I. [15 points] You will be making 20 annual contributions of $75 to a bank account (the first is made at the end of year 1) that pays an APR of 5.90% compounded semiannually. What will be the balance of the bank account at the end of year 20?

II. [10 points] I have available $5M to invest today. The continuously compounded annual interest rate is 10%. How much will I have in 6 months?

III. [15 points] One year ago XYZ stock had just run up from $12 per share to $25 per share. With a net worth of $20000, you bought $40000 worth of XYZ stock on margin at $25 per share. The call money rate (which was the rate at which your broker would lend to you) was 8.5% per annum EAR. The stock recently declared its first dividend: $1 per share. (The dividend is payable in 10 days. The ex dividend date is tomorrow.) The stock is presently trading at $27 per share. Commissions are $0.50 per share (each way), payable when you close out your position. If you close out your position today, what is your total profit or loss on the entire transaction.

IV. [30 points] Consider the following data:

<table>
<thead>
<tr>
<th>Economy</th>
<th>Probability</th>
<th>Return on Arctic Stock</th>
<th>Return on Zebra Stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>0.6</td>
<td>15%</td>
<td>30%</td>
</tr>
<tr>
<td>Bad</td>
<td>0.4</td>
<td>-10%</td>
<td>-15%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stock</th>
<th>Expected Return</th>
<th>Standard Deviation of Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arctic</td>
<td>5%</td>
<td>12.25%</td>
</tr>
<tr>
<td>Zebra</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

A. What is the expected return on Zebra?
B. What is the return standard deviation on Zebra?
C. What is the covariance between Arctic’s return and Zebra’s return?
D. What are the expected return and standard deviation of return on a portfolio that is 20% invested in Zebra and 80% in Arctic?
Practice Midterm Questions

V. [15 points] Consider the following data for QDF stock (1/29 is a Friday):

<table>
<thead>
<tr>
<th>Date</th>
<th>1/29</th>
<th>2/1</th>
<th>2/2</th>
<th>2/3</th>
<th>2/4</th>
<th>2/5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closing Price-QDF share</td>
<td>41</td>
<td>42.5</td>
<td>41.125</td>
<td>41.75</td>
<td>43.25</td>
<td>43</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dividend per QDF Share</th>
<th>Declared</th>
<th>Ex-date</th>
<th>Record Date</th>
<th>Payable Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1/20</td>
<td>2/1</td>
<td>2/4</td>
<td>3/5</td>
</tr>
</tbody>
</table>

Calculate the daily return on QDF stock (buy at the close of the previous day and sell on the close of the current day) for the following dates:
A. 2/1.
B. 2/2.
C. 2/3.

VI. [15 points] You are saving to buy a $150000 house. At the end of each of the next five years, you will deposit $5000 into a bank account. At the end of five years you will use the money as a down payment on the house. You will finance the balance of the purchase price with a thirty-year annual-payment mortgage. If the investing rate is 5% per annum EAR and the borrowing rate is 7% per annum EAR, what is the size of your annual mortgage payments (which are made at the end of each year of the mortgage).

VII. [20 points] Consider the following:

<table>
<thead>
<tr>
<th></th>
<th>Expected Return</th>
<th>Standard Deviation of Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japanese Stock Fund</td>
<td>15%</td>
<td>20%</td>
</tr>
<tr>
<td>U.S. Stock Fund</td>
<td>12%</td>
<td>10%</td>
</tr>
</tbody>
</table>

The correlation between the return on the U.S. stock fund and the Japanese stock fund is 0.2. The rate on T-bills is 5%.
A. Suppose that Sure-thing Brokers’ recommended portfolio is 70% in the U.S. stock fund and 30% in the Japanese stock fund. What is the expected return and standard deviation of return for this portfolio?
B. An investor is trying to allocate a $20000 investment between Sure-thing’s recommended portfolio and T-bills to achieve a portfolio return standard deviation of 5%. What dollar amounts should be invested in the Japanese stock fund, the U.S. stock fund and T-bills.
VIII. [30 points] Consider the following data:

<table>
<thead>
<tr>
<th></th>
<th>Expected Return</th>
<th>Standard Deviation of Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bull Fund</td>
<td>16%</td>
<td>15%</td>
</tr>
<tr>
<td>Hosem Fund</td>
<td>12%</td>
<td>8%</td>
</tr>
</tbody>
</table>

The correlation between the return on the Bull fund and the Hosem fund is 0.7. The rate on T-bills is 8%.

A. Suppose I am trying to decide whether to hold Bull in combination with T-bills or Hosem in combination with T-bills. Which should I choose? (Show calculations to support your answer.)

B. Suppose instead that I can combine Bull and Hosem into a portfolio P which I would then combine with T-bills to obtain my final portfolio.

1. What are the weights of Bull and Hosem in portfolio P?
2. What is the expected return and standard deviation of return for portfolio P?

