Lecture 5: CAPM.

I. Reading
II. Market Portfolio.
III. CAPM World: Assumptions.
IV. Portfolio Choice in a CAPM World.
V. Individual Assets in a CAPM World.
VI. Intuition for the SML (E[R_p] depending on \( \beta_{p,M} \)).
VII. CML vs SML.
VIII. Example Problem.
IX. More Intuition for the SML (E[R_p] depending on \( \beta_{p,M} \)).
X. Beta Estimation.

Lecture 5: CAPM Performance Measures and Empirical Evidence

I. Reading.
II. Performance Measurement in a Mean-variance CAPM World.
III. Testable Implications of the CAPM
IV. Limitations of CAPM Tests.
V. CAPM Empirical Evidence:
Lecture 5 Foundations of Finance

Lecture 5: CAPM.

I. Reading
   A. BKM, Chapter 9, Section 9.1.
   B. BKM, Chapter 10, Section 10.1 and 10.2.

II. Market Portfolio.
   A. Definition: The market portfolio $M$ is the portfolio of all risky assets in the economy each asset weighted by its value relative to the total value of all assets.
   B. Economy: $N$ risky assets and $J$ individuals.
   C. Weight of asset $i$ in the market portfolio ($\omega_{i,M}$) is given by:

   $$\omega_{i,M} = \frac{V_i}{V_M}$$

   where
   - $V_i$ is the market value of the $i$th risky asset;
   - $V_M = V_1 + \ldots + V_N$ is the total value of all risky assets in the economy.

   D. One Formula for the Return on the Market Portfolio:

   $$R_M = \omega_{1,M} R_1 + \ldots + \omega_{N,M} R_N$$

   where
   - $R_M$ is the return on the value weighted market portfolio;
   - $R_i$ is the return on the $i$th risky asset, $i=1,2,\ldots,N$;
Example: Suppose there are only 2 individuals and 3 risky assets in the economy.

1. Individual 1 invests $80000 in risky assets of which $40000 is in asset 1, $30000 in asset 2 and $10000 in asset 3. Individual 2 invests $20000 in risky assets of which $6000 is in asset 1, $12000 is in asset 2 and $2000 is in asset 3.

2. Return on asset 1 is 10%. Return on asset 2 is 20%. Return on asset 3 is -10%.

<table>
<thead>
<tr>
<th>Asset i</th>
<th>Individual 1</th>
<th>Individual 2</th>
<th>Market</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_{i,p1}$</td>
<td>$\omega_{i,p1}$</td>
<td>$V_{i,p2}$</td>
</tr>
<tr>
<td>1</td>
<td>40000</td>
<td>0.500</td>
<td>6000</td>
</tr>
<tr>
<td>2</td>
<td>30000</td>
<td>0.375</td>
<td>12000</td>
</tr>
<tr>
<td>3</td>
<td>10000</td>
<td>0.125</td>
<td>2000</td>
</tr>
<tr>
<td>Total</td>
<td>80000</td>
<td>1.000</td>
<td>20000</td>
</tr>
</tbody>
</table>

3. What is the market value of asset 1? $V_1 = 40000 + 6000 = 46000$.

4. What is the weight of asset 1 in the market portfolio?

$\omega_{1,M} = 46000/100000 = 0.46$.

5. What is the return on the market portfolio?

$R_M = \omega_{1,M} \times R_1 + \omega_{2,M} \times R_2 + \omega_{3,M} \times R_3 = 0.46 \times 10\% + 0.42 \times 20\% + 0.12 \times -10\% = 11.8\%$

6. What is the return on each individual’s portfolio (p1 and p2)?

1: $R_{p1} = \omega_{1,p1} \times R_1 + \omega_{2,p1} \times R_2 + \omega_{3,p1} \times R_3 = 0.5 \times 10\% + 0.375 \times 20\% + 0.125 \times -10\% = 11.25\%$

2: $R_{p2} = \omega_{1,p2} \times R_1 + \omega_{2,p2} \times R_2 + \omega_{3,p2} \times R_3 = 0.3 \times 10\% + 0.6 \times 20\% + 0.1 \times -10\% = 14\%$

