Problem Set 5 Solution: CAPM and Performance Measurement

I.  *SML and the CAPM:*

A.  In a CAPM world, all assets lie on the SML. So

\[ E[R_p] = R_f + \beta_{p,m} \{E[R_m] - R_f \} \]

\[ 20\% = 5\% + \beta_{p,m} \{15\% - 5\% \} \]

\[ \beta_{p,m} = \frac{15\%}{10\%} = 1.5. \]

B.  
1. The market has a Beta with respect to the market of 1. All assets plot on the SML including the market. So the market has the same expected return as the portfolio with a \( \beta_{p,m} \) of 1.

2. All assets lie on the SML. So an asset with a \( \beta_{p,m} = 0 \) has an expected return of:

\[ E[R_p] = R_f + \beta_{p,m} \{E[R_m] - R_f \} = 4\% + 0 \{12\% - 4\% \} = 4\%. \]

3. The expected return on the stock is given by:

\[ E[R] = R_f + \beta \{E[R_m] - R_f \} = 4\% + -0.5 \{12\% - 4\% \} = 0\%. \]

The intrinsic value of the stock is given by:

\[ V_0 = E[P_1 + D_1]/(1 + E[R]) = (41 + 3)/(1.00) = 44 \]

which is greater than its current price. So it is underpriced today.

II.  *SML vs CML in the CAPM:* Assume that the CAPM holds in the economy. The following data is available about the market portfolio, the riskless rate and two assets, A and B. Remember \( \beta_{i,m} = \sigma[R_i, R_m]/(\sigma[R_m]^2) \).

<table>
<thead>
<tr>
<th>Asset i</th>
<th>( E[R_i] )</th>
<th>( \sigma[R_i] )</th>
<th>( \beta_{i,m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>m (market)</td>
<td>0.15</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0.096</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.07</td>
<td>0.6</td>
<td></td>
</tr>
</tbody>
</table>

\( R_f = 0.10. \)

A. What is \( \beta_{i,m} \) for \( i \) equal to the market portfolio (i.e., \( \beta_{m,m} \))?

\[ \beta_{m,m} = \sigma[R_m, R_m]/(\sigma[R_m]^2) = 1. \]
B. What is the expected return on asset A (i.e., \( E[R_A] \))?

All assets plot on the SML:
\[
E[R_i] = R_f + \beta_{i,M} \{E[R_M] - R_f \}
\]

So
\[
E[R_A] = R_f + \beta_{A,M} \{E[R_M] - R_f \} = 0.10 + 1.2\{0.15-0.10\} = 0.16.
\]

C. What is the expected return on asset B (i.e., \( E[R_B] \))?

Similarly,
\[
E[R_B] = R_f + \beta_{B,M} \{E[R_M] - R_f \} = 0.10 + 0.6\{0.15-0.10\} = 0.13.
\]

D. Does asset A plot:
1. on the SML (security market line)?
   Yes.
2. on the CML (capital market line)?

Formula for the CML:
\[
E[R_i] = R_f + \sigma[R_i] \{E[R_M] - R_f \}/\sigma[R_M].
\]

For A,
\[
R_f + \sigma[R_A] \{E[R_M] - R_f \}/\sigma[R_M] = 0.10 + 0.096\{0.15-0.10\}/0.08 = 0.16 = E[R_A]
\]
as required for A to lie on the CML.

E. Does asset B plot:
1. on the SML?

   Yes.
2. on the CML?

For B,
\[
R_f + \sigma[R_B] \{E[R_M] - R_f \}/\sigma[R_M] = 0.10 + 0.07\{0.15-0.10\}/0.08 = 0.14375 > 0.13 = E[R_B]
\]
and so B does not lie on CML.

F. Could any investor be holding asset A as her entire portfolio?
   Yes since it lies on the CML.

G. Could any investor be holding asset B as her entire portfolio?
   No since it does not lie on the CML.

H. What is the correlation of asset A with the market portfolio?

Recall \( \beta_{i,M} = \rho[R_i, R_M] \sigma[R_i] / \sigma[R_M] \) which implies \( \rho[R_i, R_M] = \beta_{i,M} \sigma[R_M] / \sigma[R_i]. \)
So, for A,
\[ \rho[R_A, R_M] = \beta_{A,M} \sigma[R_M] / \sigma[R_A] = (1.2 \times 0.08)/0.096 = 1. \]

I. What is the correlation of asset B with the market portfolio?

Similarly, for B,
\[ \rho[R_B, R_M] = \beta_{B,M} \sigma[R_M] / \sigma[R_B] = (0.6 \times 0.08)/0.07 = 0.6857. \]

J. Can anything be said about the composition of asset A (i.e., what assets make up asset A)?

Since A lies on the CML, it must be some combination of the market portfolio and the riskless asset.

