I. Expected Return, Return Standard Deviation, Covariance and Portfolios:

<table>
<thead>
<tr>
<th>State</th>
<th>Probability</th>
<th>Asset A</th>
<th>Asset B</th>
<th>Riskless Asset</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom</td>
<td>0.25</td>
<td>24%</td>
<td>14%</td>
<td>7%</td>
</tr>
<tr>
<td>Normal Growth</td>
<td>0.5</td>
<td>18%</td>
<td>9%</td>
<td>7%</td>
</tr>
<tr>
<td>Recession</td>
<td>0.25</td>
<td>2%</td>
<td>5%</td>
<td>7%</td>
</tr>
</tbody>
</table>

A. What is the expected return on each asset?

\[ E[R_A] = 0.25 \times 24\% + 0.5 \times 18\% + 0.25 \times 2\% = 15.5\% \]
\[ E[R_B] = 0.25 \times 14\% + 0.5 \times 9\% + 0.25 \times 5\% = 9.25\% \]
\[ E[R_f] = R_f = 0.25 \times 7\% + 0.5 \times 7\% + 0.25 \times 7\% = 7\% \]

B. What is the standard deviation of return on each asset?

First, calculate variance

\[ \sigma_{R_A}^2 = 0.25 \times (24 \times 24) + 0.5 \times (18 \times 18) + 0.25 \times (2 \times 2) - (15.5 \times 15.5) = 307 - 240.25 = 66.75 \]
\[ \sigma_{R_B}^2 = 0.25 \times (14 \times 14) + 0.5 \times (9 \times 9) + 0.25 \times (5 \times 5) - (9.25 \times 9.25) = 95.75 - 85.5625 = 10.1875 \]
\[ \sigma_{R_f}^2 = 0.25 \times (7 \times 7) + 0.5 \times (7 \times 7) + 0.25 \times (7 \times 7) - (7 \times 7) = 49 - 49 = 0 \]

Then calculate standard deviation

\[ \sigma_{R_A} = 8.1701\% \]
\[ \sigma_{R_B} = 3.1918\% \]
\[ \sigma_{R_f} = 0\% \]

C. What is the correlation and covariance between the returns on

1. assets A and B?

Covariance:

\[ \sigma_{R_A, R_B} = 0.25 \times (24 \times 14) + 0.5 \times (18 \times 9) + 0.25 \times (2 \times 5) - (15.5 \times 9.25) = 167.5 - 143.375 = 24.125 \]

Correlation:

\[ \rho_{R_A, R_B} = \frac{\sigma_{R_A, R_B}}{\sigma_{R_A} \sigma_{R_B}} = \frac{24.125}{8.1701 \times 3.1918} = 0.9251 \]

2. asset A and the riskless asset?

Covariance:
\sigma[R_A, R_f] = 0.25 \times (24x7) + 0.5 \times (18x7) + 0.25 \times (2x7) - (15.5x7) = 108.5 - 108.5 = 0.

Correlation:
\rho[R_A, R_f] = \sigma[R_A, R_f] / \{\sigma[R_A] \sigma[R_f]\} = 0/0 which is not well defined.

3. asset B and the riskless asset?

Covariance:
\sigma[R_B, R_f] = 0.

Correlation:
\rho[R_B, R_f] = \sigma[R_B, R_f] / \{\sigma[R_B] \sigma[R_f]\} = 0/0 which is not well defined.
II. Using Dividend Yield Information: Suppose the following data is to be used by Ms Q (a risk-averse investor) to form a portfolio that consists of the small firm fund and T-bills.

\[ E[R_{\text{Small}}(t)] = 1.369\% \]
\[ \sigma[R_{\text{Small}}(t)] = 8.779\% \]
\[ E[DP(\text{start } t)] = 4.446\% \]
\[ \sigma[DP(\text{start } t)] = 1.513\% \]
\[ \sigma[DP(\text{start } t), R_{\text{Small}}(t)] = 1.967 \]

where \( DP(\text{start } t) \) is the dividend yield on the S&P 500 known at the start of month \( t \).
\( R_{\text{Small}}(t) \) is the return on the small firm fund in month \( t \).