7. But can see that the market portfolio can be formed by adding together the portfolios of the two individuals. Can think of the market portfolio as a portfolio with 80% (80000/100000) invested in individual 1’s portfolio and 20% in individual 2's portfolio. Thus, can calculate the market portfolio’s return:

$R_M = 0.8 \times R_{p1} + 0.2 \times R_{p2} = 0.8 \times 11.25\% + 0.2 \times 14\% = 11.8\%$
F. Another Formula for Market Return: The market portfolio can also be thought of as a portfolio of individuals’ risky asset portfolios where the weights are the value of each individual’s portfolio relative to the total value of all assets.

\[ R_M = \frac{W_1}{V_M} R_{p1} + \ldots + \frac{W_J}{V_M} R_{pJ} \]

where
\[ R_{pj} \] is the return on the jth individual’s risky portfolio, j=1,2,...,J;
\[ W_{pj} \] is the market value of the jth individual’s risky asset portfolio;
\[ V_M = W_1 + \ldots + W_J. \]

G. How to calculate the market value of a firm’s equity:

1. Formula:

\[ V_i = n_i p_i \]

where:
\[ n_i \] is the number of shares of equity i outstanding;
\[ p_i \] is the price of a share of i.

2. Example: IBM has 517.546M shares outstanding at a price of $143.875 at close Monday 2/24/97. So

\[ V_{IBM} = 517.546M \times 143.875 = 74461.93M. \]
III. CAPM World: Assumptions.
A. All individuals care only about expected return and standard deviation of return.
B. Individuals agree on the opportunity set of assets available.
C. Individuals can borrow and lend at the one riskfree rate.
D. Individuals can trade costlessly, can sell short any asset, face zero taxes, can hold any fraction of an asset and are price takers. This assumption is known as the perfect capital markets assumption.

IV. Portfolio Choice in a CAPM World.
A. All individuals want to hold a combination of the riskless asset and the tangency portfolio.
B. Example (cont): Suppose a CAPM wold exists in our 2 individual, 3 asset economy. The tangency portfolio invests 30% in asset 1, 50% in asset 2 and 20% in asset 3. Individual 1 invests $80000 in the tangency portfolio and individual 2 invests $20000 in the tangency portfolio.

<table>
<thead>
<tr>
<th>Asset i</th>
<th>Individual 1</th>
<th>Individual 2</th>
<th>Market</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_{i,p1}$</td>
<td>$\omega_{i,p1}$</td>
<td>$V_{i,p2}$</td>
</tr>
<tr>
<td>1</td>
<td>24000</td>
<td>0.3</td>
<td>6000</td>
</tr>
<tr>
<td>2</td>
<td>40000</td>
<td>0.5</td>
<td>10000</td>
</tr>
<tr>
<td>3</td>
<td>16000</td>
<td>0.2</td>
<td>4000</td>
</tr>
<tr>
<td>Total</td>
<td>80000</td>
<td>1.0</td>
<td>20000</td>
</tr>
</tbody>
</table>

Since both investors hold the tangency portfolio as their risky asset portfolio, can see that the market portfolio of risky assets must be the tangency portfolio.

C. Since everyone holds the same risky portfolio and the market portfolio is a weighted average of individuals’ portfolios, all individuals must be holding the market as their risky portfolio; the market portfolio is the tangency portfolio.

D. So everyone holds some combination of the value weighted market portfolio M and the riskless asset.
E. Capital Market Line (CML).

1. The CAL which is obtained by combining the market portfolio and the riskless asset is known as the Capital Market Line (CML) and has the following formula:

\[
CML: \quad \bar{R}_e = R_f + \frac{E[R_M] - R_f}{\sigma[R_M]} \sigma[R_e]
\]

where \(e\) is a portfolio that is a combination of the riskless asset and the market portfolio.

2. Portfolios that lie on the CML are known as efficient portfolios and have the following properties:
   a. Only assets which are a combination of the riskless asset and the market portfolio lie on the CML.
   b. For any individual, the portfolio she holds lies on the CML.
   c. Any portfolio on the CML has correlation of 1 with the market portfolio since it is a combination of the riskless asset and the market.
V. Individual Assets in a CAPM World.

