Lecture 4: Portfolio Management-2 Risky Assets and a Riskless Asset.

I. Reading.
II. Standard Deviation of Portfolio Return: Two Risky Assets.
III. Graphical Depiction: Two Risky Assets.
IV. Impact of Correlation: Two Risky Asset Case.
V. Portfolio Choice: the Two Risky Asset Portfolio.
VI. Portfolio Choice: Combining the Two Risky Asset Portfolio with the Riskless Asset.
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Lecture 4: Portfolio Management - N Risky Assets and a Riskless Asset

I. Reading.
II. Standard Deviation of Portfolio Return: N Risky Assets.
III. Effect of Diversification.
IV. Opportunity Set: N Risky Assets.
V. Portfolio Choice: N Risky Assets and a Riskless Asset
Lecture 4: Portfolio Management-2 Risky Assets and a Riskless Asset.

I. Reading.
   A. BKM, Chapter 8: read Sections 8.1 to 8.3.

II. Standard Deviation of Portfolio Return: Two Risky Assets.
   A. Formula:

   \[
   \sigma^2[R_p(t)] = \omega_{1,p}^2 \sigma[R_1(t)]^2 + \omega_{2,p}^2 \sigma[R_2(t)]^2 + 2 \omega_{1,p} \omega_{2,p} \sigma[R_1(t), R_2(t)]
   \]

   where
   - \( \sigma[R_1(t), R_2(t)] \) is the covariance of asset 1’s return and asset 2’s return in period t;
   - \( \omega_{i,p} \) is the weight of asset i in the portfolio p;
   - \( \sigma^2[R_p(t)] \) is the variance of return on portfolio p in period t.

   B. Example 2 (cont): Consider a portfolio formed at the start of May 2006 with 60% invested in the small firm fund and 40% in ADM.
      2. What is the portfolio’s standard deviation ignoring DP?

   \[
   \sigma^2[R_p] = \omega_{Small,p}^2 \sigma^2[R_{Small}] + \omega_{ADM,p}^2 \sigma^2[R_{ADM}] + 2 \omega_{Small,p} \omega_{ADM,p} \sigma[R_{Small}, R_{ADM}]
   \]

   \[
   = 0.6^2 \times 5.27^2 + 0.4^2 \times 8.65^2 + 2 \times 0.6 \times 0.4 \times 16.07
   \]

   \[
   \]

   \[
   \sigma[R_p] = \sqrt{\sigma^2[R_p]} = \sqrt{29.684} = 5.45.
   \]

   3. Obtain expected portfolio return using the formula on page 18 of Lecture 3.

   \[
   E[R_p] = \omega_{Small,p} E[R_{Small}] + \omega_{ADM,p} E[R_{ADM}]
   \]

   \[
   = 0.6 \times 1.25 + 0.4 \times 1.52
   \]

   \[
   = 1.36
   \]
III. Graphical Depiction: Two Risky Assets.
A. The standard deviation of return on a portfolio consisting of the small firm asset and ADM and its expected return can be indexed by the weight of the small firm asset in the portfolio. The curve is known as the portfolio possibility curve.

<table>
<thead>
<tr>
<th>$\omega_{\text{Small},p}$</th>
<th>$\omega_{\text{ADM},p}$</th>
<th>$\sigma[R_p(t)]$</th>
<th>$E[R_p(t)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.2</td>
<td>1.2</td>
<td>10.05</td>
<td>1.57</td>
</tr>
<tr>
<td>0.0</td>
<td>1.0</td>
<td>8.65</td>
<td>1.52</td>
</tr>
<tr>
<td>0.2</td>
<td>0.8</td>
<td>7.36</td>
<td>1.47</td>
</tr>
<tr>
<td>0.4</td>
<td>0.6</td>
<td>6.25</td>
<td>1.41</td>
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<tr>
<td>0.6</td>
<td>0.4</td>
<td>5.45</td>
<td>1.36</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>5.09</td>
<td>1.31</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0</td>
<td>5.27</td>
<td>1.25</td>
</tr>
<tr>
<td>1.2</td>
<td>-0.2</td>
<td>5.94</td>
<td>1.20</td>
</tr>
</tbody>
</table>

Portfolio of the Small Firm Asset and ADM: Ignoring DP
$\{\sigma[R_{\text{Small}}], E[R_{\text{Small}}]\}$ marked by + and $\{\sigma[R_{\text{ADM}}], E[R_{\text{ADM}}]\}$ marked by x
IV. Impact of Correlation: Two Risky Asset Case.