---

Foundations of Finance
Practice Midterm Answers
Prof. Anthony Lynch

I. \[ \text{EAR} = (1+0.059/2)^2 -1 = 0.05987. \]
\[ \text{FV}_{20} = 75 \times \text{FVAF}_{5.987\%,20} = 75 \times \frac{(1.05987)^{20} -1}{0.05987} = 75 \times 36.7342 = 2755.07. \]

II. \[ \text{FV}_{0.5} = 5M \times e^{0.1x0.5} = 5M \times 1.051271 = 5.2564M. \]

III.  
<table>
<thead>
<tr>
<th>Transaction</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borrow at 8.5% one year ago</td>
<td>20000</td>
</tr>
<tr>
<td>Buy 1600 shs of XYZ @ $25 one year ago</td>
<td>-40000</td>
</tr>
<tr>
<td>Repay loan ($20000 x 1.085) today</td>
<td>-21700</td>
</tr>
<tr>
<td>Sell 1600 shs of XYZ @ $27 today</td>
<td>43200</td>
</tr>
<tr>
<td>Commission (2x1600 x $0.50)</td>
<td>-1600</td>
</tr>
<tr>
<td>Total Profit</td>
<td>-100</td>
</tr>
</tbody>
</table>

So the total loss is $100.
(Dividend is not paid until tomorrow and so is irrelevant; the price run up prior to purchase is also irrelevant.)
IV. Answer.
A. Expected Return Zebra = 0.6 x 30% + 0.4 x (-15%) = 12%.
B. Variance of Return Zebra = 0.6 x (30x30) + 0.4 x (-15x-15) - (12x12) = 486.
   Std Dev of Return Zebra = 22.0454%.
C. Covariance Arctic & Zebra = 0.6 x (15x30) + 0.4 x (-10x-15) - (5x12) = 270.
D. Expected Portfolio Return = 0.8 x 5% + 0.2 x 12% = 6.4%.
   Variance of Portfolio Return
   = (0.8x0.8) x (12.25x12.25) + (0.2x0.2) x 486 + 2 x (0.8x0.2) x 270 = 201.88.
   Std Dev of Portfolio Return = 14.2084%.

V. Answer.
A. Return 2/1 = \{42.5 + 0.5 - 41\}/41 = 4.878%.
B. Return 2/2 = \{41.125 - 42.5\}/42.5 = -3.2353%.
C. Return 2/3 = \{41.75 - 41.125\}/41.125 = 1.5198%.

VI. Savings at time 5 will be $5000 x FVAF_{5%,5} = $5000 x \{[(1.05)^5-1]/0.05\} = $27628.16.
   Amount to be borrowed at time 5 is $150000 - $27628.16 = $122371.84.
   So letting C be the mortgage repayment:
   $122371.84 = C x PVAF_{7%,30} = C x \{[1-(1.07)^{-30}]/0.07\}.
   Thus, C = $122371.84 / 12.40904 = $9861.51.

VII. Let P be Sure-thing’s recommended portfolio.
A. Expected Return for P = 0.7 x 12% + 0.30 x 15% = 12.9%.
   Variance of Return for P
   = (0.7x0.7) x (10x10) + (0.3x0.3) x (20x20) + 2 x (0.7x0.3) x(0.2x10x20)
   = 49 + 36 + 16.8 = 101.8
   Standard Deviation of Return for P = 10.0896.
B. Let Q be the investor’s portfolio with a standard deviation of 5% which consists
   of portfolio P and T-bills. Let \(\omega_{p,Q}\) be the weight of portfolio P in portfolio Q.
   Know \(\sigma_{R_Q} = |\omega_{p,Q}| \sigma_{R_P}\) and since the expected return for P exceeds the T-bill
   rate, we know that the investor wants to hold a positive weight in P.
   So \(\omega_{p,Q} = 5/10.0896 = 0.4956\).
   So the weight of T-bills in Q is \(1- \omega_{p,Q}\) = 0.5044 which implies a dollar
   investment in T-bills of $20000 x 0.5044 = $10088.
   The weight of the U.S stock fund in Q is \(0.7 \omega_{p,Q}\) = 0.7 x0.4956 = 0.34692
   which implies a dollar investment in the U.S. stock fund of $20000 x 0.3469 = $6938.4.
   The weight of the Japanese stock fund in Q is \(0.3 \omega_{p,Q}\) = 0.3 x0.4956 = 0.1487
   which implies a dollar investment in the Japanese stock fund of $20000 x 0.14868 = $2973.6.

VIII. Answer.
A. Slope - CAL(Bull) = \{16 - 8\}/15 = 0.5333.
Slope - CAL(Hosem) = \( \{12 - 8\}/8 = 0.5 \).
Prefer Bull and T-bills since the associated CAL has a higher slope.