A. Importance: Why care about the expected return for an individual asset?
   1. Stock Valuation: What discount rate do we use to discount the expected cash flows from the stock?
   2. Capital Budgeting: What rate do we use as the cost of equity capital?

B. Main Result.
   1. Since the market portfolio lies on the MVF for the N risky assets, the following relationship holds for any portfolio \( p \) formed from the N risky assets and the riskless asset:

   \[
   SML: \quad \mathbb{E}[R_p] = R_f + \{\mathbb{E}[R_M] - R_f\} \beta_{p,M}
   \]

   which is known as the Security Market Line.
Lecture 5  

Foundations of Finance

C. Properties of Beta:

1. The Beta of the riskless asset is 0: $\beta_{f,M} = \sigma[R_f, R_M] / \sigma[R_M]^2 = 0$.

2. The Beta of the minimum variance portfolio uncorrelated with the market is 0: $\beta_{\{0,M\},M} = \sigma[R_{0,M}, R_M] / \sigma[R_M]^2 = 0$.

3. The Beta of the market is 1: $\beta_{M,M} = \sigma[R_M, R_M] / \sigma[R_M]^2 = 1$.

4. The Beta of a portfolio is a weighted average of the Betas of the assets that comprise the portfolio where the weights are those of the assets in the portfolio. So if the portfolio return is given by:

$$R_p = \omega_{f,p} R_f + \omega_{1,p} R_1 + \omega_{2,p} R_2 + \ldots + \omega_{K,p} R_K$$

then the portfolio’s Beta is given by

$$\beta_{p,M} = \omega_{f,p} \beta_{f,M} + \omega_{1,p} \beta_{1,M} + \omega_{2,p} \beta_{2,M} + \ldots + \omega_{K,p} \beta_{K,M} = \omega_{1,p} \beta_{1,M} + \omega_{2,p} \beta_{2,M} + \ldots + \omega_{K,p} \beta_{K,M}.$$
Lecture 5  
Foundations of Finance

VI. Intuition for the SML (E[R_i] depending on \( \beta_{p,M} \)).

A. Decomposing the Variance of the Market Portfolio.

1. It can be shown that \( \sigma^2[R_M] \) can be written as a weighted average of the covariance of the individual assets with the market portfolio:

\[
\sigma^2[R_M] = \sum_{i=1}^{N} \sum_{j=1}^{N} \omega_{i,M} \omega_{j,M} \sigma[R_i, R_j] \\
= \sum_{i=1}^{N} \omega_{i,M} \sigma[R_i, R_M]
\]

2. So \( \sigma[R_i, R_M] \) measures the contribution of asset i to \( \sigma[R_M]^2 \).

3. Since

\[
\beta_{i,M} = \frac{\sigma[R_i, R_M]}{\sigma[R_M]^2}
\]

it follows that \( \beta_{i,M} \) measures the contribution of asset i to \( \sigma[R_M]^2 \) as a fraction of the market portfolio’s variance.