A. Standard Deviation Formula.
   1. Can be rewritten in terms of correlation rather than covariance (using the definition of correlation):

   \[ \sigma^2[R_p(t)] = \omega_{1,p}^2 \sigma[R_1(t)]^2 + \omega_{2,p}^2 \sigma[R_2(t)]^2 + 2 \omega_{1,p} \omega_{2,p} \rho[R_1(t), R_2(t)] \sigma[R_1(t)] \sigma[R_2(t)] \]

   where \( \rho[R_1(t), R_2(t)] \) is the correlation of asset 1’s return and asset 2’s return in period \( t \).

   2. For a given portfolio with \( \omega_{1,p} > 0 \) and \( \omega_{2,p} > 0 \) and \( \sigma[R_1(t)] \) and \( \sigma[R_2(t)] \) fixed, \( \sigma[R_p(t)] \) decreases as \( \rho[R_1(t), R_2(t)] \) decreases.

B. Example 2 (cont):
   a. Suppose the \( E[R] \) and \( \sigma[R] \) for the small firm asset and for ADM remain the same but the correlation between the two assets is allowed to vary:
V. Portfolio Choice: the Two Risky Asset Portfolio.
   A. A risk averse investor is not going to hold any combination of the two risky assets on the negative sloped portion of the portfolio possibility curve.
      1. So the negative-sloped portion is known as the inefficient region of the curve.
      2. And the positive-sloped portion is known as the efficient region of the curve.
   B. The exact position on the efficient region that an individual holds depends on her tastes and preferences.
   C. Example 2 (cont): The portfolio possibility curve for the small firm asset and ADM can be divided into its efficient and inefficient regions.
      1. Any risk averse individual combining the small firm asset with ADM wants to lie in the efficient region: so wants to invest a positive fraction of her portfolio in ADM.

---

Portfolio of the Small Firm Asset and ADM: Ignoring DP
Efficient vs Inefficient
\{\sigma[R_{Small}], E[R_{Small}]\} marked by +, \{\sigma[R_{ADM}], E[R_{ADM}]\} marked by x

---

Small & ADM (Effic)
Small & ADM (Ineff)
VI. Portfolio Choice: Combining the Two Risky Asset Portfolio with the Riskless Asset.

A. Two-step Decision Process:

1. What is the preferred weights of the two risky assets in the risky portfolio?
   a. all risk averse individuals want access to the CAL with the largest slope; this involves combining the riskless asset with the same risky portfolio (in the graph below).
   b. this same risky portfolio is the one whose CAL is tangential to the portfolio opportunity curve; this is why is known as the tangency portfolio (denoted T).
   c. Can calculate the weight of risky asset 1 in the tangency portfolio T using the following formula:

\[
\omega_{1,T} = \frac{\sigma[R_2]^2 E[r_1] - \sigma[R_1, R_2] E[r_2]}{\{\sigma[R_2]^2 E[r_1] - \sigma[R_1, R_2] E[r_2]\} + \{\sigma[R_1]^2 E[r_2] - \sigma[R_1, R_2] E[r_1]\}}
\]

where \(r_i = R_i - R_f\) is the excess return on asset i (in excess of the riskless rate).

2. What is the preferred weights of the risky portfolio T and the riskless asset in the individual’s portfolio?
   a. the weight of T (in an individual’s portfolio \(\omega_{T,p}\) depends on the individual’s tastes and preferences.
B. Example 2 (cont): Suppose I form a portfolio (ignoring DP) of the small firm asset, ADM and T-bills.

1. What is the preferred weights of the two risky assets in the risky portfolio?
   a. Graph suggests that the risky asset portfolio I want to hold has positive weights invested in the small firm asset and in ADM (since \( p \) is between + and x on the portfolio possibility curve for the small firm asset and ADM).
   b. Can calculate the weight of the small firm asset in the tangency portfolio using the following formula:

\[
\omega_{\text{Small},T} = \frac{\sigma[R_{ADM}]^2 E[r_{Small}] - \sigma[R_{Small}, R_{ADM}] E[r_{ADM}]}{\left\{ \sigma[R_{ADM}]^2 E[r_{Small}] - \sigma[R_{Small}, R_{ADM}] E[r_{ADM}] \right\} + \left\{ \sigma[R_{Small}]^2 E[r_{ADM}] - \sigma[R_{ADM}, R_{Small}] E[r_{Small}] \right\}}
\]