B. Portfolio P is the tangency portfolio.

1. To get the weight of Bull in P use the following formula

\[
\omega_{B,P} = \frac{\sigma[R_H]^2 E[r_B] - \sigma[R_B, R_H] E[r_H]}{\{\sigma[R_H]^2 E[r_B] - \sigma[R_B, R_H] E[r_H]\} + \{\sigma[R_B]^2 E[r_H] - \sigma[R_B, R_H] E[r_B]\}}
\]

where \( r_i = R_i - R_f \) is the excess return on asset \( i \). Can calculate

\[
\sigma[R_B, R_H] = 0.7 \times 15 \times 8 = 84; \\
\sigma[R_B]^2 = 15 \times 15 = 225; E[r_B] = 8; \\
\sigma[R_H]^2 = 8 \times 8 = 64; E[r_H] = 4; \text{ to give} \\
\omega_{B,P} = \{64x8 - 84x4\}/\{\{64x8 - 84x4\} + \{225x4 - 84x8\}\} \\
= 176/176 + 228 = 0.4356.
\]

So the weight of Hosem in P is \( 1 - \omega_{B,P} \) = 0.5644

2. Expected Return for P = 0.4356 x 16% + 0.5644 x 12% = 13.7424%.
Variance of Return for P = (0.4356x0.4356) x 225 + (0.5644x0.5644) x 64 + 2 (0.4356x0.5644) x 84 = 42.693 + 20.387 + 41.303 = 104.383
Standard Deviation of Return for P = 10.2168%. 


I. [22 points] Mr X borrows a sum of money from the YZ bank on 1/1/96 at an APR of 15% compounded monthly. The loan agreement stipulates that he make monthly payments of $100 for 48 months, the first payment to made on 2/1/96. There are no up-front finance charges.

A. What is the effective annual rate for the loan?
B. What is the continuously compounded annual rate for the loan?
C. How much did Mr X borrow from the bank on 1/1/96?
D. What is the balance of the loan outstanding on 1/1/97 after the 12th payment has been made (calculate the balance exactly; do not use the rule of 78ths to estimate it).

II. [6 points] The specialist’s order book and bid and ask prices for IBM on 3/9/96 are given by the following table:

<table>
<thead>
<tr>
<th>Price</th>
<th>Limit Buy</th>
<th>Limit Sell</th>
<th>Specialist</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>100 sh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100.875</td>
<td>100 sh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100.75</td>
<td>100 sh</td>
<td></td>
<td>ask</td>
</tr>
<tr>
<td>100.625</td>
<td>100 sh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100.5</td>
<td>100 sh</td>
<td></td>
<td>bid</td>
</tr>
<tr>
<td>100.375</td>
<td>100 sh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100.25</td>
<td>100 sh</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A. If a market buy order comes in for 100 shares, at what price will the trade execute?
B. If a market sell order comes in for 100 shares, at what price will the trade execute?

III. [18 points] The following price data is available for VB stock and QM stock.

<table>
<thead>
<tr>
<th>Date</th>
<th>12/31/93</th>
<th>12/31/94</th>
<th>6/30/95</th>
<th>12/31/95</th>
</tr>
</thead>
<tbody>
<tr>
<td>VB</td>
<td>70</td>
<td>50</td>
<td>65</td>
<td>55</td>
</tr>
<tr>
<td>QM</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>18</td>
</tr>
</tbody>
</table>
At the end of 1994, with $50000 of your own money, you short sold $80000 worth of VB stock at $50 per share and used the proceeds together with your $50000 to buy QM stock at $15 per share (ignore any margin requirements associated with the short sale). The call money rate (which was the rate at which your broker would lend to you) was 5% per annum EAR. Neither stock paid any dividends in 1994 or 1995. Ignore commissions.

A. What is the 1995 return on VB (from the end of 1994 to the end 1995)?
B. What is the 2 year return on VB (from the end of 1993 to the end 1995)?
C. What is the 1995 return on QM (from the end of 1994 to the end 1995)  
D. What is the one year return on your position if you close it out at the end of 1995? 
E. What is your total dollar profit or loss on your transaction if you close it out at the end of 1995?

IV. [21 points] Consider the following:

<table>
<thead>
<tr>
<th></th>
<th>Expected Return</th>
<th>Standard Deviation of Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. Stock Fund</td>
<td>12%</td>
<td>10%</td>
</tr>
<tr>
<td>Japanese Stock Fund</td>
<td>16%</td>
<td>20%</td>
</tr>
</tbody>
</table>

The correlation between the return on the U.S. stock fund and the return on the Japanese stock fund is 0.4. The rate on T-bills is 5%.

A. Suppose that Sure-thing Brokers’ recommended portfolio is 70% in the U.S. stock fund and 30% in the Japanese stock fund. What is the expected return and standard deviation of return for this portfolio?
B. An investor allocates a $20000 investment between Sure-thing’s recommended portfolio and T-bills to achieve a portfolio return standard deviation of 6%. A positive amount is invested in Sure-thing’s recommended portfolio. What is the expected return on the investor’s investment?
C. Jack is a risk-averse investor who only cares about expected return and standard deviation of return. Jack can either hold the Japanese stock fund in combination with T-bills or the U.S. stock fund in combination with T-bills. Which of these risky assets (the Japanese stock fund or the U.S. stock fund) would Jack prefer to hold in combination with T-bills?
V. [21 points] Assume that the CAPM holds in the economy. The following data is available about the market portfolio, the riskless rate and two assets, Y and Z. Further, Tom Hyde, an individual in the economy, holds asset Y as his total portfolio of assets. The riskless rate $R_f$ is 5%. Remember $\beta_{i,M} = \frac{\sigma[R_i, R_M]}{\sigma[R_M]^2}$.