B. CAPM world:

1. All agents hold the market portfolio in combination with the riskless asset as their total portfolio.

2. So agents only care about how an individual asset i contributes to \( \sigma[R_M]^2 \) in equilibrium.

3. So \( \beta_{i,M} \) is the right measure of the riskiness of asset i.

4. So it makes sense that E[R_i] depends on \( \beta_{i,M} \).
VII. CML vs SML.
   A. All assets lie on the SML yet only efficient portfolios which are combinations of the market portfolio and the riskless asset lie on the CML
   B. How can this be?
      1. First note that since by definition
         \[ \sigma[R_p, R_M] = \rho[R_p, R_M] \sigma[R_p] \sigma[R_M] \]
         it follows that
         \[ \beta_{p,M} = \frac{\sigma[R_p, R_M]}{\sigma[R_M]^2} = \frac{\rho[R_p, R_M] \sigma[R_p] \sigma[R_M]}{\sigma[R_M]^2} = \frac{\rho[R_p, R_M] \sigma[R_p]}{\sigma[R_M]} \cdot \]
         2. Thus, the SML can be written
         \[
         \text{SML: } E[R_p] = R_f + \{E[R_M] - R_f\} \beta_{p,M}.
         \]
         \[
         \text{SML: } E[R_p] = R_f + \frac{E[R_M] - R_f}{\sigma[R_M]} \{\rho[R_p, R_M] \sigma[R_p]\}.
         \]
      3. Comparing this equation to the CML
         \[
         \text{CML: } E[R_{ef}] = R_f + \frac{E[R_M] - R_f}{\sigma[R_M]} \sigma[R_{ef}]
         \]
         it can be seen that:
         a. an asset p lies on the SML and the CML if \( \rho[R_p, R_M] = 1 \).
         b. an asset p only lies on the SML and is not a combination of the riskless asset and the market portfolio if \( \rho[R_p, R_M] < 1 \).
C. Example: Suppose the CAPM holds. Two assets G and H have the same Beta with respect to the market: $\beta_{G,M} = \beta_{H,M}$. Since all assets including G and H lie on the SML, both have the same expected return: $E[R_G] = E[R_H]$. But G is a combination of the market portfolio and the riskless asset and so lies on the CML while H lies to the right of the CML having a higher standard deviation than G: $\sigma[R_G] < \sigma[R_H]$. Further $\rho[R_G, R_M] = 1$ while $\rho[R_H, R_M] < 1.$
VIII. Example Problem. Assume that the CAPM holds in the economy. The following data is available about the market portfolio, the riskless rate and two assets, G and H. Remember $\beta_{p,M} = \sigma_{p,R_M}/(\sigma_{R_M})^2$.

<table>
<thead>
<tr>
<th>Asset i</th>
<th>$E[R_i]$</th>
<th>$\sigma[R_i]$</th>
<th>$\beta_{i,M}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M (market)</td>
<td>0.13</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>0.05</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>0.08</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

$R_f = 0.05$.

A. What is the expected return on asset G (i.e., $E[R_G]$)?

All assets plot on the SML:

$$E[R_p] = R_f + \beta_{p,M} \{E[R_M] - R_f\}$$

So

$$E[R_G] = R_f + \beta_{G,M} \{E[R_M] - R_f\} = 0.05 + 0.5 \times (0.13 - 0.05) = 0.09.$$ 

B. What is the expected return on asset H (i.e., $E[R_H]$)?

Similarly,

$$E[R_H] = R_f + \beta_{H,M} \{E[R_M] - R_f\} = 0.05 + 0.5 \times (0.13 - 0.05) = 0.09.$$ 

C. Does asset G plot:

1. on the SML (security market line)?
   Yes.

2. on the CML (capital market line)?
   Formula for the CML:

$$E[R_{cM}] = R_f + \sigma_{cM} \{E[R_M] - R_f\}/\sigma_{R_M}.$$ 

For G,

$$R_f + \sigma_{R_G} \{E[R_M] - R_f\}/\sigma_{R_M} = 0.05 + 0.05 \times (0.13 - 0.05)/0.10 = 0.09 = E[R_G]$$

as required for G to lie on the CML.

D. Does asset H plot:

1. on the SML?
   Yes.

2. on the CML?
   For H,

$$R_f + \sigma_{R_H} \{E[R_M] - R_f\}/\sigma_{R_M} = 0.05 + 0.08 \times (0.13 - 0.05)/0.10 = 0.114 > E[R_H]$$

as required for H to lie on the CML.
and so H does not lie on CML.

E. Could any investor be holding asset G as her entire portfolio?
Yes since it lies on the CML.

F. Could any investor be holding asset H as her entire portfolio?
No since it does not lie on the CML.

G. What is the correlation of asset G with the market portfolio?
Recall
\[ \beta_{p,M} = \rho_{[R_p, R_M]} \frac{\sigma_{[R_p]}}{\sigma_{[R_M]}} \]
which implies
\[ \rho_{[R_p, R_M]} = \beta_{p,M} \frac{\sigma_{[R_M]}}{\sigma_{[R_p]}}. \]
So, for G,
\[ \rho_{[R_G, R_M]} = \beta_{G,M} \frac{\sigma_{[R_M]}}{\sigma_{[R_G]}} = \frac{0.5 \times 0.10}{0.05} = 1. \]

H. What is the correlation of asset H with the market portfolio?
Similarly, for H,
\[ \rho_{[R_H, R_M]} = \beta_{H,M} \frac{\sigma_{[R_M]}}{\sigma_{[R_H]}} = \frac{0.5 \times 0.10}{0.08} = 0.625. \]

I. Can anything be said about the composition of asset G (i.e., what assets make up asset G)?
Since G lies on the CML, it must be some combination of the market portfolio and the riskless asset.