   c. Now (using Lecture 3 pages 12-15)

\[
\begin{align*}
E[r_{Small}] &= 1.25 - 0.39 = 0.86. \\
E[r_{ADM}] &= 1.52 - 0.39 = 1.13. \\
\sigma[R_{Small}]^2 &= 5.27 \times 5.27 = 27.81. \\
\sigma[R_{ADM}]^2 &= 8.65 \times 8.65 = 74.75. \\
\sigma[R_{ADM}, R_{Small}] &= 16.07.
\end{align*}
\]

\[
\omega_{\text{Small},T} = \frac{74.75 \times 0.86 - 16.07 \times 1.13}{74.75 \times 0.86 - 16.07 \times 1.13 + 27.81 \times 1.13 - 16.07 \times 0.86}
\]

\[
= 46.126 / [46.126 + 17.605] = 0.72.
\]

2. What is the preferred weights of the risky portfolio T and the riskless asset in the individual’s portfolio?
   a. Depends on the tastes and preferences of the particular individual.
3. Suppose Individual Y wants to invest 60% in the tangency portfolio (\(\bullet\)) and 40% in T-bills. What is the weight of the small firm asset and of ADM in Y’s total portfolio?
   a. Use the following formula:

   \[ \omega_{i,p} = \omega_{i,T} \omega_{T,p} \]

   where
   \( \omega_{i,T} \) is the weight of risky asset \( i \) in the tangency portfolio \( T \).
   \( \omega_{i,p} \) is the weight of risky asset \( i \) in the total portfolio \( p \).
   \( \omega_{T,p} \) is the weight of portfolio \( T \) in the total portfolio \( p \).

   b. So, the answer is:

   \[ \omega_{\text{Small},p} = \omega_{\text{Small},T} \omega_{T,p} = 0.72 \times 0.60 = 0.432. \]
   \[ \omega_{\text{ADM},p} = \omega_{\text{ADM},T} \omega_{T,p} = 0.28 \times 0.60 = 0.168. \]
VII. Applications.

A. Adding a New Stock: The two-risky-asset formulas can be used to assess the impact of adding a new stock to a portfolio or varying the weight of an existing stock in the portfolio.

1. Example 2 (cont): Above considered the impact of adding ADM to the small firm fund (ignoring DP).

B. Asset Allocation between Two Broad Classes of Assets: The two-risky-asset formulas can be used to determine how much to invest in each of two broad asset classes.

1. Example 2 (cont): If I intend to form a risky portfolio from the small firm asset and the S&P 500 (ignoring DP) and then combine that risky portfolio with the riskless asset, what weight will the small firm asset have in the risky portfolio?
2. Can see from the graph that using historical data from 1/65 to 12/04 to approximate the return distribution for 5/06, would lead a risk averse individual to hold a risky portfolio with positive weight in the small firm asset and a negative weight in the S&P 500 ($\omega_{\text{Small,T}} = 1.33$ using formula above).

3. Weight of the small firm asset in the tangency portfolio $\omega_{\text{Small,T}}$ is sensitive to the $E[R_{\text{Small}}]$:
   a. As $E[R_{\text{Small}}]$ declines so does $\omega_{\text{Small,T}}$ holding $E[R_{\text{S&P}}]$, the standard deviations and correlation fixed:

<table>
<thead>
<tr>
<th>$E[R_{\text{Small}}]$</th>
<th>$E[R_{\text{S&amp;P}}]$</th>
<th>$\omega_{\text{Small,T}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25%</td>
<td>0.94%</td>
<td>133%</td>
</tr>
<tr>
<td>1.10%</td>
<td>0.94%</td>
<td>68%</td>
</tr>
<tr>
<td>0.94%</td>
<td>0.94%</td>
<td>-2%</td>
</tr>
</tbody>
</table>

b. This sensitivity of $\omega_{\text{Small,T}}$ to changes in $E[R_{\text{Small}}]$ explains why the small firm effect is of interest to practitioners.

