Problem Set 4

I. Expected Return, Return Standard Deviation, Covariance and Portfolios (cont)

<table>
<thead>
<tr>
<th>State</th>
<th>Probability</th>
<th>Asset A</th>
<th>Asset B</th>
<th>Riskless Asset</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom</td>
<td>0.25</td>
<td>24%</td>
<td>14%</td>
<td>7%</td>
</tr>
<tr>
<td>Normal Growth</td>
<td>0.5</td>
<td>18%</td>
<td>9%</td>
<td>7%</td>
</tr>
<tr>
<td>Recession</td>
<td>0.25</td>
<td>2%</td>
<td>5%</td>
<td>7%</td>
</tr>
</tbody>
</table>

A. What is the expected return and standard deviation of return of a portfolio consisting of \( \omega \)% invested in asset A and \((1-\omega)\)% in the riskless asset when \( \omega \)% is
1. -20%?
2. 60%?
3. 120%?

B. What is the expected return and standard deviation of return of a portfolio consisting of \( \omega \)% invested in asset B and \((1-\omega)\)% in the riskless asset when \( \omega \)% is
1. -20%?
2. 60%?
3. 120%?

C. If a risk-averse investor has to decide whether to hold either asset A with the riskless asset or asset B with the riskless asset, which asset would the investor prefer to hold in combination with the riskless asset? Explain why? Do you need more information about the investor’s preferences to answer the question?

D. What is the expected return and standard deviation of return of a portfolio consisting of \( \omega \)% invested in asset A and \((1-\omega)\)% in asset B when \( \omega \)% is
1. -20%?
2. 80%?
3. 120%?
II. Using Dividend Yield Information (cont): Suppose the following data is to be used by Ms Q (a risk-averse investor) to form a portfolio that consists of the small firm fund and T-bills.

<table>
<thead>
<tr>
<th>E[R_{Small}(t)]</th>
<th>= 1.369%</th>
<th>σ[R_{Small}(t)]</th>
<th>= 8.779%</th>
</tr>
</thead>
<tbody>
<tr>
<td>E[DP(start t)]</td>
<td>= 4.446%</td>
<td>σ[DP(start t)]</td>
<td>= 1.513%</td>
</tr>
</tbody>
</table>

σ[DP(start t),R_{Small}(t)] = 1.967

where DP(start t) is the dividend yield on the S&P 500 known at the start of month t. R_{Small}(t) is the return on the small firm fund in month t.

A. Suppose it is the end of March 1997, Ms Q does not know DP and the return on T-bills for April is 0.3%.
   1. Will Ms Q short sell the small firm fund?
   2. Will Ms Q buy the small firm fund on margin?
   3. Will Ms Q buy a positive amount of both assets?

B. Suppose it is the end of March 1997, Ms Q knows that DP is 2% and the return on T-bills for April is 0.3%.
   1. Will Ms Q short sell the small firm fund?
   2. Will Ms Q buy the small firm fund on margin?
   3. Will Ms Q buy a positive amount of both assets?

C. Suppose it is the end of October 1997, Ms Q does not know DP and the return on T-bills for November is 0.4%.
   1. Will Ms Q short sell the small firm fund?
   2. Will Ms Q buy the small firm fund on margin?
   3. Will Ms Q buy a positive amount of both assets?

D. Suppose it is the end of October 1997, Ms Q knows that DP is 5% and the return on T-bills for November is 0.4%.
   1. Will Ms Q short sell the small firm fund?
   2. Will Ms Q buy the small firm fund on margin?
   3. Will Ms Q buy a positive amount of both assets?
III. *The Two Risky Asset Case:* A pension fund manager is considering three mutual funds. The first is a stock fund (S), the second is a long-term government and corporate bond fund (B), and the third is a T-bill money market fund that yields a rate of 9%. The probability distribution of the risky funds can be characterized as follows:

<table>
<thead>
<tr>
<th></th>
<th>Expected Return</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock Fund (S)</td>
<td>22%</td>
<td>32%</td>
</tr>
<tr>
<td>Bond Fund (B)</td>
<td>13%</td>
<td>23%</td>
</tr>
</tbody>
</table>

The correlation between the fund returns is 0.15.

A. Tabulate and draw the investment opportunity set of the two risky funds. Use investment proportions for the stock fund of zero to 100% in 20% increments.

B. Draw a tangent from the risk-free rate to the opportunity set. What does your graph show for the expected return and standard deviation of the optimal portfolio?

C. Solve numerically for the proportions of each asset, and for the expected return and standard deviation of the optimal risky portfolio.

D. What is the reward-to-viability ratio of the best feasible capital allocation line (CAL)?

E. You require that your portfolio yield an expected return of 15% and that it is efficient on the best feasible CAL.
   1. What is the standard deviation of your portfolio?
   2. What is the proportion invested in the T-bill fund and each of the two risky funds?

F. If you were to use only the two risky funds, and still require an expected return of 15%, what must be the investment proportions of your portfolio? Compare its standard deviation to that of the optimized portfolio in part E. What do you conclude?