J. Can anything be said about the composition of asset H?
No.
IX. More Intuition for the SML (E[R_p] depending on \(\beta_{p,M}\)).

A. Think of running a regression of \(R_p\) on \(R_M\).

\[ R_p = \mu_{p,M} + \beta_{p,M} R_M + e_{p,M} \]

1. The \(\mu_{p,M}\) and \(\beta_{p,M}\) which minimize \(E[e_{p,M}^2]\) are known as regression coefficients and are given by:

\[ \beta_{p,M} = \frac{\sigma[R_p, R_M]}{\sigma[R_M]^2}, \quad \text{and}, \quad \mu_{p,M} = E[R_p] - \beta_{p,M} E[R_M]. \]

2. So the slope coefficient from a regression of \(R_p\) on \(R_M\) is the Beta of asset \(i\) with respect to the market portfolio.

3. Further, it can be shown that \(\sigma[R_M, e_{p,M}] = 0\).

B. Decomposing the Variance of asset \(p\):

\[ \sigma[R_p]^2 = \sigma[\mu_{p,M} + \beta_{p,M} R_M + e_{p,M}]^2 = \beta_{p,M}^2 \sigma[R_M]^2 + \sigma[e_{p,M}]^2 + 2 \beta_{p,M} \sigma[R_M, e_{p,M}] = \beta_{p,M}^2 \sigma[R_M]^2 + \sigma[e_{p,M}]^2 \]

since \(\sigma[R_M, e_{p,M}] = 0\).

C. In the context of holding the market portfolio as your risky portfolio, the first term represents the undiversifiable risk of asset \(p\) while the second term represents the risk which is diversified away when asset \(p\) is held in the market portfolio.

D. It can be seen that portfolio \(p\)’s undiversifiable risk depends on \(\beta_{p,M}\).

E. Hence it makes sense that in a CAPM setting \(E[R_p]\) depends on \(\beta_{p,M}\) since every individual holds some combination of the market portfolio and the riskless asset.
X. Beta Estimation.
   A. If return distributions are the same every period, then can use a past series of returns to run regressions of $R_p$ on $R_M$ to obtain an estimate of $\beta_{p,M}$.
   B. Market Portfolio Proxy.
      1. Can not observe the return on the market portfolio.
      2. Use the S&P 500 index as a proxy.
      3. Why?
         a. S&P 500 contains 500 stocks chosen for “representativeness”.
         b. S&P 500 is value-weighted.

Example 2 (most recent 60 months): Ignoring DP.
ADM Plotted Against S&P 500. Regression of $R_{ADM}$ on $R_{S&P}$.

\[ 1.721 + 0.4484 \times R_{S&P} \]
D. Empirical evidence suggests that over time the Betas of stock move toward the average Beta of 1. For this reason, a raw estimate of Beta is often adjusted using the following formula: $\beta_{adj} = w \beta_{est} + (1-w) 1$. 
Lecture 5: CAPM Performance Measures and Empirical Evidence

I. Reading.
   A. BKM, Chapter 24, Sections 24.1-24.2.
   B. BKM, Chapter 13, Section 13.1.

II. Performance Measurement in a Mean-variance CAPM World.
   A. Relation between CAPM and the excess return market model.
      1. Excess return market model regression: Can always run the following regression for asset i:

         \[ r_i(t) = \alpha_{i,M} + \beta_{i,M} r_M(t) + \epsilon_{i,M}(t). \]

         where \( r_i(t) = R_i(t) - R_f \).

      2. The slope of the excess return market model is CAPM beta:

         \[ \beta_{i,M} = \frac{\text{cov} \left[ r_i(t), r_M(t) \right]}{\text{var} \left[ r_M(t) \right]} = \frac{\text{cov} \left[ R_i(t), R_M(t) \right]}{\text{var} \left[ R_M(t) \right]}. \]

      3. Implication of CAPM for the intercept of the excess return market model:
         a. CAPM Restriction: all assets lie on the SML

         \[ E[R_i] = R_f + \beta_{i,M} \{E[R_M] - R_f\} \quad \Rightarrow \quad E[r_i] = 0 + \beta_{i,M} E[r_M]. \]

         b. Taking expectations of the market model regression.

         \[ E[r_i] = \alpha_{i,M} + \beta_{i,M} E[r_M]. \]

         c. Thus CAPM constrains \( \alpha_{i,M} = 0 \) for all i.
B. Jensen’s Alpha.