<table>
<thead>
<tr>
<th>Asset i</th>
<th>$E[R_i]$</th>
<th>$\sigma[R_i]$</th>
<th>$\beta_{i,M}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M (market)</td>
<td>15%</td>
<td>12%</td>
<td>1</td>
</tr>
<tr>
<td>Y</td>
<td>?</td>
<td>9%</td>
<td>?</td>
</tr>
<tr>
<td>Z</td>
<td>?</td>
<td>9%</td>
<td>0.25</td>
</tr>
</tbody>
</table>

A. What is the expected return on asset Z (i.e., $E[R_Z]$)?
B. What is the correlation of asset Z with the market portfolio?
C. What is the expected return on asset Y (i.e., $E[R_Y]$)?
D. What is the Beta of asset Y with respect to the market portfolio (i.e., $\beta_{Y,M}$)?

VI. [12 points] Suppose Tom and Joan both form portfolios using 3 risky assets Microsoft, ADM and Nike and the riskless asset. Both agree on the return distributions for the 4 assets and only care about expected portfolio return and standard deviation of portfolio return. Tom decides to hold the following weights in the 4 assets:

<table>
<thead>
<tr>
<th>Asset</th>
<th>Riskless Asset</th>
<th>ADM</th>
<th>Microsoft</th>
<th>Nike</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>40%</td>
<td>10%</td>
<td>20%</td>
<td>30%</td>
</tr>
</tbody>
</table>

Joan decides to hold 70% in the riskless asset. Can you determine the weights of the 3 risky assets in her portfolio? If so, what are the weights of the 3 risky assets in her portfolio? If not, describe what is similar about the portfolios that Joan and Tom decide to hold and what is different?
I.  
   A. \( r_1 = \text{APR}_{12}/12 = 15%/12 = 1.25\%; \quad r_{12} = (1+r_1)^{12}-1 = (1+0.0125)^{12}-1 = 16.075\%. \)
   B. \( r'_{12} = \ln(1+r_{12}) = \ln(1+0.16075) = 14.9066\%. \)
   C. \( V_{48}^0 = 100 \times \text{PVAF}_{1.25\%,48} = 100 \times \frac{1-(1+0.0125)^{-48}}{0.0125} = 3593.15. \)
   D. \( V_{36}^{12} = 100 \times \text{PVAF}_{1.25\%,36} = 100 \times \frac{1-(1+0.0125)^{-36}}{0.0125} = 2884.73. \)

II.  
   A. 100.75 - lower of ask and lowest limit sell.
   B. 100.625 - higher of bid and highest limit buy.

III.  
   A. \( R_{VB}(95) = \frac{55-50}{50} = 10\%. \)
   B. \( R_{VB}(94-95) = \frac{55-70}{70} = -21.428\%. \)
   C. \( R_{QM}(95) = \frac{18-15}{15} = 20\%. \)
   D. \( \omega_{VB, p} = -80000/50000 = -1.6; \quad \omega_{QM, p} = (80000+50000)/50000 = 1 - \omega_{VB, p} = 2.6; \)
   \( R_p(95) = \omega_{VB, p} \times R_{VB}(95) + \omega_{QM, p} \times R_{QM}(95) = -1.6 \times 10\% + 2.6 \times 20\% = 36\%. \)
   E. Profit \( p(\text{end 95}) = R_p(95) \times V_p(\text{end 94}) = 0.36 \times 50000 = 18000. \)

IV.  
   A. \( E[R_p] = 0.7 \times 12\% + 0.3 \times 16\% = 13.2\%. \)
   \( \sigma^2[R_p] = (0.7 \times 0.7) \times (10 \times 10) + (0.3 \times 0.3) \times (20 \times 20) + 2 \times (0.7 \times 0.3) \times (0.4 \times 10 \times 20) \)
   \( = 49 + 36 + 33.6 = 118.6; \quad \sigma[R_p] = 10.8904. \)
   B. \( \sigma[R_y] = \omega_{p,I} \times \sigma[R_p]; \) so 6\% = \( \omega_{p,I} \times 10.804\%; \) so \( \omega_{p,I} = 0.55094. \)
   C. \( E[R_y] = \omega_{p,I} \times E[R_p] + (1 - \omega_{p,I}) \times R_f = 0.5509 \times 13.2\% + 0.4491 \times 5\% = 9.5177\%. \)

