
   Yes since it lies on the CML.

G. Could any investor be holding asset B as her entire portfolio?
   No since it does not lie on the CML.

H. What is the correlation of asset A with the market portfolio?

Recall $\beta_{i,M} = \rho[R_i, R_M] \sigma[R_i] / \sigma[R_M]$ which implies $\rho[R_i, R_M] = \beta_{i,M} \sigma[R_M] / \sigma[R_i]$. 

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So, for A,
\[ \rho[R_A, \ R_M] = \beta_{A,M} \sigma[R_M] / \sigma[R_A] = \frac{1.2 \times 0.08}{0.096} = 1. \]

I. What is the correlation of asset B with the market portfolio?

Similarly, for B,
\[ \rho[R_B, \ R_M] = \beta_{B,M} \sigma[R_M] / \sigma[R_B] = \frac{0.6 \times 0.08}{0.07} = 0.6857. \]

J. Can anything be said about the composition of asset A (i.e., what assets make up asset A)?

Since A lies on the CML, it must be some combination of the market portfolio and the riskless asset.

K. Can anything be said about the composition of asset B?

No.