K. Can anything be said about the composition of asset B?

No.

III. **Performance Measurement.** The following information is to be used to evaluate the performance of the Bull Fund and the Boom Fund.

<table>
<thead>
<tr>
<th>i</th>
<th>E[R_i]</th>
<th>\sigma[R_i]</th>
<th>\sigma[R_i, R_{S&amp;P}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P</td>
<td>15</td>
<td>20</td>
<td>400</td>
</tr>
<tr>
<td>Bull</td>
<td>17</td>
<td>30</td>
<td>440</td>
</tr>
<tr>
<td>Boom</td>
<td>19</td>
<td>40</td>
<td>460</td>
</tr>
<tr>
<td>Riskfree</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

A. Calculate the Sharpe ratio for
1. the S&P 500 index fund.
   \[ \text{Sharpe}_{S&P} = E[r_{S&P}] / \sigma[R_{S&P}] = (15-5)/20 = 0.5. \]
2. the Bull fund.
   \[ \text{Sharpe}_{Bull} = E[r_{Bull}] / \sigma[R_{Bull}] = (17-5)/30 = 0.4. \]
3. the Boom fund.
   \[ \text{Sharpe}_{Boom} = E[r_{Boom}] / \sigma[R_{Boom}] = (19-5)/40 = 0.35. \]

B. Calculate Jensen’s alpha for
1. the S&P 500 index fund.
   \[ \beta_{S&P,S&P} = \text{cov} [r_{S&P}(t), r_{S&P}(t)]/\text{var} [r_{S&P}(t)] = 1 \]
   \[ \alpha_{S&P,S&P} = E[r_{S&P}(t)] - \beta_{S&P,S&P} E[r_{S&P}(t)] = (15-5) - 1 \times (15-5) = 0. \]
2. the Bull fund.

\[ \beta_{Bull,S&P} = \frac{\text{cov} \left[ r_{Bull}(t), r_{S&P}(t) \right]}{\text{var} \left[ r_{S&P}(t) \right]} \]
\[ = \frac{440}{202} = 1.1 \]

\[ \alpha_{Bull,S&P} = E\left[ r_{Bull}(t) \right] - \beta_{Bull,S&P} \cdot E\left[ r_{S&P}(t) \right] \]
\[ = (17 - 5) - 1.1 \times (15-5) = 1. \]

3. the Boom fund.

\[ \beta_{Boom,S&P} = \frac{\text{cov} \left[ r_{Boom}(t), r_{S&P}(t) \right]}{\text{var} \left[ r_{S&P}(t) \right]} \]
\[ = \frac{460}{202} = 1.15 \]

\[ \alpha_{Boom,S&P} = E\left[ r_{Boom}(t) \right] - \beta_{Boom,S&P} \cdot E\left[ r_{S&P}(t) \right] \]
\[ = (19 - 5) - 1.15 \times (15-5) = 2.5. \]

C. An investor who only cares about the mean and standard deviation of her portfolio’s return is trying to decide which of these funds to hold in combination with T-bills. Which fund should the investor choose?

The investor should choose Bull Fund rather than Boom Fund since \( \text{Sharpe}_{Bull} > \text{Sharpe}_{Boom} \). However, the investor would rather hold an S&P 500 index fund instead of either of these funds because \( \text{Sharpe}_{S&P} \) is higher than \( \text{Sharpe}_{Bull} \) or \( \text{Sharpe}_{Boom} \).

D. An investor who only cares about the mean and standard deviation of her portfolio’s return is considering combining Bull with the S&P 500 index fund (the market portfolio) and the riskfree asset. Will Bull’s weight be positive, negative or zero in the investor’s portfolio?

Bull’s weight will be positive since \( \alpha_{Bull,S&P} > 0 \).

E. An investor who only cares about the mean and standard deviation of her portfolio’s return is considering combining the Boom fund with the S&P 500 index fund (the market portfolio) and the riskfree asset. Will Boom’s weight be positive, negative or zero in the investor’s portfolio?

Boom’s weight will be positive since \( \alpha_{Boom,S&P} > 0 \).