V.  
   A. Z plots on SML; so \( E[R_Y] = R_f + \{E[R_M] - R_f \} \beta_{Z,M} = 5\% + \{15-5\}% \times 0.25 = 7.5\%. \)
   B. \( \rho[R_y, R_M] = \beta_{Z,M} \times \sigma[R_M]/\sigma[R_y] = 0.25 \times 12/9 = 0.3333. \)
   C. Since Hyde holds \( Y \) as his total portfolio, \( Y \) lies on CML; so \( E[R_Y] = R_f + \{E[R_M] - R_f \} \sigma[R_Y]/\sigma[R_M] = 5\% + \{15-5\}% \times 9/12 = 12.5\%. \)
   D. Y plots on SML; so 12.5\% = \( R_f + \{E[R_M] - R_f \} \beta_{Y,M}; \) and \( \beta_{Y,M} = (12.5-5)/(15-5) = 0.75. \)

VI.  
Both Joan and Tom hold \( R_f \) and the same Tangency portfolio of the three risky assets. Joan has 30\% in T and Tom has 60\%. Can use Tom’s portfolio to determine weights of risky assets in the Tangency portfolio: \( \omega_{ADM,Tom} = \omega_{ADM,T} \times \omega_{T,Tom} \) so \( \omega_{ADM,T} = \omega_{ADM,Tom} / \omega_{T,Tom} = 0.1 / 0.6 = 0.1667; \omega_{Ms,T} = \omega_{Ms,Tom} / \omega_{T,Tom} = 0.2 / 0.6 = 0.3333; \omega_{Nk,T} = 0.3 / 0.6 = 0.5. \) Can now get weights of the three risky assets in Joan’s portfolio: \( \omega_{ADM,J} = \omega_{ADM,T} \times \omega_{T,J} = 0.1667 \times 0.3 = 0.05; \omega_{Ms,J} = 0.3333 \times 0.3 = 0.1; \omega_{Nk,J} = 0.5 \times 0.3 = 0.15. \)
Instructions: 80 minutes. Closed book. You are permitted use of a financial calculator and one sheet (8½" x 11") of notes. Answer all questions. Each question carries the number of points indicated. Total points is 100. Show all work on the exam paper itself. (If you wish, you can indicate which calculator entries you made to get your answer.) If you do not have or can not calculate a number that you need, make an assumption and proceed with the rest of the question. No credit will be given for illegible, unsupported or ambiguous answers.

Name: ____________________________________________________________

Student #: _________________________________________________________

I pledge my honor that I have not violated the Stern Student Code of Conduct in the completion of this exam.

Signature: _________________________________________________________
I. [16 points] Joe and Jane’s new daughter Joanne has just been born. Joe and Jane would like to open a bank account today to pay for their daughter’s education. The school that they have selected for Joanne has annual fees of $5000 payable on Joanne’s birthday during each year that she attends the school. Her first year’s fees would be paid on her 6th birthday and her last year’s fees would be paid on her 17th birthday. The APR of their bank account is 12% compounded quarterly.

A. What is the effective annual rate (EAR) being offered by their bank account?
Joe and Jane’s new daughter Joanne has just been born. Joe and Jane would like to open a bank account today to pay for their daughter’s education. The school that they have selected for Joanne has annual fees of $5000 payable on Joanne’s birthday during each year that she attends the school. Her first year’s fees would be paid on her 6th birthday and her last year’s fees would be paid on her 17th birthday. The APR of their bank account is 12% compounded quarterly.

B. How much must they deposit today to be able to exactly pay the 12 annual payments of $5000 out of their account.
II. [6 points] The following data is available for asset L:

<table>
<thead>
<tr>
<th>State</th>
<th>Good</th>
<th>Bad</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>Return on Asset L</td>
<td>25%</td>
<td>-5%</td>
</tr>
</tbody>
</table>

A. What is the expected return on asset L?

B. What is the variance of the return on asset L?
III. [8 points] Consider the following data for DFL stock (3/5 is a Friday):

<table>
<thead>
<tr>
<th>Date</th>
<th>3/5</th>
<th>3/8</th>
<th>3/9</th>
<th>3/10</th>
<th>3/11</th>
<th>3/12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closing Price-DFL share</td>
<td>41</td>
<td>42.5</td>
<td>41.125</td>
<td>41.75</td>
<td>43.25</td>
<td>43</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dividend per DFL Share</th>
<th>Declared</th>
<th>Ex-date</th>
<th>Record Date</th>
<th>Payable Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>2/20</td>
<td>3/9</td>
<td>3/12</td>
<td>3/28</td>
</tr>
</tbody>
</table>

Calculate the daily return on DFL stock (buy at the close of the previous day and sell on the close of the current day) for the following dates:


B. 3/11.
IV. [15 points] The following price data is available for VB stock and QM stock.

<table>
<thead>
<tr>
<th>Date</th>
<th>12/31/93</th>
<th>6/30/94</th>
<th>12/31/94</th>
<th>12/31/95</th>
</tr>
</thead>
<tbody>
<tr>
<td>VB</td>
<td>70</td>
<td>63</td>
<td>80.5</td>
<td>90</td>
</tr>
<tr>
<td>QM</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>18</td>
</tr>
</tbody>
</table>

At the end of 1993, using $50000 of your own money, you bought $70000 worth of VB stock at $70 per share on margin. The call money rate (which was the rate at which your broker would lend to you) was 5% per annum EAR. Neither stock paid any dividends in 1994 or 1995. Ignore commissions.