C. International Diversification: The two-risky-asset formulas can also be used when deciding how much to invest in an international equity fund and how much in a U.S. based fund.
Lecture 4: Portfolio Management - N Risky Assets and a Riskless Asset

I. Reading.
   A. BKM, Chapter 8, Sections 8.4 and 8.5 and Appendix 8.A.

II. Standard Deviation of Portfolio Return: N Risky Assets.
   1. Formula.

   \[
   \sigma^2[R_p(t)] = \sum_{i=1}^{N} \sum_{j=1}^{N} \omega_{i,p} \omega_{j,p} \sigma[R_i(t), R_j(t)]
   \]

   where
   - \(\sigma[R_i(t), R_j(t)]\) is the covariance of asset i’s return and asset j’s return in period t;
   - \(\omega_{i,p}\) is the weight of asset i in the portfolio p;
   - \(\sigma^2[R_p(t)]\) is the variance of return on portfolio p in period t.

   2. The formula says that \(\sigma^2[R_p(t)]\) is equal to the sum of the elements in the following N x N matrix.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>j</th>
<th>...</th>
<th>N-1</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\omega_{1,p}) (\omega_{1,p}) (\sigma[R_1, R_1])</td>
<td>(\omega_{2,p}) (\omega_{2,p}) (\sigma[R_2, R_2])</td>
<td>(\omega_{3,p}) (\omega_{3,p}) (\sigma[R_3, R_3])</td>
<td>(\omega_{N-1,p}) (\omega_{N-1,p}) (\sigma[R_{N-1}, R_{N-1}])</td>
<td>(\omega_{N,p}) (\omega_{N,p}) (\sigma[R_N, R_N])</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(\omega_{2,p}) (\omega_{1,p}) (\sigma[R_2, R_1])</td>
<td>(\omega_{2,p}) (\omega_{2,p}) (\sigma[R_2, R_2])</td>
<td>(\omega_{3,p}) (\omega_{3,p}) (\sigma[R_3, R_3])</td>
<td>(\omega_{N-1,p}) (\omega_{N-1,p}) (\sigma[R_{N-1}, R_{N-1}])</td>
<td>(\omega_{N,p}) (\omega_{N,p}) (\sigma[R_N, R_N])</td>
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</tr>
<tr>
<td>...i...</td>
<td>(\omega_{i,p}) (\omega_{i,p}) (\sigma[R_i, R_i])</td>
<td>(\omega_{i,p}) (\omega_{i,p}) (\sigma[R_i, R_i])</td>
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<td>(\omega_{i,p}) (\omega_{i,p}) (\sigma[R_i, R_i])</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N-1</td>
<td>(\omega_{N-1,p}) (\omega_{N-1,p}) (\sigma[R_{N-1}, R_{N-1}])</td>
<td>(\omega_{N-1,p}) (\omega_{N-1,p}) (\sigma[R_{N-1}, R_{N-1}])</td>
<td>(\omega_{N-1,p}) (\omega_{N-1,p}) (\sigma[R_{N-1}, R_{N-1}])</td>
<td>(\omega_{N-1,p}) (\omega_{N-1,p}) (\sigma[R_{N-1}, R_{N-1}])</td>
<td>(\omega_{N,p}) (\omega_{N,p}) (\sigma[R_N, R_N])</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>(\omega_{N,p}) (\omega_{N,p}) (\sigma[R_N, R_N])</td>
<td>(\omega_{N,p}) (\omega_{N,p}) (\sigma[R_N, R_N])</td>
<td>(\omega_{N,p}) (\omega_{N,p}) (\sigma[R_N, R_N])</td>
<td>(\omega_{N,p}) (\omega_{N,p}) (\sigma[R_N, R_N])</td>
<td>(\omega_{N,p}) (\omega_{N,p}) (\sigma[R_N, R_N])</td>
<td></td>
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</tbody>
</table>

a. Notice that there are \(N^2\) terms.

b. The diagonal elements are the variance terms since \(\sigma^2[R_i(t)] = \sigma[R_i(t), R_i(t)]\); so there are \(N\) variance terms and \((N-1)N\) covariance terms.
Notice that this formula specializes to the formula used above for the two asset case:

$$\sigma^2[R_\rho(t)] = \omega_{1,\rho}^2 \sigma[R_1(t)]^2 + \omega_{2,\rho}^2 \sigma[R_2(t)]^2 + 2 \omega_{1,\rho} \omega_{2,\rho} \sigma[R_1(t), R_2(t)]$$