1. The excess return market model intercept $\alpha_{i,M}$ is known as Jensen’s alpha:
   a. $\alpha_{i,M}>0$ implies asset $i$ lies above the SML and so is underpriced.
   b. $\alpha_{i,M}=0$ implies asset $i$ lies on the SML and so is correctly priced.
   c. $\alpha_{i,M}<0$ implies asset $i$ lies below the SML and so is overpriced.

2. Note that Jensen’s alpha can be calculated:

$$\alpha_{i,M} = E[r_i(t)] - \beta_{i,M} E[r_M(t)].$$

3. Jensen’s alpha measures the performance of an asset as part of a CAPM-optimal portfolio of $R_f$ and the market portfolio.

4. So Jensen’s alpha can be used to measure the performance of a mutual fund as an individual asset in a CAPM world.

5. Moreover, if an investor is combining the asset into a portfolio with the market portfolio and the riskfree then:
   a. $\alpha_{i,M}>0$ implies the asset has a positive weight in the portfolio.
   b. $\alpha_{i,M}=0$ implies the asset has a zero weight; the portfolio consists of the market and the riskfree.
   c. $\alpha_{i,M}<0$ implies the asset has a negative weight in the portfolio.
C. Sharpe ratio.
1. Earlier, investor’s used the slope of the Capital Allocation Line to decide which risky asset to hold in combination with $R_f$.
2. The slope of the Capital Allocation Line for risky asset $i$ is given by:

\[
\text{slope}[\text{CAL}_i] = \frac{|E[R_i] - R_f|}{\sigma[R_i]}.
\]

3. The slope of the Capital Allocation Line (without the absolute value) is known as the Sharpe ratio for asset $i$:

\[
\text{Sharpe}_i = \frac{E[r_i]}{\sigma[R_i]}.
\]

4. So the Sharpe ratio measures the performance of a fund as the only risky asset the investor holds (in combination with T-bills).
D. Example:
1. Evaluate Small and Value using 40 years of data ending 12/04, ignoring DP, taking $R_f = 0.39\%$, and S&P 500 as the market proxy.
2. Know (using Lecture 3 pp.12-15)
   \[E[r_{Small}] = 1.25 - 0.39 = 0.86.\]
   \[E[r_{Value}] = 1.23 - 0.39 = 0.84.\]
   \[E[r_{S&P}] = 0.94 - 0.39 = 0.55.\]
   \[\sigma[r_{Small}] = 5.27.\]
   \[\sigma[r_{Value}] = 5.67.\]
   \[\sigma[r_{S&P}] = 4.38.\]
   \[\sigma[r_{Small}, R_{S&P}] = 19.38.\]
   \[\sigma[r_{Value}, R_{S&P}] = 18.23.\]
3. Sharpe ratios: same as CAL slopes calculated in Lecture 4
   a. Sharpe_{Small} = \frac{E[r_{Small}]}{\sigma[r_{Small}]} = \frac{0.86}{5.27} = 0.163;
   b. Sharpe_{Value} = \frac{E[r_{Value}]}{\sigma[r_{Value}]} = \frac{0.84}{5.67} = 0.148;
   c. Sharpe_{S&P} = \frac{E[r_{S&P}]}{\sigma[r_{S&P}]} = \frac{0.55}{4.38} = 0.126;
   \text{Sharpe}_\text{Small} > \text{Sharpe}_\text{Value} > \text{Sharpe}_\text{S&P}.
4. Jensen’s alpha:
   a. First need to calculate Beta:
      \[\beta_{Small,S&P} = \frac{\text{cov}[r_{Small}(t), r_{S&P}(t)]}{\text{var}[r_{S&P}(t)]}\]
      \[= \frac{19.38}{4.38^2} = 1.010\]
      \[\beta_{Value,S&P} = \frac{\text{cov}[r_{Value}(t), r_{S&P}(t)]}{\text{var}[r_{S&P}(t)]}\]
      \[= \frac{18.23}{4.38^2} = 0.950\]
   b. Then can calculate Jensen’s alpha:
      \[\alpha_{Small,M} = E[r_{Small}(t)] - \beta_{Small,S&P} E[r_{S&P}(t)]\]
      \[= 0.86 - 1.010 \times 0.55 = 0.30 > 0\]
      \[\alpha_{Value,M} = E[r_{Value}(t)] - \beta_{Value,S&P} E[r_{S&P}(t)]\]
      \[= 0.84 - 0.950 \times 0.55 = 0.32 > 0\]
   c. Both Small Firms and Value Firms performed well over this 40 year period relative to the CAPM’s SML: not supportive of CAPM.
E. Morningstar reports both Jensen’s alpha and the Sharpe ratio for each mutual fund.
### Vanguard Value Index VIVAX