A. What is the 1994 return on VB (from the end of 1993 to the end 1994)?

B. What is the one year return on your position if you close it out at the end of 1994?
### Data Restated:

<table>
<thead>
<tr>
<th>Date</th>
<th>12/31/93</th>
<th>6/30/94</th>
<th>12/31/94</th>
<th>12/31/95</th>
</tr>
</thead>
<tbody>
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<td>VB</td>
<td>70</td>
<td>63</td>
<td>80.5</td>
<td>90</td>
</tr>
<tr>
<td>QM</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>18</td>
</tr>
</tbody>
</table>

At the end of 1993, using $50000 of your own money, you bought $70000 worth of VB stock at $70 per share *on margin*. The call money rate (which was the rate at which your broker would lend to you) was 5% per annum EAR. Neither stock paid any dividends in 1994 or 1995. Ignore commissions.

C. What is your total dollar profit or loss on your position if you close it out at the end of 1994?
V. [21 points] Consider the following:

<table>
<thead>
<tr>
<th>Fund</th>
<th>Expected Return</th>
<th>Standard Deviation of Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500 Fund</td>
<td>12%</td>
<td>10%</td>
</tr>
<tr>
<td>Emerging Market Fund</td>
<td>16%</td>
<td>20%</td>
</tr>
</tbody>
</table>

The correlation between the return on the S&P 500 fund and the return on the Emerging market fund is 0.3. The rate on T-bills is 10%.

A. Suppose that Bull Broker’s recommended portfolio is 60% in the S&P 500 fund and 40% in the Emerging market fund. What is the expected return and standard deviation of return for this portfolio?
Data Restated:

<table>
<thead>
<tr>
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<td>20%</td>
</tr>
</tbody>
</table>

The correlation between the return on the S&P 500 fund and the return on the Emerging market fund is 0.3. The rate on T-bills is 10%.

B. An investor allocates a $10000 investment between Bull Broker’s recommended portfolio and T-bills to achieve a portfolio return standard deviation of 7%. A positive amount is invested in Bull Broker’s recommended portfolio. What is the expected return on the investor’s investment?
Data Restated:

<table>
<thead>
<tr>
<th></th>
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<th>Standard Deviation of Return</th>
</tr>
</thead>
<tbody>
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</table>

The correlation between the return on the S&P 500 fund and the return on the Emerging market fund is 0.3. The rate on T-bills is 10%.

C. Jill is a risk-averse investor who only cares about expected return and standard deviation of return. Jill can either hold the Emerging market fund in combination with T-bills or the S&P 500 fund in combination with T-bills. Which of these risky assets (the Emerging market fund or the S&P 500 fund) would Jill prefer to hold in combination with T-bills?
VI. [12 points] Suppose the following data is to be used by Ms Q (a risk-averse investor) to form a portfolio that consists of the small firm fund and T-bills.

\[
E[R_{Small}(t)] = 1.5 \quad \sigma[R_{Small}(t)] = 8 \quad E[DP=start(t)] = 4.375 \quad \sigma[DP=start(t)] = 1.2
\]

\[
\mu_{Small,DP} = -2 \quad \phi_{Small,DP} = 0.8
\]

where DP(start t) is the dividend yield on the S&P 500 known at the start of month t. 
\( R_{Small}(t) \) is the return on the small firm fund in month t. 
\( \mu_{Small,DP} \) is the intercept and \( \phi_{Small,DP} \) is the slope coefficient from a regression of \( R_{Small}(t) \) (dependent variable) on DP(start t).

A. Suppose it is the end of March 1997, Ms Q knows that DP(start Apr) is 3 and the return on T-bills for April is 0.5%.

1. What is the expected April return on the small firm fund given DP(start Apr)?

2. Will Ms Q short sell the small firm fund? Why or why not?
VI cont Data Restated: Suppose the following data is to be used by Ms Q (a risk-averse investor) to form a portfolio that consists of the small firm fund and T-bills.

\[ E[R_{Small}(t)] = 1.5\% \quad \sigma[R_{Small}(t)] = 8 \quad E[DP(start t)] = 4.375 \quad \sigma[DP(start t)] = 1.2 \]

\[ \mu_{Small,DP} = -2 \quad \phi_{Small,DP} = 0.8 \]

where \( DP(\text{start } t) \) is the dividend yield on the S&P 500 known at the start of month \( t \). 
\( R_{Small}(t) \) is the return on the small firm fund in month \( t \) (expressed as a %).
\( \mu_{Small,DP} \) is the intercept and \( \phi_{Small,DP} \) is the slope coefficient from a regression of \( R_{Small}(t) \) (dependent variable) on \( DP(\text{start } t) \).

B. Suppose it is the end of October 1997, Ms Q knows that \( DP(\text{start Nov}) \) is 5 and the return on T-bills for November is 0.4%.