<table>
<thead>
<tr>
<th></th>
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<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>$\omega_{1,\rho} \omega_{2,\rho}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma[R_1, R_1]$</td>
<td>$\sigma[R_1, R_2]$</td>
</tr>
<tr>
<td>2</td>
<td>$\omega_{2,\rho} \omega_{1,\rho}$</td>
<td>$\omega_{2,\rho} \omega_{2,\rho}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma[R_2, R_1]$</td>
<td>$\sigma[R_2, R_2]$</td>
</tr>
</tbody>
</table>
III. Effect of Diversification.
   A. Consider an equal weighted portfolio (So $\omega_{i,p} = 1/N$ for all i.). For example, when $N=2$, an equal weighted portfolio has 50% in each asset.
   B. Suppose all assets have the same $E[R] = \bar{R}$ and $\sigma[R] = \sigma$ and have returns which are uncorrelated. Then, for the equal weighted portfolio:
      1. $N=2$:
         \[
         E[R_p(t)] = \frac{1}{2} E[R_1(t)] + \frac{1}{2} E[R_2(t)] = \bar{R}.
         \]
         \[
         \sigma[R_p(t)]^2 = \left(\frac{1}{2}\right)^2 \sigma[R_1(t)]^2 + \left(\frac{1}{2}\right)^2 \sigma[R_2(t)]^2 = \frac{1}{2} \sigma^2.
         \]
      2. $N=3$:
         \[
         E[R_p(t)] = \frac{1}{3} E[R_1(t)] + \frac{1}{3} E[R_2(t)] + \frac{1}{3} E[R_3(t)] = \bar{R}.
         \]
         \[
         \sigma[R_p(t)]^2 = \left(\frac{1}{3}\right)^2 \sigma[R_1(t)]^2 + \left(\frac{1}{3}\right)^2 \sigma[R_2(t)]^2 + \left(\frac{1}{3}\right)^2 \sigma[R_3(t)]^2 = \frac{1}{3} \sigma^2.
         \]
      3. Arbitrary $N$:
         \[
         E[R_p(t)] = \bar{R}.
         \]
         \[
         \sigma[R_p(t)]^2 = \frac{\sigma^2}{N}.
         \]
      4. As $N$ increases:
         a. the variance of the portfolio declines to zero.
         b. the portfolio’s expected return is unaffected.
      5. This is known as the effect of diversification.
C. Suppose all assets have the same $\sigma[R] = \sigma$ and have returns which are correlated.

1. Formulas for expected return and standard deviation of return for the equal weighted portfolio can be written:

$$E[R_p(t)] = \text{average expected return}$$

$$\sigma^2[R_p(t)] = \sigma^2 \left[ \frac{1}{N} 1 + (1 - \frac{1}{N}) \text{average correlation} \right]$$

where

$$\text{average expected return} = \frac{1}{N} \sum_{i=1}^{N} E[R_i(t)]$$

$$\text{average correlation} = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \rho[R_i(t), R_j(t)].$$

2. As $N$ increases:
   a. Expected portfolio return is unaffected.
   b. Variance of portfolio return:
      (1) Expressed as a fraction of firm variance, portfolio variance converges to the average pairwise correlation between assets.

3. Shows the benefit of diversification depends on the correlation between the assets.

4. Can see that assets with low correlation maximize the diversification benefits.
D. Suppose assets have non-zero covariances and differing expected returns and standard deviations.

1. Formulas for expected portfolio return and standard deviation can be written:

\[ E[R_p(t)] = \text{average expected return} \]

\[ \sigma^2[R_p(t)] = \frac{1}{N} \text{average variance} + (1 - \frac{1}{N}) \text{average covariance} \]

where

\[
\text{average expected return} = \frac{1}{N} \sum_{i=1}^{N} E[R_i(t)]
\]

\[
\text{average variance} = \frac{1}{N} \sum_{i=1}^{N} \sigma[R_i(t)]^2
\]

\[
\text{average covariance} = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \sigma[R_i(t), R_j(t)].
\]

2. As \(N\) increases:
   a. Expected portfolio return is unaffected.
   b. Variance of portfolio return:
      (1) First term (the unique/ firm specific/ diversifiable/ unsystematic risk) goes to zero.
      (2) Second term (the market/ systematic/ undiversifiable risk) remains.
         (a) When the assets are uncorrelated (the case above in III.B), this second term is zero.
IV. Opportunity Set: N Risky Assets.
   A. Set of Possible Portfolios.
      1. No longer a curve as in the two asset case.
      2. Instead, a set of curves.
   B. Minimum Variance Frontier.
      1. Since individuals are risk averse, can restrict attention to the set of portfolios with the lowest variance for a given expected return.
      2. This curve is known as the minimum variance frontier (MVF) for the risky assets.
      3. Every other possible portfolio is dominated by a portfolio on the MVF (lower variance of return for the same expected return).
      4. Example 2 (cont): Ignoring DP. The basic shape of the MVF is the same as that for the MVF for three of funds in this example (S&P 500, small firm fund and govt bond fund) which is graphed below.
      5. Further, risk averse individuals would never hold a portfolio on the negative sloped portion of the MVF; so can restrict attention to the positive sloped portion. This portion is known as the efficient frontier.