A. What is the intercept and slope coefficients from a regression of \( R_{\text{Small}}(t) \) (dependent variable) on \( DP(\text{start } t) \)?

Slope: \( \phi_{\text{Small}, DP} = \frac{\sigma[R_{\text{Small}}(t), DP(\text{start } t)]}{\sigma[DP(\text{start } t)]^2} = \frac{1.967}{(1.513 \times 1.513)} = 0.859 \)

Intercept: \( \mu_{\text{Small}, DP} = E[R_{\text{Small}}(t)] - \phi_{\text{Small}, DP} E[DP(\text{start } t)] = 1.369 - 0.859 \times 4.446 = -2.451 \).

B. What is the standard deviation of the residual from the regression of \( R_{\text{Small}}(t) \) on \( DP(\text{start } t) \)?

Defining \( e_{\text{Small}, DP}(t) \) to be the residual from the regression of \( R_{\text{Small}}(t) \) on \( DP(\text{start } t) \), then

\[ \sigma[R_{\text{Small}}(t)]^2 = \phi_{\text{Small}, DP}^2 \sigma[DP(\text{start } t)]^2 + \sigma[e_{\text{Small}, DP}(t)]^2. \]

So
\[ \sigma[e_{\text{Small}, DP}(t)]^2 = \sigma[R_{\text{Small}}(t)]^2 - \phi_{\text{Small}, DP}^2 \sigma[DP(\text{start } t)]^2 \]
\[ = 8.779^2 - 0.859^2 \times 1.513^2 = 75.381 \]
and \( \sigma[e_{\text{Small}, DP}(t)] = 8.682 \).

C. Suppose it is the end of March 1997, Ms Q does not know \( DP \) and the return on T-bills for April is 0.3%.
1. What is the April return on the small firm fund that Ms Q expects?
\[ E[R_{\text{Small}}(t)] = 1.369\% \]
2. What is the volatility of the April return on the small firm fund, given Ms Q’s information?
\[ \sigma[R_{\text{Small}}(t)] = 8.779\% \]

D. Suppose it is the end of March 1997, Ms Q knows that \( DP \) is 2% and the return on T-bills for April is 0.3%.
1. What is the April return on the small firm fund that Ms Q expects?
\[
\mu_{\text{Small,DP}} + \varphi_{\text{Small,DP}} \cdot \text{DP(start Apr)} = -2.451 + 0.859 \times 2 = -0.733\%.
\]

2. What is the volatility of the April return on the small firm fund, given Ms Q’s information?
Since Ms Q observes DP, the volatility of the April return on the small firm fund is the volatility of the regression residual:
\[
\sigma_{\text{e_{Small,DP}(t)}} = 8.682\%.
\]

E. Suppose it is the end of October 1997, Ms Q does not know DP and the return on T-bills for November is 0.4%.
1. What is the November return on the small firm fund that Ms Q expects?
2. What is the volatility of the April return on the small firm fund, given Ms Q’s information?

The answer to this question is the same as for part C.

F. Suppose it is the end of October 1997, Ms Q knows that DP is 5% and the return on T-bills for November is 0.4%.
1. What is the November return on the small firm fund that Ms Q expects?
\[
\mu_{\text{Small,DP}} + \varphi_{\text{Small,DP}} \cdot \text{DP(start Nov)} = -2.451 + 0.859 \times 5 = 1.844\%.
\]
2. What is the volatility of the April return on the small firm fund, given Ms Q’s information?
Since Ms Q observes DP, the volatility of the April return on the small firm fund is the volatility of the regression residual:
\[
\sigma_{\text{e_{Small,DP}(t)}} = 8.682\%.
\]