1. What is the expected November return on the small firm fund given \( DP(\text{start Nov}) \)?

2. Will Ms Q short sell the small firm fund? Why or why not?
VII. [22 points] Assume that the CAPM holds in the economy. The following data is available about the market portfolio, the riskless rate and two assets, W and X. The riskless rate $R_f$ is 4%. Remember $\beta_{i,M} = \sigma[R_i, R_M]/(\sigma[R_M]^2)$.

<table>
<thead>
<tr>
<th>Asset</th>
<th>$E[R_i]$</th>
<th>$\sigma[R_i]$</th>
<th>$\beta_{i,M}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M (market)</td>
<td>?</td>
<td>10%</td>
<td>?</td>
</tr>
<tr>
<td>W</td>
<td>16%</td>
<td>12%</td>
<td>1.0</td>
</tr>
<tr>
<td>X</td>
<td>?</td>
<td>6%</td>
<td>0.6</td>
</tr>
</tbody>
</table>

A. What is the Beta of the market portfolio (i.e., $\beta_{M,M}$)?

B. What is the expected return on the market portfolio (i.e., $E[R_M]$)?

C. What is the expected return on asset X (i.e., $E[R_X]$)?
Data Restated:
Assume that the CAPM holds in the economy. The riskless rate $R_f$ is 4%. Remember $\beta_{i,M} = \frac{\sigma[R_i, R_M]}{(\sigma[R_M])^2}$.

<table>
<thead>
<tr>
<th>Asset i</th>
<th>$E[R_i]$</th>
<th>$\sigma[R_i]$</th>
<th>$\beta_{i,M}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M (market)</td>
<td>?</td>
<td>10%</td>
<td>?</td>
</tr>
<tr>
<td>W</td>
<td>16%</td>
<td>12%</td>
<td>1.0</td>
</tr>
<tr>
<td>X</td>
<td>?</td>
<td>6%</td>
<td>0.6</td>
</tr>
</tbody>
</table>

D. What is the correlation of asset X with the market portfolio?

E. Does asset W lie on the Capital Market Line (CML)? Explain why or why not.
VII cont  

*Data Restated:*
Assume that the CAPM holds in the economy. The riskless rate $R_f$ is 4%. Remember

$$\beta_{i,M} = \frac{\sigma[R_i, R_M]}{(\sigma[R_M])^2}.$$

<table>
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</tr>
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<td>16%</td>
<td>12%</td>
<td>1.0</td>
</tr>
<tr>
<td>X</td>
<td>?</td>
<td>6%</td>
<td>0.6</td>
</tr>
</tbody>
</table>

F. Would a portfolio with 40% in asset X and 60% in asset W lie on the Capital Market Line (CML)? Explain why or why not?
Foundations of Finance Midterm Exam Spring 1997 Answer Key
Prof. Anthony Lynch

I. 
A. \( \text{r}_{1/4} = \text{APR}_{4}/4 = 12\%/4 = 3\%; \text{EAR} = \left(1 + \text{r}_{1/4}\right)^{4} - 1 = (1+0.03)^{4} - 1 = 12.55\%. \)
B. \( V_{5} = 5000 \times \text{PVAF}_{12.55\%, 12} = 5000 \{1-(1+0.1255)^{-12}/0.1255\} = 30198.35. \)
\( V_{0} = V_{5} \times \text{PVIF}_{1.255\%, 5} = 30198.35 \left(1+0.1255\right)^{-5} = 16720.74. \)

II. 
A. \( \text{E}[R_{L}] = p\{G\} \text{R}_{L}\{G\} + p\{B\} \text{R}_{L}\{B\} = 0.4 \times 25\% + 0.6 \times (-5\%) = 7\%. \)
B. \( \sigma^{2}[R_{L}] = p\{G\} (R_{L}\{G\} - \text{E}[R_{L}])^{2} + p\{B\} (R_{L}\{B\} - \text{E}[R_{L}])^{2} = 0.4 \times (25-7)^{2} + 0.6 \times (-5-7)^{2} = 216. \)

III. 
A. \( \text{Return } 3/9 = \frac{41.125 + 0.8 - 42.5}{42.5} = -1.353\%. \)
B. \( \text{Return } 3/11 = \frac{43.25 - 41.75}{41.75} = +3.593\%. \)

IV. 
A. \( \text{R}_{VB}(94) = \frac{(80.5 - 70)}{70} = 15\%. \)
B. Buying VB on margin: so other asset is the riskless asset.
\( \omega_{VB,p} = 70000/50000 = 1.4; \omega_{f,p} = -20000/50000 = -0.4; \)
\( \text{R}_{p}(94) = \omega_{VB,p} \text{R}_{VB}(94) + \omega_{f,p} \text{R}_{f}(94) = 1.4 \times 15\% + (-0.4) \times 5\% = 19\%. \)
C. \( \text{Dollar-Profit}_{p,\text{end 94}} = \text{R}_{p}(94) \text{V}_{p,\text{end 93}} = 0.19 \times 50000 = 9500. \)