![Minimum Variance Frontier MVF for S&P 500, Small and Govt Bond: Ignoring DP](image)
C. Adding risky assets.
   1. Adding risky assets to the opportunity set always causes the minimum variance frontier to shift to the left in \( \{\sigma[R], E[R]\} \) space. Why?
      a. For any given \( E[R] \), the portfolio on the MVF for the subset of risky assets is still feasible using the larger set of risky assets.
      b. Further, there may be another portfolio which can be formed from the larger set and which has the same \( E[R] \) but an even lower \( \sigma[R] \).
   2. Example 2 (cont): Ignoring DP. MVF for the S&P 500, the small firm fund, the value firm fund and the govt bond fund is to the left of the MVF for the S&P 500, the small firm fund and the govt bond fund excluding the value firm fund. This happens even though the value firm fund has an \( \{\sigma[R], E[R]\} \) denoted by \( \times \) which lies to the right of the MVF for the 3 funds excluding the value firm fund.
3. Example 2 (cont): Ignoring DP. MVF for all 6 stock assets (including the 3 stock funds or portfolios: S&P 500 fund, small firm fund, and value firm fund) is to the left of the MVF for the 3 individual stocks (ADM, IBM, WAG).
V. Portfolio Choice: N Risky Assets and a Riskless Asset
   A. The analysis for the two risky asset and a riskless asset case applies here.
      1. Any risk averse individual combines the riskless asset with the risky portfolio whose Capital Allocation Line has the highest slope.
      2. That risky portfolio is on the efficient frontier for the N risky assets and is known as the tangency portfolio (⊕): calculating the weights of assets in the tangency portfolio can be performed via computer.
      3. All risk averse individuals want to hold this tangency portfolio in combination with the riskless asset. The associated Capital Allocation Line is the efficient frontier for the N risky assets and the riskless asset.
      4. Only the weights of the tangency portfolio and the riskless asset in an individual’s portfolio depend on the individual’s tastes and preferences.
      5. Example 2 (cont): Ignoring DP. If individuals can form a risky portfolio from the 7 assets and combine that risky portfolio with T-bills, then all individuals will hold ⊕ as their risky portfolio. The weights of ⊕ and T-bills in an individual’s portfolio will depend on that individual’s tastes and preferences.

Minimum Variance Frontier MVF for the 7 Assets with and without T-bills: Ignoring DP
{σ(Ri), E[Ri]}s marked by x (stocks) and +

![Minimum Variance Frontier MVF](image_url)
B. Adding risky assets to the set of available risky assets:
1. shifts the MVF for the risky assets to the left.
2. allows investors access to a CAL with a higher slope.
3. increases the utility of any individuals (in the absence of transaction costs).
4. Example 2 (cont): Ignoring DP. The slope of the CAL available using all 7 risky assets is higher than that for the CAL available using only the 3 individual stocks.

Efficient Frontier for the 3 Stocks and for the 7 Assets with and without T-bills: Ignoring DP 
\{\sigma(R_i), E(R_i)\}s marked by x (stocks) and +
C. Transaction Costs.

1. When the transaction costs associated with forming portfolios increase with the number of assets in the portfolio, there may be some optimal number of assets to have in the portfolio.

2. In this case, assets are added to the portfolio until the benefits from adding one more asset are offset by the associated increase in transactions costs.

3. Example 2 (cont): Ignoring DP. If an investor has used the 4 funds and T-bills to form a portfolio, the benefit from adding the 3 individual stocks appears small (see the graph below). If the investor faces significant fixed costs to start trading individual stocks (find a broker, open a brokerage account, ...) then the individual may prefer not to trade individual stocks.

![Graph of Efficient Frontier for the 4 Funds and the 7 Assets with and without T-bills: Ignoring DP](image-url)