#### Volatility Measurements
- **Mean**
- **Standard Deviation**

#### Modern Portfolio Theory Statistics

<table>
<thead>
<tr>
<th>Metric</th>
<th>Standard Index</th>
<th>Best Fit Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>89</td>
<td>97</td>
</tr>
<tr>
<td>R-Squared</td>
<td>1.00</td>
<td>1.01</td>
</tr>
<tr>
<td>Beta</td>
<td>4.08</td>
<td>-0.18</td>
</tr>
<tr>
<td>Alpha</td>
<td>8.24</td>
<td>19.34</td>
</tr>
</tbody>
</table>

Trailing 3-Yr through 04-30-06 | *Trailing 5-Yr through 04-30-06

### Vanguard Growth Index VIGRX

#### Volatility Measurements
- **Mean**
- **Standard Deviation**

#### Modern Portfolio Theory Statistics

<table>
<thead>
<tr>
<th>Metric</th>
<th>Standard Index</th>
<th>Best Fit Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>88</td>
<td>11.46</td>
</tr>
<tr>
<td>R-Squared</td>
<td>1.04</td>
<td>8.62</td>
</tr>
<tr>
<td>Beta</td>
<td>-3.24</td>
<td>11.46</td>
</tr>
<tr>
<td>Alpha</td>
<td>1.00</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Trailing 3-Yr through 04-30-06 | *Trailing 5-Yr through 04-30-06

See Fund Family Data →
III. Testable Implications of the CAPM
   A. Market portfolio is the tangency portfolio.
   B. All assets lie on the SML: so variation in expected returns is fully explained by linear variation in Beta with respect to the market.

IV. Limitations of CAPM Tests.
   A. Tests always use some kind of proxy for the market portfolio.
      1. Market Portfolio is the value weighted portfolio of all assets which is unobservable (Roll [1977]’s critique).
   B. Tests only use a subset of all available assets.
      1. If the CAPM holds, every asset lies on the SML but not every asset is used in testing.

V. CAPM Empirical Evidence:
   A. Fama and French [1992].
      1. Two sets of 100 portfolios:
         a. First set: within each size decile form 10 portfolios on the basis of Beta with respect to the market.
         b. Second set: within each size decile form 10 portfolios on the basis of book-to-market.
      2. Results:
         a. Average return varies inversely with size (holding Beta fixed) but hardly varies with Beta (holding size fixed): inconsistent with CAPM.
         b. Average return varies inversely with size (holding book-to-market fixed) and varies positively with book-to-market (holding size fixed): suggests that average returns vary across stocks with both size and book-to-market.
   B. Fama and French [1993].
      1. 25 portfolios:
         a. quintile break-points calculated based on size and book-to-market.
         b. form 25 value-weighted portfolios based on these breakpoints.
      2. Run excess return market model regressions.
      3. Results:
         a. Positive and significant Jensen’s alphas (as high as 0.57% per month) for high book-to-market portfolios; negative Jensen’s alphas (as low as -0.22% per month) for low book-to-market portfolios.
         b. Jensen’s alpha increasing going from large firm to small firm quintiles holding the book-to-market quintile fixed.
   C. Conclusion: Results imply market proxy is not on the MVF for the individual stocks.