V. 
A. \( \text{E}[R_{p}] = 0.6 \times 12\% + 0.4 \times 16\% = 13.6\%. \)
\( \sigma^{2}[R_{p}] = (0.6 \times 0.6) \times (10 \times 10) + (0.4 \times 0.4) \times (20 \times 20) + 2 \times (0.6 \times 0.4) \times (0.3 \times 10 \times 20) = 36 + 64 + 28.8 = 128.8; \sigma[R_{p}] = \sqrt{128.8} = 11.3490\%. \)
B. \( \sigma[R_{f}] = \omega_{p,1} \sigma[R_{f}]; \text{ so } 7\% = \omega_{p,1} \times 11.3490\%; \text{so } \omega_{p,1} = 0.6168. \)
\( \text{E}[R_{f}] = \omega_{p,1} \text{E}[R_{f}] + (1 - \omega_{p,1}) \text{R}_{f} = 0.6168 \times 13.6\% + 0.3832 \times 10\% = 12.2205\%. \)
C. \( \text{slope-CAL}[S&P] = (12\%-10\%)/10\% = 0.2; \text{slope-CAL}[EM] = (16\%-10\%)/20\% = 0.3; \text{slope-CAL}[EM] > \text{slope-CAL}[S&P] \text{ so prefer EM and T-bills. } \)

VI. 
A. Know DP(start Apr) = 3.
1. \( \text{E}[R_{\text{Small}(\text{Apr})}] = \mu_{\text{Small},\text{DP}} + \varphi_{\text{Small},\text{DP}} \text{DP(start Apr)} = -2 + 0.8 \times 3 = 0.4\% \)
2. \( \text{E}[R_{\text{Small}(\text{Apr})}] < \text{R}_{f}(\text{Apr}) = 0.5\% \text{ so Mrs Q wants to short-sell Small to lie on the positive-sloped portion of Small’s capital allocation line.} \)
B. Know DP(start Nov) = 5.
1. \( \text{E}[R_{\text{Small}(\text{Nov})}] = \mu_{\text{Small},\text{DP}} + \varphi_{\text{Small},\text{DP}} \text{DP(start Nov)} = -2 + 0.8 \times 5 = 2.0\% \)
2. \( \text{E}[R_{\text{Small}(\text{Nov})}] > \text{R}_{f}(\text{Nov}) = 0.4\% \text{ so Mrs Q does not want to short-sell Small so she can lie on the positive-sloped portion of Small’s CAL.} \)

VII. 
A. \( \beta_{MM} = 1. \)
B. \( \text{M lies on SML. W lies on SML. } \beta_{MM} = \beta_{WW} = 1. \text{ So E}[R_{M}] = E[R_{W}] = 16\%. \)
C. \( \text{X plots on SML; so E}[R_{X}] = \text{R}_{f} + \{E[R_{M}] - \text{R}_{f}\} \beta_{XM} = 4\% + \{16-4\}\% \times 0.6 = 11.2\%. \)
D. \( \rho[R_{X}, R_{M}] = \beta_{XM} \sigma[R_{M}]/\sigma[R_{X}] = 0.6 \times 10/6 = 1. \)
E. \( \text{W lies on CML if and only if } \rho[R_{W}, R_{M}] = 1. \text{ But } \rho[R_{W}, R_{M}] = \beta_{WM} \sigma[R_{M}]/\sigma[R_{W}] \)
= 1\times\frac{10}{12} = 0.833<1. \ So \ W \ does \ not \ lie \ on \ CML.

F. \ X \ lies \ on \ CML \ since \ \rho[R_X,R_M] = 1. \ W \ does \ not \ lie \ on \ CML. \ So \ \rho[R_W,R_M] < 1. \ Thus, \ any \ portfolio, \ p, \ of \ X \ and \ W, \ with \ a \ non-zero \ weight \ in \ W, \ must \ have \ \rho[R_p,R_M] < 1. \ Thus, \ a \ portfolio \ with \ 40\% \ in \ X \ and \ 60\% \ in \ W \ does \ not \ lie \ on \ the \ CML. \ (Note \ that \ since \ X \ lies \ on \ the \ CML, \ \rho[R_W,R_M] = \rho[R_W,R_X] = 0.833. \ Can \ use \ this \ to \ calculate \ the \ standard \ deviation \ of \ the \ portfolio.)

X \ lies \ on \ CML. \ So \ X \ consists \ of \ the \ riskless \ asset \ and \ the \ market. \ W \ does \ not \ lie \ on \ the \ CML. \ So \ W \ does \ not \ consist \ of \ the \ riskless \ asset \ and \ the \ market. \ Thus, \ a \ portfolio \ of \ X \ and \ W \ cannot \ consist \ of \ the \ riskless \ asset \ and \ the \ market. \ Thus, \ a \ portfolio \ with \ 40\% \ in \ X \ and \ 60\% \ in \ W \ does \ not \ lie \ on \ the \ CML.